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Resonance frequencies of the human skull *in vivo*

Bo Håkansson, Anders Brandt, and Peder Carlsson
Department of Applied Electronics, Chalmers University of Technology, S-412 96 Gothenburg, Sweden

Anders Tjellström
ENT Department, Sahlgren Hospital, S-413 45 Gothenburg, Sweden

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Patients with skin penetrating titanium implants in the temporal bone, for attachment of bone-anchored hearing aids, have made it possible to investigate the free-damped natural frequencies (resonance frequencies) of the human skull *in vivo*. The resonance frequencies of the skull of six subjects were investigated. The resonance frequencies were extracted from two frequency response functions (acceleration/force) measured on each subject: One point measurement where the force and acceleration were both measured at the same point, and one transcranial measurement where the acceleration was measured contralaterally. Between 14 and 19 resonance frequencies were identified for each subject in the frequency range 500 Hz to 7.5 kHz. The two lowest resonance frequencies were found to be on the average 972 (range 828–1164) and 1230 (range 981–1417) Hz. The relative damping coefficients of all resonances were found to be between 2.6 and 8.9%. Due to the relatively high damping coefficients, it is assumed that the resonance frequencies do not significantly affect bone conducted sound. In the transcranial measurements, however, a few large antiresonances were found which may affect bone-conducted sound. Intersubject variations were large, probably due to individual variations in skull geometry and in mechanical parameters. The results were shown to be consistent with previous results obtained on dry skulls. No obvious correlation between lowest resonance frequency and skull size was found.

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INTRODUCTION

Several investigations of the resonance frequencies of the human skull in the audiofrequency range have previously been made, theoretical as well as experimental. The main reasons for these investigations have usually been either to gain better understanding of head injuries, or to gain knowledge of hearing by bone conduction. This report will focus on the latter issue, however it is hoped that the results can also be applicable in gaining understanding of head injuries.

Previous experimental investigations of the resonance frequencies of the human skull have been made on living subjects, as well as on cadavers and dry skulls. However, the vibrations of the skull have never been measured directly on the skull bone of living subjects. Measurements on living subjects have been made on the surface of the skin, while measurements directly on the bone have been made either on cadavers or on dry skulls. Due to the attenuating effect of the skin, especially at high frequencies, knowledge of the true vibrations of the human skull bone *in vivo* is poor.

Most of the previous investigations of the vibrational characteristics of the human skull have been concentrated on the mechanical impedance, and have presented only one or two of the lowest resonance frequencies. Furthermore, serious doubts were raised by Khalil *et al.* (1979) as to whether the methods used to support the heads under investigation, in many cases, may have affected the free resonance frequencies of the skull.

Békésy (1951) reported the two lowest resonance frequencies to be 800 and 1600 Hz, based on measurements on living subjects excited by a vibrating piston. Franke (1956) measured the skin impedance on living subjects, but was unable to extract the resonance frequencies of the human skull from these data. The fact that the resonance frequencies of the human skull cannot be extracted from the skin impedance was confirmed by Flottorp and Solberg (1976), and by Håkansson *et al.* (1986). Franke also measured the vibrations on a dry skull and reported the lowest resonance frequency to be 820 Hz. For the same skull filled with gelatin, he reported the frequency to be 500 Hz. From measurements on a cadaver, Franke reported the two lowest resonance frequencies to be 600 and 900 Hz.

So far, Khalil *et al.* (1979) are the only ones, to our knowledge, who have presented a comprehensive investigation of the resonance frequencies of the human skull. They investigated two dry skulls (male and female) by experimental modal analysis in the frequency range 20 Hz to 5 kHz. Both resonance frequencies and associated mode shapes were reported. In total, they found 11 resonance frequencies for the male skull, and six for the female skull. To estimate resonance frequencies valid for living subjects, Khalil *et al.* extrapolated the resonance frequencies they had obtained on the dry skulls, taking into account known and estimated differences in mechanical parameters between dry and living bone, as well as the effects of the brain and soft tissues. The validity of this estimation, however, was not confirmed in that paper.

