Programming a Quantum Computer

Dominic Moylett

Quantum Engineering Centre for Doctoral Training,
University of Bristol,
dm1905@bristol.ac.uk

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1   What is Quantum Computing?

Quantum physics is a collection of bizarre behaviours that happen to really small particles. The nature of quantum physics makes simulations of particles at this small level really hard, even on today's fastest computers.

Quantum computing came about after a number of proposals by physicists such as Richard Feynman. They suggested that in order to make computers able to simulate quantum systems, we should build computers that take advantage of the same effects that make them hard to simulate on classical computers. These computers run on a very different model of computation to our current laptops, and can lead to more powerful machines in turn. In the decades that have followed since then, academics at many institutions – including the University of Bristol – have studied how quantum physics can benefit our technology, and how we can realise these benefits in practice.

In this workshop, we aim to highlight some of the strange effects of quantum physics, and how they can allow our computers to be faster and more powerful than otherwise. To do so, we will simulate a small quantum computer using the Quantum Computing Playground.

All you need to get started is a laptop with Google Chrome installed. Open up Google Chrome and go to the website http://quantumplayground.net/. Click "Playground" in the menu. This will load the playground with a default script. Delete this script and click on the "2D+Phase" icon in the top left of the screen. We are now ready to explore the world of quantum computers!

2   Quantum Bits

A classical computer is made up of bits: 1s and 0s that represent data. Quantum computers store data in quantum bits, or *qubits*. We write qubits as $|1\rangle$ and $|0\rangle$. These symbols are called *kets*[1], and are used to describe quantum states.

In the Quantum Playground, we define qubits using the command `VectorSize`, which we write at the top of our code. This

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[1]This notation was invented by Paul Dirac, one of the best-known quantum physicists and originally born in Bristol.

specifies the number of qubits we will be using, and the playground initialises these qubits all to the value $|0\rangle$ Try running the code below by typing it into the text box, pressing "Compile" and then pressing "Run":

```
VectorSize 6
```

This line initialises 6 qubits, all in the $|0\rangle$ state. We can see this on the left, where we see a black image with one white square in the bottom right hand corner. This image describes our state. Each square of this image represents a quantum state, and hovering over a square with the mouse will show us the quantum state, and numbers labelled Re and Im. The number Re is 0 for all quantum states except for the state $|0\rangle$, where Re is 1. We will see what these numbers mean in the next sections.

3   Measurement

While images like this showing off our complete quantum state are nice, we cannot look at quantum states this way in practice. Instead, we have to *measure* our qubits. We can do this using the `MeasureBit` command:

```
VectorSize 6
MeasureBit 0
Print "Qubit 0 is |" + measured_value + ">"
```

The second line measures the qubit and puts the result in the `measured_value` variable. The third line displays the result.

From this we get the output "Qubit 0 is $|0\rangle$". So, if we set all our qubits to $|0\rangle$ and measure one of them, we find that the measured qubit is in the $|0\rangle$ state.

4   Quantum Gates

But data is only one half of computation. We also need be able to perform operations. Classical computers do this with logic gates, such as the NOT gate, which flips a bit: $NOT(0) = 1$ and $NOT(1) = 0$.

In quantum computing, we also have logic gates, which are called quantum gates. One example is the X gate, which is the

quantum equivalent of a classical NOT gate: $X|0\rangle = |1\rangle$ and $X|1\rangle = |0\rangle$. Let's try this in the playground:

```
1  VectorSize 6
2  SigmaX 0
```

Now our image is black with one white square denoting an Re number of 1 for the $|1\rangle$ state. Our qubit has flipped from $|0\rangle$ to $|1\rangle$. We can also check by measuring:

```
1  VectorSize 6
2  SigmaX 0
3  MeasureBit 0
4  Print "Qubit 0 is |" + measured_value + ">"
```

Now when we measure our qubit, we find that the result is $|1\rangle$. We can also check the other direction by applying X to our qubit twice before measureing it, and find that $XX|0\rangle = X|1\rangle = |0\rangle$.

