

# WebFlood

Interactive Shallow-Water Simulations  
in City Environments

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Supervisor:  
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# Structure

- GPGPU with WebGL
- Related Work & Models
- Shallow-Water Implementation
- Results
  - 2008 Flooding of Iowa
  - Classical Dam-Break Experiment
- Conclusion

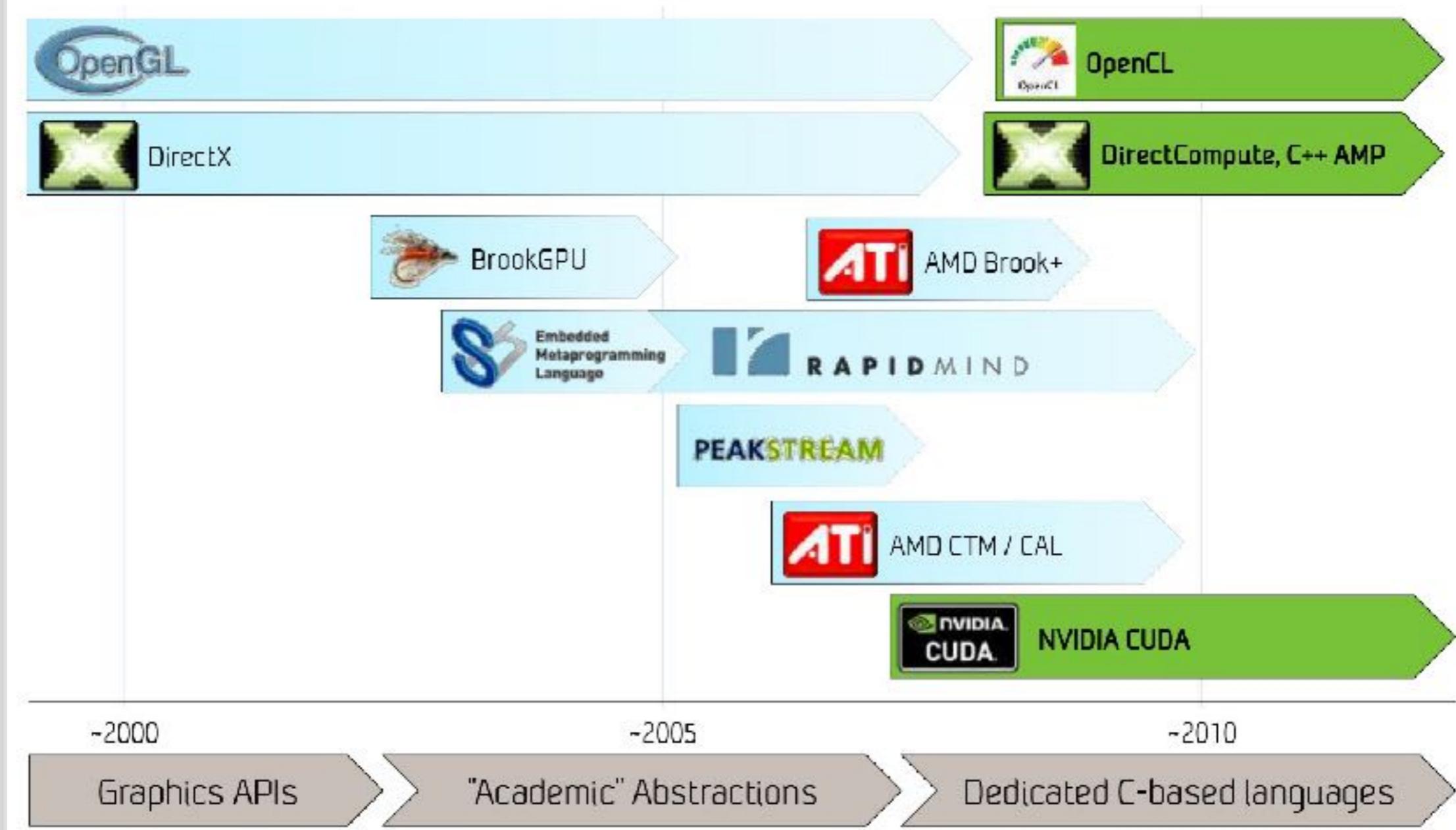
# Makeshift GPGPU

# GPGPU in a browser

- WebGL widely supported
- Functionality level: OpenGL ES 2.0
- WebCL exists, but highly experimental

“How did people survive  
before CUDA/OpenCL?”

# GPU Programming: From Academic Abuse to Industrial Use



*From: André R. Brodtkorb  
“Compact Stencils for the Shallow Water Equations  
on Graphics Processing Units”*

computation problem



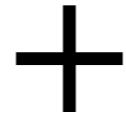
graphics API

# Basic Technique



## Framebuffer Object (FBO)

Texture  
can be read from



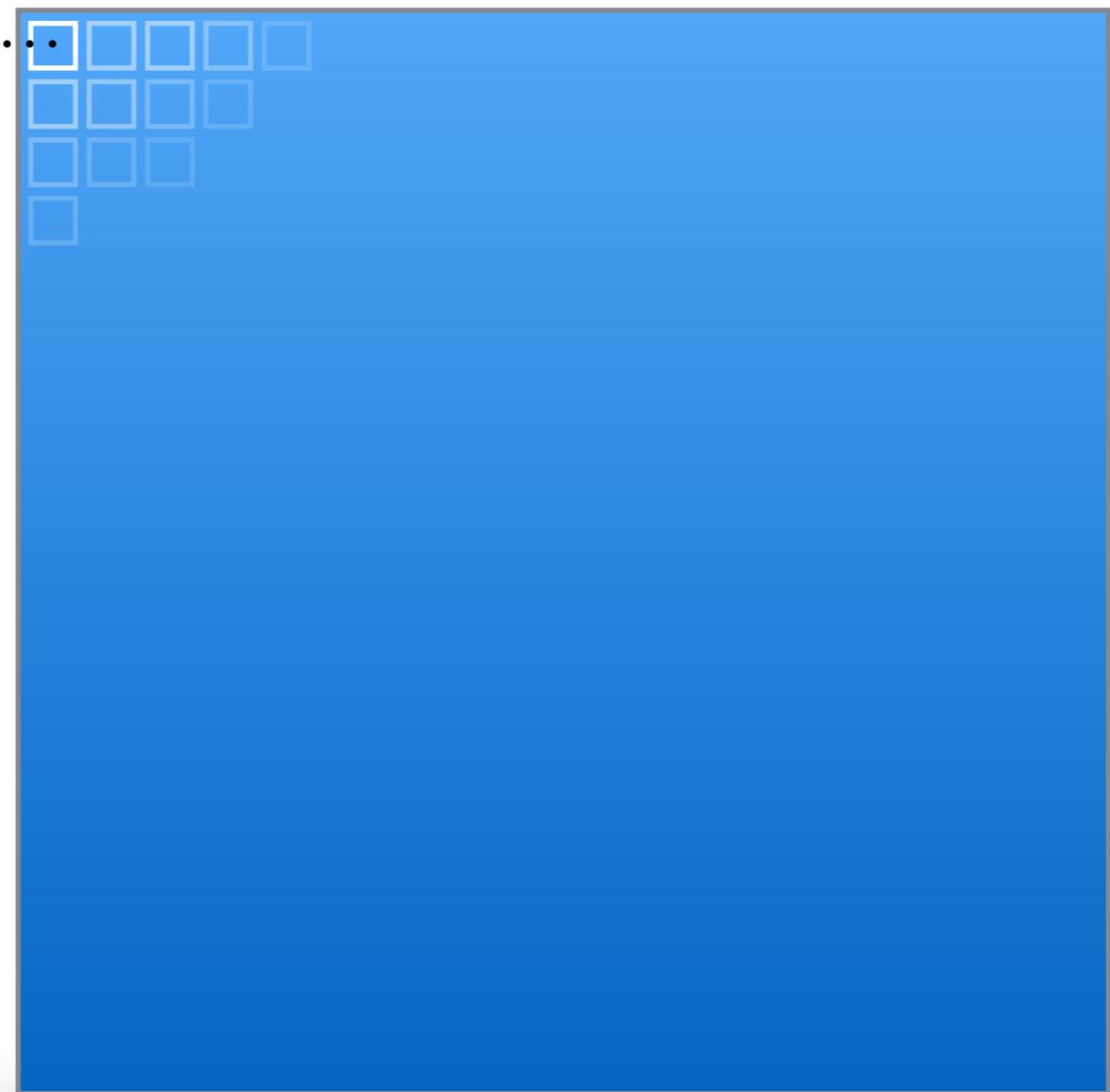
Render Target  
can be written to

# Basic Technique

**red** .....  
**green**  
**blue**  
**alpha**

4 data values  
per pixel  
(ulnt or Float)

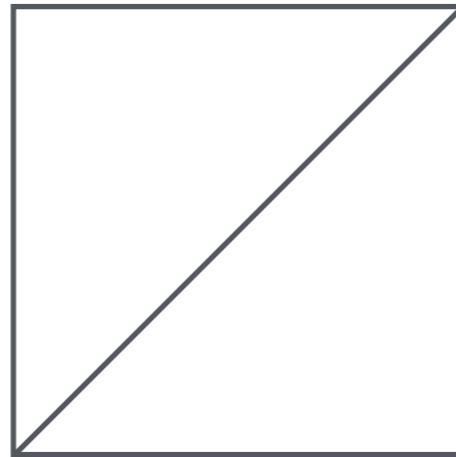
**Encode simulation  
data in pixels**



# Basic Technique



FBO 1  
initial  
simulation state



Flat Geometry  
fills entire screen

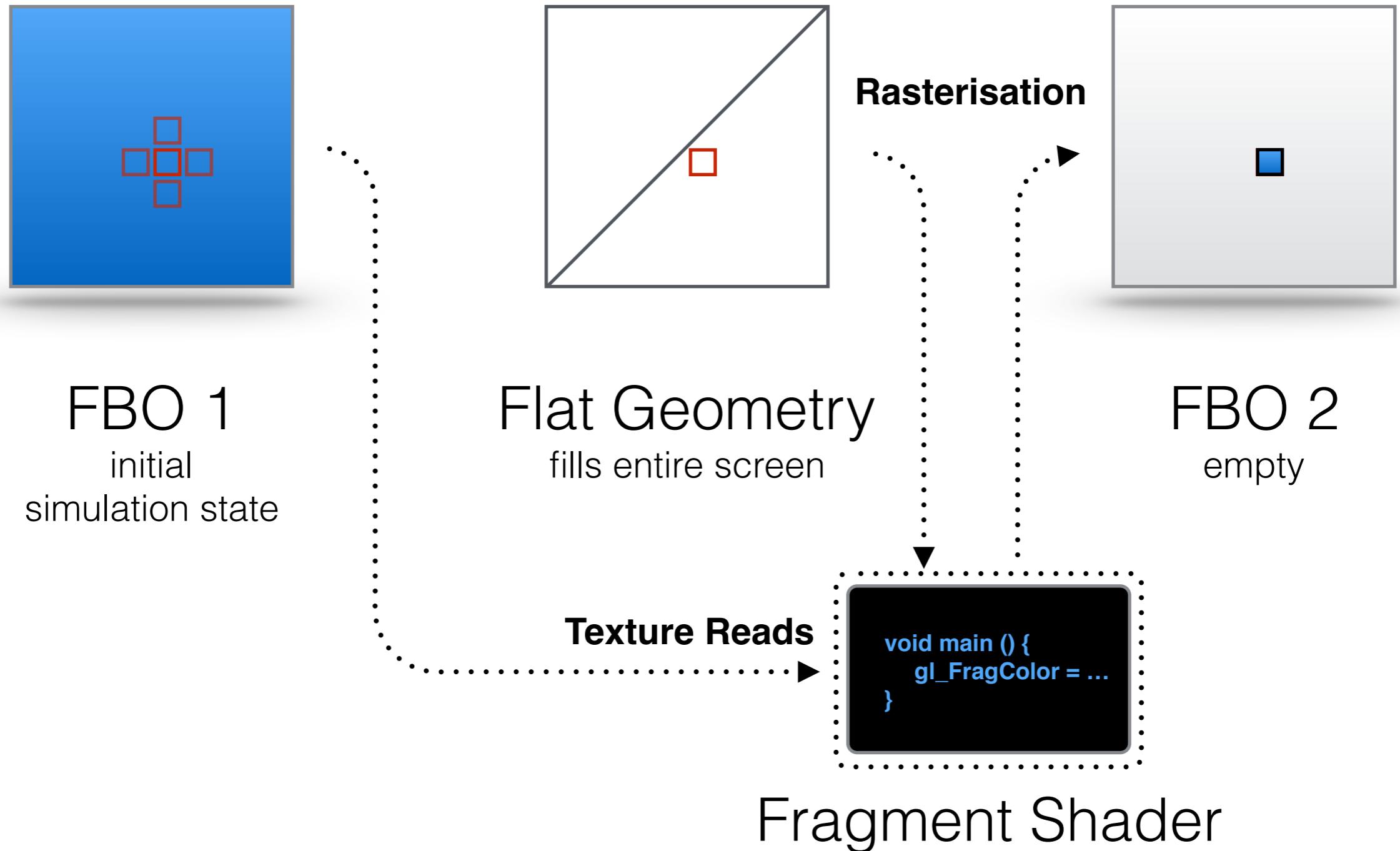


FBO 2  
empty

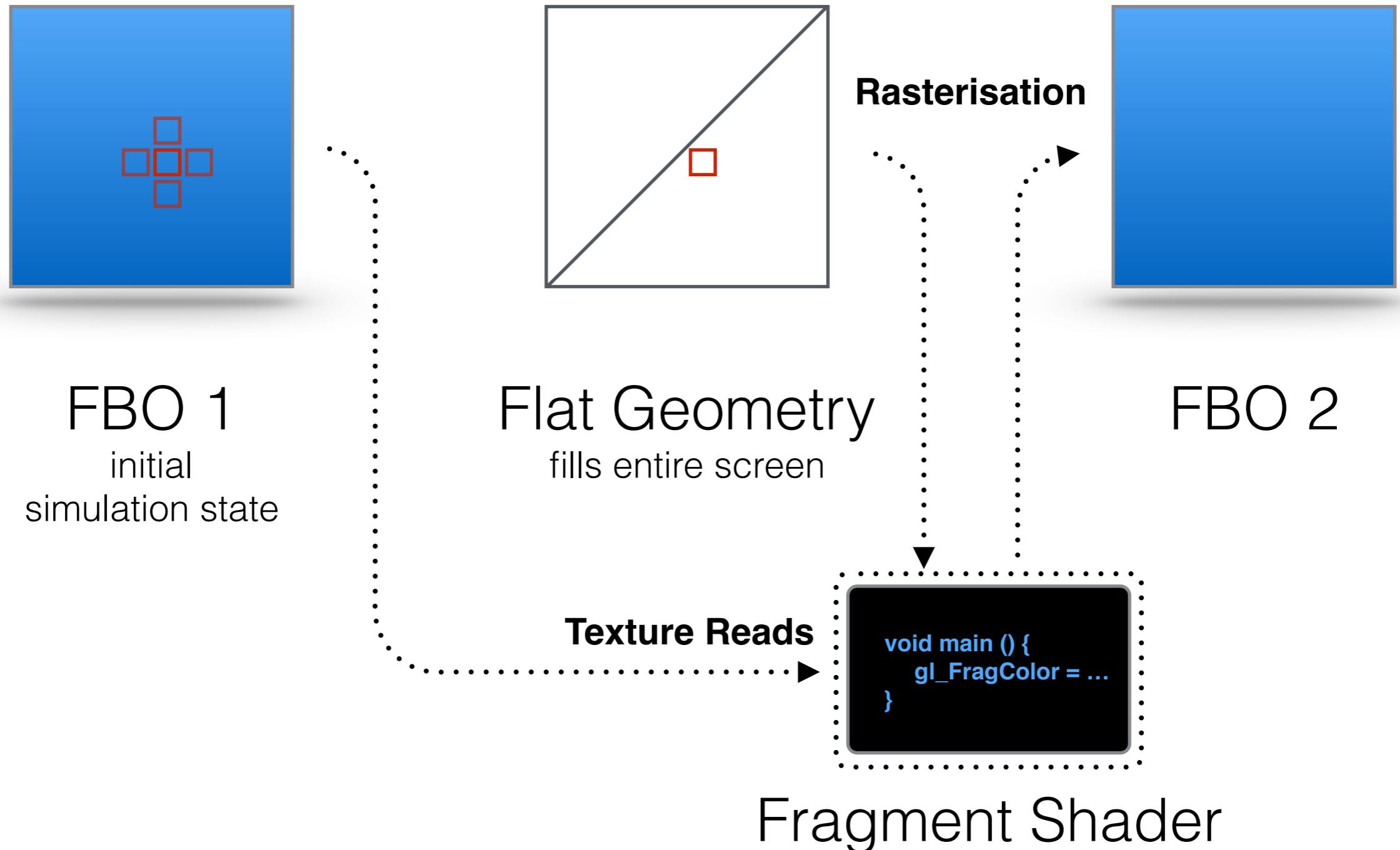
```
void main () {  
    gl_FragColor = ...  
}
```