Khalil *et al.* also observed that the intersubject varia-

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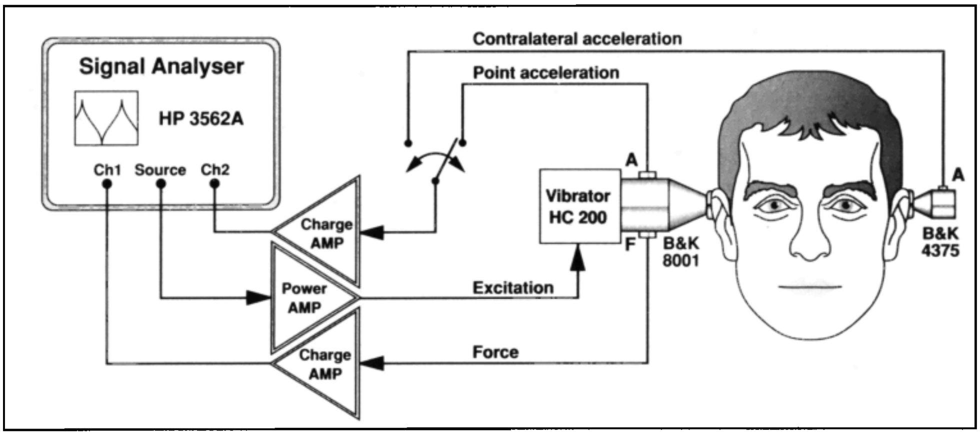

FIG. 1. Measurement setup consisting of gauges (impedance B&K 8001 and accelerometer B&K 4375), vibrator (HC 200), amplifiers, and a 2-channel FFT signal analyzer (HP 5423) as major parts. The excitation signal is generated by the signal analyzer and is fed to the vibrator via a power amplifier. The signal analyzer computes the frequency responses: Point acceleration/excitation force or contralateral acceleration/excitation force.

tions in resonance frequencies and in mode shapes were large. They suggested this was due to individual differences in skull geometry and mechanical parameters.

Håkansson *et al.* (1986) reported large variations between subjects in the mechanical point impedance of the human skull. In their study, they found the first two resonance frequencies of the skull to be on the average 1000 and 1500 Hz with standard deviations of 200 and 270 Hz, respectively. These resonance frequencies were obtained from the mechanical point impedance, measured on skin-penetrating titanium fixtures on seven subjects. In that study it was stated that the resonance frequencies are not affected by the attachment of vibrator and gauges to the skull, except for local compliance at high frequencies (>7.5 kHz) and bending at low frequencies (<500 Hz).

The aims of the present study are to

(1) identify as many free resonance frequencies as possible;

(2) determine the relative damping coefficients of the resonance frequencies; and

(3) investigate whether there is a simple relation between skull geometry and resonance frequencies.

I. MEASUREMENT SETUP

As shown by the block diagram of the measurement setup in Fig. 1, a Hewlett-Packard dynamic signal analyzer, model 3562A, was utilised for the sampling and processing of the signals. The skulls were excited by a miniature vibrator of the type used in the bone-anchored hearing aid HC 200, through a Brüel & Kjær (B&K 8001) impedance head. The vibrator was fed with bandlimited random noise generated by the noise generator in the signal analyzer and amplified by a power amplifier (B&K 2706). For the point measurements, the force and acceleration were both measured by the impedance head. For the transcranial measurements, the response acceleration was measured with a B&K 4375 accelerometer. For both point and transcranial measurements, the force and the acceleration signals were fed into the analyser via battery operated charge amplifiers (B&K 2651 and B&K 2635, respectively).

The impedance head and the accelerometer were attached to the skull via adaptors which were threaded onto the skin-penetrating titanium abutment as is shown in Fig. 2. Each adaptor was rigidly attached to the titanium abutment by tightening two screws. Two photographs show the vibrator and impedance head unit [Fig. 3(a)], and the contralaterally placed accelerometer [Fig. 3(b)]. The total mass of the impedance head, vibrator, and adaptor was 55 g, whereas the accelerometer and adaptor weighed 4.5 g.

The excitation level was set to produce a sound level as high as possible without causing discomfort. Due to the characteristics of the vibrator used for the excitation, the force level obtained varied between −100 and −30 dB [relative to 1 N^2/Hz] rms force power spectrum, which corresponds to normal hearing levels.

To obtain sufficient frequency resolution, the total frequency band was divided into three different intervals: 0–2.5, 2.5–5, and 5–7.5 kHz. This yielded a frequency resolution of 4.69 Hz. The lower frequency limit in the analysis was 500 Hz because the inertial mass of the miniature vibrator significantly reduces the force input below this frequency, making the signal-to-noise ratio too poor. That this does not reduce the quality of the study can be as-

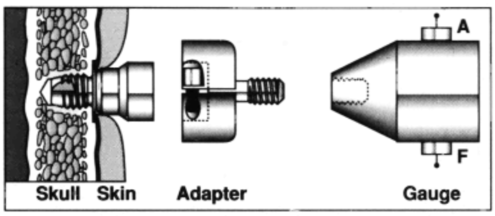

FIG. 2. Illustration of the adaptor used for rigid attachment of the gauge (impedance head or accelerometer) to the titanium implant. From left: Titanium implant, adaptor, and gauge. The adaptor squeezes (tightened with two transversely directed screws) over the outer cylindrical part of the implant.

