MT207 - Numerical Methods

Using Multigrid to Clear Images

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We take full responsibility of the work done in the project of Numerical Analysis course titled "Using Multigrid to Clear Images". We solemnly declare that the work presented in the report is done solely by us with no significant help from any other person; however, small help wherever taken is duly acknowledged. We have also written the complete report by ourselves.

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Executive Summary

In this report, we briefly discuss the applications & underlying principles of multigrid, along with studies on clearing the details of images, especially those with high contrast. We explore the use of numerical methods such as Jacobi method and Laplacian edge detection, in the multigrid implementation. We explain how iterating and interpolating an image to and from its coarse and fine grids, can effectively be used to smoothen it out from its original form.

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Chapter 1

Introduction

In the field, vigilance is the key to survival. This is one of the ideas behind the topic of our project: detecting objects in low-quality images.

Low-quality images can occur at the most crucial of moments, for example when photographing a object moving at high speed. It is after these moments have passed that we must learn to make do with the data we have collected.

Consider a scenario in which a fighter jet has been photographed, but it is not the most clearly visible due to interference while taking the photo. In order to see it clearly, the image of the fighter jet must be restored - but the moment has passed, and we cannot just photograph the jet again. This is where numerical methods can be of helpful to us.
We will use the multigrid method, which incorporates numerical methods, to detect the object in the image.

Chapter 2

Other studies

Algorithm #1:

1. Edge Detection by search-based methods, that involve computing a measure of edge strength, usually a first-order derivative expression such as the gradient magnitude, and then searching for local directional maxima of the gradient magnitude using a computed estimate of the local orientation of the edge, usually the gradient direction.
2. Then sending the results of Edge Detection, into Gauss-Seidel iteration for smoothening of the image
3. Then passing Gauss-Seidel result into the multi grid iterations to get the result.

Algorithm #2:

1. To increase the precision of edge detection, several subpixel techniques had been proposed, including curve-fitting, moment-based, reconstructive, and partial area effect methods.
2. Sending Edge detection result into Gaussian Smoothing, applying Gaussian method iteratively.
3. Applying multigrid iteratively on the result from Gaussian smoothing.

Chapter 3

Our Work

When an image has high contrast (i.e. its values are not smoothened), the details in the high-contrast areas will be difficult to make out. This problem represents the loss of clarity and detail in images; which we are aiming to recover using multigrid.

The multigrid method was initially developed for the solution of elliptic partial differential equations[1] for which near optimum convergence characteristics have been demonstrated (i.e. computational time directly proportional to grid size), has gained a wide acceptance within the CFD community and has become an essential acceleration technique for solving industrial type flow problems[2] . Multigrid strategies for the incompressible Navier-Stokes equations using pressure-based methods, such as SIMPLE[3], as smoothers have shown substantial increase in convergence rate and improvement in overall robustness for both staggered and collocated grids. In these algorithms, a pressure correction equation is used to enforce mass conservation by correcting both the pressure and the momentum satisfying velocity field. Moreover, being elliptic in nature, the use of a multigrid technique in solving the incompressible pressure correction equation[4] is expected to decrease its computational time in

[1] Poussin, F. V., "An Accelerated Relaxation Algorithm for Iterative Solution of Elliptic Equations," SIAM Journal of Numerical Analysis, vol. 5, pp. 340-351, 1968

[2] Brandt, A. and Dinar, N. "Multigrid Solutions to flow problems," Numerical Methods for Partial Differential Equations, Part S., pp., 43-147, Academic Press, New York, 1979

[3] Patankar, S.V. and Spalding, D.B., "A Calculation Procedure for Heat, Mass and Momentum Transfer in Three-Dimensional Parabolic Flows," International Journal of Heat and Mass Transfer, vol. 15, pp. 1787-1806, 1972

[4] Gjesdal, T. and Lossius, M.E.H.,"Comparison of Pressure Correction Smoothers for Multigrid Solution of Incompressible Flow," International Journal for Numerical Methods in Fluids, vol. 25,

comparison with a single grid method. This elliptic form is very different from the hyperbolic form assumed by the equation when derived for all speed compressible flows [22,23]. Nevertheless, multigrid methods have been successfully employed in the simulation of high-speed compressible flows, with density-based algorithms as smoothers[5].

