Take Home Midterm 2, AM212, November 23-24 2024.

Problem 1: For each of the following 3 ODEs,

- Plot the numerical solution for. $\epsilon = 0.1$, $\epsilon = 0.01$ and $\epsilon = 0.001$
- Explain in a few words what of method you plan to use to solve this asymptotically and why, based on the numerical solution
- Find the lowest order uniformly convergent analytical approximation to the solution for small positive ϵ
- Compare the numerical and analytical solutions for $\epsilon = 0.01$.

ODE A:

$$\frac{d^2f}{dt^2} = -f - \epsilon f^2 \left(\frac{df}{dt}\right) \quad \text{with } f(0) = 1, \frac{df}{dt}(0) = 0 \tag{1}$$

ODE B:

$$\frac{d^2f}{dt^2} = -f + \epsilon f \left(\frac{df}{dt}\right)^4, \quad \text{with } f(0) = 1, \frac{df}{dt}(0) = 0 \tag{2}$$

ODE C:

$$\epsilon\frac{d^2f}{dt^2} + \frac{df}{dt} + (t+1)f = 0 \quad \text{with } f(0) = 1, f(1) = 2 \tag{3}$$

Problem 2: Find the eigenvalues and eigenfuntions of this eigenvalue problem, in the limit where the eigenvalue λ is very large

$$\frac{d^2f}{dx^2} + \lambda(x+1)^2 f = 0 \quad \text{with } f(1) = 0, f(2) = 0 \tag{4}$$