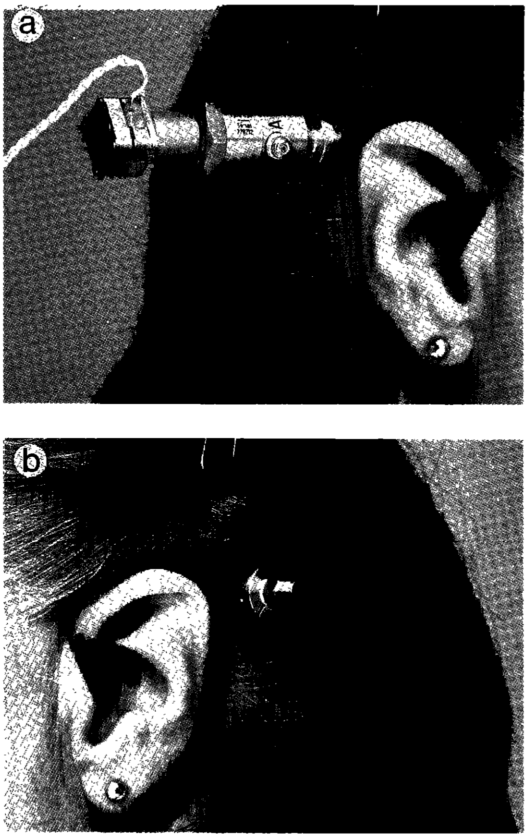

FIG. 3. Photographs showing: (a) vibrator, impedance head, and adaptor used for attachment to the titanium abutment; (b) the contralaterally placed accelerometer.

sumed since most previous investigations (Khalil *et al.*, 1979; Håkansson *et al.*, 1986) as well as a separate investigation within the present study, have found no free resonance frequencies below 800 Hz. To reduce the variance of the estimated frequency response functions, 50 consecutive records were averaged. This resulted in a total measurement time of approximately 16 s for each frequency response.

II. METHOD OF RESONANCE FREQUENCY ESTIMATION

When investigating resonance frequencies of a mechanical structure, it is important to separate *free* resonance frequencies and *forced* resonance frequencies. The free resonance frequencies are determined by the properties of the structure. These resonance frequencies are given by the eigenvalues in theoretical analysis. In an experimental investigation of a structure, however, one has to apply an external excitation at at least one point of the structure and apply transducers for response measurements at one or more points. This connection between external devices and the structure will affect the measurement results. Resonances caused by interaction between the external devices and the structure are called *forced* resonances. The forced resonances are affected by the properties of the external devices and the local properties of the structure, as well as by the coupling between them. Håkansson *et al.* (1986) thoroughly investigated the point impedance of the skull bone by a method similar to the one used in the present study. They reported that the forced resonances are found at frequencies below 800 Hz and that the free resonance frequencies are found above 800 Hz. In this report *resonance frequency* is used interchangeably with *damped natural frequency*.

Determining resonance frequencies and damping coefficients of lightly damped structures with sufficiently separated resonance frequencies can be made by simple methods—assuming viscous damping and using single degree of freedom (SDOF) curve fit. To determine the resonance frequencies and damping coefficients from frequency response data obtained from a complicated structure having insufficiently separated resonance frequencies which all are heavily damped, such as the human skull, is associated with significant difficulties and requires more sophisticated curve fit algorithms. The most important factors which will influence the validity of a general model of a complicated structure with high damping are mode coupling, type of damping (viscous or structural), and proportional damping. Fortunately, in this paper we are only interested in determining the resonance frequencies and damping coefficients. Hence, only the denominator of the transfer function is of interest and the discussion of proportional damping is irrelevant. In the present paper, the multiple degree of freedom (MDOF) curve fit algorithm included in the signal analyzer HP 3562, was used. This algorithm computes the weighted least-square fit of a rational polynomial to the measured frequency response function (Adcock, 1987). This MDOF curve fit algorithm assumes viscous damping and that assumption could be a source for errors in the frequency and damping estimates since the damping coefficients are in the range of 2% to 9%, see Sec. IV. However, we have chosen this MDOF curve fit algorithm since this is the most powerful (fits polynomials with up to 30 poles and zeros) one of those we have access to.

Assuming linearity, the dynamic behavior of a mechanical structure can be characterized by the transfer function

$$H(s)=A(s)/F(s), \tag{1}$$

where $A(s)$ and $F(s)$ are the Laplace transforms of response acceleration and excitation force, respectively. The force can be applied to any part of the system, and the response acceleration can be measured anywhere on the structure.

