On the Modeling of Long Arc in Still Air and Arc Resistance Calculation

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***Abstract*—An important macroscopic arc parameter, describing its complex nature is the *arc resistance*. It can be easily calculated by using the well-known Warrington formula. Authors investigated the results of Warrington's tests. By taking into account the conditions under which they are obtained (e.g., inaccurate measurement devices), it is unquestionable that these results are today highly empirical and not accurate and general enough. Laboratory testing provided in the high-power test laboratory FGH-Mannheim (Germany), in which long high current arcs are initiated, was the basis for the research results presented in the paper. Based on the analysis of laboratory-recorded arc voltage and current waveforms, the new arc model is derived. An example of arc computer simulation using the new model is given. Based on the new arc model, a new approach to arc resistance calculation is presented. The new formula for arc resistance is compared with the old Warrington formula.**

***Index Terms*—Arc resistance, laboratory testing, long arc in still air, modeling, simulation.**

I. Introduction

ARC discharge is encountered in the everyday use of power equipment. Permanent faults in a transformer, machine, cable, or transmission line always involve an arc. Whenever a circuit breaker is opened while currying a current, an arc strikes between its separating contacts. The arc existing at the fault point is a *high-power long arc in still air*. It has not the same properties as an arc existing in circuit breakers. All arcs have a highly complex nonlinear nature, influenced by a number of factors. An arc can be considered as an element of electrical power system having a resistive nature (i.e., as a pure resistance). Due to its nonlinear nature, the modeling of arc is additionally a complex task. Some models [1]–[3] are developed, but they are not practical enough from the application point of view. Typical applications in power system protection are autoreclosure [4], distance, and directional protection [5], etc. From the short-circuit studies and its accuracy point of view, the consideration of fault arc is unavoidable. From the power quality point of view, an arc can be considered as a source of harmonics so its investigation (modeling, simulation, features derivation, etc.) is an important and challenging task today.

In the case of short-circuits occurring on lines within medium- and high-voltage networks, the distance protection has to locate precisely the fault location for a selective interruption of the fault. In most cases (over 90%), short circuits in networks are followed with an arc (arcing faults), so the arc voltage arising at the fault point disturbs the impedance evaluation (i.e., the fault location). In other words, an arc is a source of errors in the fault location process if it is not taken into the consideration when locating the fault. To avoid these errors, the well-known Warrington formula [6] for arc resistance calculation is used.

Empirically obtained results play an important role in investigating the nature of the electrical arc. One of the earliest experimental studies considering the long arc in still air is presented in [7] and [8].

In this paper, laboratory tests provided in the high-power test laboratory FGH-Mannheim (Germany) are described and used in derivation of arc model, arc features, and formula for arc resistance calculation.

In the paper, first Warrington results are analyzed and discussed. Second, the results obtained in FGH-Mannheim are presented and a new arc model is derived. Third, based on the new arc model, a new formula for arc resistance is derived. Finally, the new formula is compared with the Warrington formula.

II. Discussion on Warrington Formula

In [6], Warrington presented his remarkable results of field tests on the high-voltage systems of the New England and the Tennessee Electric Power Company. Through these tests, he investigated the influence of arc resistance on protective devices and derived his well known and widely applied general formula for arc resistance calculation

$$U_a = E_a L = \frac{K}{I^n} L \tag{1}$$

where U_a is arc voltage (V), E_a is arc voltage gradient (V/ft, or V/m), $1\ \mathrm{ft} = 0.3048\ \mathrm{m}$, L is arc length (ft, m), I is arc root mean square (rms) current (A), and K and n are unknown constants. The unknown parameters K and n are estimated from measurements.

In Fig. 1, the third figure from [6], scanned and incorporated into this paper, is presented. In this figure, the measured arc voltage gradient E_a expressed in (kV/ft), is presented over currents in amperes. Here, only the selected measurement set is depicted. By this, the bad measurements are omitted. In [6], it is not explained how the bad measurements are omitted from consideration. In the same figure, a curve defining the relationship between E_a and current is plotted. The curve is obtained using the following parameters included in (1): $K = 8750$ and $n = 0.4$ $(1/n = 2.5)$. These parameters are valid if the arc length is expressed in (feet). In Fig. 1, in the Warrington formula given below the graph, the arc voltage is expressed in (kV).