Fragment Shader

# Basic Technique



# Basic Technique



# Basic Technique

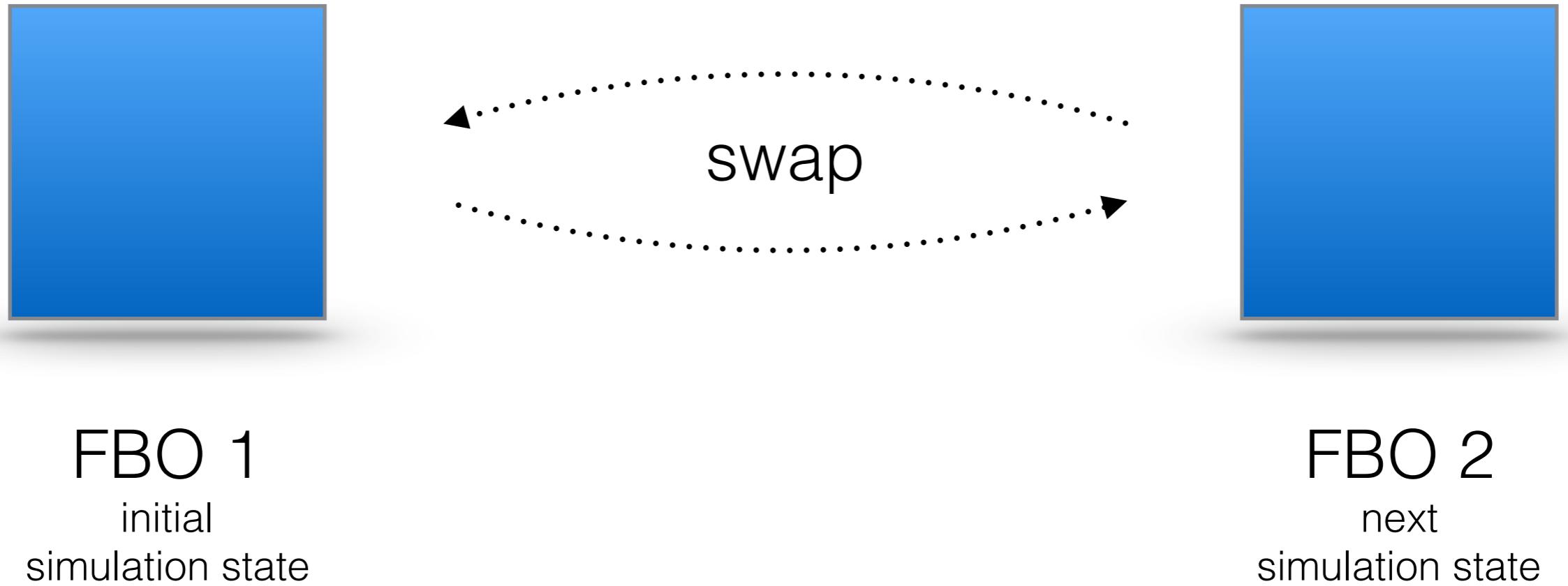


FBO 1  
initial  
simulation state



FBO 2  
next  
simulation state

# Basic Technique



# Related Work

# Computational Fluid Dynamics

$$\nabla \cdot \vec{u} = 0$$

$$\frac{D\vec{u}}{Dt} + \frac{1}{\rho} \nabla p = \vec{g}$$

Incompressible & Inviscid  
Navier-Stokes Equations

$$\nabla \cdot \vec{u} = 0$$

$$\frac{D\vec{u}}{Dt} + \frac{1}{\rho} \nabla p = \boxed{\vec{g}}$$

1. Apply External Forces

$$\nabla \cdot \vec{u} = 0$$

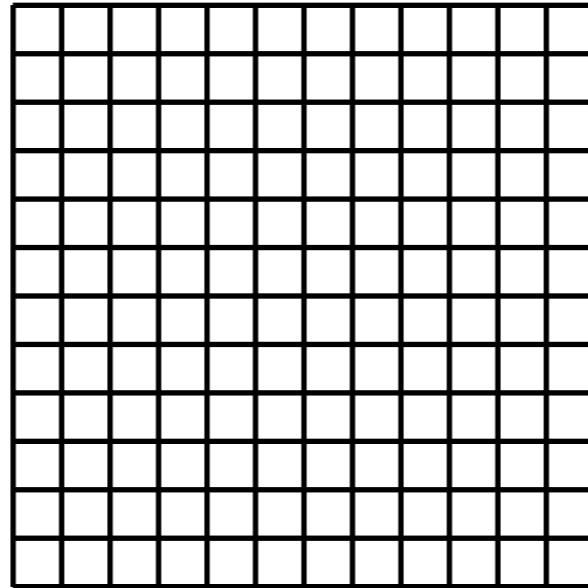
$$\boxed{\frac{D\vec{u}}{Dt}} + \frac{1}{\rho} \nabla p = \vec{g}$$

2. Advection  
(move quantities along velocity field)

$$\boxed{\nabla \cdot \vec{u} = 0}$$

$$\frac{D\vec{u}}{Dt} + \boxed{\frac{1}{\rho} \nabla p} = \vec{g}$$

3. Solve for Incompressibility



# Grid-Based Methods

- “Eulerian Viewpoint”
- Changes of quantities at grid points are observed
- Fluid = Field
- NSE discretised, solved using finite differences
  - Advection inexact and unstable
  - Incompressibility requires solution to Poisson equation

# Numerical Calculation of Time-Dependent Viscous Incompressible Flow of Fluid with Free Surface

FRANCIS H. HARLOW AND J. EDDIE WELCH

*Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico*

(Received 16 April 1965; final manuscript received 3 September 1965)

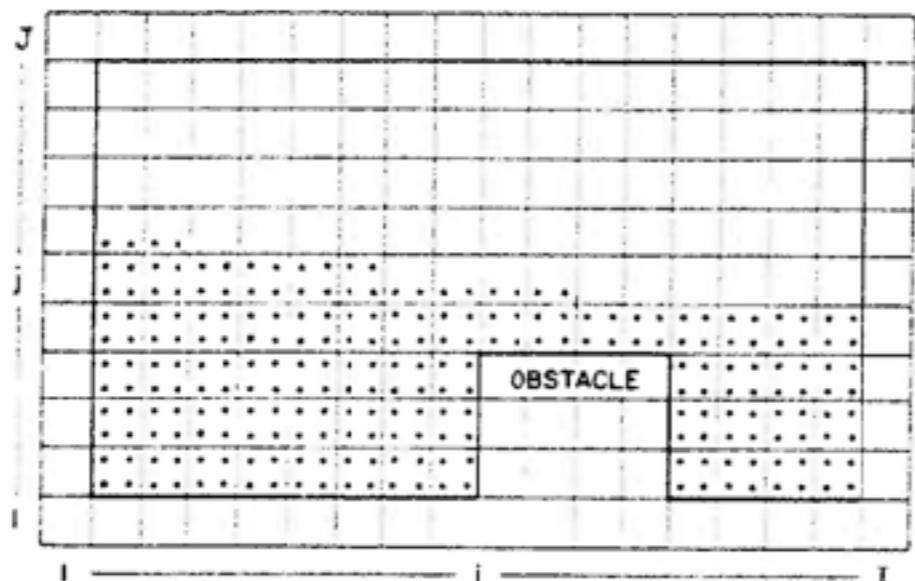


FIG. 1. Sketch of typical mesh and marker-particle layout.  
An actual calculation would be much more finely resolved.

Marker Particles

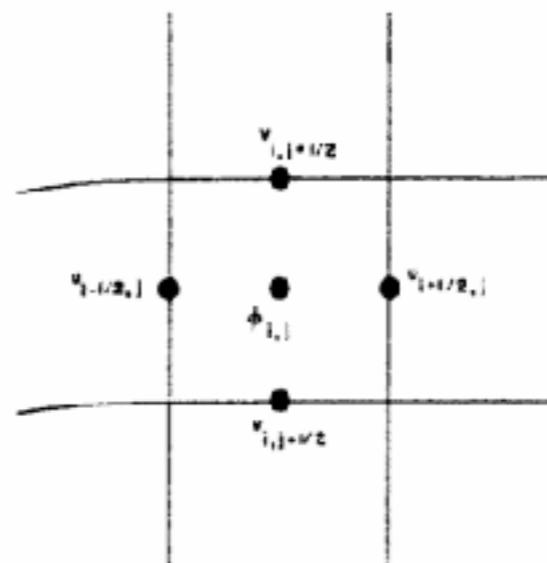
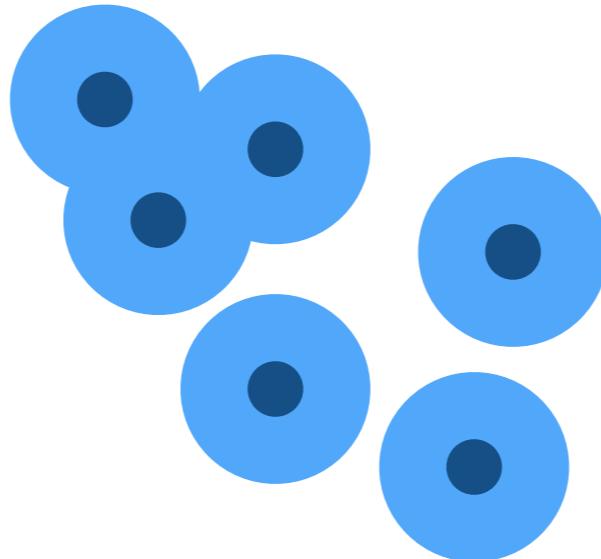


FIG. 2. Field variable value placement about a computational cell. Velocities are defined at cell boundaries while pressures are defined at cell centers.

MAC-Grid



# Particle-Based Methods

- “Lagrangian Viewpoint”
- Particles are followed along their motion through the fluid
- Fluid = Collection of Particles
- Advection obvious and exact
- How to define fluid,  $\nabla$  and  $\nabla^2$  from particle properties?
- Incompressibility problematic

# Smoothed particle hydrodynamics: theory and application to non-spherical stars

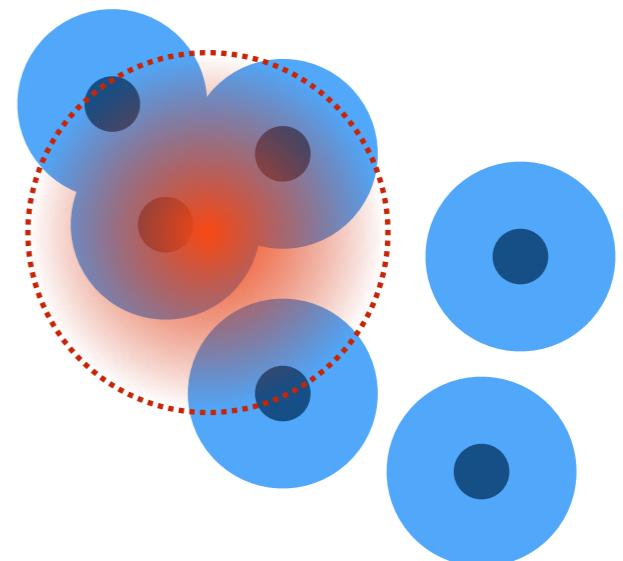
R. A. Gingold and J. J. Monaghan<sup>\*</sup> *Institute of Astronomy,  
Madingley Road, Cambridge, CB3 0HA*

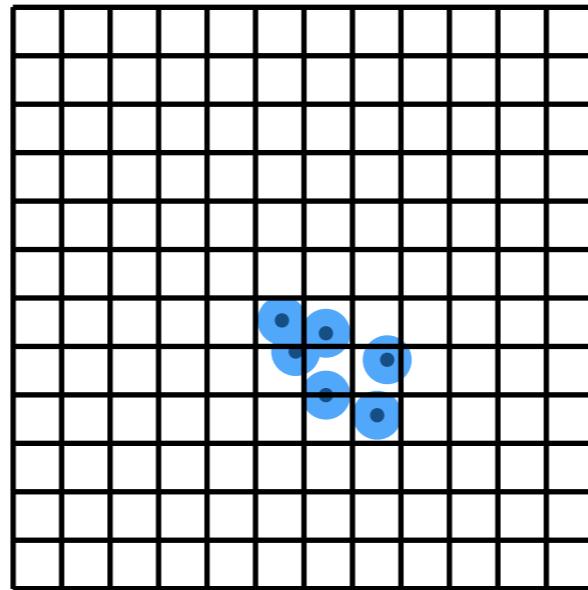
Received 1977 May 5, in original form February 2

$$A(x) = \sum_i m_i \frac{A_i}{\rho_i} W(x - x_i, h)$$

$$\nabla A(x) = \sum_i m_i \frac{A_i}{\rho_i} \nabla W(x - x_i, h)$$

$$\nabla^2 A(x) = \sum_i m_i \frac{A_i}{\rho_i} \nabla^2 W(x - x_i, h)$$





# Hybrid Methods

**Stable Fluids** Jos Stam, 1999

- Lagrangian advection, otherwise grid-based
- Unconditionally stable

**Lattice Boltzmann Method**

- Average microscopic kinetics = macroscopic behaviour
- Converges to Navier-Stokes

# Specialisations

## Realtime Fluid Animation

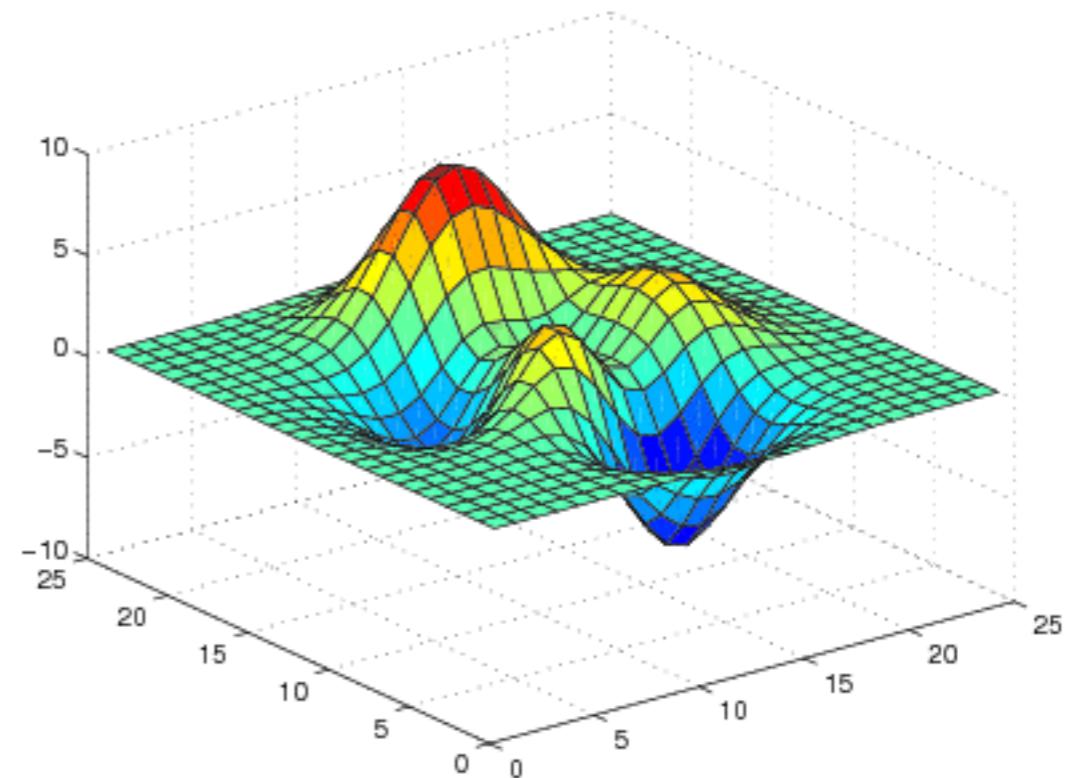
- Performance and Plausibility
- All Methods: Parallelisable!
- Realtime 3D: Achievable
- 2D Shallow-Water
  - less expensive
  - maps even better to GPU

## Flood Simulation

- Correctness
- Offline Simulation:  
Acceptable
- Vast Simulation Domains
- Optimise for topography  
with irregular meshes
- 2D Shallow-Water

# Shallow-Water

- Simplification of 3D Navier-Stokes
- Water Columns Hydrostatic Pressure
  - no vertical velocities
  - horizontal velocities equal over the whole column
- Water Height = Pressure
  - 2D Height Field Representation

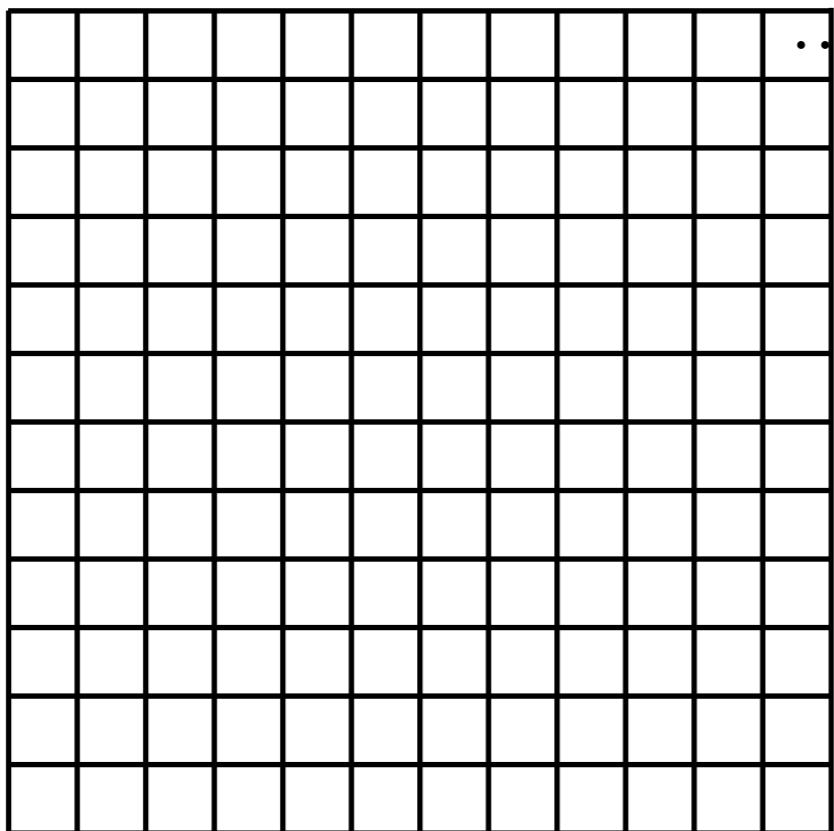


$$\frac{\partial \eta}{\partial t} + \vec{v} \cdot \nabla \eta = -\eta \nabla \cdot \vec{v}$$
$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} = g \nabla h$$