This paragraph will show how the resonance frequencies and damping coefficients were extracted from the poles of $H(s)$. The transfer function can be written as a sum of independent second-order subsystems as shown in Eq. (2) where the numerator Φ_n for simplicity is assumed to be real (the numerator does not affect the estimates of resonance frequencies and damping coefficients). Each subsystem is normally referred to as a mode.

$$H(s)=-\omega^2 \sum_{n=1}^{N} \frac{\Phi_n}{s^2+s\,2\zeta_n\omega_n+\omega_n^2}. \tag{2}$$

The pole pair corresponding to the nth mode are, hence,

$$s_{1,2}=-\zeta_n\omega_n\pm j\omega_n\sqrt{1-\zeta_n^2}=\mathrm{Re}\pm j\ \mathrm{Im}, \qquad (3)$$

where ω_n is the undamped resonance frequency in radians and ζ_n is the relative damping coefficient. The free-damped natural frequencies f_{r_n} in Hz of that mode correspond to the imaginary part of the poles divided by 2π, i.e.,

$$f_{r_n}=\mathrm{Im}/2\pi. \qquad (4)$$

If ω_n is substituted in Eq. (3), then the damping coefficient is given by

$$\zeta_n=100/\sqrt{1+(\mathrm{Im}/\mathrm{Re})^2}\%. \qquad (5)$$

For more details in these matters, see Ewins (1986).

The number of zeros and poles of $H(s)$ was chosen manually by inspecting the magnitude/phase and Nyquist plots of the frequency response (Kennedy and Pancu, 1947). Extra poles and zeros were added to compensate for residual terms outside the fitted region. To reduce the effect of residual terms, each fit was computed over a frequency span as large as possible, taking into account the quality of the frequency response. Before accepting the fitted curve, the magnitude of the measured frequency response and the magnitude of the fitted curve were compared. A maximum deviation of 0.5 dB in magnitude between the curves was allowed, except for frequencies in the vicinity of antiresonances, where the coherence was low. To further improve the estimate of the resonance frequencies and damping coefficients, two different frequency response measurements were made: One point measurement, where the force and acceleration were measured at the same point, and one transcranial measurement, where the force and acceleration were measured on opposite sides of the skull. Each resonance frequency and damping coefficient was calculated as the mean of the corresponding frequencies and damping coefficients obtained from both the point and transcranial frequency response functions.

A. Accuracy of resonance frequency estimates

The total measurement setup was calibrated by measurements on a rigid mass of 50 g. The calibration factors were taken so as to give correct accelerance values at 1 kHz.

The estimation of resonance frequencies and damping coefficients can be affected by several errors, of which the following three are considered to be the most serious:

(a) random errors due to poor signal-to-noise ratios;
(b) bias errors (frequency dependent) caused by the transducers; and
(c) the model assumes viscous damping.

Bias errors that are frequency independent, such as calibration errors, do not affect the estimation of resonance frequencies or damping coefficients, since the bias errors simply produce a constant factor in the transfer function and do not affect the poles.

1. Random errors

As a quality factor of the estimated frequency response function, the coherence function $\gamma^2(f)$ was used. For both the point and the transcranial measurements, the coherence function was observed generally to exceed 0.98 for frequencies above 800 Hz. For the transcranial measurements, however, the coherence values were sometimes below 0.98 for frequencies around antiresonances. Between 500 and 800 Hz, the coherence function exceeded 0.8 for all frequencies in both measurements.

For frequencies above 800 Hz, measurement accuracies were estimated to be within $\pm 5\%$ in magnitude and $\pm 3°$ in phase for the point frequency response functions, and $\pm 6\%$ and $\pm 3°$ for the transcranial frequency response functions. These values were based on a worst case coherence value of 0.98 and a confidence interval of 95%, as well as a match error between the channels in the analyzer, given by the manufacturer to be within ± 0.1 dB in amplitude and $\pm 0.5°$ in phase. In the low-frequency region between 500 and 800 Hz, the measurement accuracies were estimated to be within $\pm 12\%$ in magnitude and $\pm 7°$ in phase because of the lower coherence value (worst case 0.8) in this region. Further details on error analysis can be found in Bendat and Piersol (1980).

2. Bias errors

Measurements with impedance heads are biased because of several factors. For point measurements on the human skull, Håkansson and Carlsson (1987) reported that bias errors, other than the one caused by the mass above the force gauge, are negligible. This mass was compensated for by postprocessing point measurement data in the present study.

3. Viscous versus structural damping

A model based on the assumption that the damping is viscous will give errors when the resonances of the structure are heavily damped and not well separated. In such cases, a model based on structural damping would be better. As mentioned previously, we do not have access to a curve fit algorithm involving structural damping at present and therefore cannot determine the error in the estimates of the resonance frequencies and damping coefficients.

III. SUBJECTS

Six voluntary subjects with bilateral titanium fixtures were investigated. The fixtures were situated in the temporal bone approximately normal to the skull surface. The

TABLE I. Six subjects: Sex, age, skull circumference, and skull width between the titanium fixtures.