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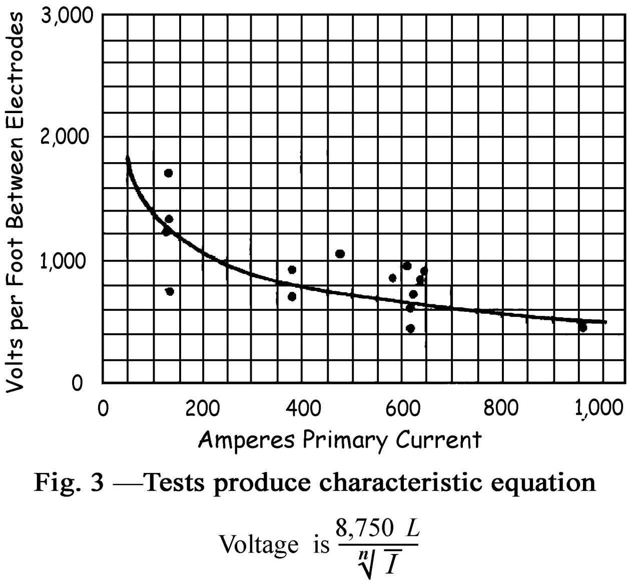

Fig. 1. Original measurements and results obtained by Warrington [6].

In other words, from the selected measurement set, Warrington determined parameters K and n, and by using (1), obtained the curve showing the relationship between arc voltage gradient and arc current. By including K and n into (1), one obtains the following formula for the arc voltage

$$U_a = \frac{8750}{\sqrt[2.5]{I}} L = \frac{8750}{I^{0.4}} L\,[(\mathrm{V/ft})\ \mathrm{ft}] = \frac{28\,688.5}{I^{0.4}} L\,[(\mathrm{V/m})\ \mathrm{m}]. \tag{2}$$

From (2), the next equation for arc resistance follows:

$$R_a = \frac{U_a}{I} = \frac{28\,688.5}{I^{1.4}} L\ [\Omega/\mathrm{m}] \tag{3}$$

where voltage is in volts (V), current in amperes (A), and arc length in meters (m).

In [6], a table with all measurements obtained by Warrington is given. Based on the full measurement set from [6], in this paper, parameters K and n are estimated and new estimated curves E_{a} over i_{a} are derived. In Fig. 2, both the full measurement set and the estimated curve for arc voltage gradient are presented. Parameters estimated in this case are $K = 3460.49$ and $n = 0.225\,333$ $(1/n = 4.437\,877)$.

Both parameters are essentially different from the parameters obtained by Warrington.

By observing Fig. 2, it is obvious that for one current, follow several various values for arc voltage gradient. This variety is probably the consequence of arc elongation occurring during the tests.

Under the assumption that some measurements were not correct (i.e., that some of them could be treated as bad data), in this paper, a reduced measurement set is selected and presented in Fig. 3. From the reduced measurement set, the following unknown parameters are estimated: $K = 11\,387.4$ and $n = 0.427\,663$ $(1/n = 2.338\,289)$. The new curve for E_{a} is depicted in the same plot.

In Fig. 4, the full measurement set from [6] and three arc voltage gradient curves (the Warrington, the full, and the reduced measurement set curves) are presented.

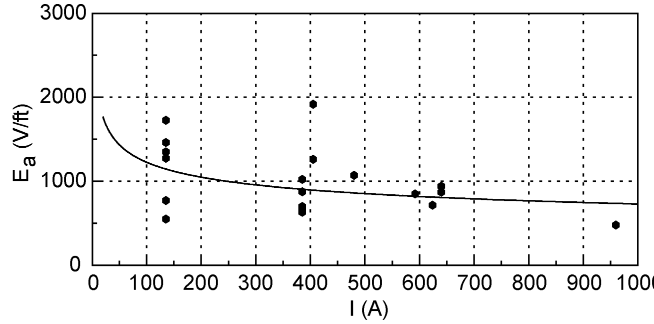

Fig. 2. Full measurement set and estimated arc voltage gradient curve.