# Grid-Based Shallow-Water

Makeshift GPGPU Implementation

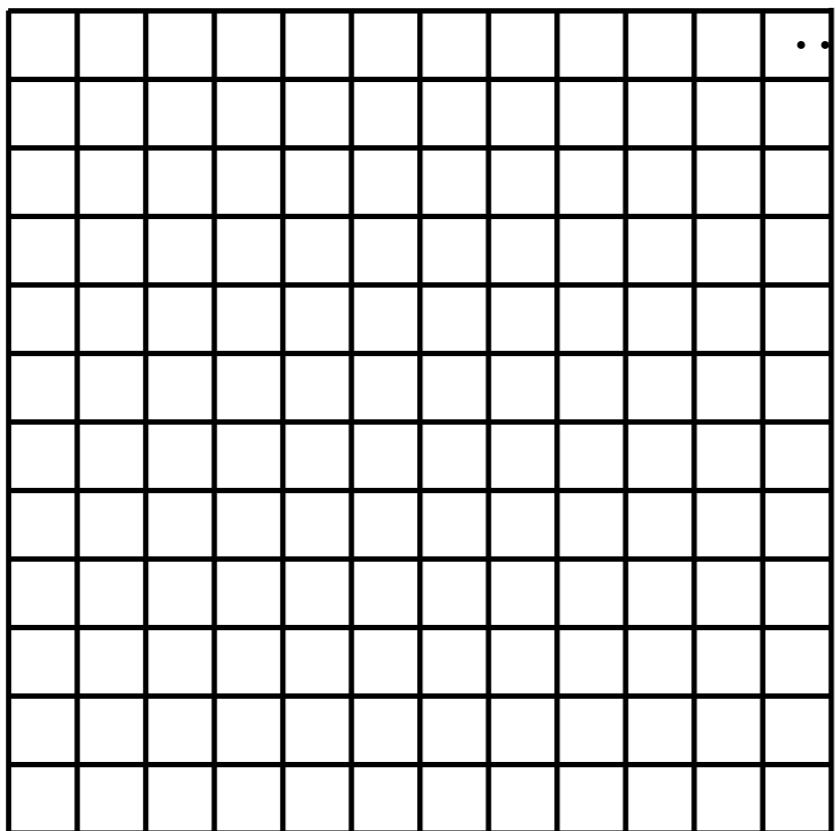
# Data Representation



..... water velocity  
water height  
terrain height

Simulation Grid

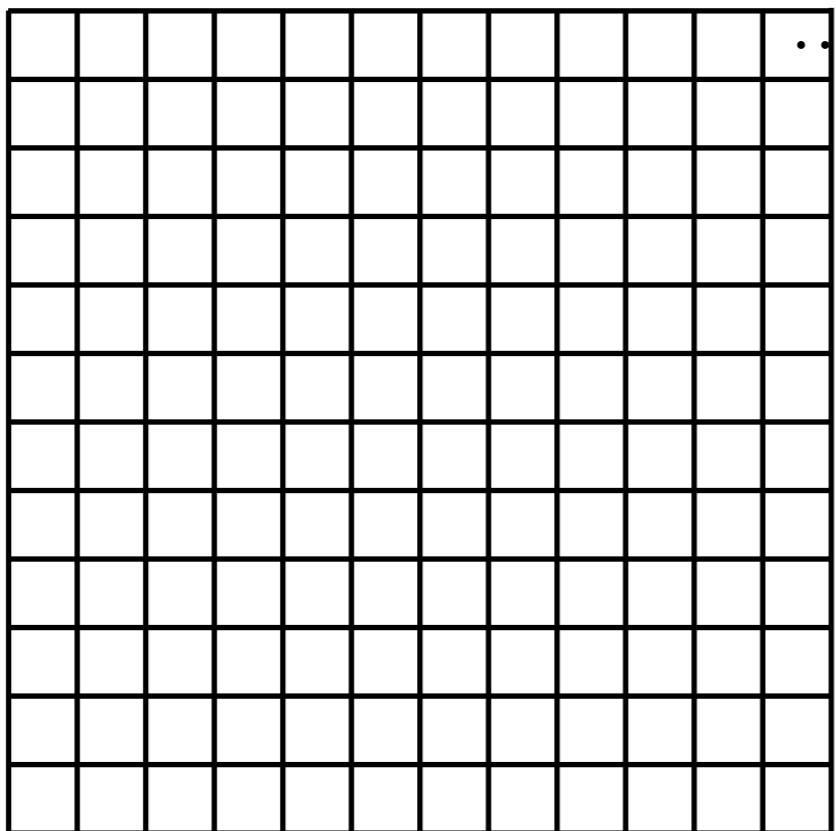
# Data Representation



..... water velocity (x)  
..... water velocity (y)  
..... water height  
..... terrain height

Simulation Grid

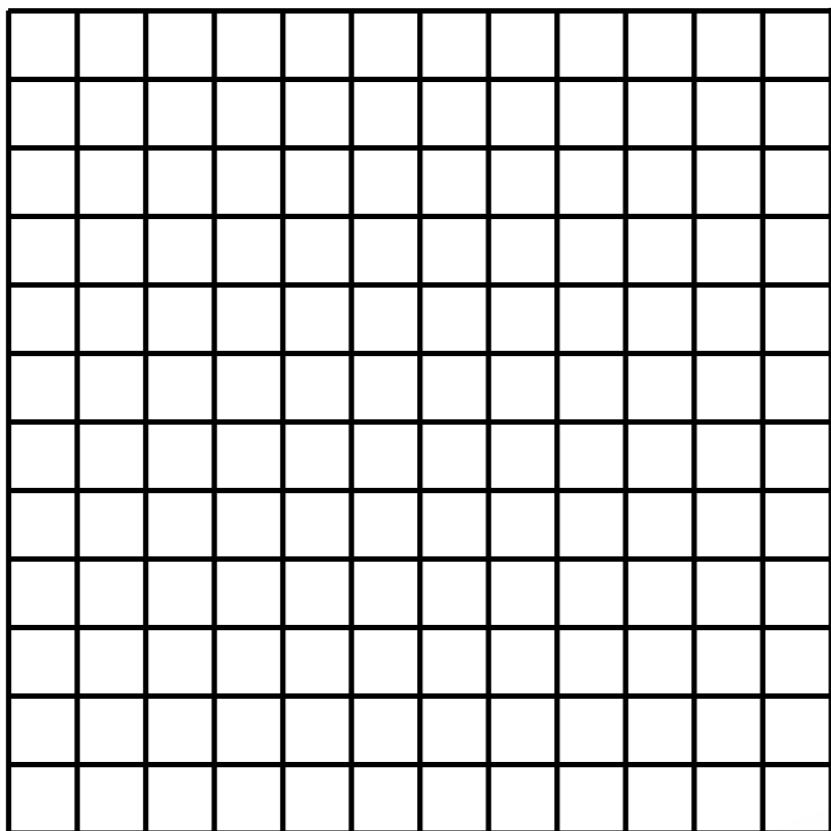
# Data Representation



Simulation Grid

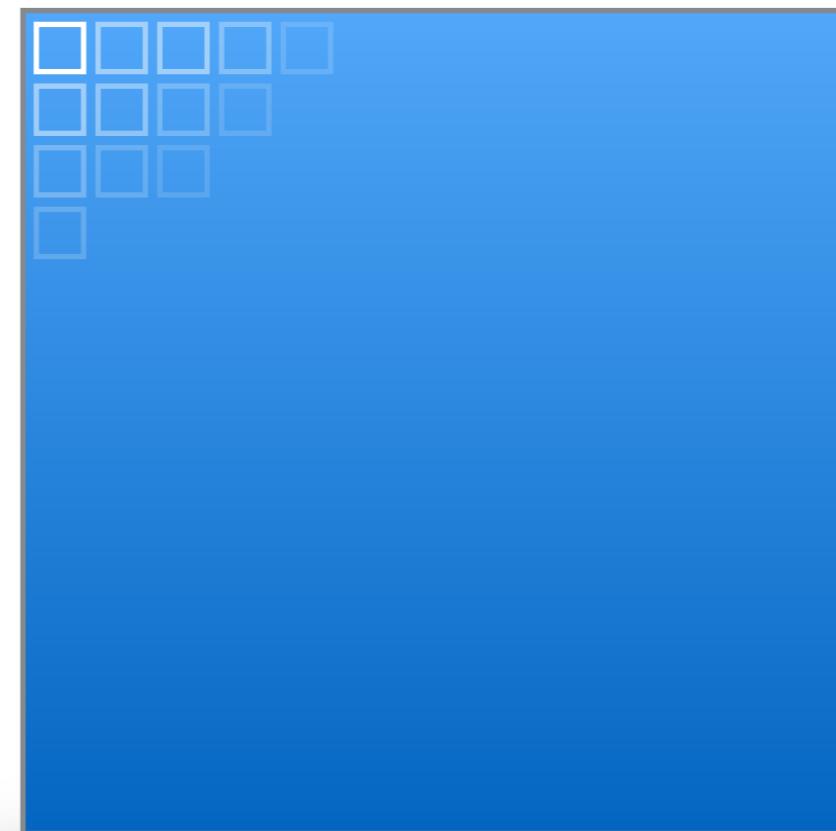
..... water velocity (x) **red**  
..... water velocity (y) **green**  
..... water height **blue**  
..... terrain height **alpha**

# Data Representation



Simulation Grid

=



FBO

# Simulation Steps

Advection

Height Update

Velocity Update

# Advection

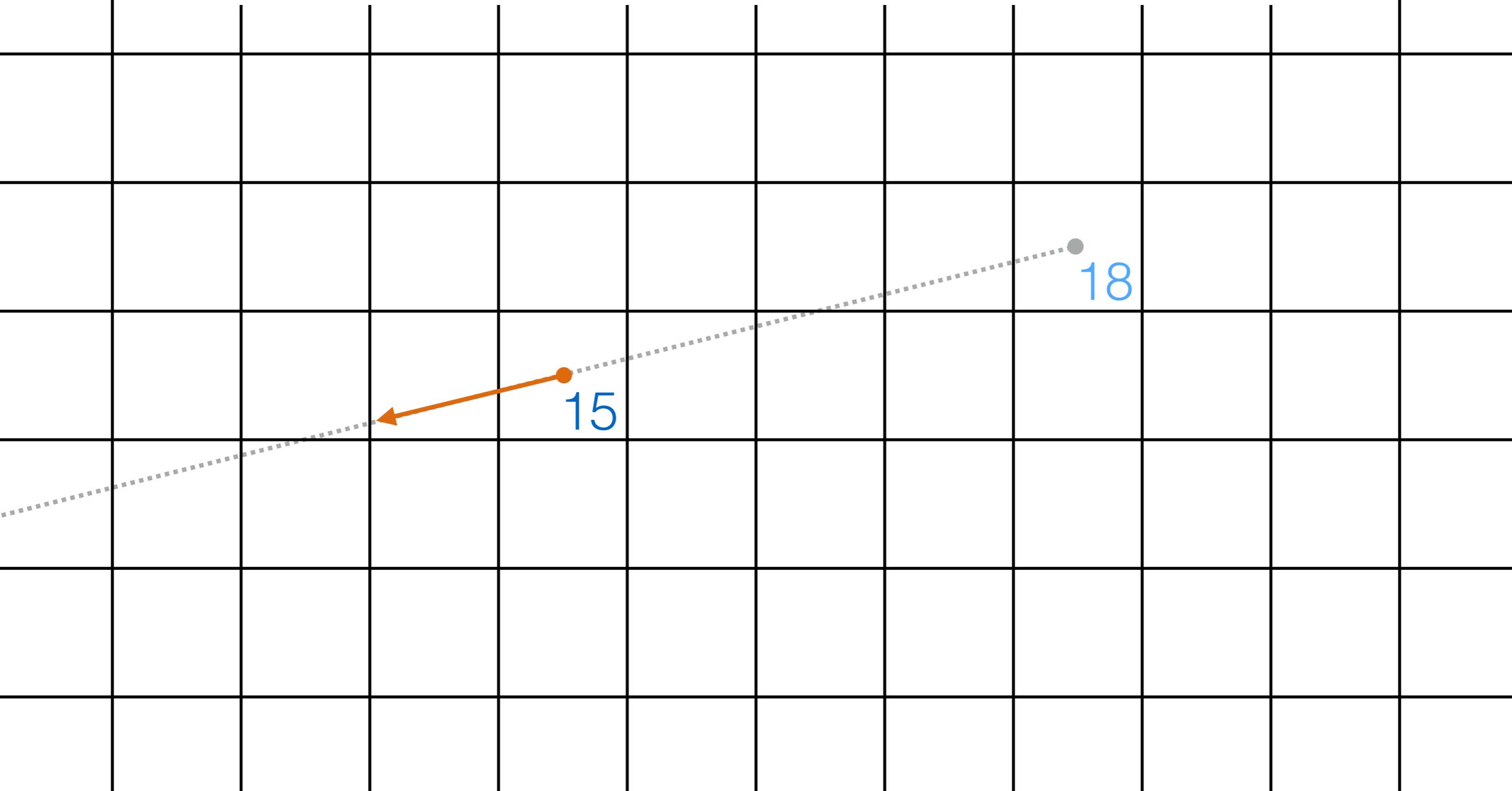
Move quantities along velocity field

$$\frac{\partial \eta}{\partial t} + \boxed{\vec{v} \cdot \nabla \eta} = -\eta \nabla \cdot \vec{v}$$
$$\frac{\partial \vec{v}}{\partial t} + \boxed{\vec{v} \cdot \nabla \vec{v}} = g \nabla h$$

