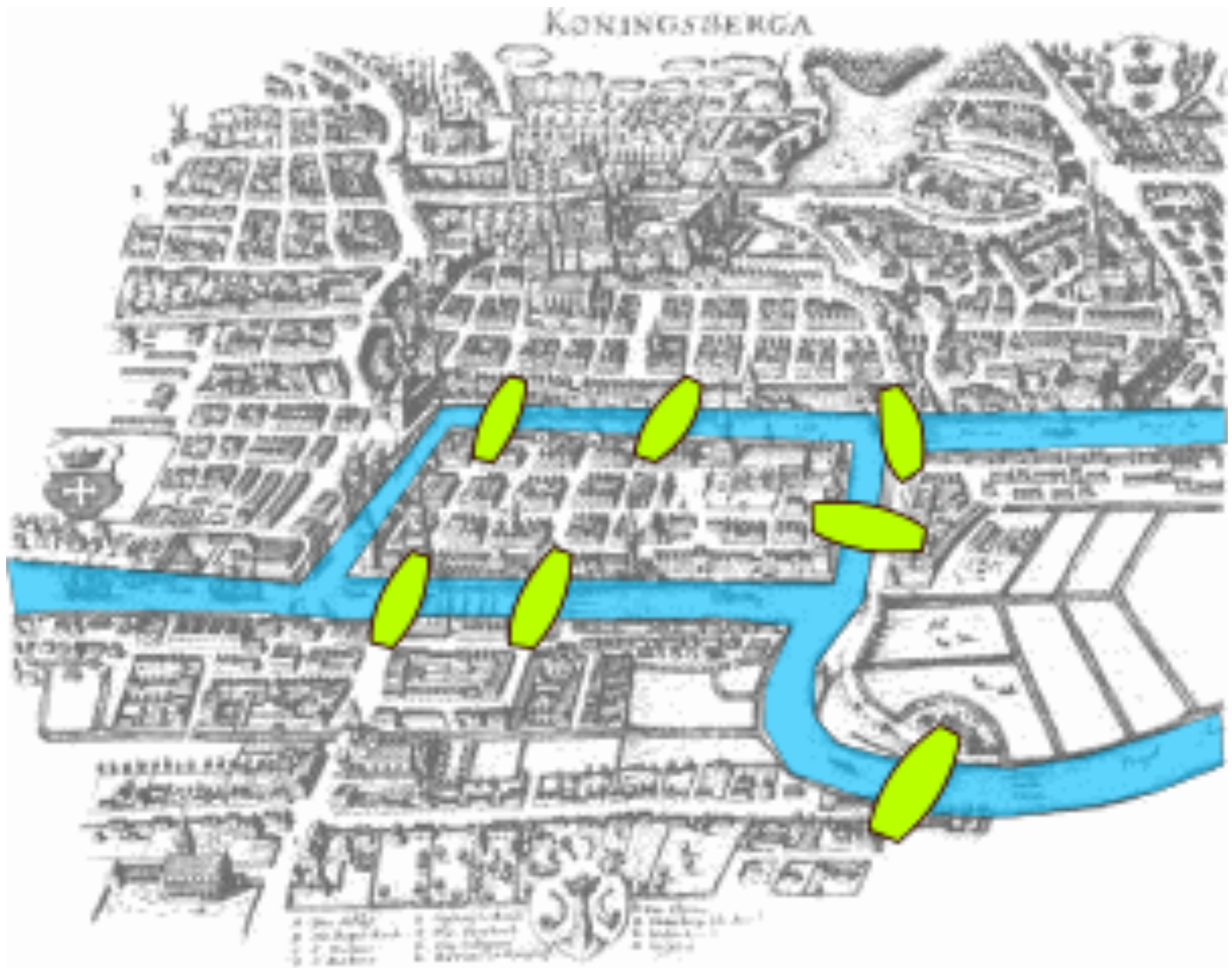
Introduction à l’informatique CM7

Antonio E. Porreca
https://aeporreca.org/teaching

Problème des sept ponts de Königsberg



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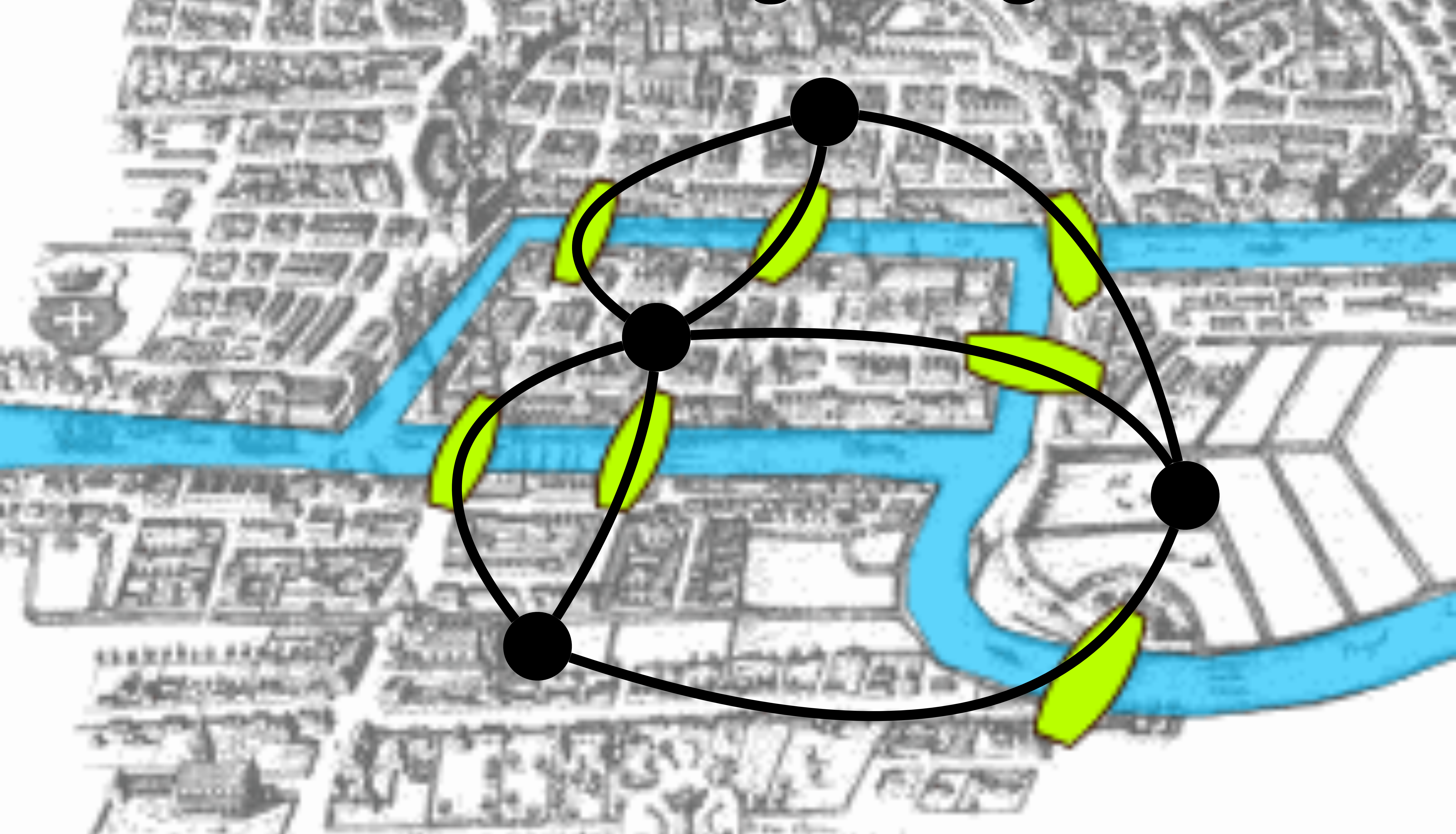