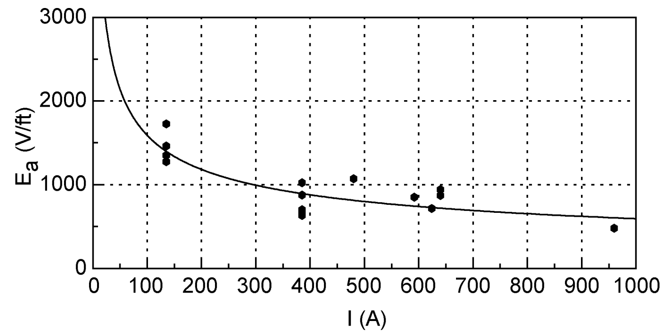

Fig. 3. Reduced measurement set and estimated arc voltage gradient curve.

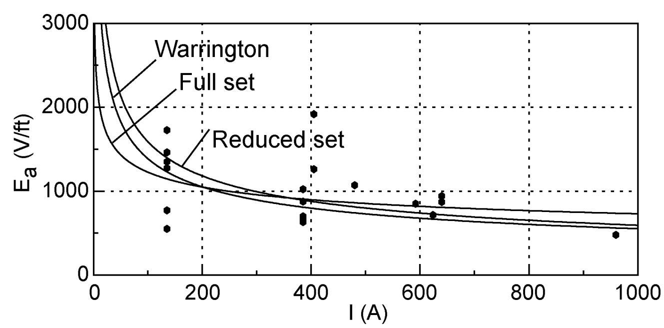

Fig. 4. Full measurement set from [6] and three arc voltage gradient curves.

It is obvious that two new independent and different equations for arc voltage resistance calculation can be now obtained.

From the above results, the following observations regarding Warrington field tests and results are formulated.

1) Measurement devices used during Warrington testing were inaccurate, so the conclusions derived are not reliable enough.
2) During the arc life, the arc length has been changed. These changes are not considered when Warrington formula is derived.
3) A criterion used in [6] by which some bad data are rejected (i.e., omitted from the consideration), is not described. It seems that the selection of the measurements processed is provided quite arbitrarily. The use of known standard robust estimators, not sensitive to bad data, should solve this task.
4) The methodology how Warrington formula is derived is not mentioned in the text.
5) The range of arc currents observed is extremely small (<1 kA) compared to real short-circuit currents.

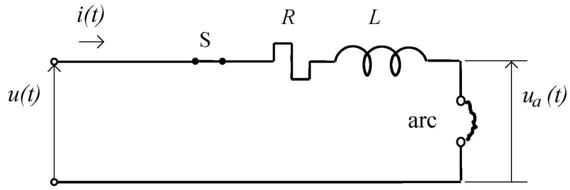

Fig. 5. Laboratory test circuit.

Fig. 6. Insulator chain with an arc.

6) Warrington formulas cannot be accepted as correct, so the new formulas should be derived.

The sixth observation *that Warrington formula is not correct*, as well as the fact that *the formula is not derived by analyzing a wide range of currents* (the expected short-circuit currents are reaching today values over 50 kA), motivated authors to investigate the possibilities for deriving a new formula for arc resistance. The new formula should be used as an alternative to the Warrington one.

In order to derive a new formula for arc resistance, a new mathematical model for arc is derived. It is based on the investigation of arc voltage and current recorded in a high-power test laboratory. These two important research steps are presented in the next paper section.