5 Superposition

Another example of a single qubit gate is the Hadamard gate, H. Let's see what this gate does to a qubit in the $|0\rangle$ state:

```
1  VectorSize 6
2  Hadamard 0
```

This now produces white squares for the states $|0\rangle$ and $|1\rangle$, both with Re numbers of $0.707107 \simeq \frac{1}{\sqrt{2}}$. Let's measure this a few times to see what happens:

```
1  VectorSize 6
2  Hadamard 0
3  MeasureBit 0
4  Print "Qubit 0 is |" + measured_value + ">"
```

Running this multiple times, we see that some times we get $|0\rangle$ and other times we get $|1\rangle$. This is called a superposition, where measuring the qubit could give $|0\rangle$ or $|1\rangle$, and it is impossible to predict which result it will be. This is already a very useful reason for utilising quantum effects in technology, as classical computers are not good at generating numbers purely at random. And this technology is already being used for applications from security and simulations to gambling and gaming.

But how likely are we to measure $|0\rangle$ or $|1\rangle$? This is decided by the Re and Im^2 numbers we have seen. These numbers form the *amplitude*s of the state. When we measure a qubit, the probability of measuring a state with amplitude a is $|a|^2$, where $|a|$ is the absolute function[3]. In our case, the probability of measuring $|0\rangle$ and $|1\rangle$ are both $|\frac{1}{\sqrt{2}}|^2 = (\frac{1}{\sqrt{2}})^2 = \frac{1}{2}$, so half the time we see a $|0\rangle$ and half the time we see a $|1\rangle$. Because the amplitude of each state is $\frac{1}{\sqrt{2}}$, we say that the state is:

$$H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

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[2]For this workshop, we will only focus on the Re numbers and the Im numbers will be 0.

[3]The absolute function $|x|$ converts any negative numbers to positive numbers: $|-1| = |1| = 1$

So we can generate a random bit based on the result of this state. Can we generate more random bits by measuring it multiple times? Let's try:

```
1  VectorSize 6
2  Hadamard 0
3  MeasureBit 0
4  Print "Qubit 0 is |" + measured_value + ">"
5  MeasureBit 0
6  Print "Qubit 0 is |" + measured_value + ">"
```

After running this a few times, it seems clear that the first measurement is random, but the second measurement always matches the first. This is true even if we measure it three, four or a hundred times. This is because once we have measured a qubit in a superposition, that qubit has *collapsed* into either $|0\rangle$ or $|1\rangle$, and the randomness has now been lost. So we need to generate a new superposition for each random bit we want.

But we have only been looking at the $|0\rangle$ state. What happens when we try applying H to a qubit in the $|1\rangle$ state?

```
1  VectorSize 6
2  SigmaX 0
3  Hadamard 0
4  MeasureBit 0
5  Print "Qubit 0 is |" + measured_value + ">"
```

This seems to produce the same result as before: The qubit is in a superposition, and when we measure it the particle collapses into $|0\rangle$ or $|1\rangle$. So does this produce the same result as $H|0\rangle$? Remove the last two lines of code and look at the image. Now the square for the $|0\rangle$ state is white with an amplitude of $\frac{1}{\sqrt{2}}$, but the $|1\rangle$ state is light blue, with an amplitude of $-\frac{1}{\sqrt{2}}$. Thus this state is:

$$H|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

This explains why the results when measuring look so similar, as the probability of getting $|0\rangle$ or $|1\rangle$ is still $\frac{1}{2}$[4].

Like the X gate, applying the Hadamard gate twice will produce the original qubit: $HH|0\rangle = H\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) = |0\rangle$ and $HH|1\rangle = H\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) = |1\rangle$. See the Further reading section for a formal proof of this. In fact, all quantum gates are reversible: After applying a quantum gate to a qubit, there is always another gate we can apply to get back to the original qubit.