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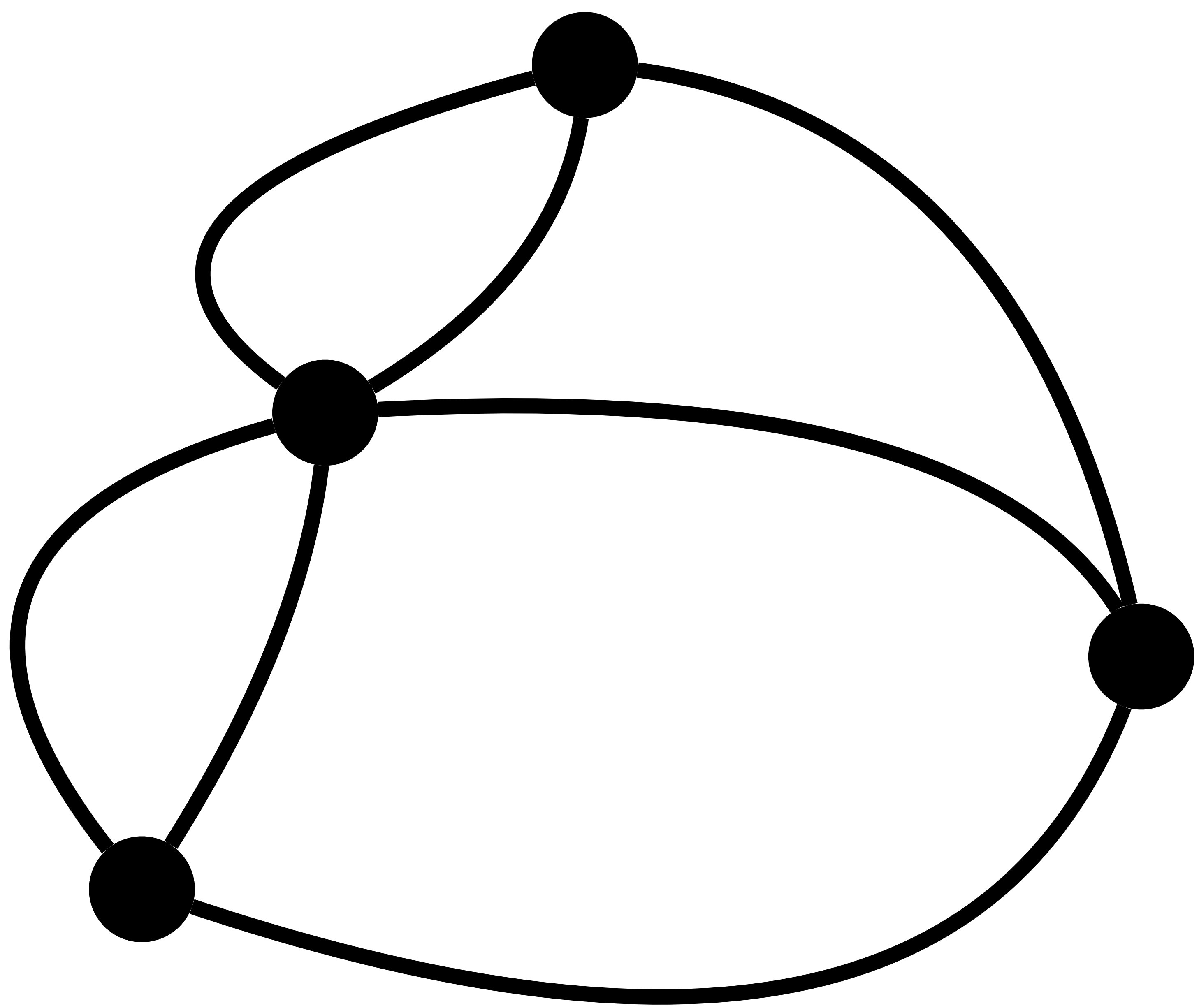

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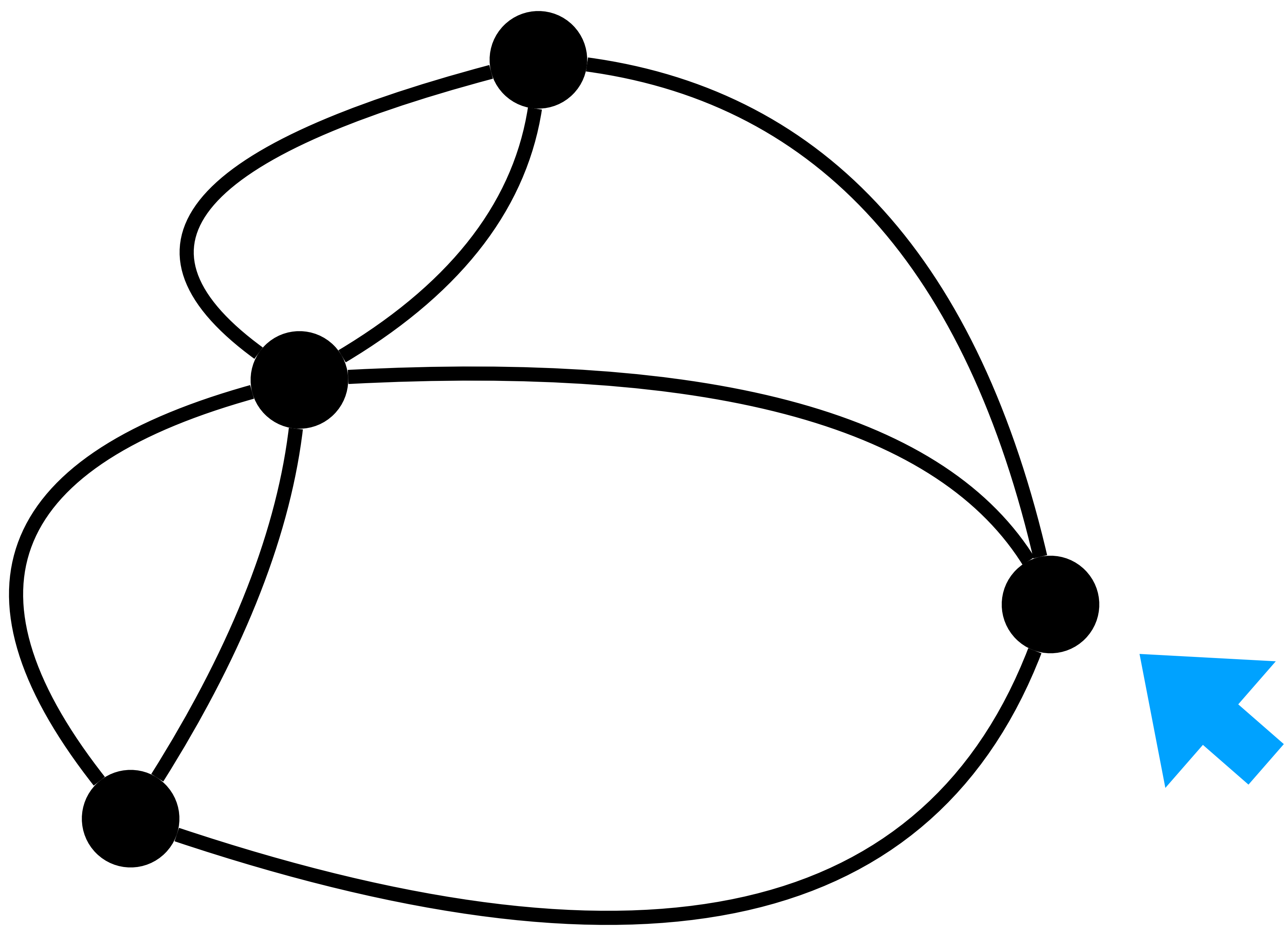