III. Laboratory Tests in High-Power Test Laboratory

The nature of arc has been investigated in the high-power test laboratory FGH-Mannheim (Germany) where a series of laboratory tests is provided. Voltage $u(t)$, current $i(t)$, and arc voltage $u_{\mathrm{a}}(t)$ are digitized from the simplified laboratory test circuit depicted in Fig. 5. All data are digitized with the sampling frequency of 0.166 MHz. The arc between arcing horns of a vertical insulator chain is initiated by means of a fuse wire, when switch S in Fig. 2 is closed. The distance between electrodes is changed in the range of 0.17–2 m. On arc initiation (i.e., immediately after melting and evaporating of the fuse wire), the arc voltage was defined with the values determined by distance between the horns.

In Fig. 6, a 2-m insulator chain from high-power test laboratory FGH-Mannheim with an arc is presented.

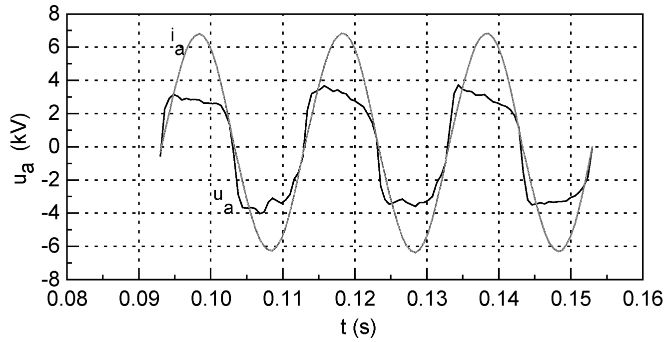

Fig. 7. Recorded arc voltage and current.

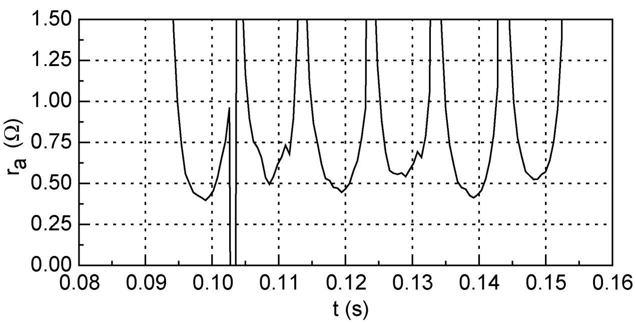

Fig. 8. Time-varying instantaneous arc resistance.

In Fig. 7, the recorded arc voltage $u_{\mathrm{a}}(t)$ and current $i(t)$, which is at the same time the arc current $i_{\mathrm{a}}(t)$, are depicted. Arc voltage and arc current are in phase. This fact confirms the resistive arc nature. The instantaneous electrical arc resistance r_{a} obtained as $r_a(t) = u_a(t)/i_a(t)$ is presented in Fig. 8.

IV. Modeling of Long Arc in Still Air

Modeling of long arc in still air attracted the attention of many authors in the past. Dynamic properties of an a-c arc can be represented by differential equations [1]–[3], given in the general form as $\mathrm{d}g_a/\mathrm{d}t = f(g_a, u_a, i_a, \mathit{\Pi}, t)$, where g_{a} is the time-varying arc conductance and $\mathit{\Pi}$ is a set of model parameters. The main problem here is the selection of the set $\mathit{\Pi}$. The unknown parameters must be estimated from test data.

By observing the arc voltage and current waveforms plotted in Fig. 3, it can be concluded that the voltage has a *distorted rectangular form*. Additionally, it is in phase with its current. Thus, the arc model can be represented through the following equation:

$$u_{a0}(t) = \left(U_a + U_b \frac{I_0}{i_b(t)} + R_\delta \left| i_b(t) \right| \right) \operatorname{sgn}(i_a) + \xi \tag{4}$$

where $u_{\mathrm{a0}}(t)$ and $i_{\mathrm{a}}(t)$ are voltage and current signals of an arc having the constant length L_0. By this, $U_{\mathrm{a}}, U_{\mathrm{b}}, I_0$ $(I_0 \neq 0)$, and R_δ are parameters, defining the shape of the arc voltage, and

$$i_b(t) = \left\{ \begin{array}{ll} I_0 & |i_a(t)| < I_0 \\ |i_a(t)| & |i_a(t)| \geq I_0 \end{array} \right\}. \tag{5}$$