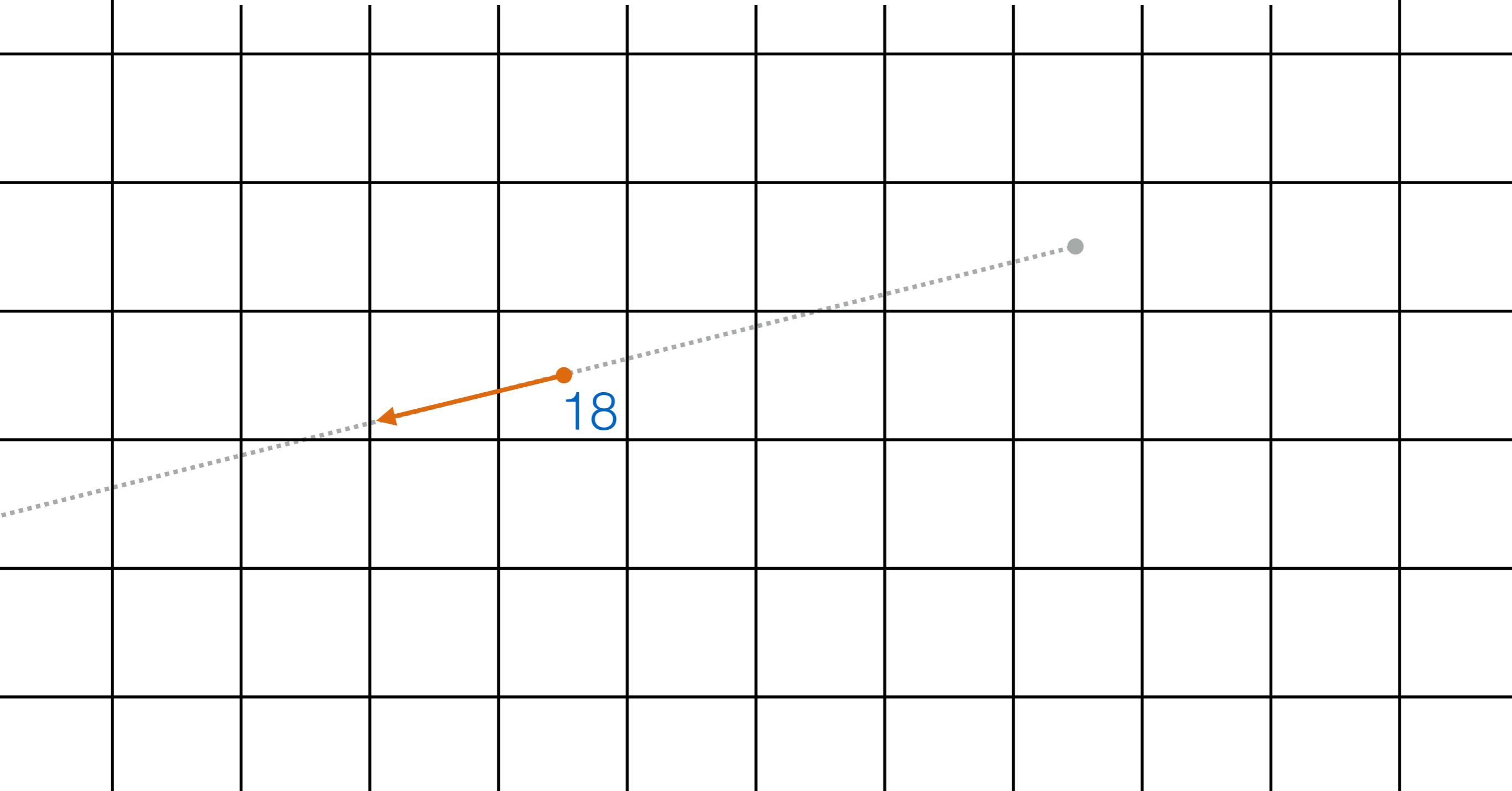
# Semi-Lagrangian Advection

Trace a particle backwards in time  
Copy value from origin

# Semi-Lagrangian Advection



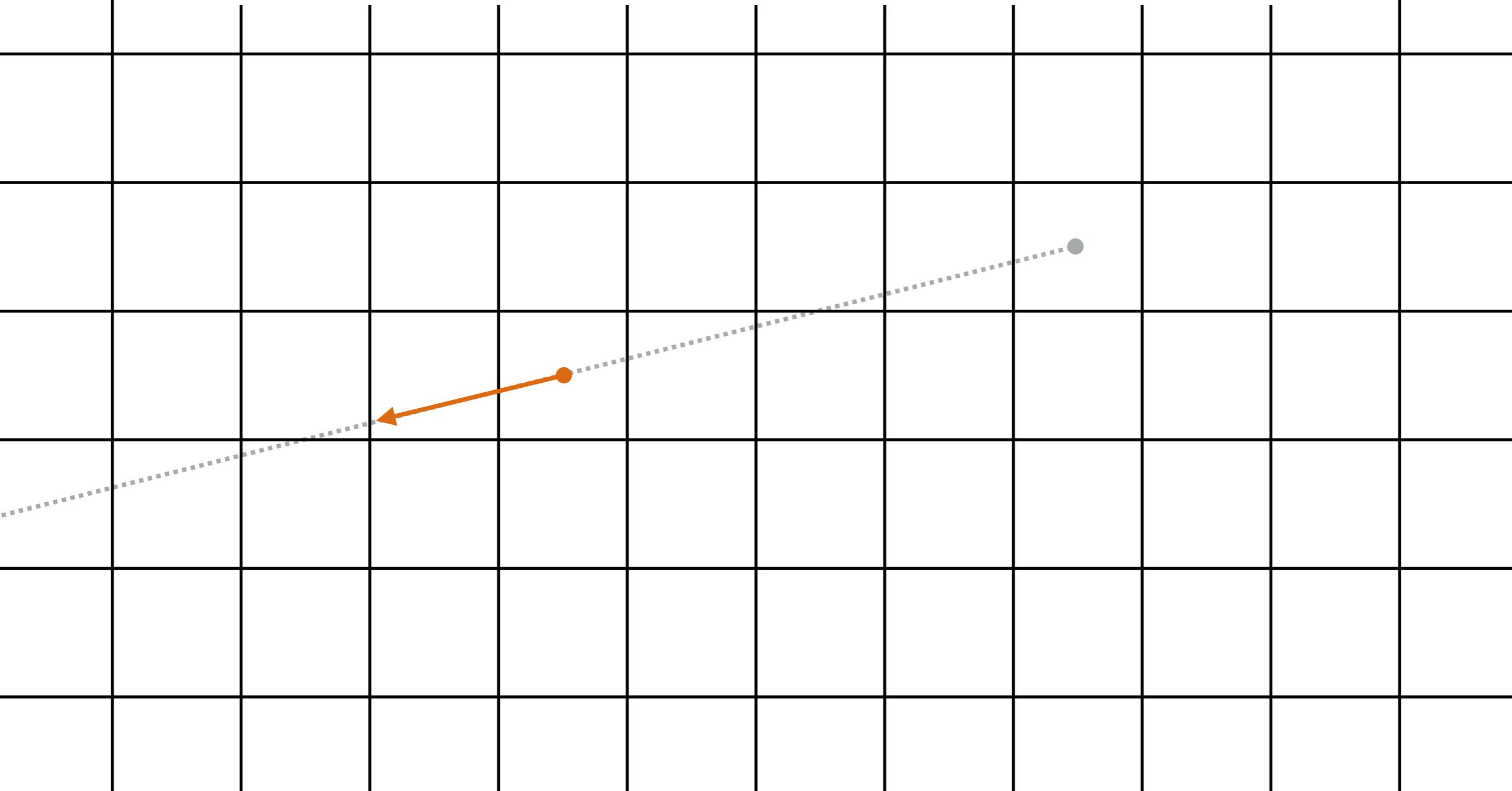
# Semi-Lagrangian Advection



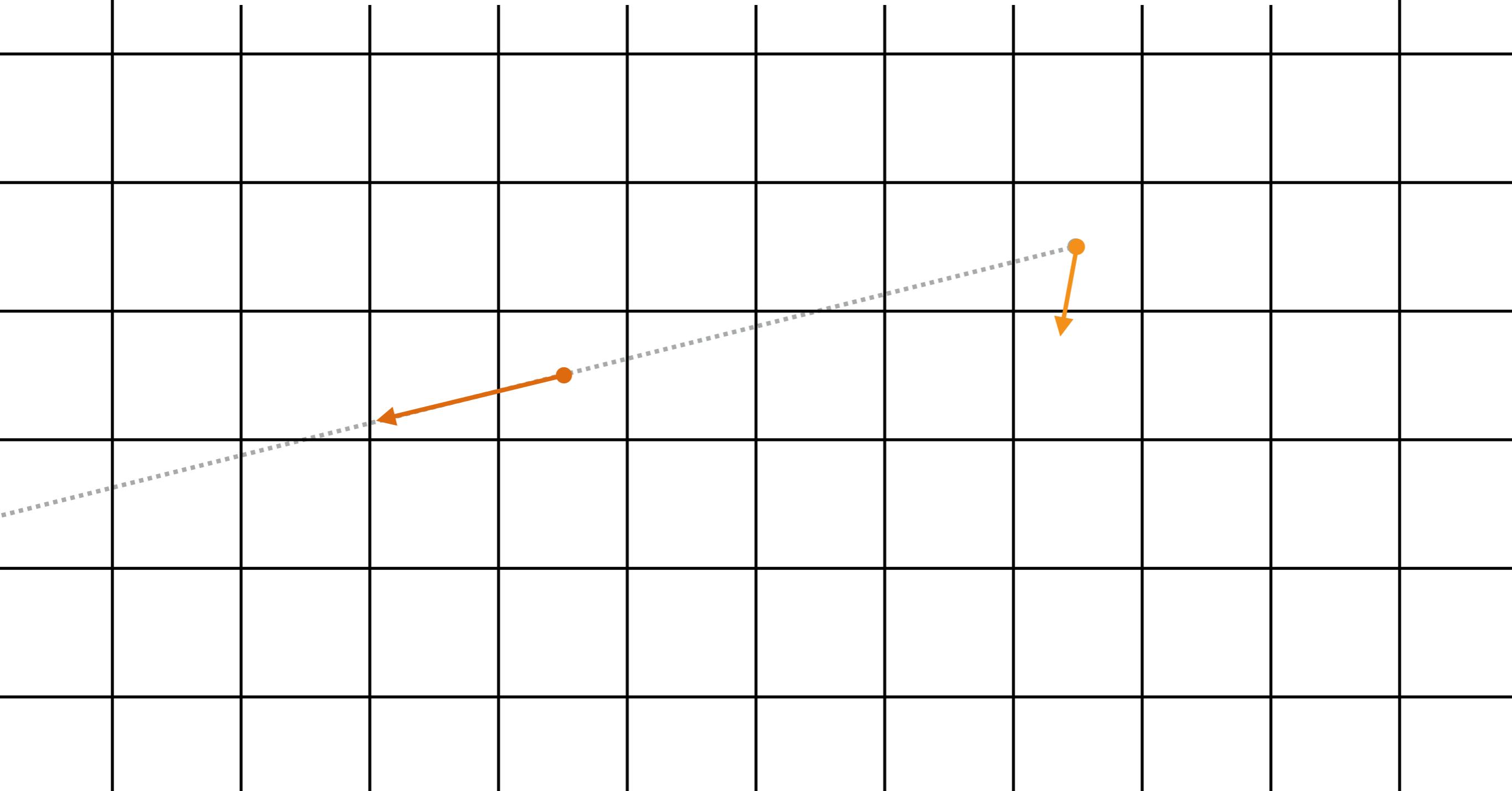
# Semi-Lagrangian Advection



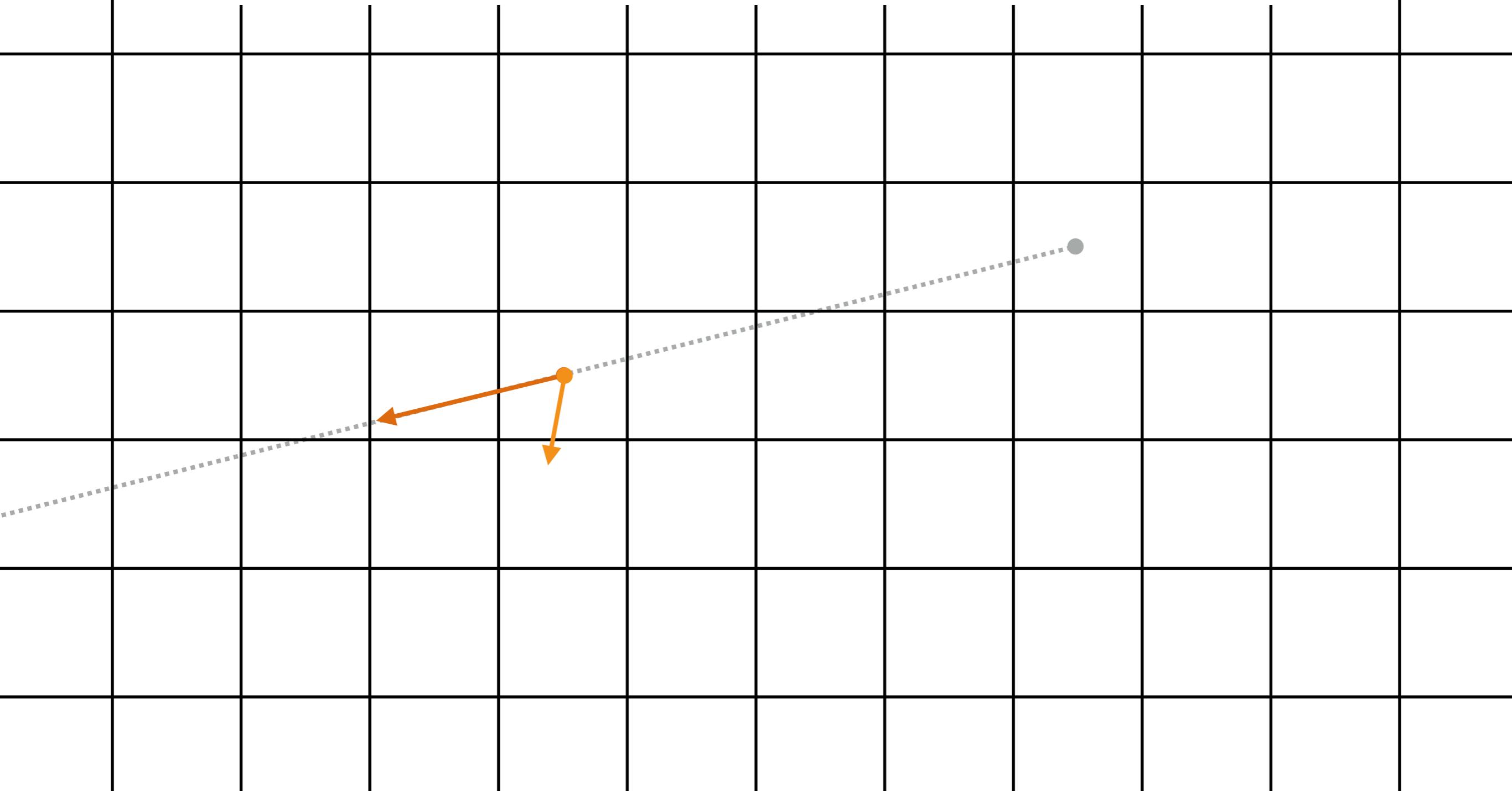
# Semi-Lagrangian Advection



# Semi-Lagrangian Advection



# Semi-Lagrangian Advection



# Semi-Lagrangian Advection



# Semi-Lagrangian Advection

```
1 vec4 simulationStep() {
2     vec4 here = simData(pos);
3
4     vec4 origin = simData(pos - dt * v(here));
5     float newHeight = H(origin);
6     vec2 newVelocity = v(origin);
7
8     return vec4(newVelocity, newHeight, T(here));
9 }
```

# Height & Velocity Update

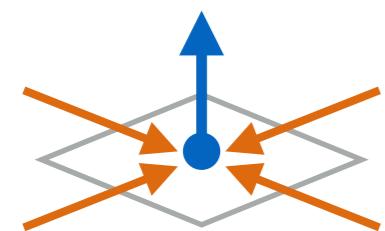
Advection only models movement, but not pressure

$$\frac{\partial \eta}{\partial t} + \vec{v} \cdot \nabla \eta = -\eta \nabla \cdot \vec{v}$$
$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} = g \nabla h$$

# Height Update

$$\frac{\partial \eta}{\partial t} + \vec{v} \cdot \nabla \eta = -\eta \nabla \cdot \vec{v}$$

- In/Decrease water height based on **divergence** of velocity field
- Divergence: finite differences of  $\pm 1$  cell, x and y separately



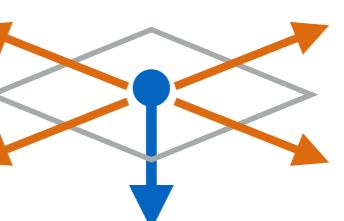
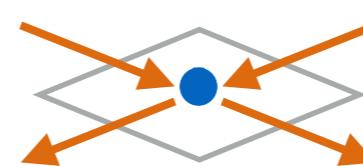
Inflow

$$\nabla \cdot \vec{v} < 0$$



Equilibrium

$$\nabla \cdot \vec{v} = 0$$



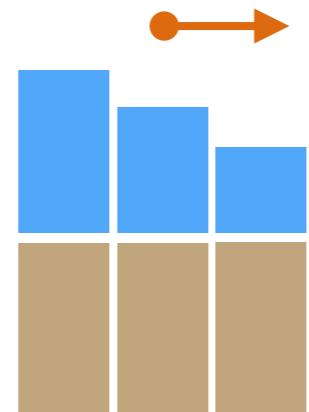
Outflow

$$\nabla \cdot \vec{v} > 0$$

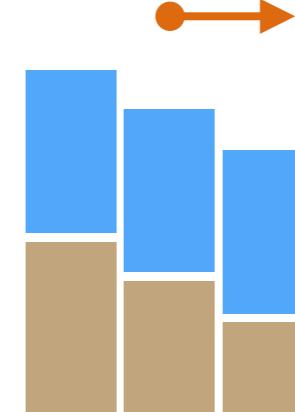
# Velocity Update

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} = g \nabla h$$

- Accelerate water velocity based on **slope** of water + terrain height
- Slope: finite differences of  $\pm 1$  cell



Water Slope



Terrain Slope

# Modifications

# Modifications For Friction

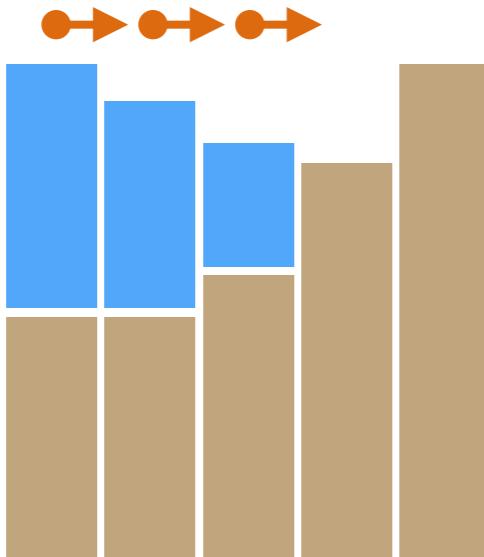
- “Friction Slope” that counteracts accelerating slope of water
- Depends on Water Velocity & Height, Local Manning Coefficient  $n$

$$\vec{S}_f = n^2 \frac{\vec{v} \cdot |\vec{v}|}{h^{\frac{4}{3}}}$$

Friction Slope according to Manning’s Law

# Modifications For Wetting/Drying

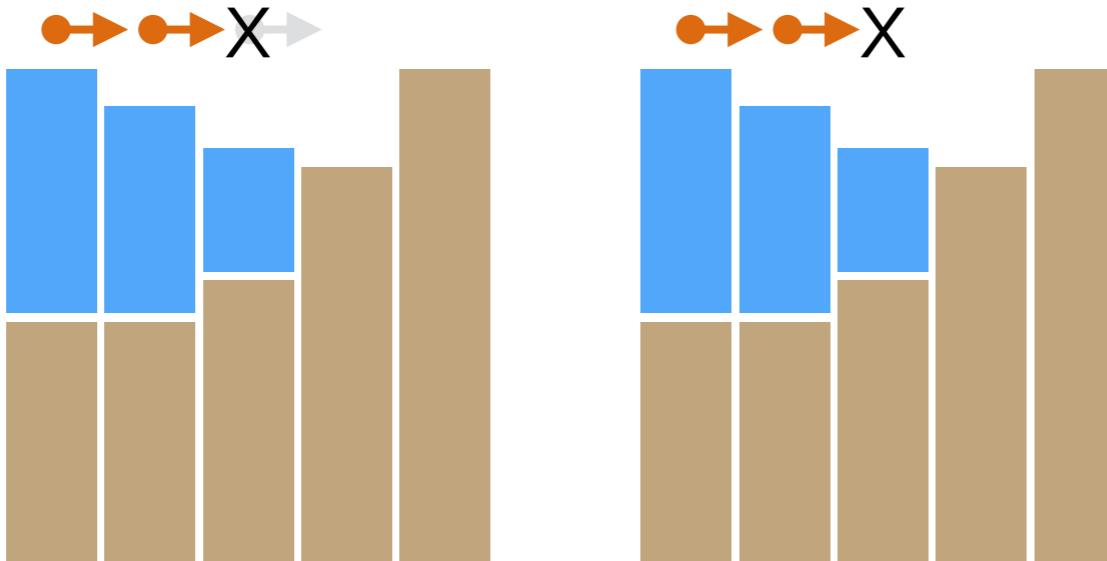
- Special boundary conditions at dry/wet front
- Dry/wet front needs to propagate realistically



wave towards shore

# Modifications For Wetting/Drying

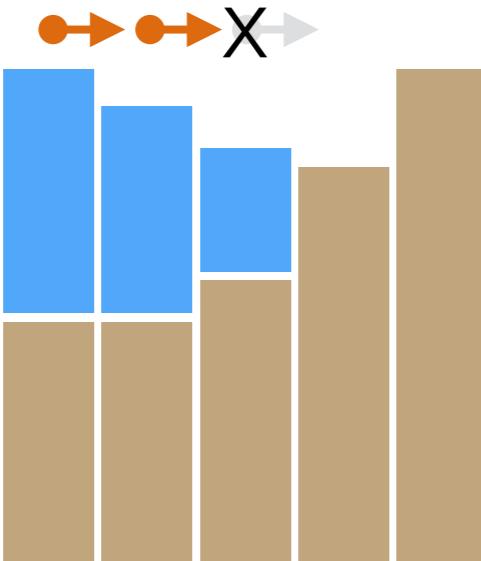
- Special boundary conditions at dry/wet front
- Dry/wet front needs to propagate realistically