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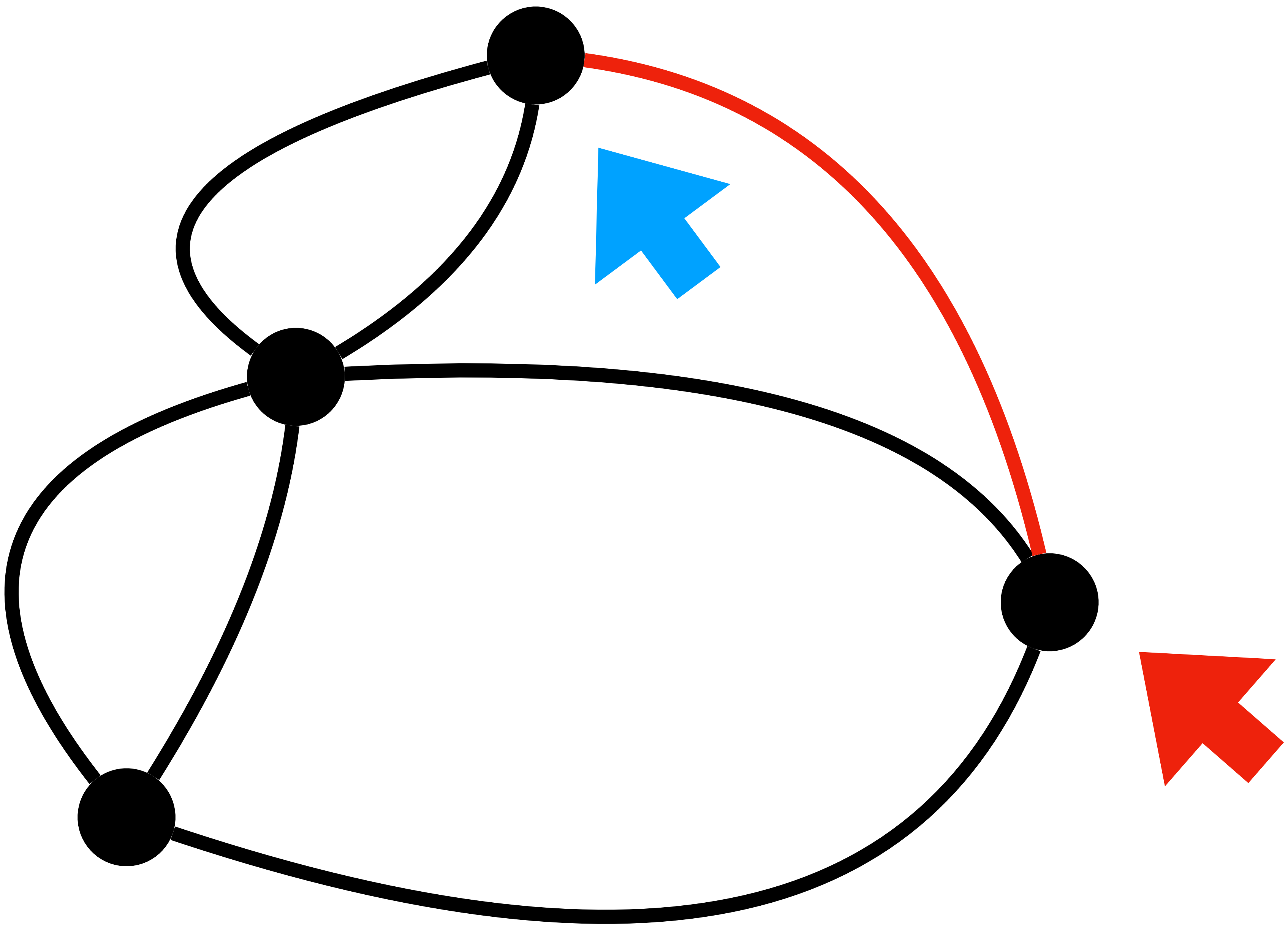

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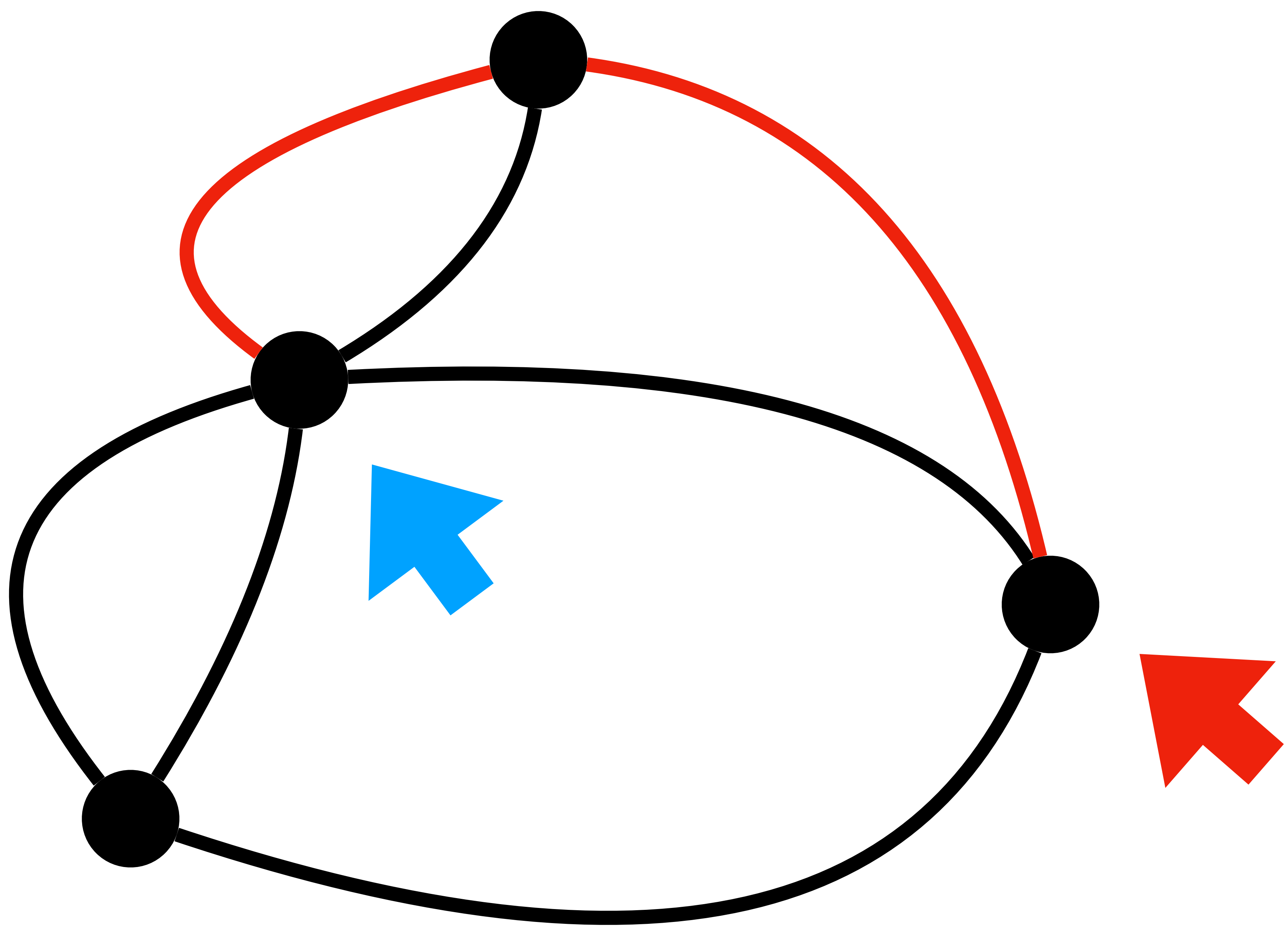