Subject	Sex	Age (years)	Skull circ.[a] (cm)	Skull width[a] (cm)
a	f	72	52	17
b	m	72	57	19
c	m	52	55	18
d	f	72	55	14
e	f	53	55	17
f	f	63	54	18

[a]Measured value reduced by 1 cm to compensate for the skin.

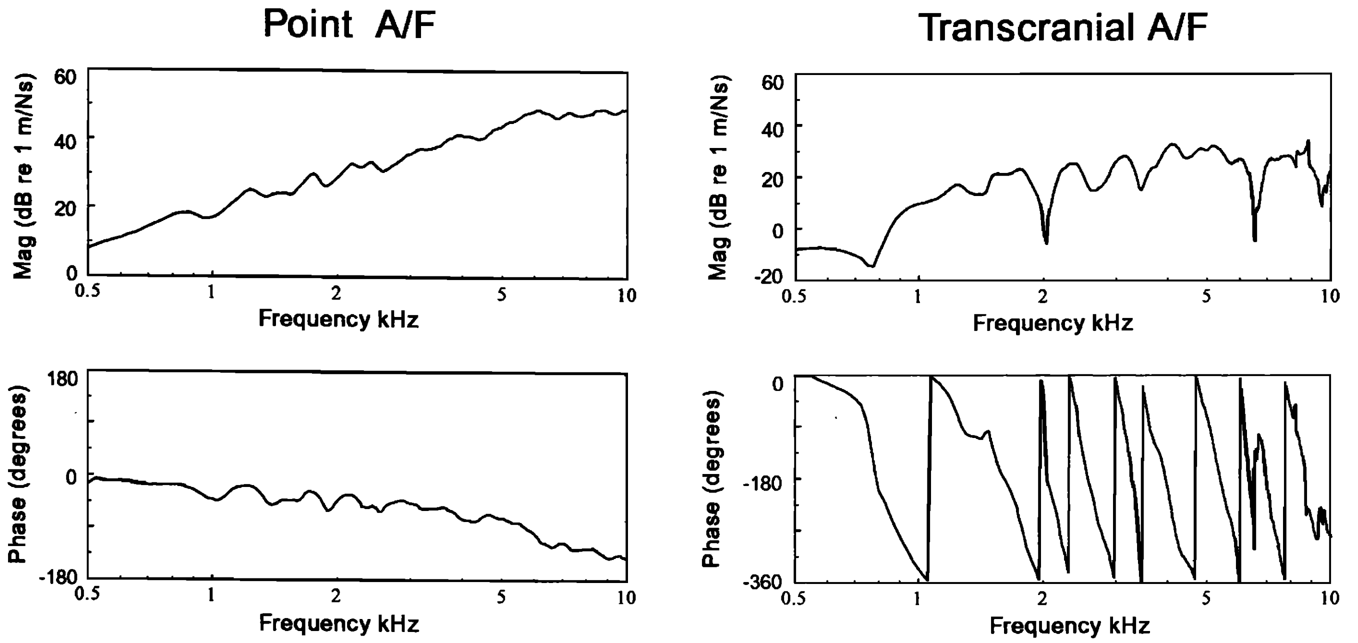

FIG. 4. The magnitude and phase response of the point A/F (accelerance) and transcranial A/F (accelerance) of the skull of subject (d). The frequency resolution was 4.69 Hz and the data cover frequencies from 500 Hz to 10 kHz (logarithmic scale).

age and sex of the different subjects, as well as their skull circumference and the distance between the fixtures, i.e., the skull width, are presented in Table I. The circumference was measured horizontally with a tape measure at the height of the forehead. The distance between the titanium fixtures was measured as the shortest distance across the the skull. As these distances are measured outside the skin, they were reduced by one cm to yield the approximate distance at the level of the skull bone. All subjects had normal skull anatomy.

IV. RESULTS

Frequency response functions (acceleration/force) for one typical subject (d) are shown in Fig. 4, for the point and transcranial measurements, respectively. By visual inspection of the curves, it can be concluded that the skull resonances (the peaks) have a small influence on the overall characteristics in the point measurement, whereas their influence on the transfer data is somewhat greater. Two sharp dips (antiresonances) can be seen in the transcranial data and their existence and influence are treated in the discussion.

In Fig. 4 it can also be seen that at low frequencies the phase seems to be approximately constant. This implies that the human skull behaves like a rigid body at frequencies below the first resonance, as has been reported by Håkansson *et al.* (1986).