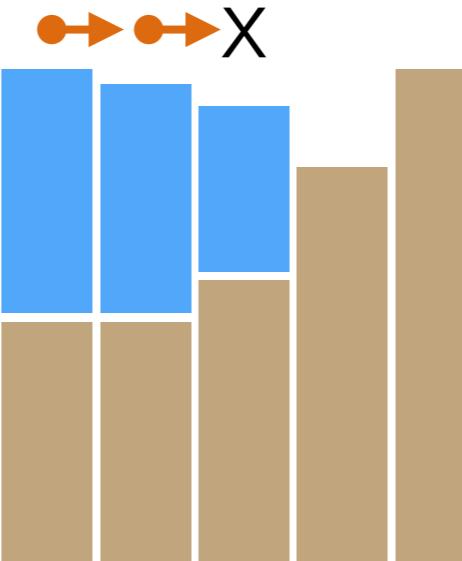
set velocity to 0

# Modifications For Wetting/Drying

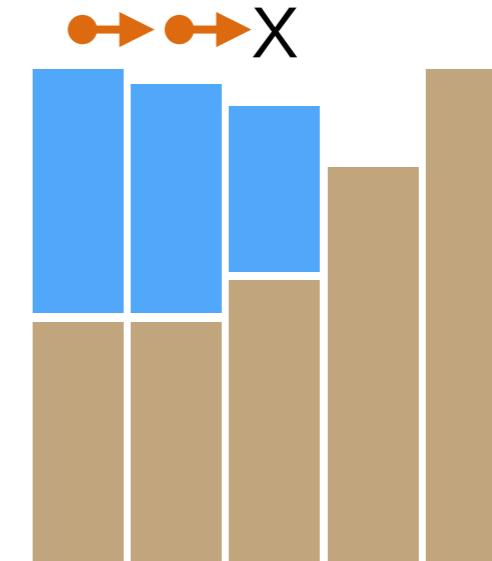
- Special boundary conditions at dry/wet front
- Dry/wet front needs to propagate realistically



set velocity to 0

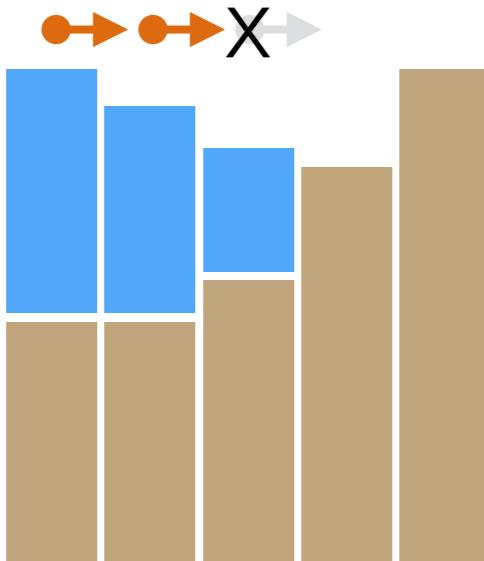


water builds up  
because of velocity  
convergence

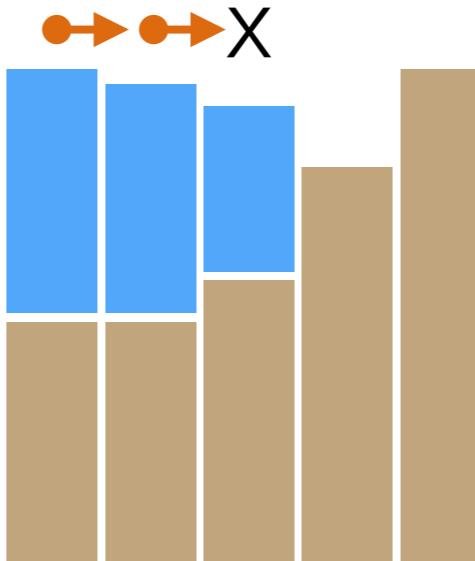


# Modifications For Wetting/Drying

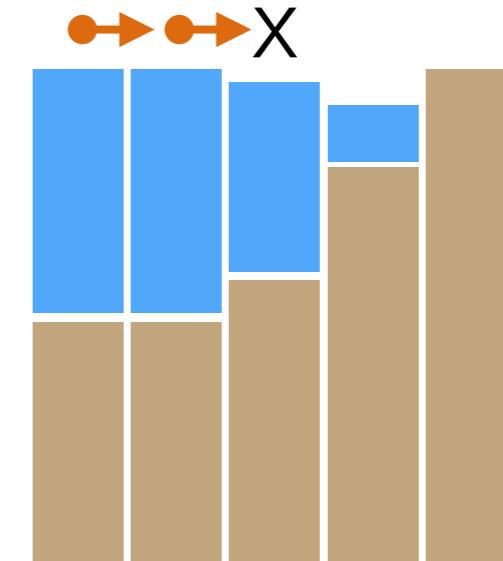
- Special boundary conditions at dry/wet front
- Dry/wet front needs to propagate realistically



set velocity to 0



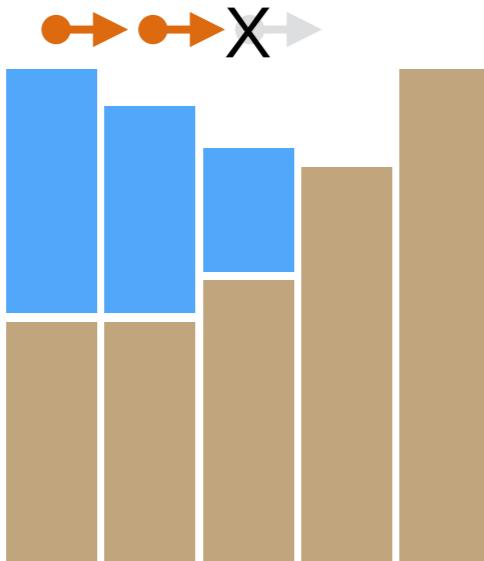
water builds up  
because of velocity  
convergence



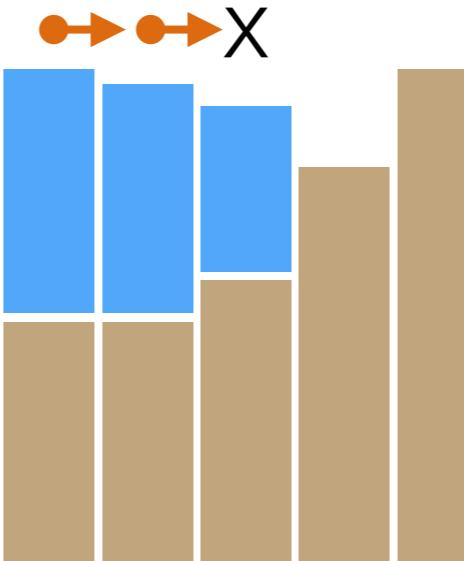
threshold reached  
neighbouring cell  
is wetted

# Modifications For Wetting/Drying

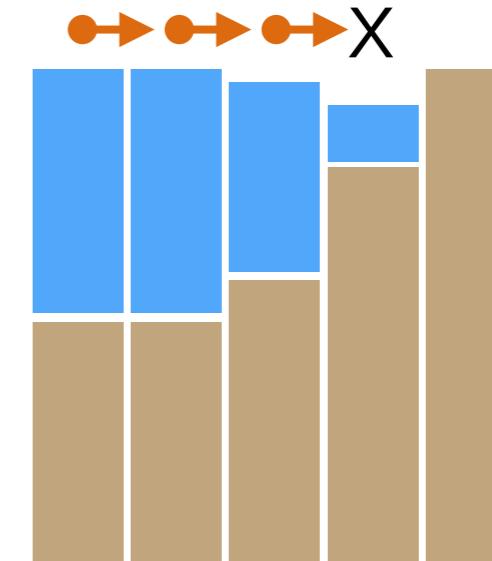
- Special boundary conditions at dry/wet front
- Dry/wet front needs to propagate realistically



set velocity to 0



water builds up  
because of velocity  
convergence



threshold reached  
neighbouring cell  
is wetted

# Application & Results



# 2008 Flooding of Iowa

Image source: <https://iowachaser.files.wordpress.com>



# Iowa City

River through Urban Core  
Interesting Topography

Image source: <http://www.ryersonaircraft.com/>

# Good Data Availability

## (in response to flooding)

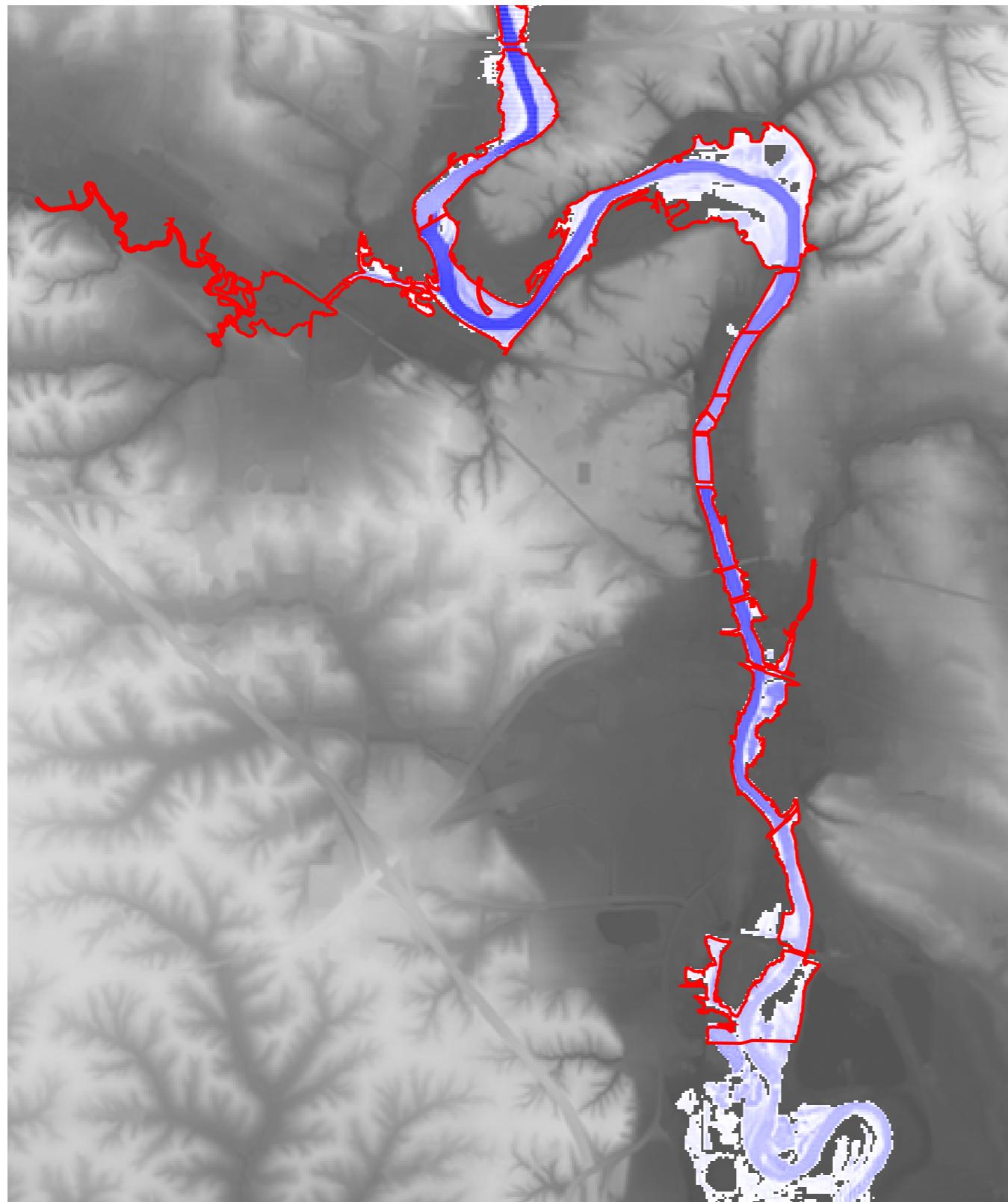


- LiDAR digital elevation maps (0.5m<sup>2</sup> resolution)
- Inundation maps for many real and simulated scenarios