In (4), sgn is a sign function and $\xi(t)$ is zero-mean Gaussian noise. The value of U_{a} can be obtained as the product of arc-voltage gradient E_{a} and the actual arc length L_{a} (i.e., the flashover length of a suspension insulator string, or

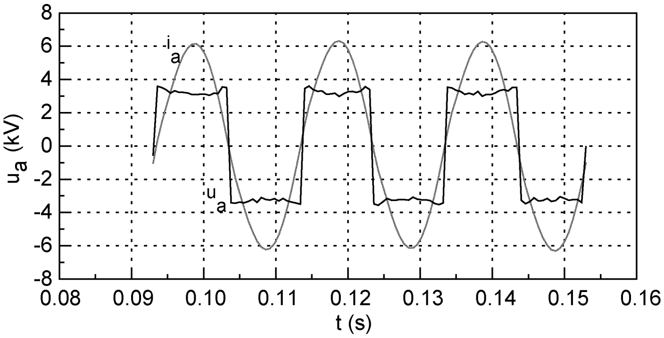

Fig. 9. Simulated arc voltage and arc current.

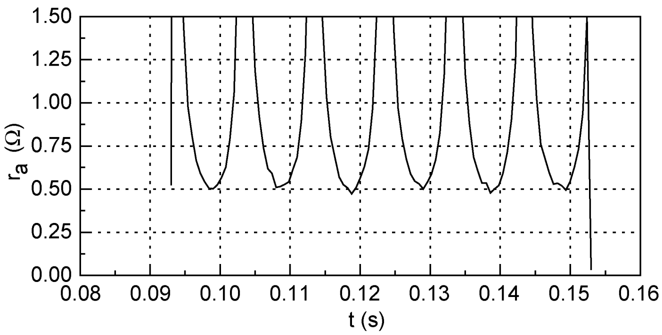

Fig. 10. Time-varying simulated arc resistance.

the flashover length between conductors). In (4), the term $U_b I_0 / i_b(t)$ models the arc ignition voltage, whereas the term $R_\delta |i_b(t)|$ is an additional quasi-linear part determined by the arc current. Due to simplicity, parameter R_δ will be called *the quasi arc resistance*, but it is just a small part of the actual arc resistance, mainly determined by the value of $U_{\rm a}$.

The transients in the circuit from Fig. 5 are simulated by using the EMTP software package presented in [9]. The arc model parameters selected were $U_{\rm a} = 3\,\rm kV$, $U_{\rm b} = 0.5\,\rm kV$, $I_0 = 2\,\rm kA$, and $R_\delta = 0.2\ \Omega$. In Fig. 9, the simulated arc voltage $u_{\rm a}(t)$ and current $i_{\rm a}(t)$ are, respectively, presented. The corresponding time-varying arc resistance is plotted in Fig. 10.

As a measure of the degree of linear relationship between the arc voltage signals presented in Figs. 7 and 9, the correlation coefficient r is calculated. Its value of $r = 0.94$ confirms that the arc model presented is very realistic.

V. Steady Conditions Properties of an Arc

From the electrical properties of an arc under steady conditions (volt-ampere characteristic) point of view, a number of equations are derived from the experimental studies. The best known is that obtained by Ayrton [7]

$$U_a = A + BL + \frac{C + DL}{I} \tag{6}$$

where A is the anode/cathode voltage drop, B is the voltage gradient, C has the dimension of power, and D has the dimension of the rate of power change over the arc length.

If in (6), L is made sufficiently large (the long arcs case), the terms involving parameters A and C may be neglected, and the characteristic equation becomes approximately

$$U_a = \left(B + \frac{D}{I}\right) L. \tag{7}$$

If in (7) current is sufficiently large (the high current long arcs case), the arc voltage becomes a function only of the arc length, according to the following equation:

$$U_a = BL. \tag{8}$$

Here, parameter B represents the voltage gradient in the arc column. It is almost independent of arc current, so the long high current arc voltages are essentially determined by the arc length L. Over the range of currents 100 A to 20 kA, the average arc voltage gradient lies between 1.2 and 1.5 kV/m [8], [10], [11]. In [7], it is shown that for long arcs, almost all the total arc voltage drop appears across the arc column.