You might wonder at this point what other quantum states we can possibly make with a single qubit. The answer is that we can make any qubit of the form $a|0\rangle + b|1\rangle$, where $|a|^2 + |b|^2 = 1$.

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[4]$|-\frac{1}{\sqrt{2}}|^2 = (\frac{1}{\sqrt{2}})^2 = \frac{1}{2}$

6 Multiple Qubits

Multiple qubits act similarly to multiple classical bits, where we can apply quantum gates to the individual qubits. We represent these qubits in the *tensor product*. For example, suppose the first qubit is $|0\rangle$ and the second is $|1\rangle$. We write this state as $|0\rangle \otimes |1\rangle$, and often shorten it to $|01\rangle$.

So for example, we can apply an X gate to the first and second qubits as follows:

```
VectorSize 6
SigmaX 0
SigmaX 1
MeasureBit 0
Print "Qubit 0 is |" + measured_value + ">"
MeasureBit 1
Print "Qubit 1 is |" + measured_value + ">"
```

And we find that they are in the $|11\rangle$ state, similarly to when we were acting only on individual qubits. Likewise, if we applied X to only qubit 0, then we would find that they were in the $|10\rangle$ state. So applying a quantum gate to one qubit doesn't affect the other.

If we apply a Hadamard gate to both qubits, we would find that they were in their own superpositions and would produce random measurements:

```
VectorSize 6
Hadamard 0
Hadamard 1
MeasureBit 0
Print "Qubit 0 is |" + measured_value + ">"
MeasureBit 1
Print "Qubit 1 is |" + measured_value + ">"
```

Like single qubits, we can produce any superposition over two qubits. Thus our state can be any form of $a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$ where $|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$.

7 Entanglement

Not all classical gates are single bit. The classical AND gate for example, takes two classical bits as input and produces one classical bit as output.

We also have multiple qubit quantum gates. One of particular importance is the Controlled-NOT gate, or $CNOT$, which takes a control qubit as the first argument and a target qubit as the second. Let's see what happens when the control qubit is $|0\rangle$:

```
VectorSize 6
CNOT 0, 1
MeasureBit 0
Print "Qubit 0 is |" + measured_value + ">"
MeasureBit 1
Print "Qubit 1 is |" + measured_value + ">"
```

It seems that $CNOT|00\rangle = |00\rangle$. If we were to apply an X gate to qubit 1 before the $CNOT$ we would find that

$CNOT|01\rangle = |01\rangle$. So when the control qubit is $|0\rangle$, the $CNOT$ gate does nothing. What if the control qubit is $|1\rangle$?

```
VectorSize 6
SigmaX 0
CNOT 0, 1
MeasureBit 0
Print "Qubit 0 is |" + measured_value + ">"
MeasureBit 1
Print "Qubit 1 is |" + measured_value + ">"
```

Now our target qubit has been flipped, so we can see that $CNOT|10\rangle = |11\rangle$. And again, if we were to apply an X gate to our target qubit too, we would find that $CNOT|11\rangle = |10\rangle$. So if our control qubit is $|0\rangle$, then our $CNOT$ gate does nothing, but if our control qubit is $|1\rangle$, then $CNOT$ applies an X gate to the target qubit.

But these aren't the only possible states our control qubit can take. What if our control qubit is in a superposition?

```
VectorSize 6
Hadamard 0
CNOT 0, 1
MeasureBit 0
Print "Qubit 0 is |" + measured_value + ">"
MeasureBit 1
Print "Qubit 1 is |" + measured_value + ">"
```

This produces a very different result. It now appears that our measurement of qubit 0 is random, but our measurement of qubit 1 is always the same state as qubit 0. If we measure qubit 1 first; then our measurement of qubit 1 will be random, but our measurement of qubit 0 will always be the same state as qubit 1. It seems that our qubits are in a superposition together, and if one collapses then the other collapses with it. We write this state as:

$$CNOT\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}|0\rangle\right) = \frac{CNOT|0\rangle|0\rangle + CNOT|1\rangle|0\rangle}{\sqrt{2}}$$
$$= \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

