UVOD U DIFERENCIJALNU GEOMETRIJU

FORMULE

Krivulje

- Frenetov dvobrid (krivulje u $\mathbb{R}^2$)
  - PDL
$$T(s) = c'(s) = (x'(s), y'(s)), \quad N(s) = (-y'(s), x'(s))$$
  - Opći parametar
$$T(t) = \frac{\dot{c}(t)}{||\dot{c}(t)||} = \frac{(x'(t), y'(t))}{||\dot{c}(t)||}, \quad N(t) = \frac{(-y'(t), x'(t))}{||\dot{c}(t)||}$$
- Frenetove formule (PDL krivulje u $\mathbb{R}^2$)
$$T' = \kappa N, \quad N' = -\kappa T$$
- Zakrivljenost krivulje u $\mathbb{R}^2$
$$\kappa(t) = \frac{\det(\dot{c}(t), \ddot{c}(t))}{\|\dot{c}(t)\|^3}$$
- Središte oskulacijske kružnice
$$S(t) = c(t) + \frac{N(t)}{\kappa(t)}$$
- Frenetov trobrid (krivulje u $\mathbb{R}^3$)
  - PDL
$$T(s) = c'(s), \quad N(s) = \frac{c''(s)}{||c''(s)||}, \quad B(s) = T(s) \times N(s) = \frac{c'(s) \times c''(s)}{||c'(s) \times c''(s)||}$$
  - Opći parametar
$$T(t) = \frac{\dot{c}(t)}{\|\dot{c}(t)\|}, \quad B(t) = \frac{\dot{c}(t) \times \ddot{c}(t)}{\|\dot{c}(t) \times \ddot{c}(t)\|}, \quad N(t) = B(t) \times T(t)$$
- Frenetove formule (krivulje u $\mathbb{R}^3$)
$$T' = \|\dot{c}\|\kappa N, \quad N' = \|\dot{c}\|(-\kappa T + \tau B), \quad B' = -\|\dot{c}\|\tau N$$
- Fleksija i torzija krivulje u $\mathbb{R}^3$
$$\kappa(t) = \frac{\|\dot{c}(t) \times \ddot{c}(t)\|}{\|\dot{c}(t)\|^3}, \quad \tau(t) = \frac{\det(\dot{c}(t), \ddot{c}(t), \dddot{c}(t))}{\|\dot{c}(t) \times \ddot{c}(t)\|^2}$$

Plohe

- Gaussova zakrivljenost
$$K(p) := \det S_p = k_1(p)k_2(p), \quad K = \frac{LN - M^2}{EG - F^2}$$
  pri čemu su $k_1(p), k_2(p)$ glavne zakrivljenosti.
- Srednja zakrivljenost
$$H(p) := \frac{1}{2}\mathrm{tr} S_p = \frac{k_1(p) + k_2(p)}{2}, \quad H = \frac{EN - 2FM + LG}{2(EG - F^2)}$$

- Weingartenova funkcija
$$W^2 = ||\mathbf{x}_u \times \mathbf{x}_v||^2, \quad W^2 = EG - F^2 > 0$$
- Prva fundamentalna forma
$$I = Edu^2 + 2Fdudv + Gdv^2$$
- Koeficijenti prve fundamentalne forme
$$E := \mathbf{x}_u^2, \quad F := \mathbf{x}_u \cdot \mathbf{x}_u, \quad G := \mathbf{x}_v^2$$
- Druga fundamentalna forma
$$II = Ldu^2 + 2Mdudv + Ndv^2$$
- Koeficijenti druge fundamentalne forme
$$L := S_p(\mathbf{x}_u) \cdot \mathbf{x}_u, \quad M := S_p(\mathbf{x}_u) \cdot \mathbf{x}_v = S_p(\mathbf{x}_v) \cdot \mathbf{x}_u, \quad N := S_p(\mathbf{x}_v) \cdot \mathbf{x}_v$$
$$\begin{aligned} L &= n \cdot \mathbf{x}_{uu} = \frac{1}{W} \det(\mathbf{x}_{uu}, \mathbf{x}_u, \mathbf{x}_v) \\ M &= n \cdot \mathbf{x}_{uv} = \frac{1}{W} \det(\mathbf{x}_{uv}, \mathbf{x}_u, \mathbf{x}_v) \\ N &= n \cdot \mathbf{x}_{vv} = \frac{1}{W} \det(\mathbf{x}_{vv}, \mathbf{x}_u, \mathbf{x}_v) \end{aligned}$$
- Normalna zakrivljenost
$$k_n(v_p) = \frac{S_p(v_p) \cdot v_p}{v_p \cdot v_p}, \quad k_n(v_p) = \frac{II(v_p)}{I(v_p)}$$
- Eulerov teorem
$$k_n(v_p) = k_1 \cos^2 \alpha + k_2 \sin^2 \alpha, \quad \text{za } v_p = \cos \alpha e_1 + \sin \alpha e_2$$
pri čemu su e_1, e_2 jedinični glavni vektori, k_1, k_2 glavne zakrivljenosti.
- ODJ za krivulje zakrivljenosti u karti
$$\begin{vmatrix} \left(\frac{dv}{dt}\right)^2 & -\left(\frac{du}{dt}\right)\left(\frac{dv}{dt}\right) & \left(\frac{du}{dt}\right)^2 \\ E & F & G \\ L & M & N \end{vmatrix} = 0$$
- ODJ za asimptotske krivulje u karti
$$L\left(\frac{du}{dt}\right)^2 + 2M\left(\frac{du}{dt}\right)\left(\frac{dv}{dt}\right) + N\left(\frac{dv}{dt}\right)^2 = 0$$
- Neka su $c_1(t) = \mathbf{x}(u_1(t), v_1(t))$, $c_2(t) = \mathbf{x}(u_2(t), v_2(t))$. Tada je kut među njima u njihovoj točki presjeka $c_1(t_1) = c_2(t_2)$ jednak
$$\cos \varphi = \frac{\dot{c}_1(t_1) \cdot \dot{c}_2(t_2)}{\|\dot{c}_1(t_1)\|\|\dot{c}_2(t_2)\|} = \frac{Eu_1'u_2' + F(u_1'v_2' + v_1'u_2') + Gv_1'v_2'}{\sqrt{Eu_1'^2 + 2Fu_1'v_1' + Gv_1'^2}\sqrt{Eu_2'^2 + 2Fu_2'v_2' + Gv_2'^2}}$$
- Površina dijela plohe
$$P = \int_U \sqrt{EG - F^2}\, dudv$$