VI. New Formula for Arc Resistance

In this section, a new formula for arc resistance calculation is derived. By this, a classical definition of electrical resistance in ac circuits is used.

Let us assume that arc voltage $u(t)$ and current $i(t)$ are modeled as follows:

$$u(t) = U_a \mathrm{sgn}\,[i(t)] \tag{9}$$

$$i(t) = \sqrt{2} I \sin \omega t. \tag{10}$$

Equation (9) represents the simplified (4). By this, the effects occurring around the current zero crossing and current maximum are neglected.

The electrical resistance R of an element belonging to an ac circuit is defined as

$$RI^2 = P = \frac{1}{T}\int_0^T p(t)dt = \frac{1}{T}\int_0^T u(t)i(t)dt \tag{11}$$

where I is the root mean square (rms) of current and $p(t)$ is the instantaneous power. By including (9) and (10) into (11), one obtains

$$RI^2 = \frac{2U_a}{T}\int_0^{\frac{T}{2}} i(t)dt. \tag{12}$$

Since

$$\int_0^{\frac{T}{2}} i(t)dt = \sqrt{2} I \int_0^{\frac{T}{2}} \sin\,\omega t\,dt = \frac{2\sqrt{2} I}{\omega} \tag{13}$$

equation (13) becomes

$$RI^2 = P = \frac{2U_a}{T}\frac{2\sqrt{2}I}{\omega} = \frac{2\sqrt{2} I U_a}{\pi}. \tag{14}$$

From (14), the explicit expression for the arc resistance follows:

$$R = \frac{2\sqrt{2}U_a}{\pi I}. \tag{15}$$

Let us now suppose that there exists the following linear relationship between the arc voltage magnitude and the arc voltage gradient [see (8)]:

$$U_a = E_a L. \tag{16}$$

By combining (15) and (16), one obtains

$$R = \frac{2\sqrt{2}}{\pi}\frac{E_a L}{I}. \tag{17}$$

Equation (17) is the new formula for arc resistance calculation. It requires a suitable selection of the value/expression for the arc voltage gradient E_{a}. In the open literature, the following values/expressions for E_{a} calculation are used: a) in accordance with (8) and from Lit. [8], [10], [11] $E_a = (1200 \div 1500)\ (\mathrm{V/m})$ and b) in accordance with (7) and from Lit. [12] $E_a = 950 + 5000/I\ (\mathrm{V/m})$, where I is expressed in amperes (A). By this, one obtains the following two new equations for arc resistance calculation:

$$R_1 = (1080.4 \div 1350.5)\frac{L}{I} \tag{18}$$

$$R_2 = \left(\frac{855.3}{I} + \frac{4501.6}{I^2}\right)L. \tag{19}$$

In (18), the constant 1080.4 follows if $E_{\mathrm{a}} = 1200\ \mathrm{V/m}$, whereas the constant 1350.5 follows if $E_{\mathrm{a}} = 1500\ \mathrm{V/m}$.

In the next section, formulas (18) and (19) will be compared with Warrington formula

$$R_W = \frac{28\,688.5}{I^{1.4}}L. \tag{20}$$

The comparison has been provided for a wide range of arc currents.