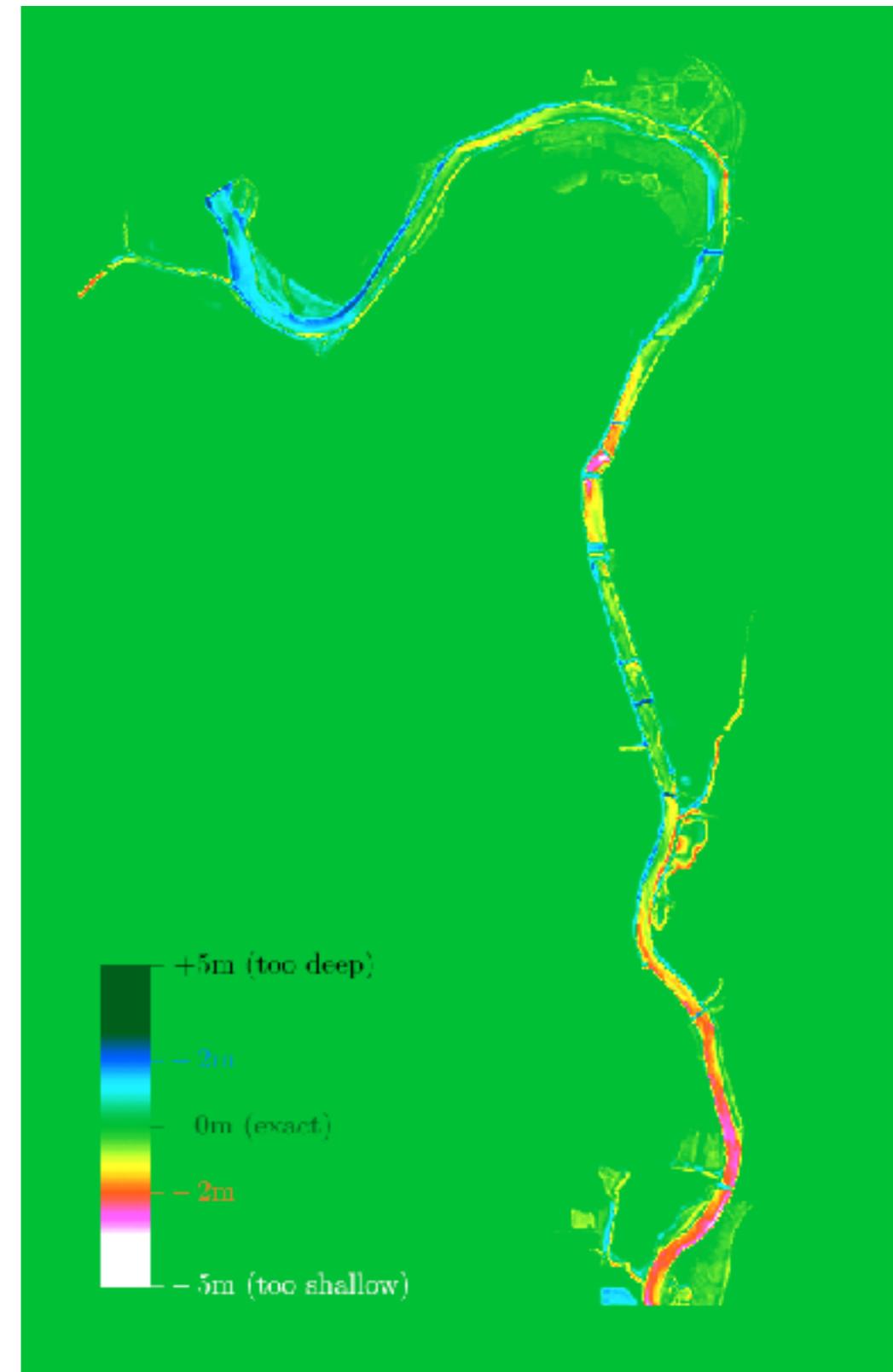
# Inundation Maps Demo

WebFlood Demo Iowa

# Source Height 7.5 / 25.5ft River Stage

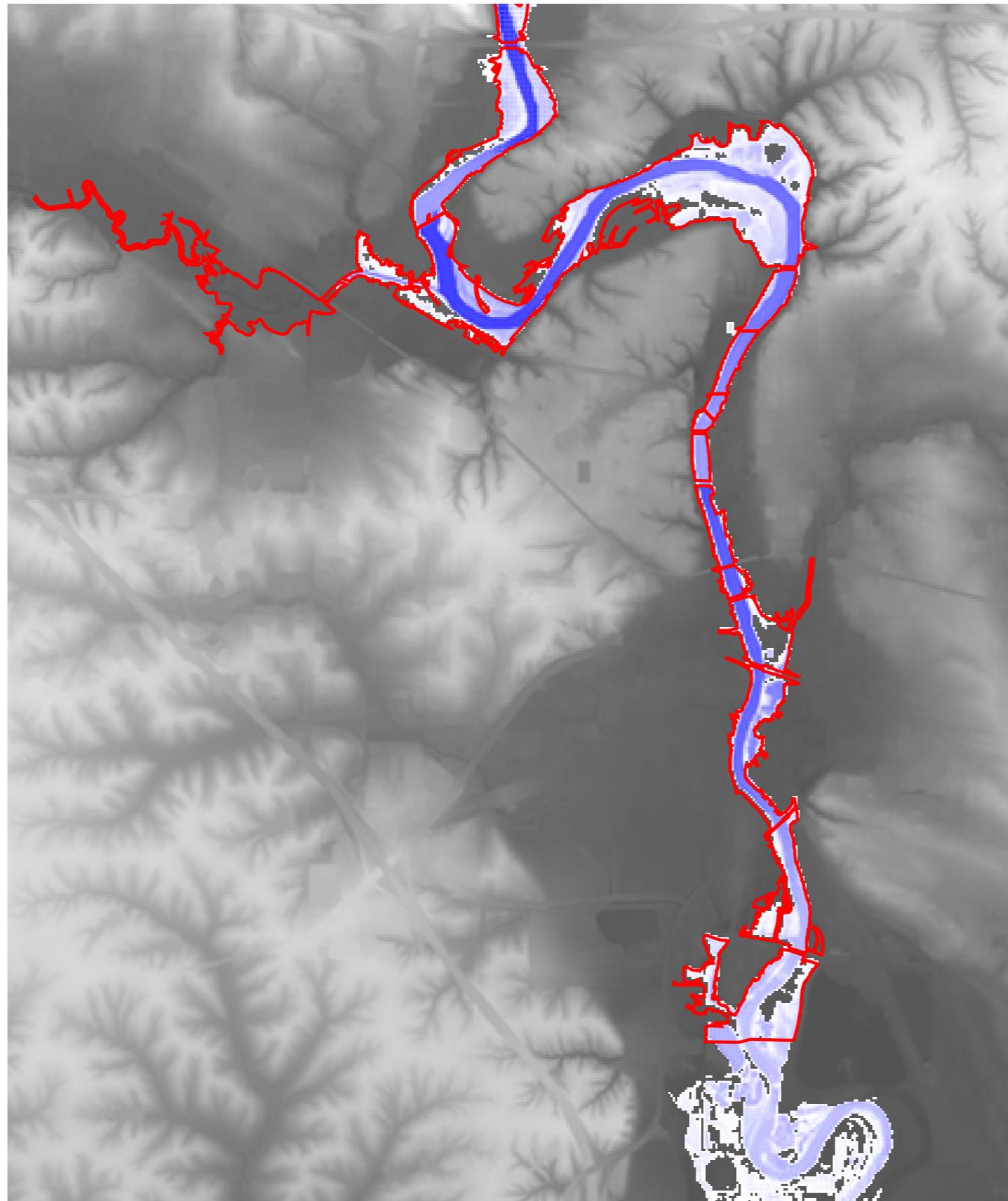


Inundation Extents

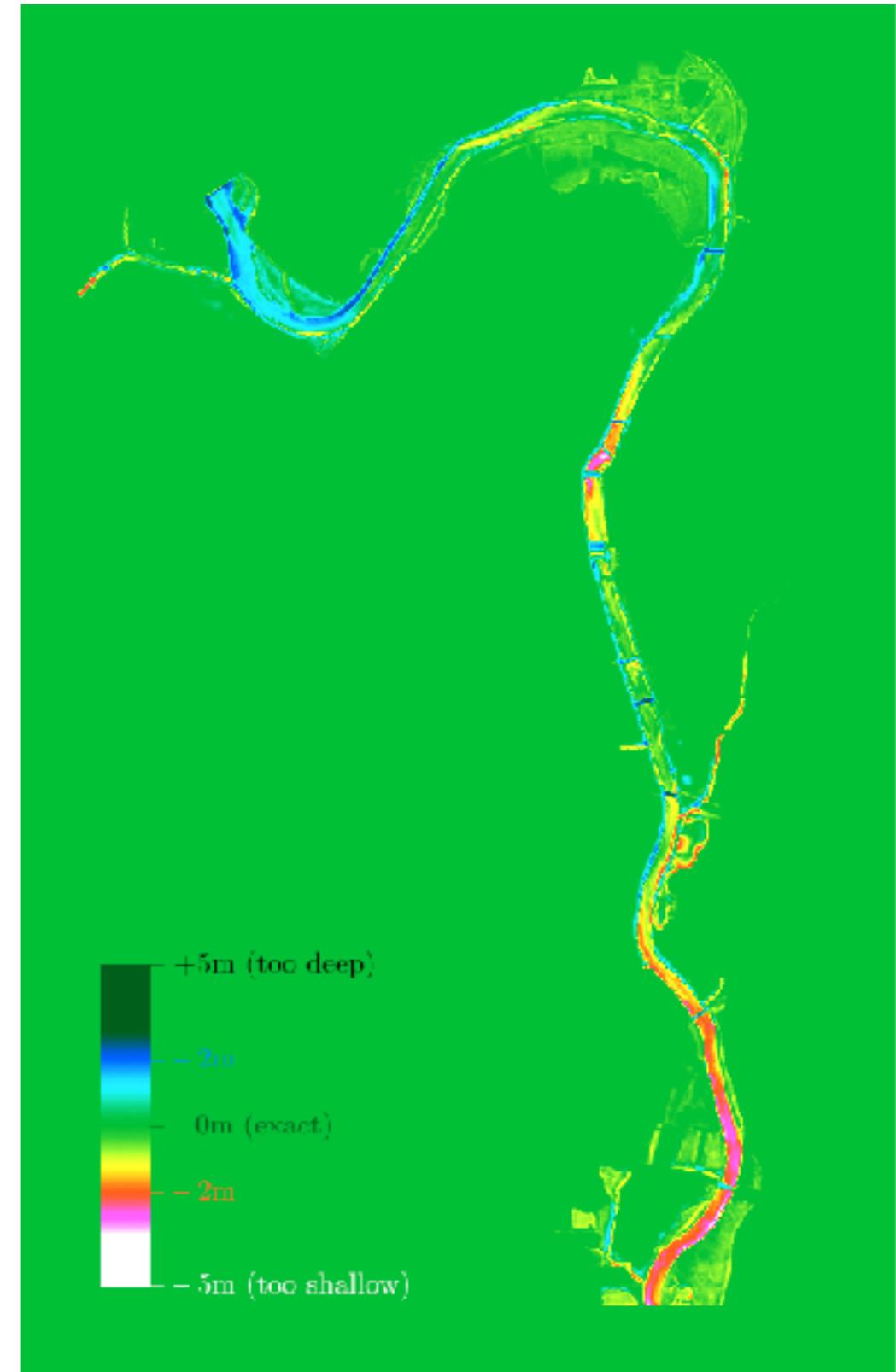


Inundation Depth Error

# Source Height 7.7 / 26.5ft River Stage

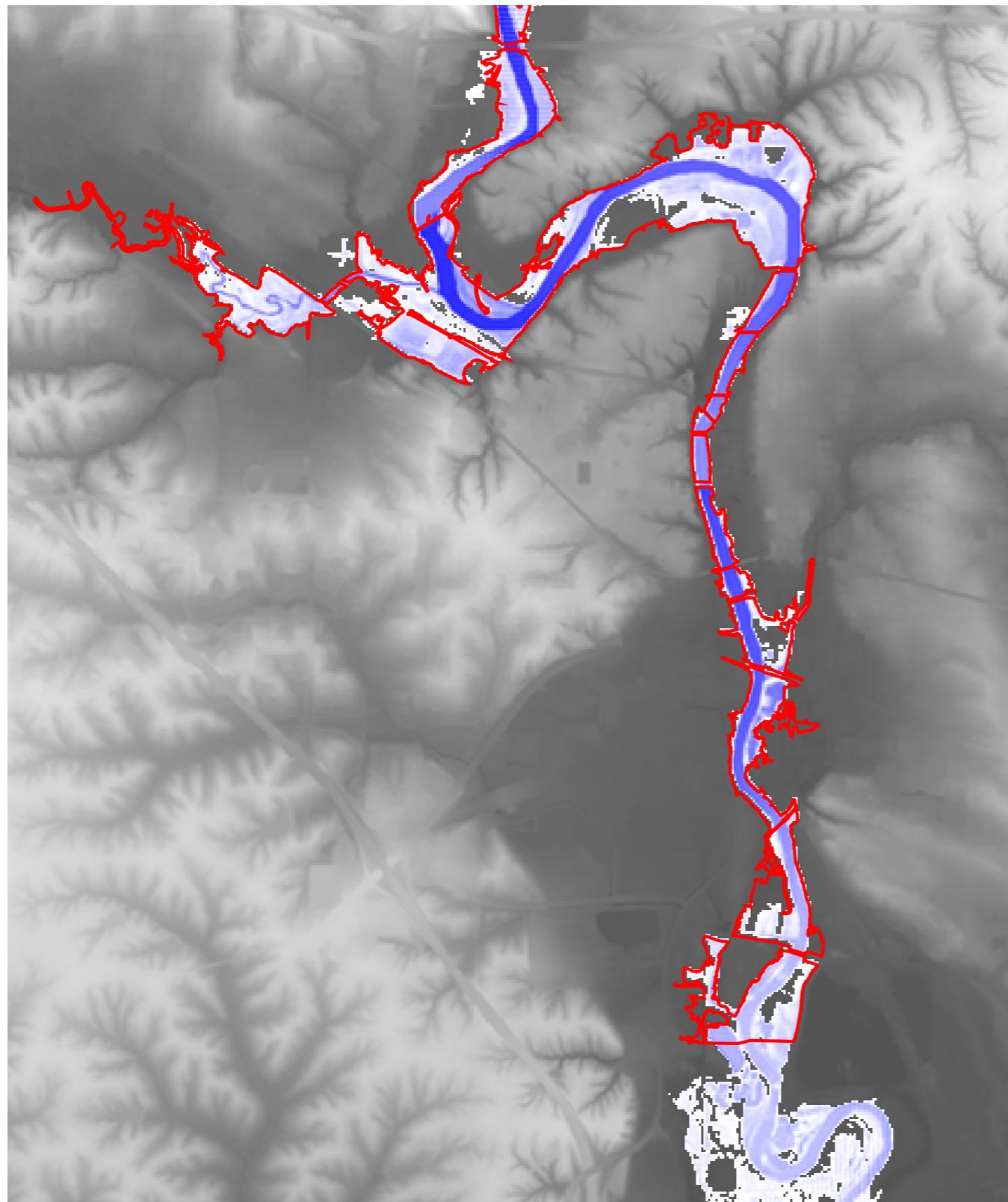


Inundation Extents

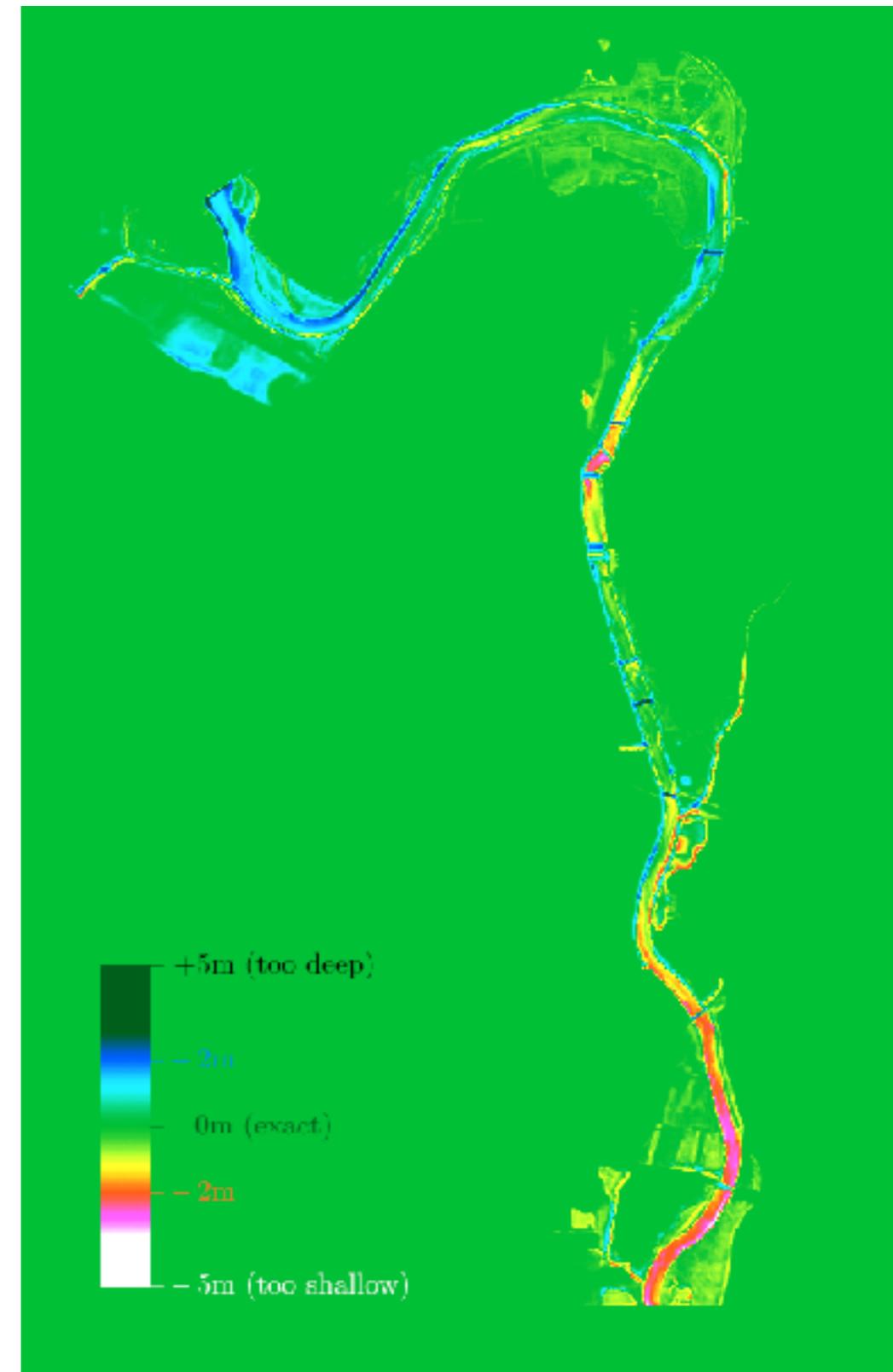


Inundation Depth Error

# Source Height 7.9 / 27.5ft River Stage

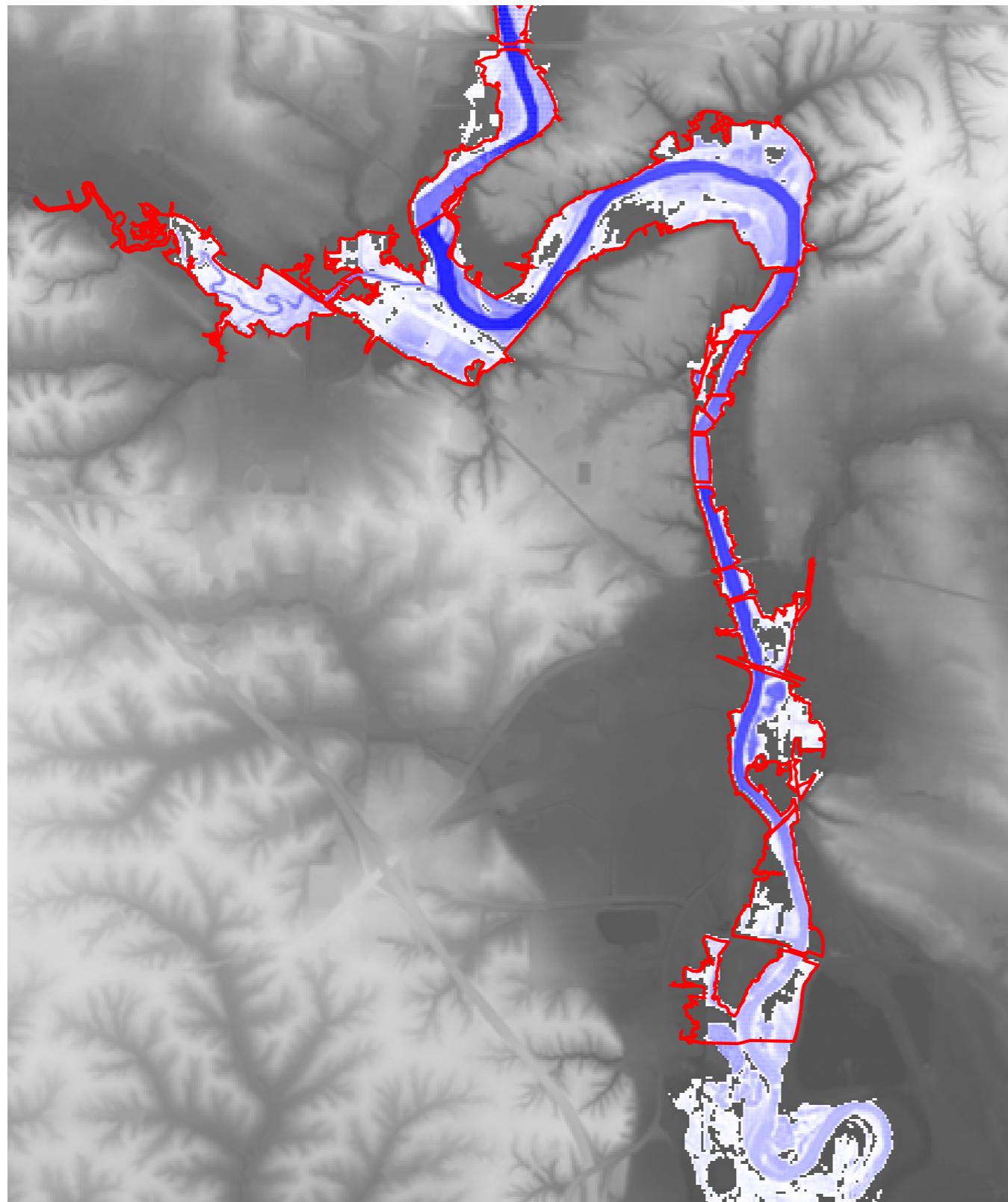


Inundation Extents

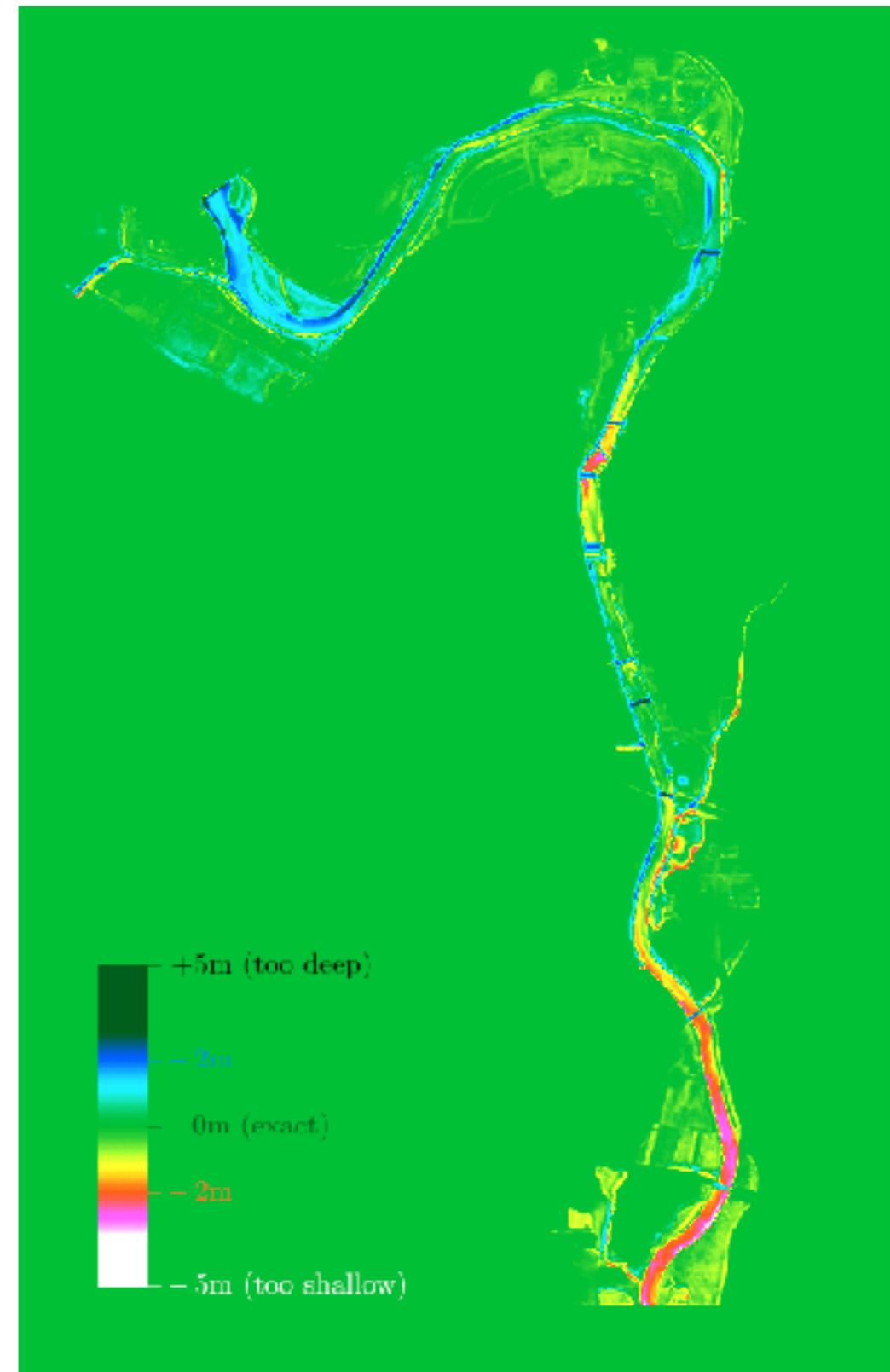


Inundation Depth Error

# Source Height 8.1 / 28.5ft River Stage

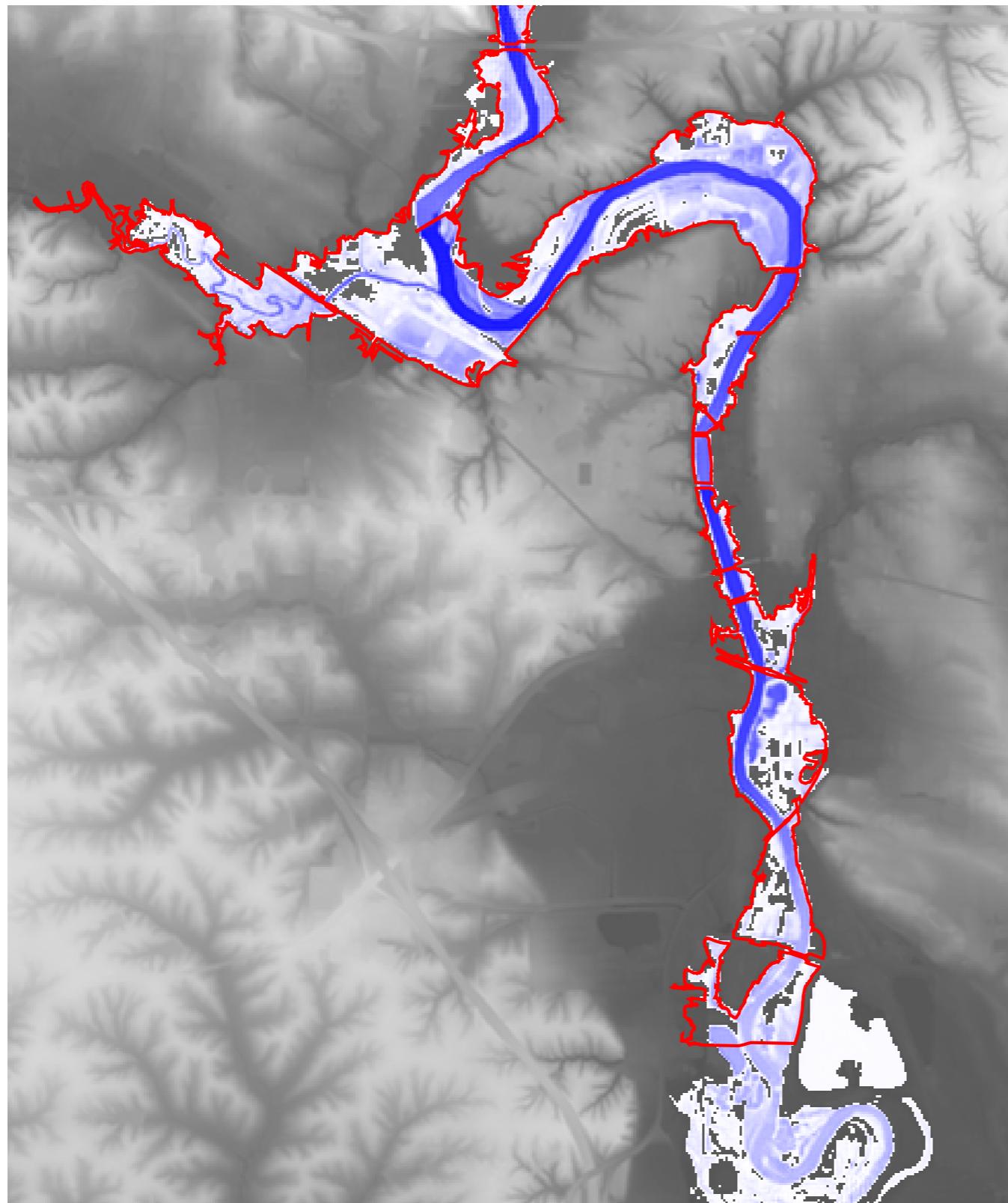


Inundation Extents

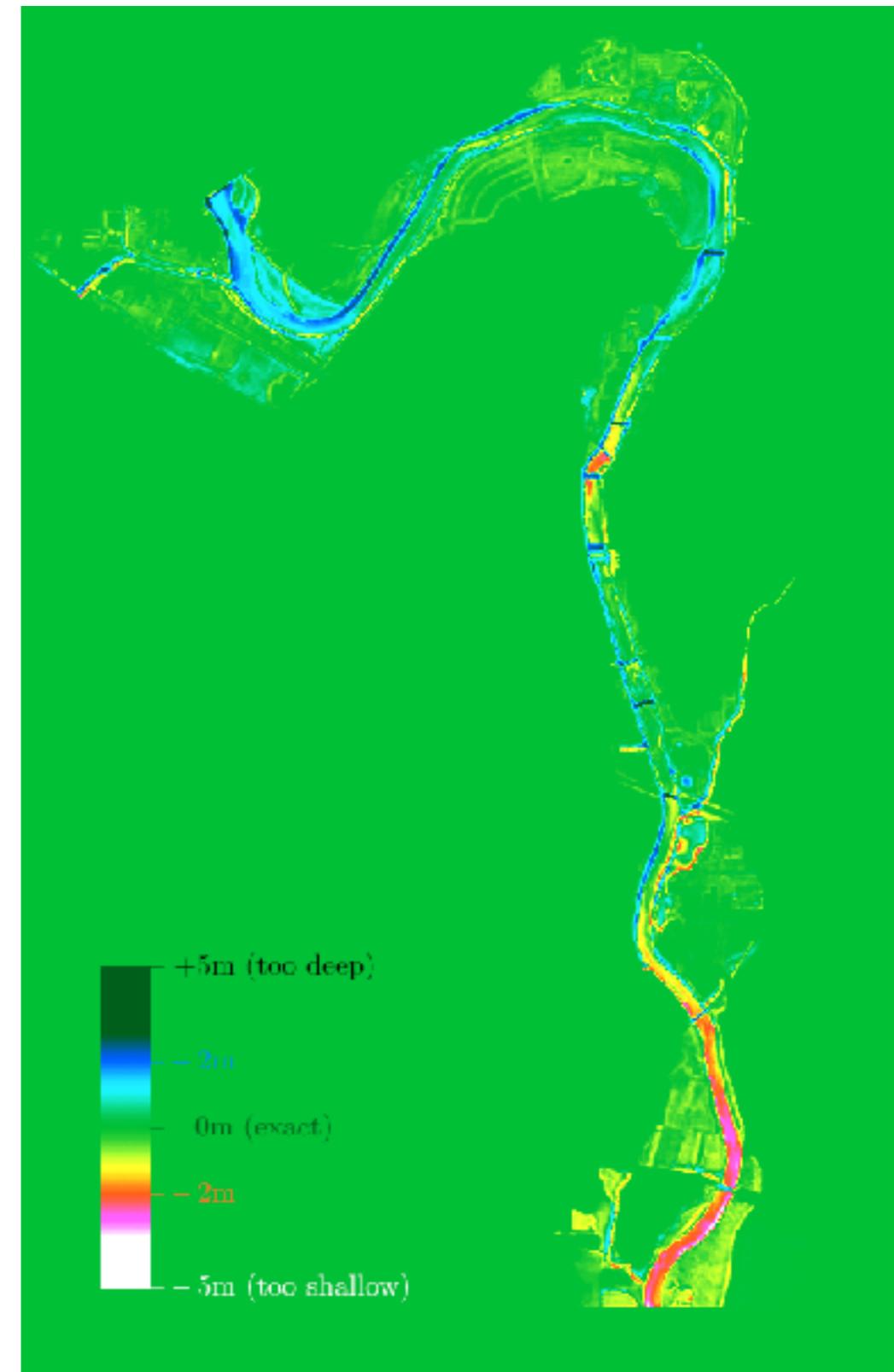


Inundation Depth Error