The steps of applying an iteration of multigrid are:

1. **Pre-smoothing:** Perform iterations over the coarse grid
2. **Interpolating residual:** Interpolate from the coarse grid to a fine grid
3. **Smoothing:** Perform iterations over the fine grid
4. **Interpolating error:** Interpolate from the fine grid back to a coarse grid

For performing iterations, we implemented Jacobi method, whilst for interpolation we applied Laplacian edge detection.

pp. 393-405, 1997

[5] SIAM Journal of Numerical Analysis, vol. 5, pp. 340-351, 1968.

Chapter 4

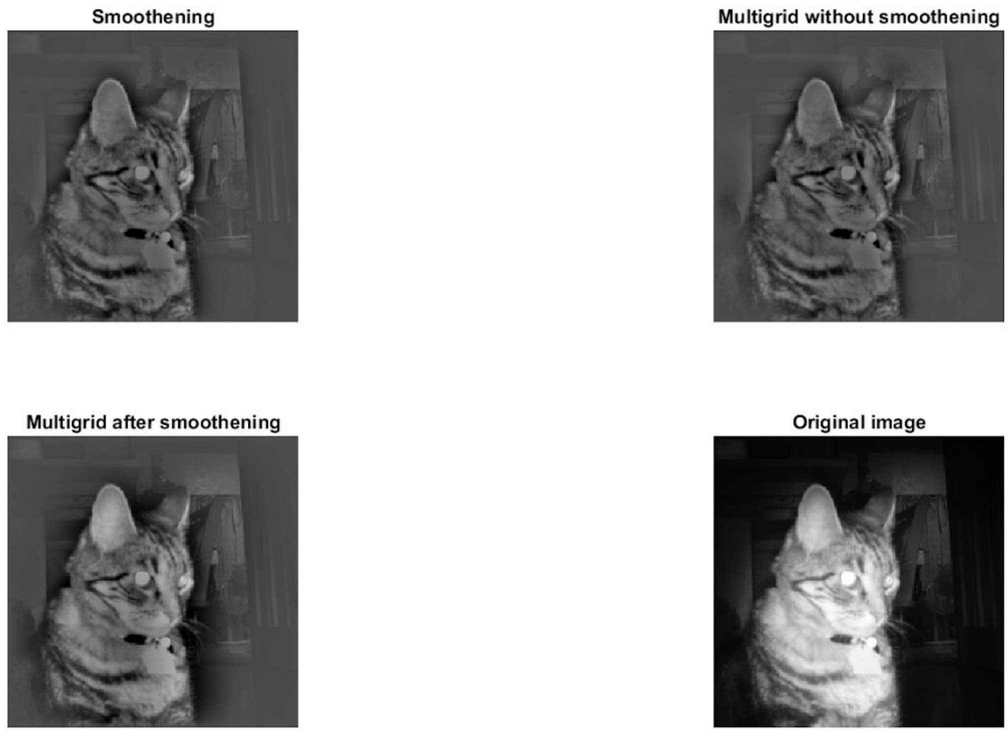
Results

After implementing and applying multigrid, we observed that details in unrefined images became more clearly visible, particularly in the higher-contrast areas (ie. those areas that had values close to or equal to the extremities at 0 or 1).

All the diagrams used in this section were obtained as output (for their respective inputs) from our multigrid program written in MATLAB.

In the comparison of the images below, the background details are strikingly more visible in the image on the right - such as the curtain and fan to the right side of the cat. Some details were not really visible at all in the image on the left, but became apparent after applying multigrid - most prominently, the shelf located above the cat's head. Also, the cat's fur appears slightly finer and more detailed in the image on the right - this is thanks to the smoothening of the multigrid process.

The following image demonstrates a comparison of various steps involved in the multigrid process:



Here is what each of the images represents:

Top left:	Output after 200 iterations of Jacobi method.
Top right:	Output after 10 multigrid iterations.
Bottom left:	Output after 100 multigrid iterations were applied to top left image.
Bottom right:	Original image.

The bottom left image becomes the clearest and most detailed of all 4 versions of the image.

The image to the left, demonstrates the output of the Laplacian edge detection as applied to the same image. This filter was used to make the details of the edges clearer, especially in high-contrast areas. When combined to form the multigrid-enhanced image, these edges stood out over the smoothened version.

Chapter 5

Conclusion and Future Work

This project is a useful application of numerical methods in the most crucial of areas, such as aviation and security, where clear images are a necessity. The multigrid is an efficient solution to clearing up images, to look at their finer details.

We learned about Laplacian edge detection and the smoothing properties of Jacobi iterations, which we used in this project.
We can extend multi grid to Multi-Grid Based Multi-Focus Image Fusion Using Watershed Algorithm.

We value the knowledge and understanding we have gained from working on it, and hope that our work is of benefit to others.

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