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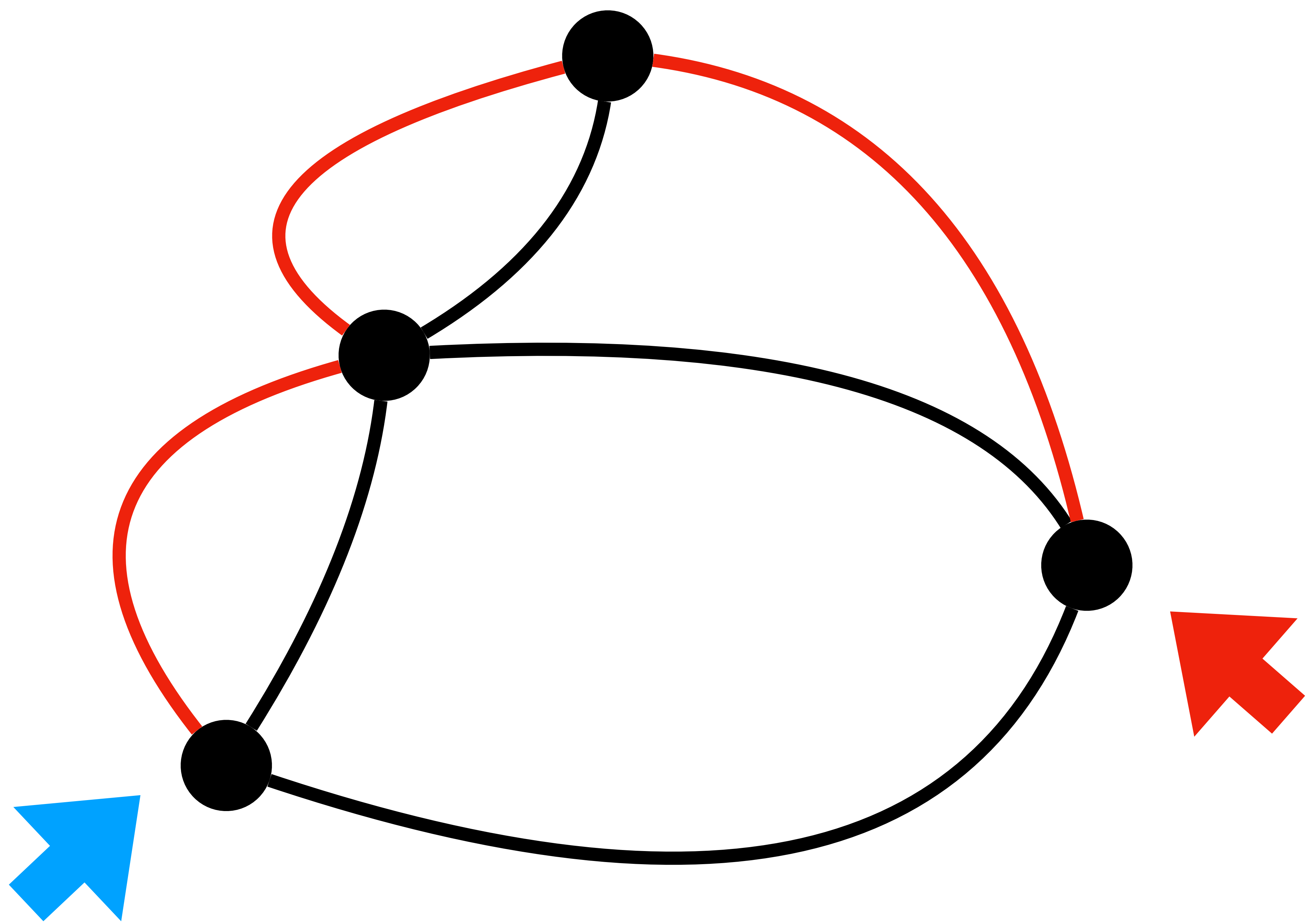

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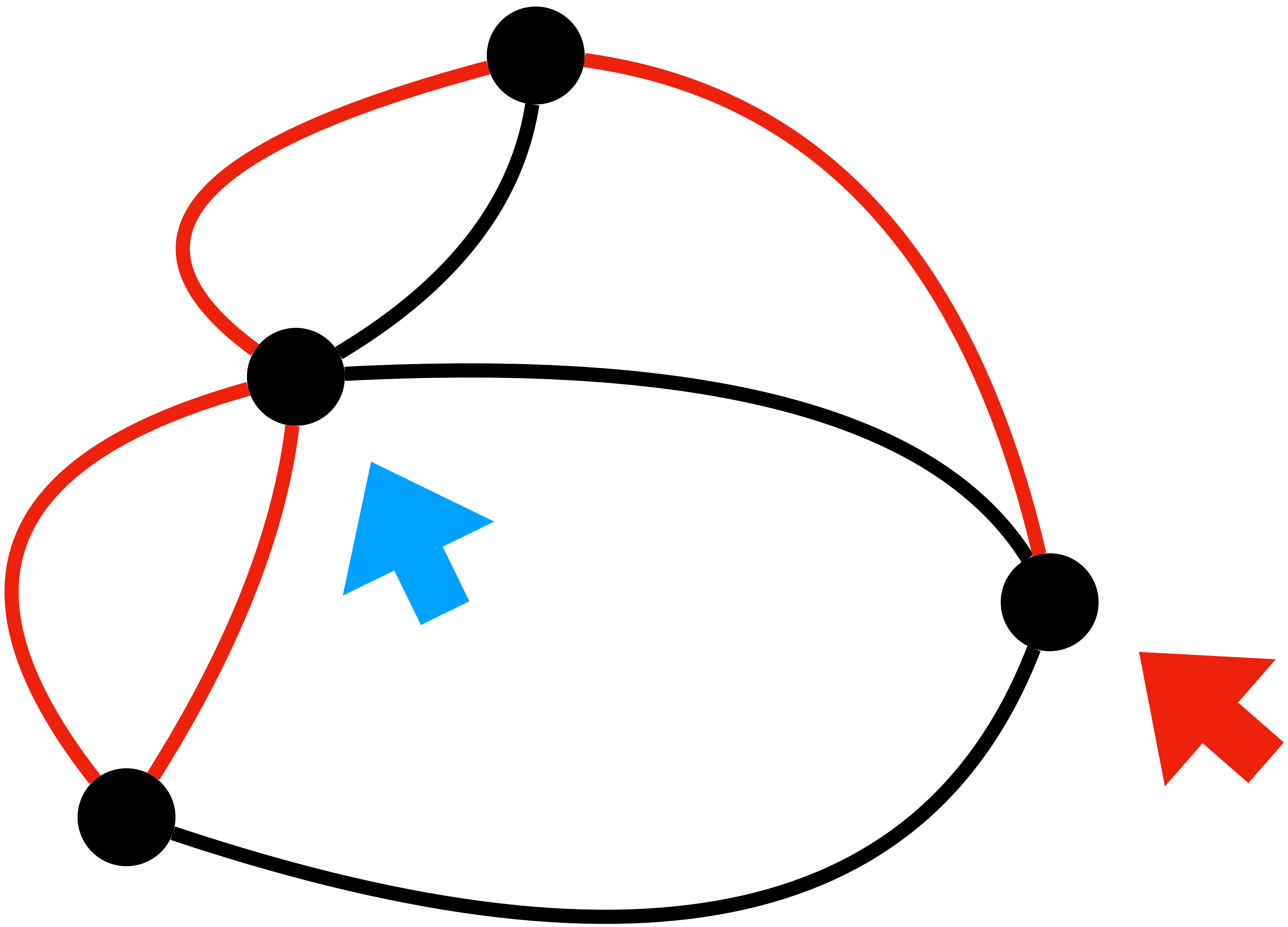