# Source Height 8.3 / 29.5ft River Stage

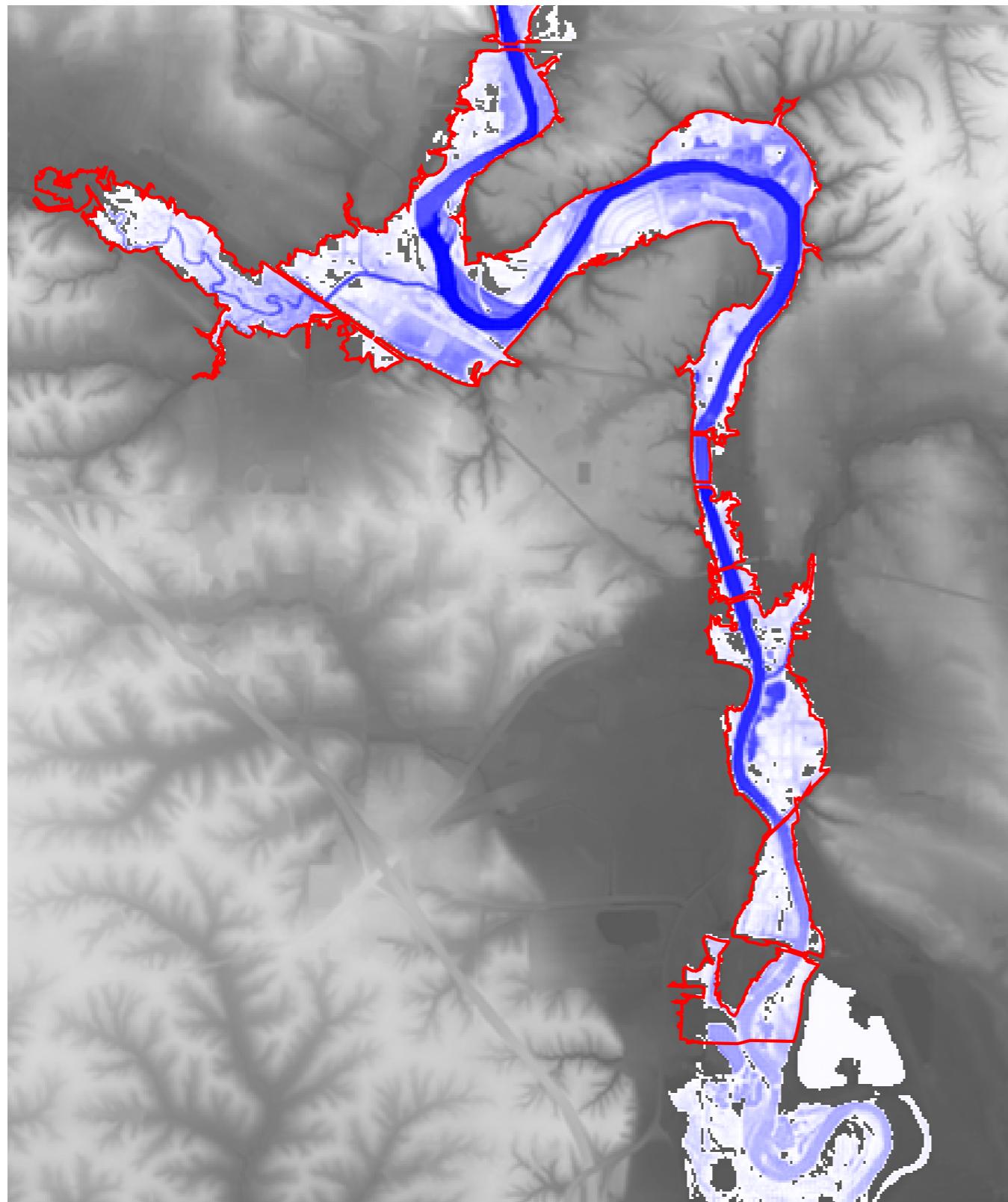


Inundation Extents

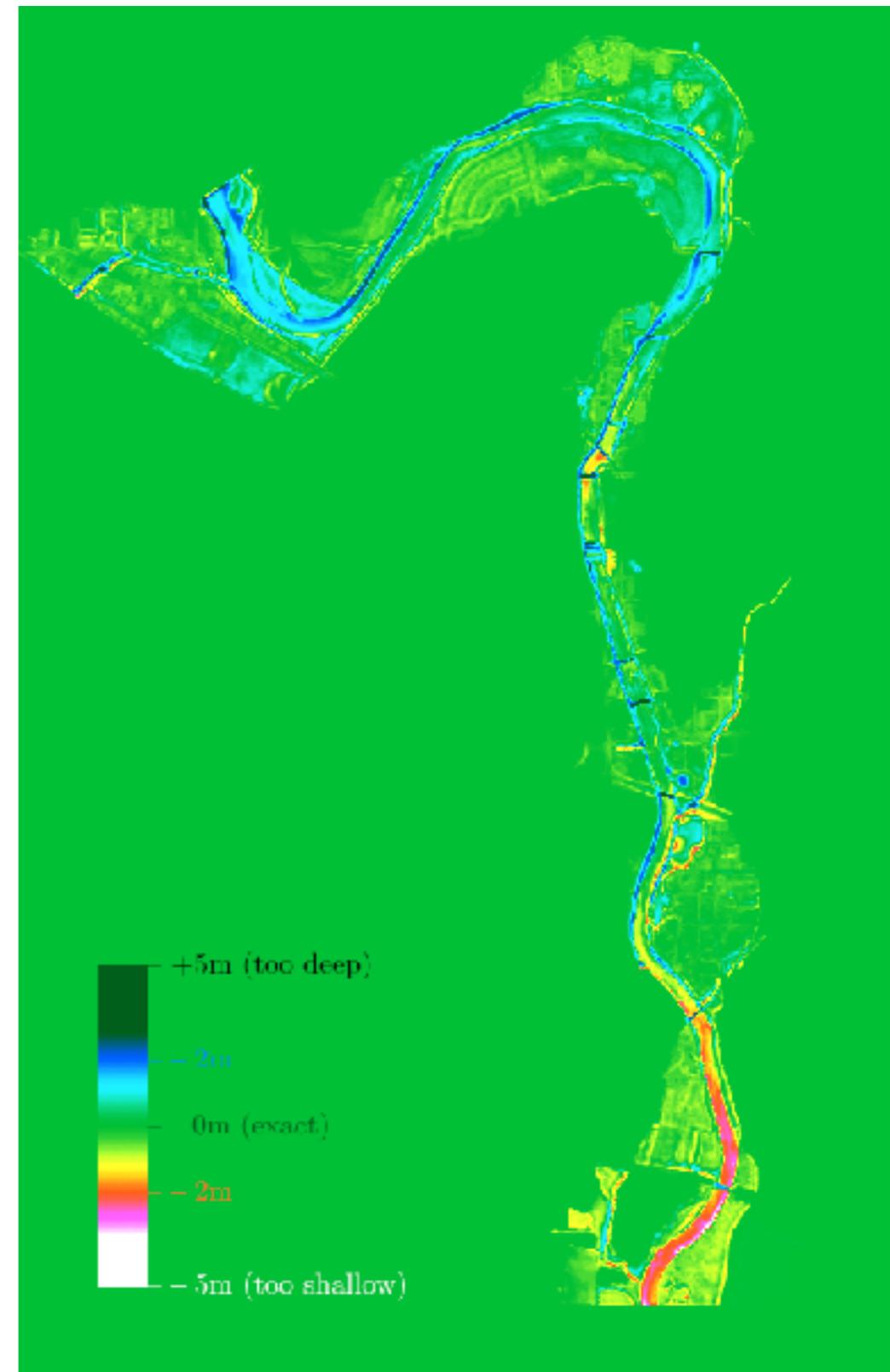


Inundation Depth Error

# Source Height 8.9 / 31.5ft River Stage

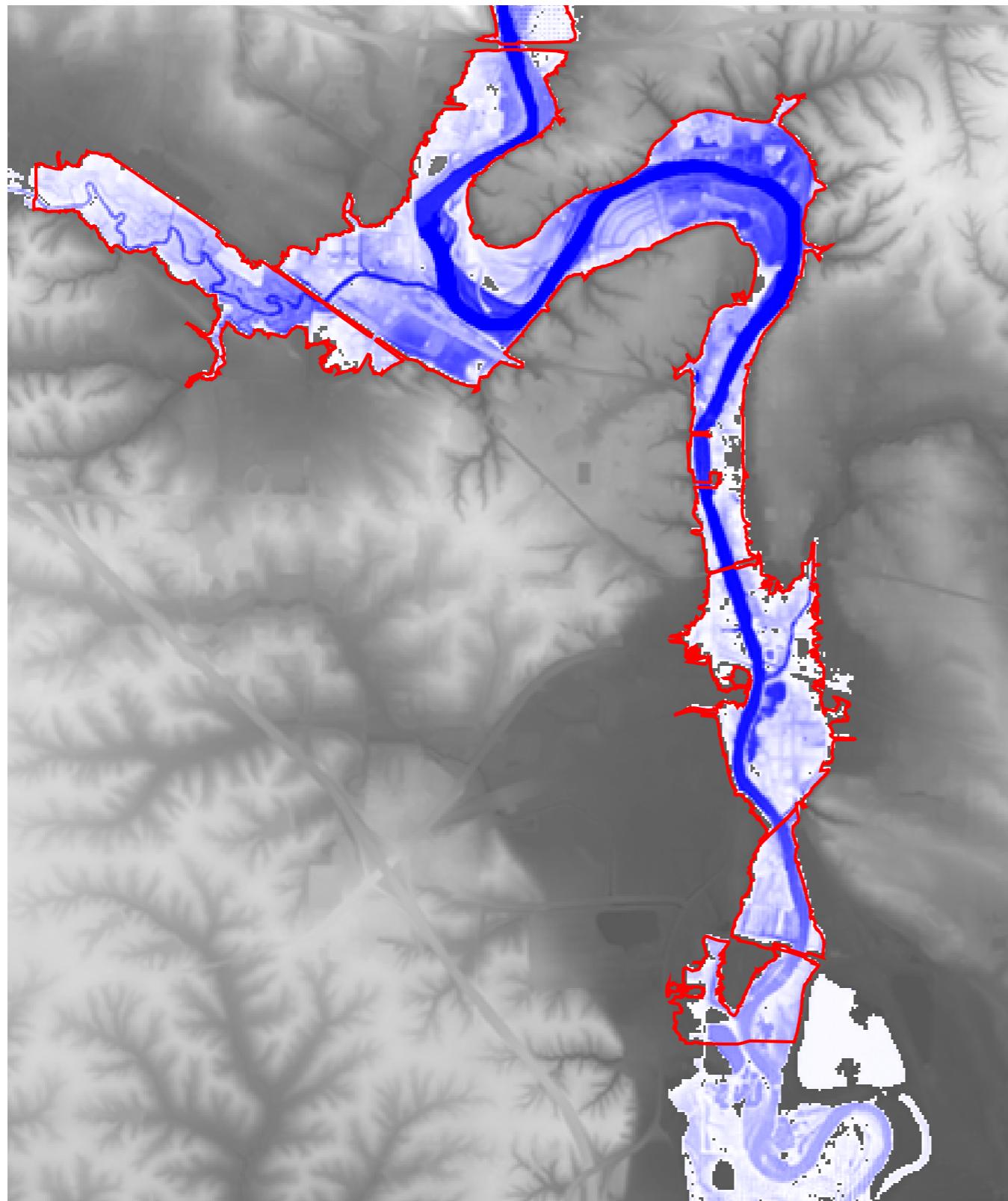


Inundation Extents

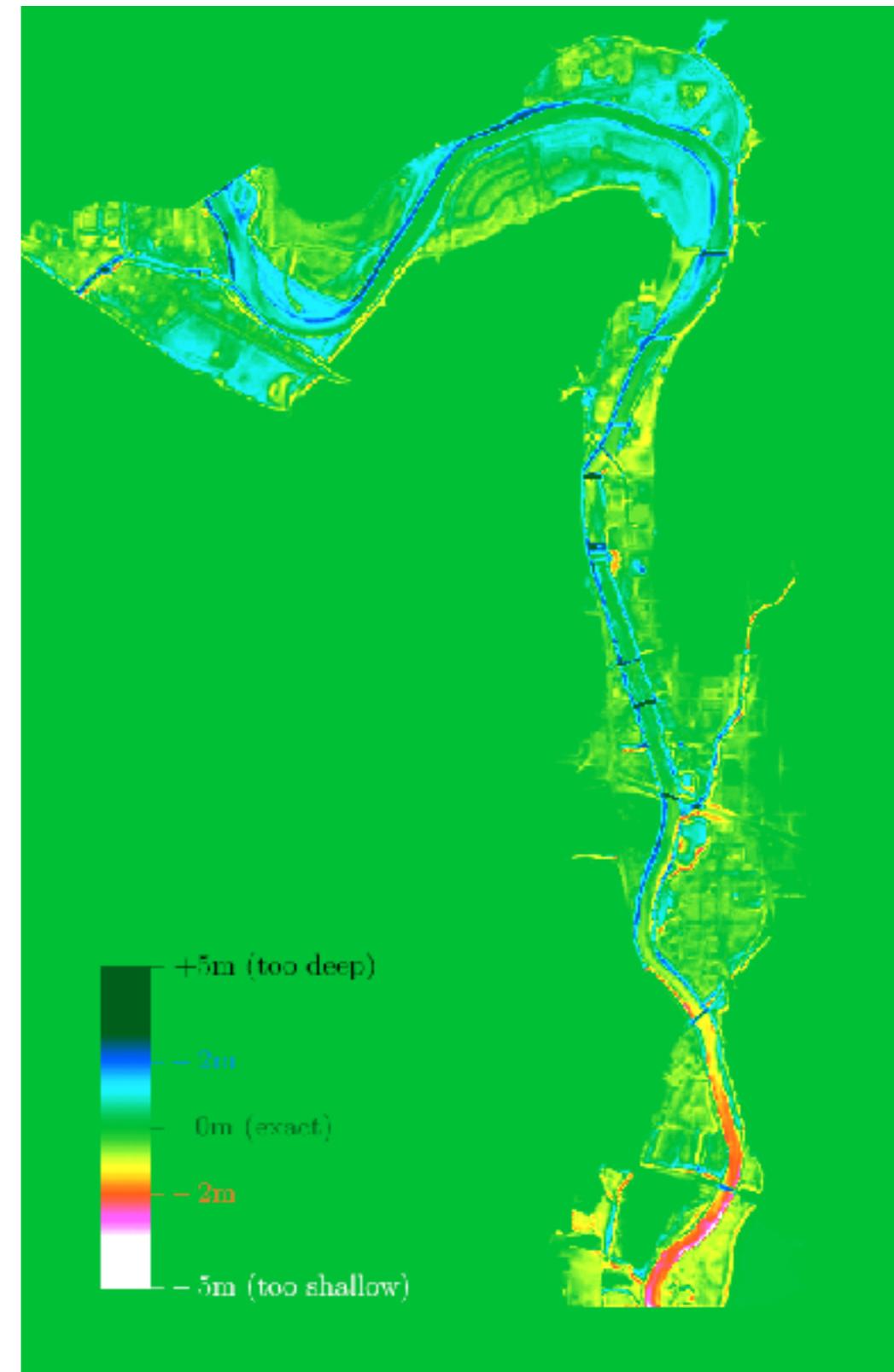


Inundation Depth Error

# Source Height 9.5 / 34.0ft River Stage

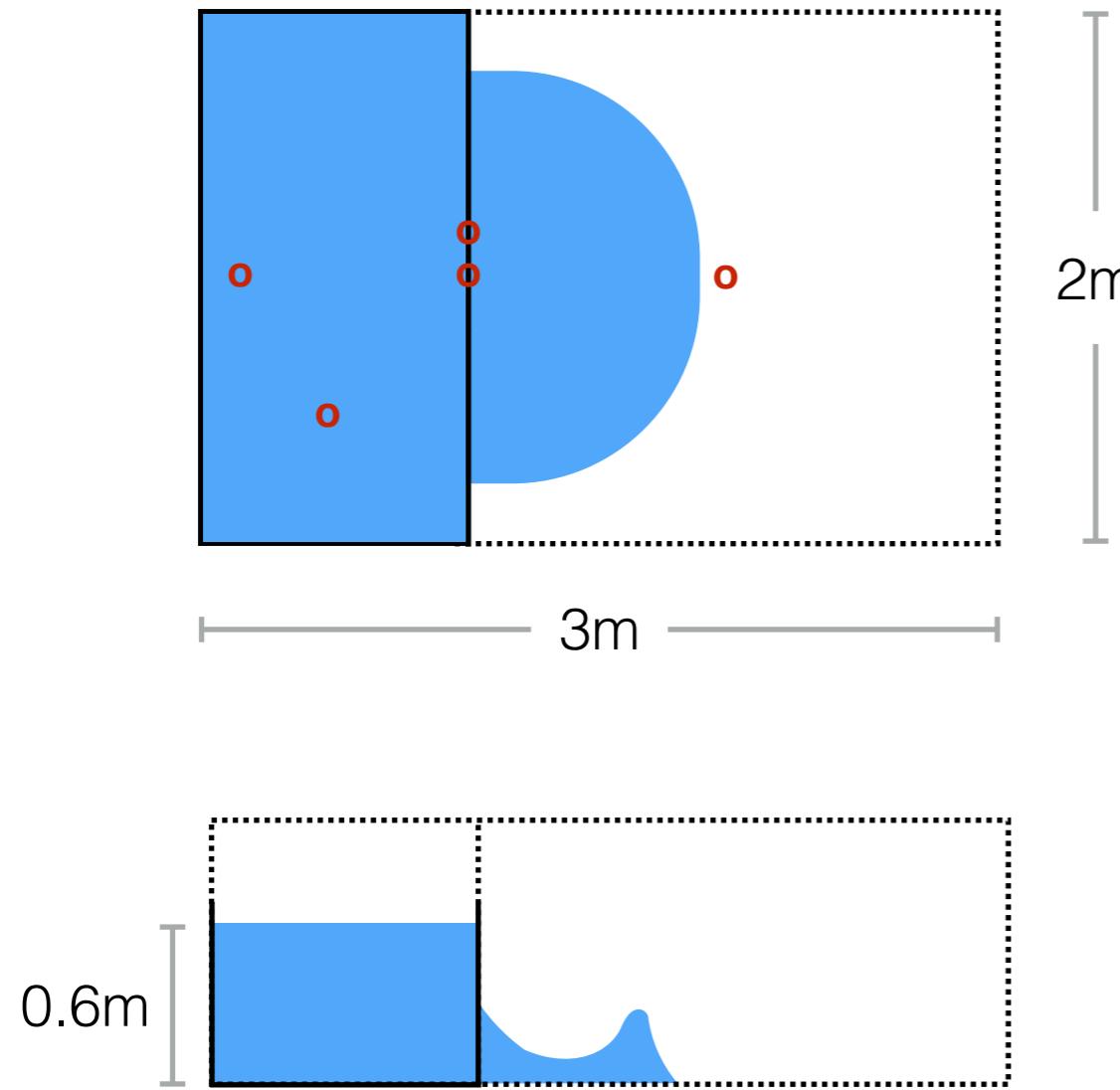


Inundation Extents



Inundation Depth Error

# Dam-break experiment of Fraccarollo & Toro (1995)



- Classic Experiment
- Good Measured and Analytical Reference Data

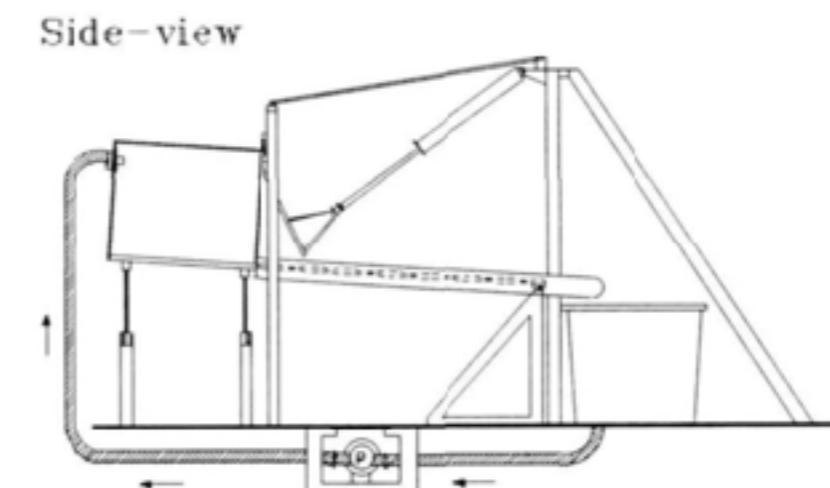
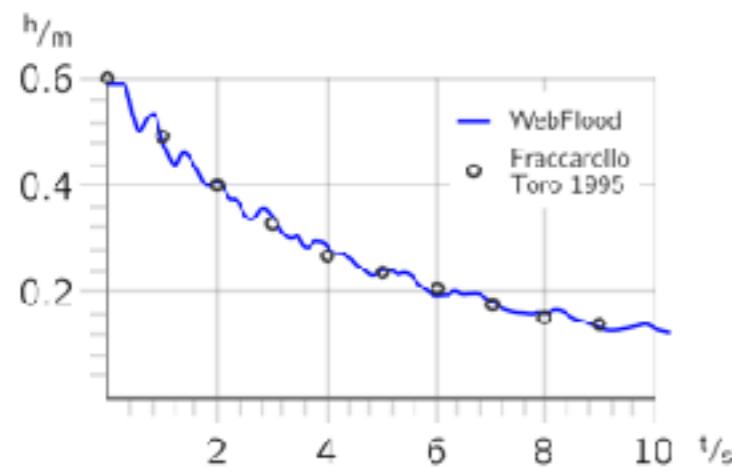
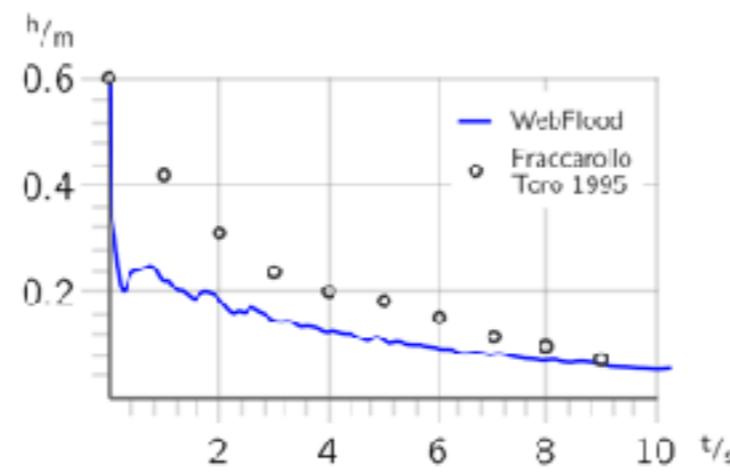


Fig.1. Schematic diagrams of the experimental set-up.

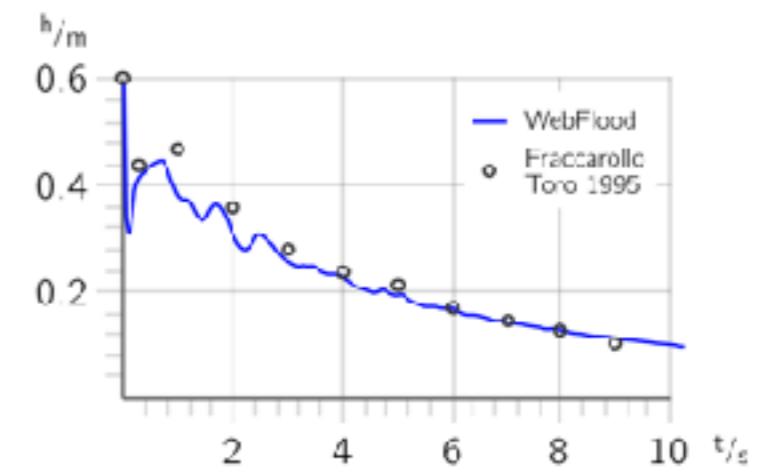
WebFlood Demo Dam-Break