Between 14 and 19 resonance frequencies were found in the frequency range investigated, 500 Hz to 7.5 kHz, for the different subjects, as presented in Fig. 5. The first resonance frequency ranged from 828 to 1164 Hz and the second from 981 to 1417 Hz. The mean of the first resonance frequency was 972 Hz with a standard deviation of 119 Hz; for the second resonance, the values were 1230 and 148 Hz, respectively. The corresponding relative damping coefficients, which are presented in Fig. 6, were found to

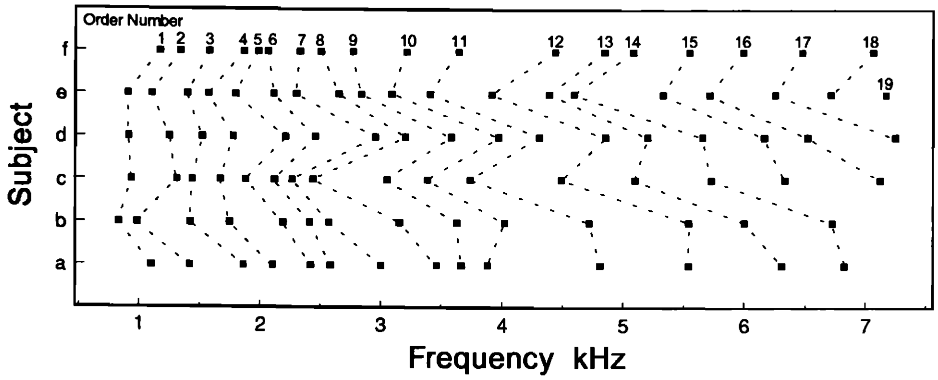

FIG. 5. Resonance frequencies for each patient (a)–(f). The "order numbers" are the consecutive numbers of the resonance frequencies detected in this study. The vertical dashed lines are introduced only for the purpose of facilitating a comparison between subjects.

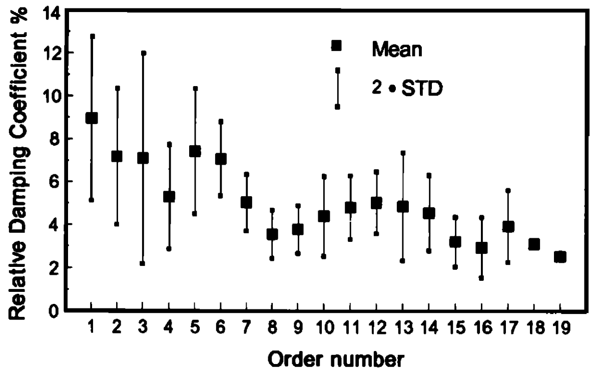

FIG. 6. Relative damping coefficients presented as mean and standard deviation for each order number (consecutive order in which the resonances were detected). The number of subjects involved in each order number is given as "N" in Table II.

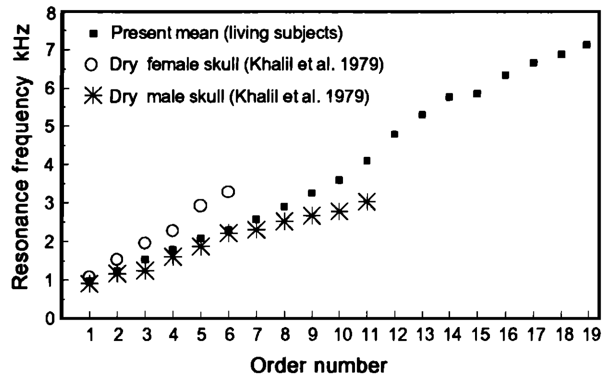

FIG. 7. Comparison between the mean values of the resonance frequencies obtained in this study and those found by Khalil *et al.* (1979) on dry skulls. The resonance frequencies on dry skulls were extrapolated by Khalil *et al.* to compensate for differences between dry and living bone tissue.

range from 2.6% to 8.9% with an overall average of 5%.

Mean and standard deviations of the resonance frequencies and the relative damping coefficients are presented in Table II in order to facilitate future comparisons with other investigations. The "order numbers" in Table II, as well as in Figs. 5, 6, and 7, are the consecutive numbers of the resonance frequencies detected in this study, introduced for convenience, and should not be confused with the mode numbers used in classical modal analysis.

V. DISCUSSION

The excitation force, as already mentioned, was low at low frequencies, which resulted in low coherence values for frequencies below 500 Hz. Previous investigations (Khalil *et al.*, 1979; Håkansson *et al.*, 1986) have not revealed any resonance frequencies of the human skull for frequencies lower than 500 Hz. To investigate this statement further, measurements for frequencies between 100 and 500 Hz were made on two subjects with a vibrator modified to emphasise low frequencies (added inertial mass). These measurements confirmed that no free resonance frequencies of the human skull were to be found at frequencies below 500 Hz.