This effect is called *entanglement*, and is one of the strangest results of quantum mechanics. It is the effect that led Albert Einstein to describe quantum mechanics as "Spooky action at a distance". Entangled states seem to collapse at exactly the same time, regardless of distance between the two qubits. One qubit could be at the At-Bristol Science Centre and the other at the Digimakers Moon Base[5], and their collapse would still be at the same time. Classically this simultaneous collapse should be impossible, as that would require one qubit to send a message to the other faster than the speed of light, breaking the laws of relativity.

While we haven't been able to entangle qubits between the At-Bristol Science Centre and the Digimakers Moon Base, physicists at a number of institutions, most famously Delft University of Technology, have managed to successfully entangle qubits and

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[5]Construction pending.

witness these effects before light could have travelled from one qubit to the other. So there must be something more to quantum.

8   Deutsch's Algorithm

We are now ready to see one problem where quantum computers can truly outperform classical ones, first proposed by David Deutsch. Suppose we have a classical circuit f, which takes one bit as input and returns one bit as output. In other words, $f(0)$ is either 0 or 1 and $f(1)$ is either 0 or 1. Deutsch's problem is to determine if $f(0) = f(1)$.

On a classical computer, we would need two queries to the function f: One to figure out what $f(0)$ is and one to figure out what $f(1)$ is. But can we do better with a quantum version of this circuit?

Deutsch's Algorithm can be implemented as follows:

```
1  VectorSize 6
2  SigmaX 1
3  Hadamard 0
4  Hadamard 1
5  //Insert quantum version of circuit here
6  Hadamard 0
7  MeasureBit 0
8  Print "Qubit 0 is |" + measured_value + ">"
```

For line 5, we pick what operation to run based on what we want $f(0)$ and $f(1)$ to be. This quantum operator is called U_f, and is formally stated as $U_f|x\rangle|y\rangle = |x\rangle|y \oplus f(x)\rangle$, where $\oplus$ is the exclusive-OR (XOR) gate. It might seem difficult to implement a quantum gate like this, but we can actually do it very easily, based on what the function f is.

Case 1: If you want $f(0) = 0$ and $f(1) = 0$, then delete line 5.

Case 2: If you want $f(0) = 0$ and $f(1) = 1$, then replace line 5 with:

```
1  CNOT 0, 1
```

Case 3: If you want $f(0) = 1$ and $f(1) = 0$, then replace line 5 with:

```
1  CNOT 0, 1
2  SigmaX 1
```

Case 4: Or if you want $f(0) = 1$ and $f(1) = 0$, then replace line 5 with:

```
1  SigmaX 1
```

If we run the full program, we find that if $f(0) = f(1)$ (cases 1 and 4) then qubit 0 is in the $|0\rangle$ state, and if $f(0) \neq f(1)$ (cases 2 and 3) then qubit 0 is in the $|1\rangle$ state. Thus we have solved the problem with only one query of f. David Deutsch and Richard

Jozsa[6] later showed that a similar problem requires many queries on a classical computer, but just one on a quantum computer.

A formal proof is in the Further Reading, but to give some intuition about this effect, after applying the Hadamard to each qubit, our two qubits are in the state $\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right) \otimes \left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)$. Now, note that in cases 1 and 4, we do nothing to qubit 0, so after applying U_f, qubit 0 is still in the state $\frac{|0\rangle+|1\rangle}{\sqrt{2}}$. But in cases 2 and 3, we apply a $CNOT$. With our two qubits in their current state, the $CNOT$ does change qubit 0, making it become $\frac{|0\rangle-|1\rangle}{\sqrt{2}}$. So the Hadamard in line 8 converts qubit 0 to $|0\rangle$ if $f(0) = f(1)$, and $|1\rangle$ if $f(0) \neq f(1)$ – recall from Section 5 that $H\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right) = |0\rangle$ and $H\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right) = |1\rangle$.