VII. Comparison Between Warrington and New Formulas

Three formulas: the Warrington formula (20) and two new formulas (18) and (19), derived in this paper, are compared by changing the rms values of arc current in the expressions for arc resistances, for the in advance assumed the constant arc length. Here, it is assumed that an 1-m-long arc is analyzed $(L = 1\ \mathrm{m})$. The current rms values are changed in a wide range: from 100 to 50 000 A. By using formulas (18), (19), and (20), the arc resistances R_{W}, $R_{1,1200\ \mathrm{V/m}}$ (R_1 for $E_{\mathrm{a}} = 1200\ \mathrm{V/m}$), $R_{1,1500\ \mathrm{V/m}}$ (R_1 for $E_{\mathrm{a}} = 1500\ \mathrm{V/m}$) and R_2 are calculated and clearly presented in Fig. 11. By observing Fig. 11, it can be concluded that in some ranges of currents, R_{w} is greater than R_1 and R_2, and vice versa. In Fig. 12, the curves depicted in Fig. 11 are zoomed and presented for currents between 2 and 7 kA, so the points at which "the new arc resistances" are equal to the Warrington resistance are observable. These are: 3.633 kA (for $R_{1,1200\ \mathrm{V/m}}$), 2.079 kA (for $R_{1,1500\ \mathrm{V/m}}$), and 6.515 kA (for R_2).

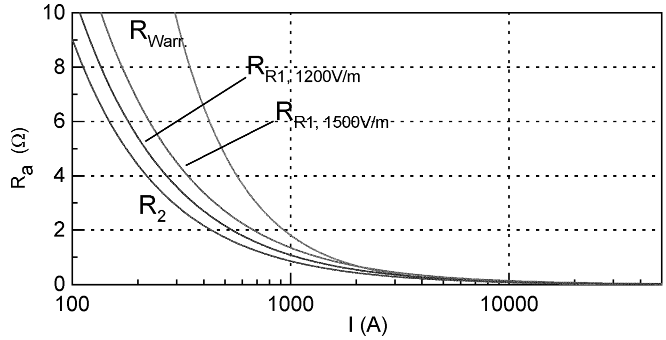

Fig. 11. Resistances obtained using Warrington and new formulas for $L = 1\ \mathrm{m}$.

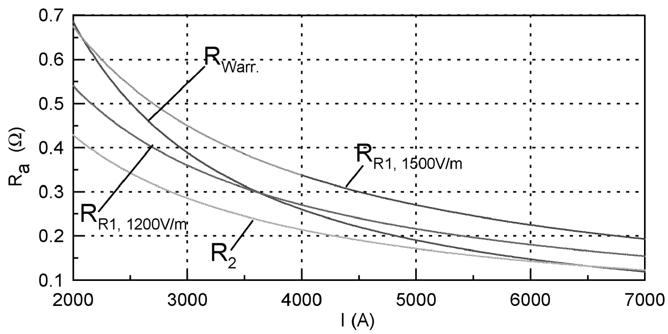

Fig. 12. Curves from Fig. 9 for $2\ \mathrm{kA} < I < 7\ \mathrm{kA}$.

In addition to the aforementioned analysis, an extra proof of the quality of new formulas has been provided. By this, both the simulated and laboratory obtained signals are processed. In the case of laboratory signals from Fig. 7, it has been concluded that arc resistances obtained by using the new formulas lies in the range of 0.53–0.65 Ω and 0.62 Ω, whereas the Warrington formula delivered 0.502 Ω. By this, the exact value was 0.59 Ω. Through the computer simulation, it has been concluded that new formulas deliver the more precise arc resistances for currents in the range of 0.5–50 kA. The equal values are obtained for currents from the crossing points (see Fig. 12).

VIII. Conclusion

Through the investigation of Warrington results, it is concluded that his well-known formula for arc resistance calculation is not correct. Based on the experimental testing in the high-power test laboratory FGH-Mannheim (Germany), the new dynamic arc model is presented and an example of arc computer simulation using the new model is given. A high correlation $(r = 0.94)$ between the simulated and laboratory recorded signals is obtained. Further, a new formula for arc resistance is derived. The new formula requires a suitable selection of arc voltage gradient value. Two approaches for arc voltage gradient are presented, so that two new formulas are derived. New formulas are compared with the Warrington formula. In some ranges of currents, the obvious differences are detected. By this, for both the laboratory and simulated signals, it is proved that the new formulas deliver better results than the old Warrington formula.

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