TABLE II. Mean and standard deviations (STD) of resonance frequencies and damping coefficients with respect to the order number.

		Resonance frequency		Damping coefficient	
Order number[a]	N	Mean (Hz)	STD (Hz)	Mean (%)	STD (%)
1	6	972	119	8.9	3.8
2	6	1230	148	7.2	3.2
3	6	1532	159	7.1	4.9
4	6	1785	169	5.3	2.4
5	6	2076	217	7.4	2.9
6	6	2287	203	7.1	1.7
7	6	2568	308	5.0	1.3
8	6	2899	389	3.6	1.1
9	6	3253	381	3.8	1.1
10	6	3590	377	4.4	1.9
11	6	4101	543	4.8	1.5
12	6	4793	596	5.0	1.4
13	6	5304	660	4.9	2.5
14	6	5766	807	4.6	1.8
15	4	5841	419	3.2	1.2
16	4	6336	539	3.0	1.4
17	3	6656	429	4.0	1.7
18	2	6883	176	3.1	0.1
19	1	7165	...	2.6	...

[a]Order number=consecutive order in which the resonances were detected.

Not all of the resonance frequencies were found in both the point and transfer frequency response curves. This occurs when the response or excitation point is situated on or near a nodal line, i.e., on a point which is not moving for that particular mode. The term "near" a modal line implies that the particular mode at a particular point could be either too weak and/or too damped to be detected in the FRF because of a poor signal-to-noise level. If the resonance is heavily damped, one could conclude that such resonances have no significance for bone conduction. Another observation is that some frequencies obtained for the same order number differ conspicuously in the point and transfer frequency response functions. There are many structural mechanical phenomena which can cause this discrepancy. One obvious explanation is that the resonance frequencies are actually caused by two different modes of vibration. Therefore, the resonance frequencies for the point and transfer frequency response functions have been arranged so that the difference between the resonance frequencies that are given the same order number does not exceed ±10% of the point resonance value.

The upper frequency limit in this analysis was 7.5 kHz. In the original measurements presented by Brandt (1989), the frequency range from 7.5 to 10 kHz was also included in the analysis. Another 2–4 resonance frequencies were found in this range, but they are not presented here because it is unclear whether some of them may have been caused by the attachment of the transducers.

The results show large variations between subjects. Not only do the resonance frequencies vary between the subjects, but also the resonance frequency spacing differs.

TABLE III. Wavelengths calculated from the first resonance frequency and the phase velocity.

Subject	Resonance frequency (Hz)	Phase velocity[a] (m/s)	Wavelength Real (cm)	Wavelength Norm[b] (cm)
a	1100	260	24	24
b	828	367	44	22
c	934	383	41	20.5
d	907	172	19	19
e	901	544	60	20
f	1164	297	26	26

[a]Phase velocities according to Brandt, 1989.
[b]*Norm* means divided by a factor of 2 in patients b and c, and a factor of 3 in patient e.

This is illustrated in Fig. 5 where the difference in consecutive resonance frequencies is illustrated by the spaces between the dashed lines (one for each order number).

To investigate further the relation between the first resonance frequency and the skull geometry, the wavelength λ was calculated using the phase velocities of the transcranial sound transmission reported by Brandt (1989). The phase velocities reported in that study were obtained from the average slope of the phase response between input and output acceleration when the skull was excited by random noise (giving the time delay) and the distance between the mastoid bones. The phase velocities from that study are presented in Table III. The wavelength λ is defined by the relation

$$\lambda = c/f, \tag{6}$$

where c denotes the phase velocity and f denotes the frequency. In Table III the wavelengths calculated according to Eq. (6) using the value of the first resonance frequency and the phase velocities of each subject are given. It was found that the wavelength at the first resonance is a multiple of approximately 20 cm which is close to the width of the skull, or a third of the circumference. The variations in the integer values between the subjects can probably be explained by different mode shapes at the first resonance. Khalil *et al.* (1979) found that the mode shapes were unique for each of the two dry skulls in their investigation and the data found here seems to verify this. The correlations between normalized wavelength and width and circumference were found to be low (0.06 and 0.47, respectively). This indicates that there is no simple relation between the first skull resonance and skull size. We conclude, therefore, that the differences in resonance frequencies between subjects are probably due to geometrical variations, as well as to variations in mechanical properties of the skull bone. The same results were reported by Khalil *et al.* who proposed the same plausible causes.

An interesting result is that the resonance frequencies obtained in this study seem to be consistent with the extrapolated results obtained from measurements on two dry skulls presented by Khalil *et al.* (1979). They obtained the extrapolated frequencies by dividing the values obtained on dry skulls by the factor 1.53 which was deduced in their paper. The frequency range used in their investigation was 20 Hz to 5 kHz. The consistency is depicted in Fig. 7 where our mean frequencies and their extrapolated frequencies are plotted versus mode number.