Of course, in this case we can see the code that make up U_f and thus figure out if $f(0) = f(1)$ by seeing if there is a $CNOT$ in the code. But Deutsch's algorithm also works if we cannot see the code within the quantum operator U_f. If we are just given a black box which we are told implements U_f then we can still figure out if $f(0) = f(1)$.

This problem doesn't sound very useful, but there are many more practical problems which quantum computers can do better than classical computers:

- Charles Bennett and Stephen Wiesner showed that you can send two classical bits from one person to another by just sending one qubit from one to the other through a technique called superdense coding.

- Searching through an unsorted list of n items for a particular item requires n classical queries – one for each item. Lov Grover showed that we can do this with $\sqrt{n}$ queries on a quantum computer.

- Peter Shor showed that you can efficiently find the prime factors of a number on a quantum computer. This problem is believed to be hard classically, as finding an efficient solution would break the RSA encryption scheme.

9   Where is my Quantum Computer?

You can buy one right now! D-Wave Systems already sell quantum computers, but these computers are only built for solving a very specific problem called quantum annealing. But they are on the market, and Google are currently seeing how useful the quantum annealer they bought from D-Wave is for their business.

But what is more interesting in the Physics community is when will we see a *universal quantum computer* – a computer that can perform any quantum computation – on the market. And the answer is we don't know, but you can use one right now! The Centre for Quantum Photonics at the University of Bristol connected their two qubit universal quantum computer to the Internet, in what they call the Quantum in the Cloud project. You

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[6]Richard Jozsa is another famous physicist who was previously a lecturer here in Bristol.

can find out more about this work here: `http://www.bristol.ac.uk/physics/research/quantum/outreach/qcloud/`

But two qubits is not very useful: When are we going to get many qubit universal quantum computers? Again, we don't know, but potentially soon: John Martinis at the University of California, Santa Barbara is hoping to develop a 100 qubit universal quantum computer in 2-3 years; Microsoft Research predicts a quantum computer in ten years; and others have predicted between five and forty years. Regardless of when it comes, one thing is certain: There's a lot of excitement surrounding quantum computers right here and now.

10   Further Reading

I first heard about quantum computing from a booklet written by Ashley Montanaro at the University of Bristol, which explains many of the concepts discussed here from a theoretical perspective, including a formal proof of Deutsch's Algorithm. That booklet is available here: `http://www.cs.bris.ac.uk/~montanar/gameshow.pdf`

Ashley also recently gave a guest lecture on quantum computing in general, the slides of which are available here:`http://www.maths.bris.ac.uk/~csxam/teaching/history.pdf`

Juan Miguel Arrazola at the University of Waterloo has been writing a collection of blog posts explaining quantum mechanics to everyone:

- Part one is here: `https://uwaterloo.ca/institute-for-quantum-computing/blog/post/anyone-can-understand-quantum-mechanics-part-1`

- Part two is here: `https://uwaterloo.ca/institute-for-quantum-computing/blog/post/anyone-can-understand-quantum-mechanics-part-2`

- And part three will be coming soon, so make sure to keep an eye on their blog.

Note that while these are the easier aspects of quantum mechanics to understand, a lot of quantum mechanics in general rely on topics that are taught at GCSE or A Level Mathematics, so be careful diving straight in. But if you want to try and get ahead, some very useful mathematical concepts to learn include:

- Trigonometry ($\sin$, $\cos$, trigonometric identities...)

- Complex numbers ($e^{i\theta} = \cos\theta + i\sin\theta$, complex conjugates...)

- Linear algebra (Matrices, vectors, eigenvalues & eigenvectors...)

All of these concepts come up on a regular basis in quantum physics, so if you want more of a reason to study these concepts, now's your chance.