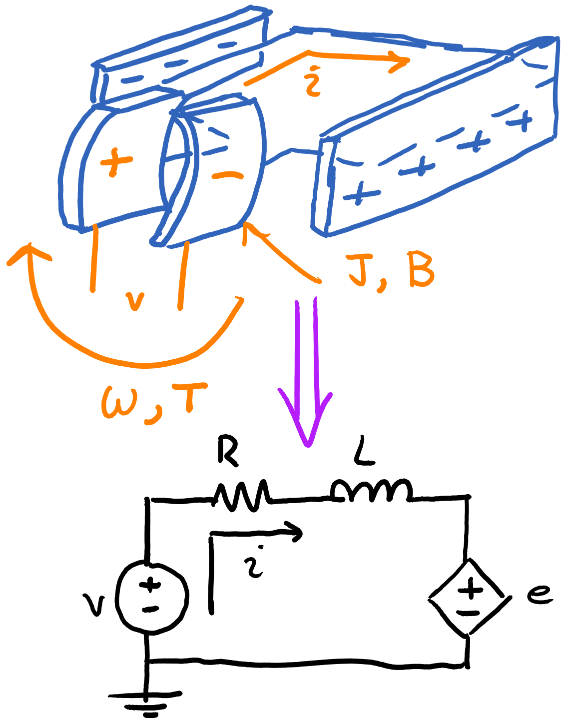
DC Motor

Wednesday, January 29, 2014 5:32 PM

DC MOTOR



The torque (T) is related linearly to the armature current (I)

$$T = K_T i \qquad (1)$$

The Backemf (E) is related linearly to the angular velocity (ω)

$$e = K_E \omega \qquad (2)$$

1st derivative of position

From Kirchoffs voltage law

$$V = Ri + L\frac{di}{dt} + e = Ri + L\frac{di}{dt} + K_E \omega \qquad (3)$$

From Newtons Mechanics

$$K_T i = J\dot{\omega} + B\omega \qquad (4)$$

ASIDE: LAPLACE TRANSFORM

The Laplace Transform is an integral manipulation that transforms a time-domain input, $f(t)$, to a frequency domain output, $F(s)$.

$$F(s) = \int_{-\infty}^{\infty} f(t)e^{-st}\,dt \qquad (5)$$

Here, $s = \sigma + j\omega$. σ is a real number, $j = \sqrt{-1}$ and ω is the frequency in rad/s.

The laplace transform is a linear operator, to allow it to work $f(t)$ must be linear.

Take the Laplace transform of (3) & (4)

$$V = RI + sLI + K_E\Omega \qquad (6)$$

$$K_T I = sJ\Omega + B\Omega \qquad (7)$$

Rewrite (6) & (7)

$$V = I(sL + R) + K_E\Omega \qquad (8)$$

$$I = \frac{\Omega}{K_T}(sJ + B) \qquad (9)$$

(8) w/ (9)

$$V = \frac{\Omega}{K_T}(sJ+B)(sL+R) + K_E\Omega$$

$$K_T V = \Omega(sJ+B)(sL+R) + K_E K_T$$

OUTPUT

$$\frac{\Omega}{V} = \frac{K_T}{(sJ+B)(sL+R) + K_E K_T} = G(s) \left[\frac{\text{rad/s}}{V}\right] \qquad (10)$$

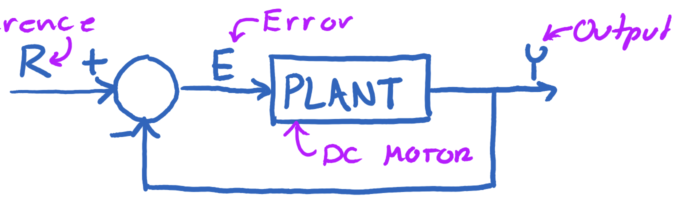
INPUT

TRANSFER FUNCTION

- Transfer functions are great for SISO systems.
- The roots of the denominator are called poles, the roots of the numerator are called zeros.
- If the real part of the poles are positive the system is unstable.
- # Poles ≥ # of Zeros

Closed Loop System

The closed loop system



The resulting closed loop dynamics are written as (11)

$$\frac{Y}{R} = \frac{G(s)}{1+G(s)} = \frac{\dfrac{K_T}{(sJ+B)(sL+R)+K_E K_T}}{1+\dfrac{K_T}{(sJ+B)(sL+R)+K_E K_T}}$$

$$= \frac{K_T}{(sJ+B)(sL+R)+K_E K_T} \cdot \frac{(sJ+B)(sL+R)+K_E K_T}{(sJ+B)(sL+R)+K_E K_T+K_T}$$

$$= \frac{K_T}{(sJ+B)(sL+R)+K_E K_T+K_T} \qquad (11)$$