From Fig. 4 it can be seen that the damping of the resonances is relatively high. Even though it was not included in the aims of this study, perhaps the most important conclusion of this paper is that the resonances do not significantly affect hearing by bone conduction. The transcranial response in Fig. 4, however, is characterized by several sharp dips (antiresonances). These antiresonances may cause a significant reduction in the sound transmission. The explanation for these dips could be that the response point is situated on a nodal line or that the sound transmission through the skull bone follows different paths (i.e., through the apex and through the base). These paths could have different phase responses such that cancellation may occur (180° phase shift).

In the present study, the excitation of the subjects can cause vibrations at the excitation point in other directions than those normal to the skull. This may occur because the axis of the titanium fixture is not directed straight at the mass center of the skull. Khalil *et al.* (1979) reported that the vibration levels in their investigation, in directions other than those normal to the skull surface at the excitation point, were less than 10% of the levels normal to the skull. The consistency of the number of resonance frequencies obtained in their study and in the present study seems to indicate that only a small number of the reported resonance frequencies in the present study can be due to forced phenomena.

To make sure that the weight of the accelerometer at the contralateral point did not seriously disturb the vibrations in the transcranial measurements, a measurement was made on one subject with an additional mass of approximately 2 g attached between the socket and the accelerometer. This measurement showed differences in the frequency response of 2 dB at the most. A curve fit made on the frequency response data resulted in maximum differences of less than $\pm 5\%$ in the resonance frequencies compared with the results obtained without additional mass.

Some lightly damped resonances were seen at frequencies above 5 kHz for subjects (c) and (f) in the point curves, and for subjects (b), (c), and (f) in the transcranial curves. Håkansson *et al.* (1986) reported the local compliance on titanium fixtures of the type used here to be approximately 100×10^{-9} m/N. Combining this compliance with the mass of the accelerometer and the adaptor, 4.5 g, yields a resonance frequency of approximately 7.5 kHz. Moreover, as the damping of these resonances was seen to be largely affected by changes of the attachment of the transducers, it seems reasonable to assume that these resonances are due to interaction between the measurement transducers and the local compliance in the attachment points.

Measurements were also made on one subject to investigate the reproducibility of the measurements. This subject was investigated twice, with an interval of 4 months, and the differences between the frequency response functions were found to be within $\pm 5\%$ in magnitude. A subsequent curve fit resulted in resonance frequency differences less than $\pm 2\%$.

One fundamental assumption made when defining the resonance frequencies of a mechanical system is that the system under investigation is linear. Many previous investigators have stated the mechanical point impedance as approximately linear for the vibration levels considered here, and for frequencies ranging from 100 Hz to 10 kHz (Smith and Suggs, 1976; Corliss and Coidan, 1955; Flottorp and Solberg, 1976; Khalil *et al.*, 1979; Håkansson *et al.*, 1986; and Brandt, 1989).

VI. CONCLUSIONS

Rigid anchorage of bilateral titanium fixtures into the human skull bone has made it possible to investigate the free resonance frequencies of the human skull on six subjects *in vivo*.

Between 14 and 19 resonance frequencies were found in the frequency range investigated from 500 Hz to 7.5 kHz. The free resonance frequencies were found to vary considerably between different subjects. The lowest free skull resonance frequency was found to range from 828 to 1164 Hz with a mean of 972 Hz. The second resonance frequency ranged from 981 to 1417 Hz and was, on the average, 1230 Hz.

The relative damping coefficients of the modes were found to be between 2.6% and 8.6% in general. It was concluded that due to the high relative damping of the modes of vibration, the effect of the resonances on hearing by bone conduction is negligible. However, in the transcranial transmission of bone conducted sound sharp antiresonances occur, which probably significantly affect hearing by bone conduction.

No obvious relation between skull size (width and circumference) and first resonance frequency was found. It is therefore probable that other properties, such as bone thickness and stiffness, as well as size, determine the resonance frequencies of the human skull.

Measurements of frequencies between 100 and 500 Hz on two subjects did not reveal any resonance frequencies in that frequency range. The absence of free resonance frequencies of the skull below 800 Hz, together with the flat phase of the transcranial frequency response, indicates that the skull behaves like a rigid body at low frequencies.

The resonance frequencies obtained from the present *in vivo* measurements are consistent with previous results obtained on dry skulls after correction of the frequencies for known differences in mechanical parameters and the effect of added soft tissues and the brain (Khalil *et al.*, 1979). This suggests that more thorough investigations of the vibrational characteristics of dry skulls may yield a better understanding of, for example, the sound propagation to the cochlea. Such investigations may prove to be very valuable since they are extremely difficult to make *in vivo*.

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