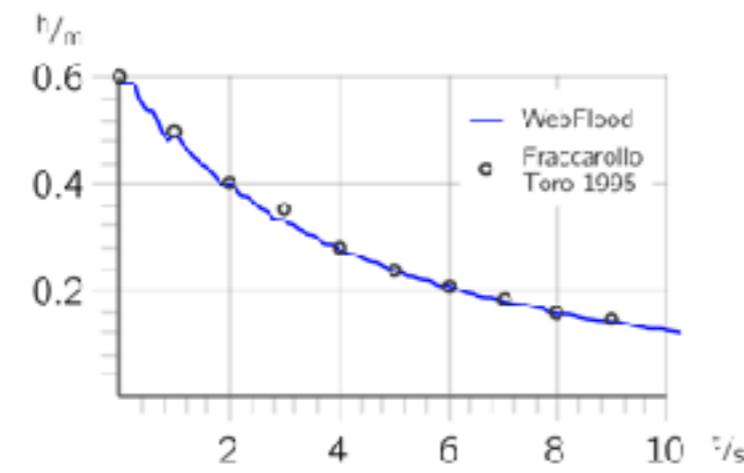
(a) Point -5A



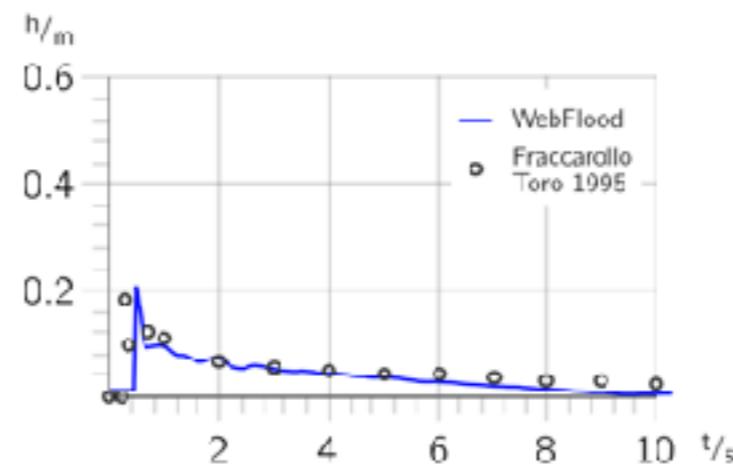
(b) Point 4



(c) Point 0



(d) Point C



(e) Point 8A

# Conclusion

# Possible Applications

- Public Information Systems
- Flood Response Decision-Making
- Simulation Aided Education

# Improvements

- Damage Modelling/Prediction
- Optimisation (simulation domain: sparse!)
- More Rigorous Implementation
  - Full Mass Conservation
  - Better Stability
  - Less Artefacts

# WebGL for GPGPU

- GPU → CPU Communication Difficult
  - synchronous, encoding issues
- Missing Features
  - Geometry Shaders
  - Transform Feedback
  - Array Textures
  - ... coming in WebGL 2!

Interactivity can make  
even complex things  
understandable

# Bonus WebFlood Demo

Thank you  
for your attention

# Questions