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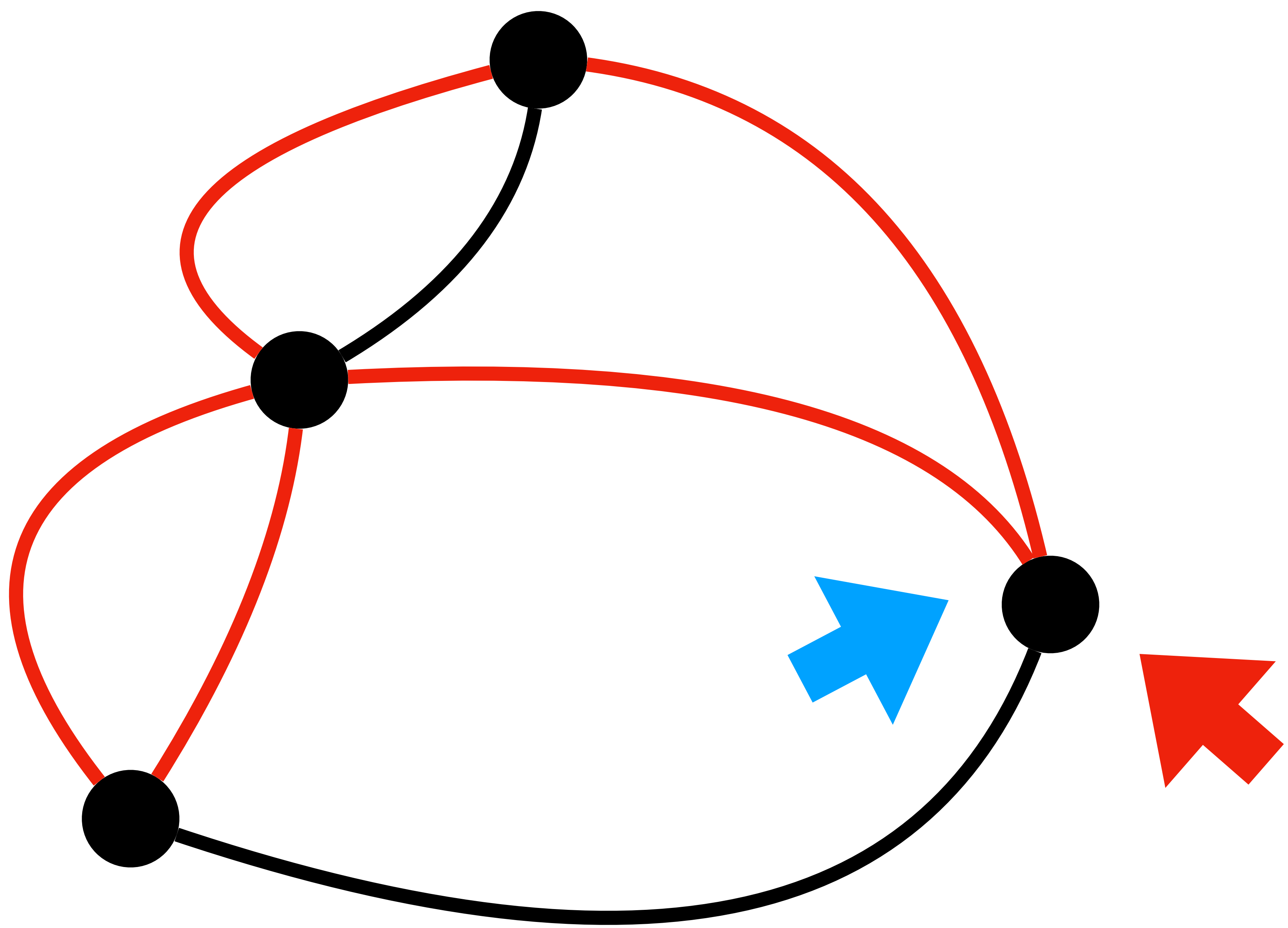

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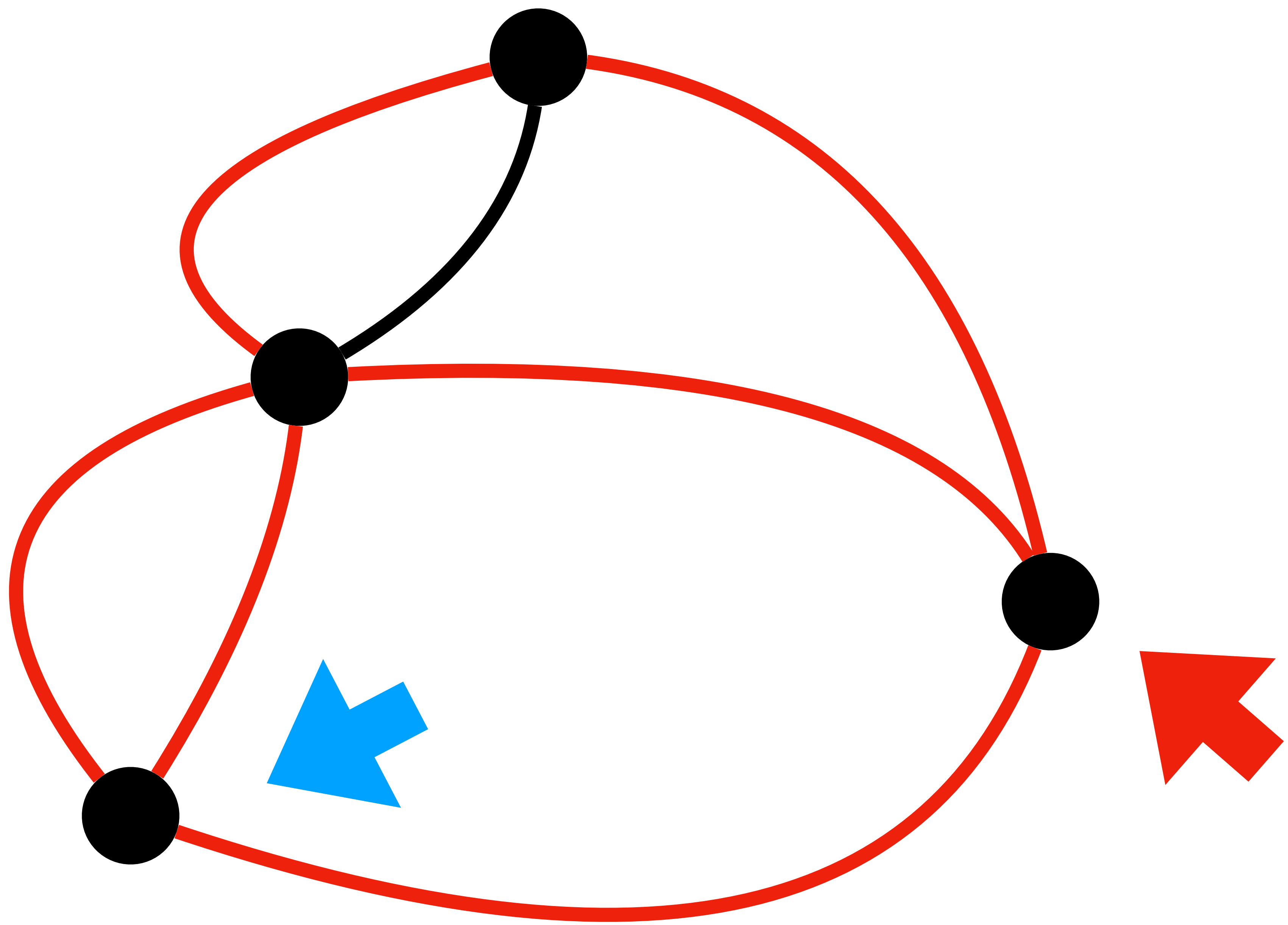

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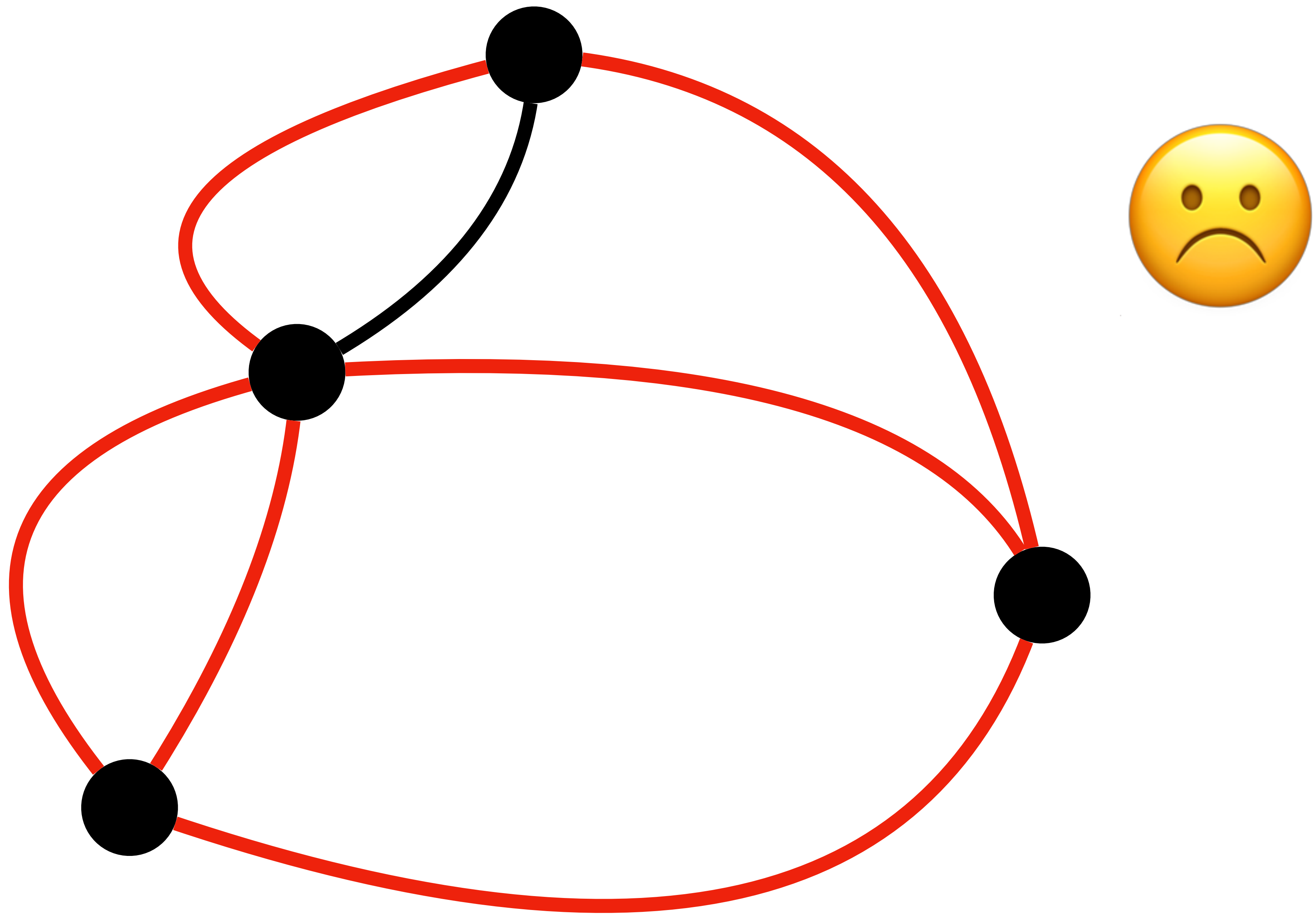

Problème des sept ponts de Königsberg

← Leonhard Euler

Théorème d’Euler (1736)

Un graphe non orienté connexe admet un cycle
qui traverse chaque arête une et une seule fois
(un « cycle eulérien ») si et seulement si
chaque sommet a un nombre pair
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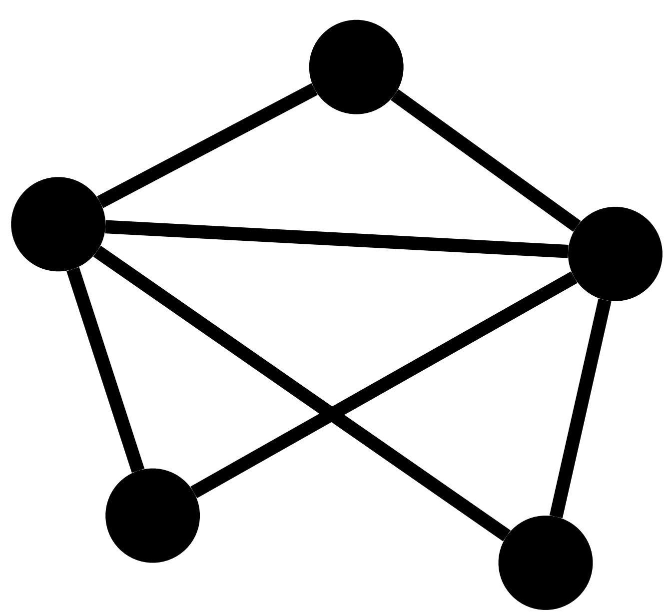

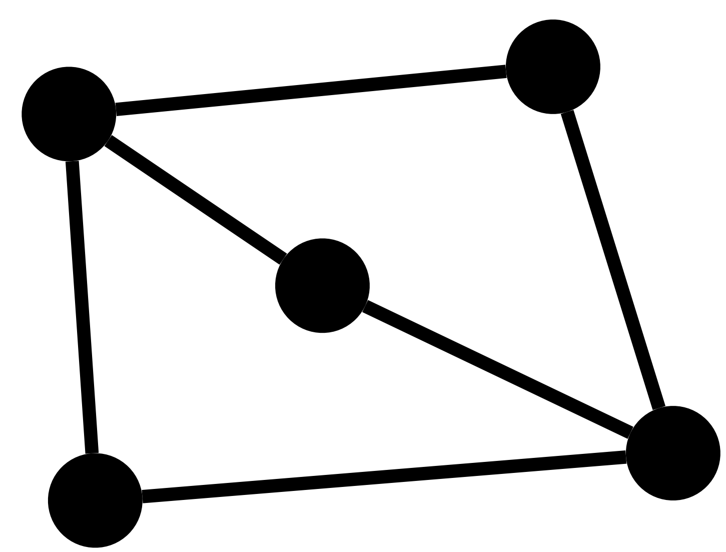

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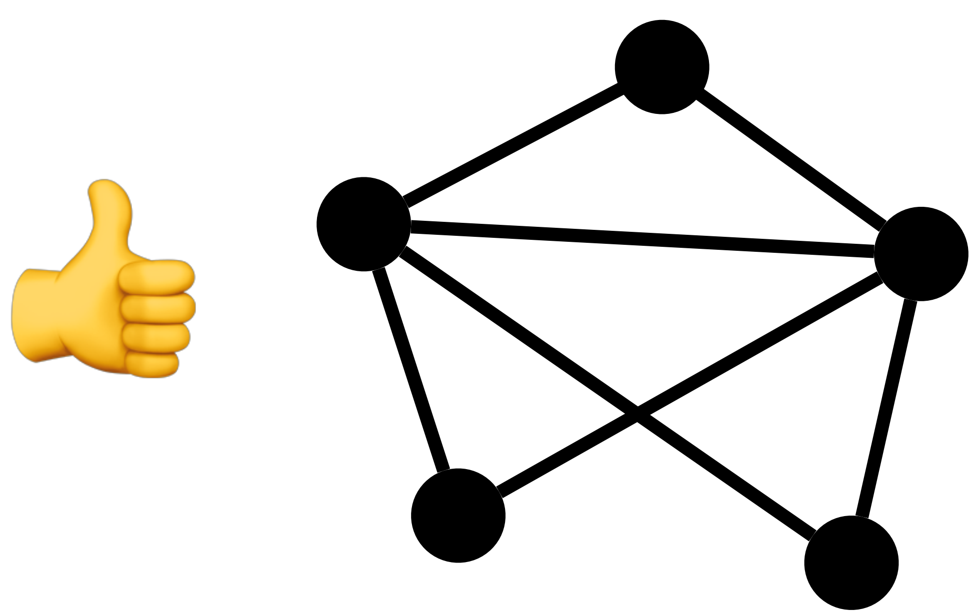

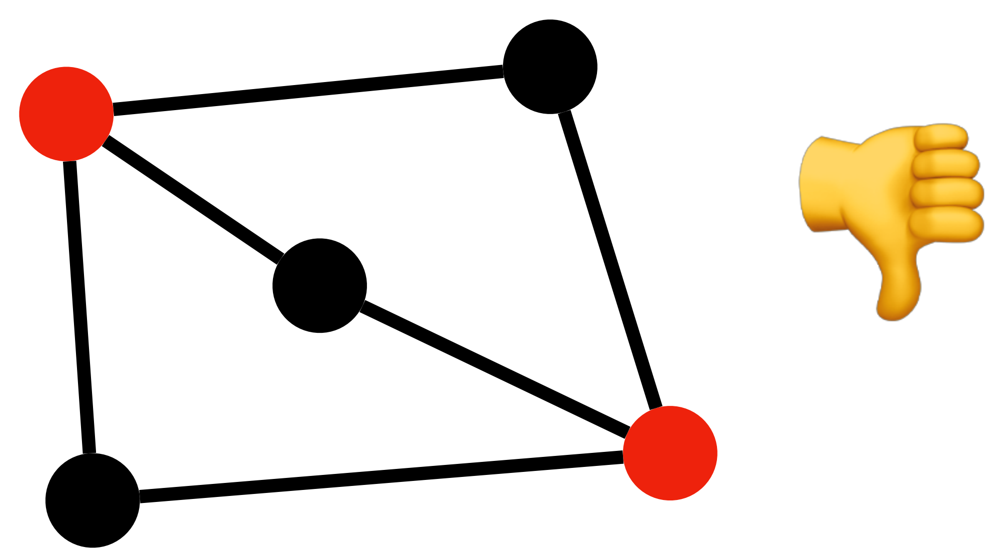

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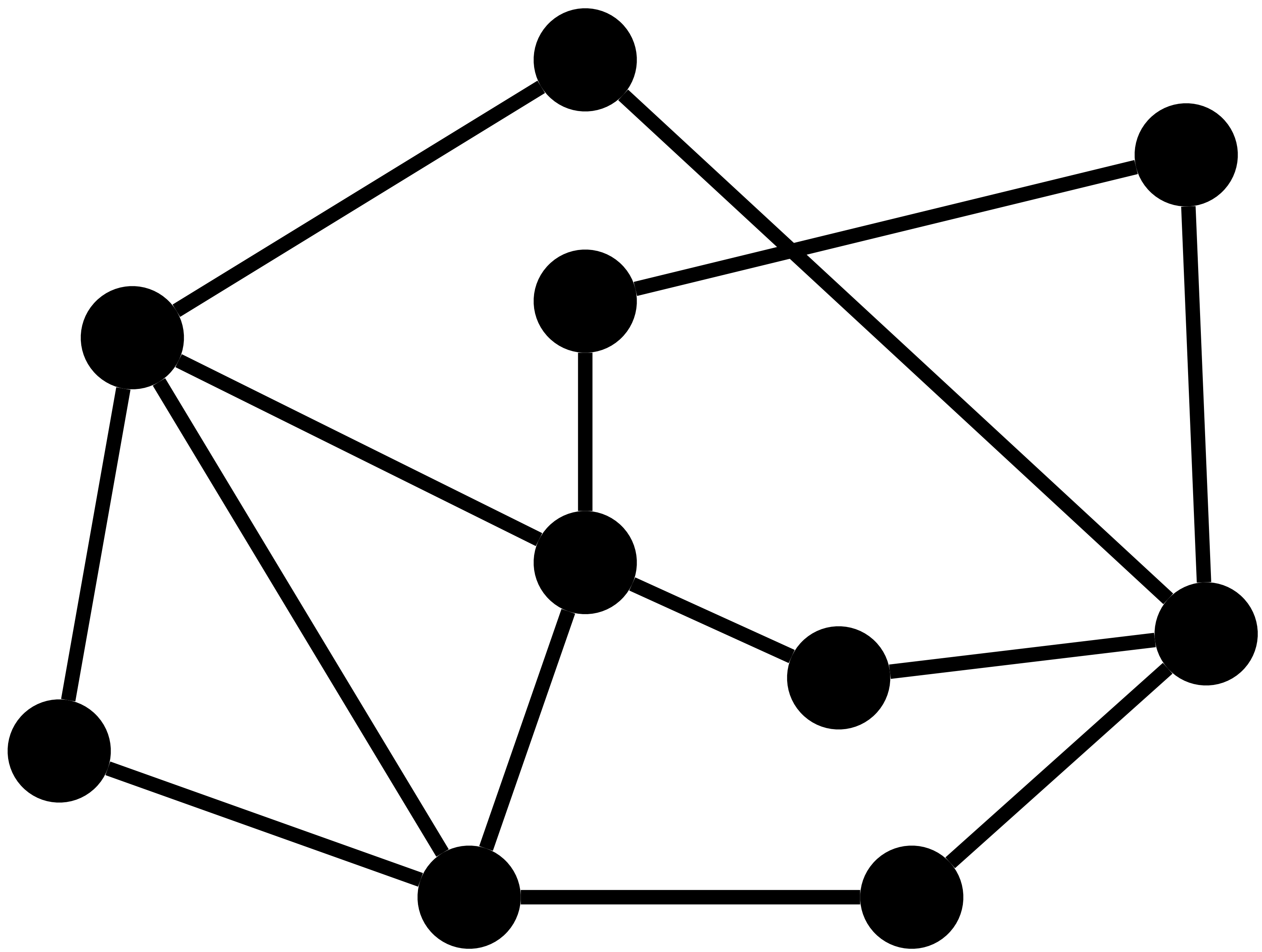

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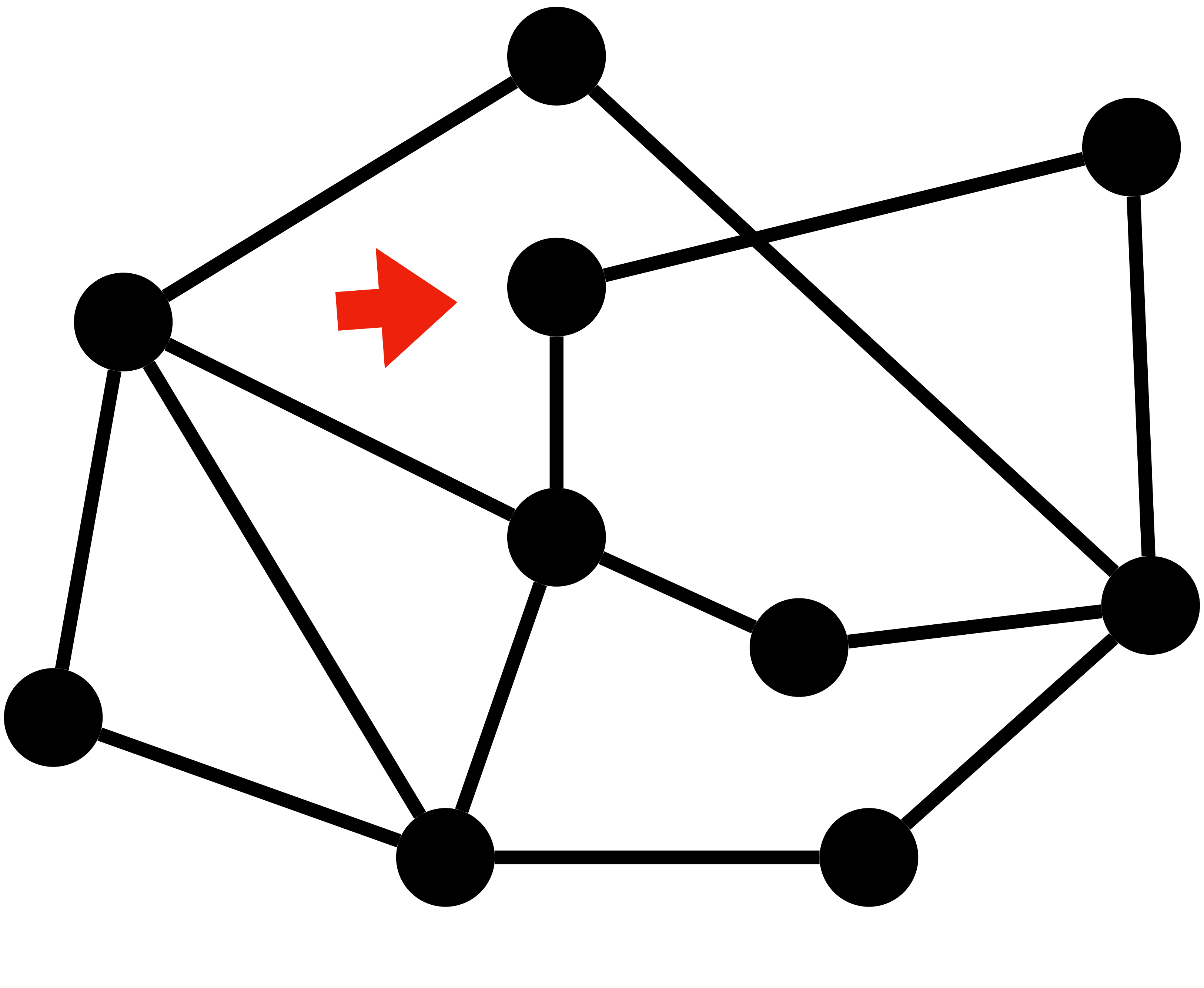

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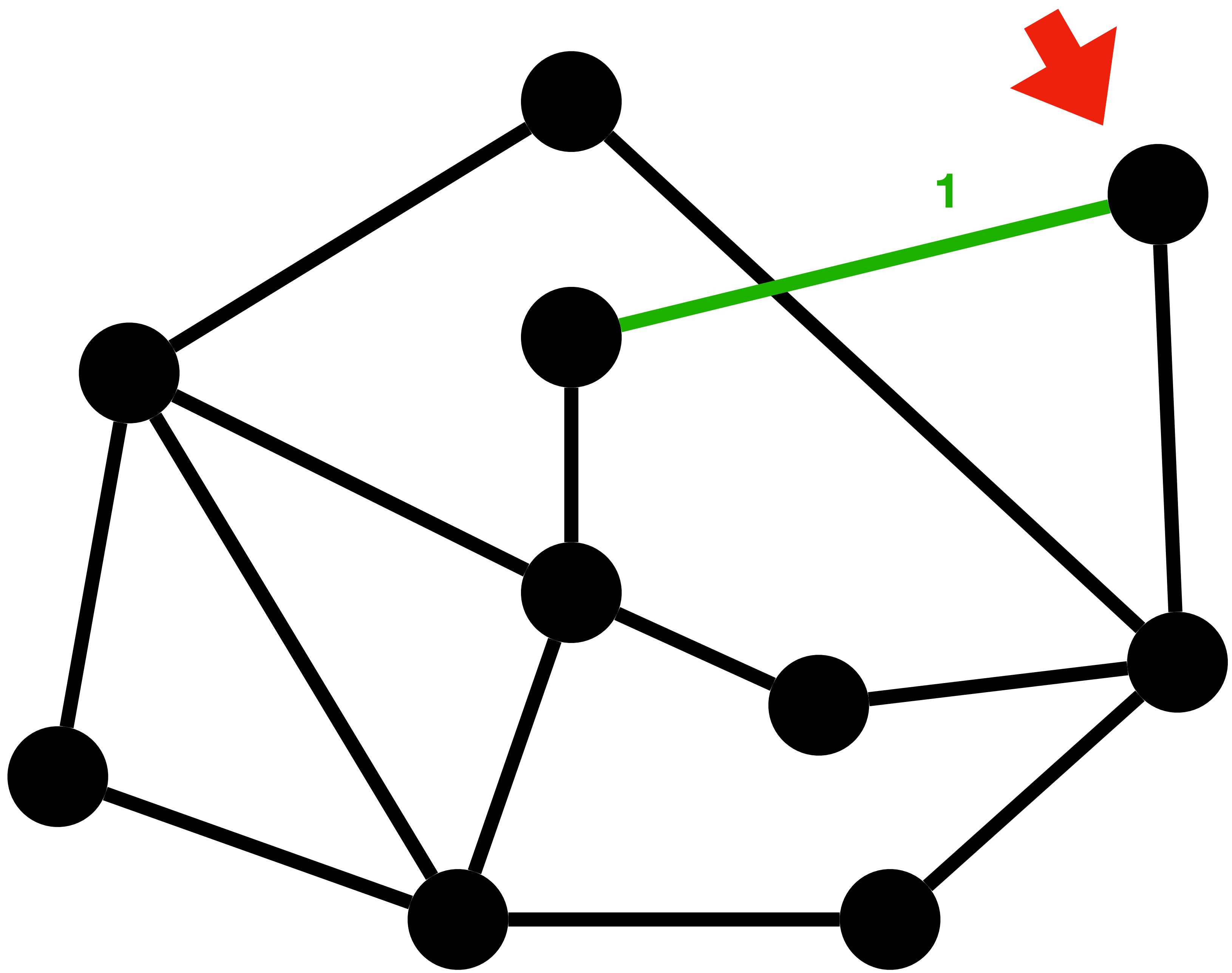

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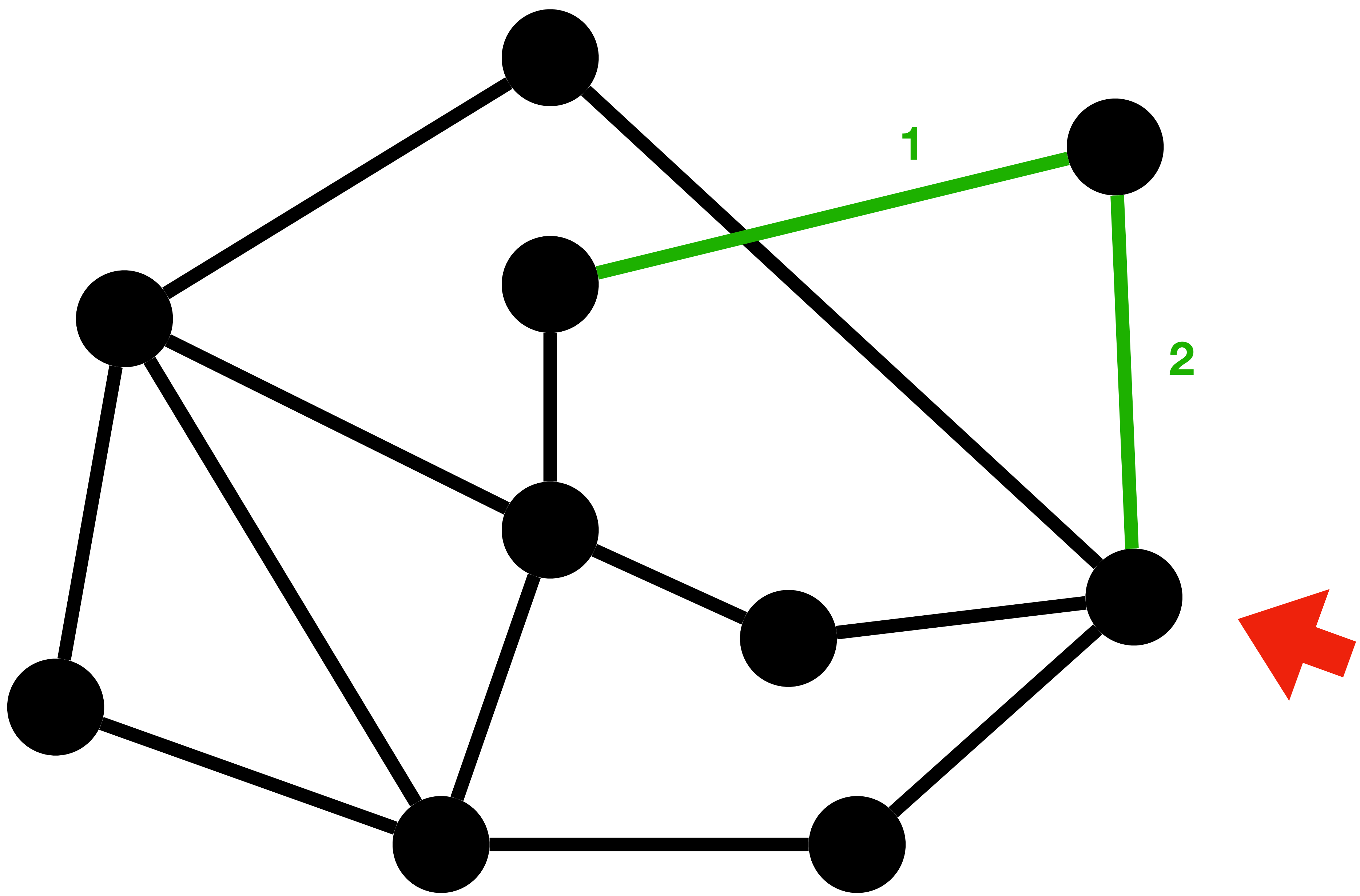

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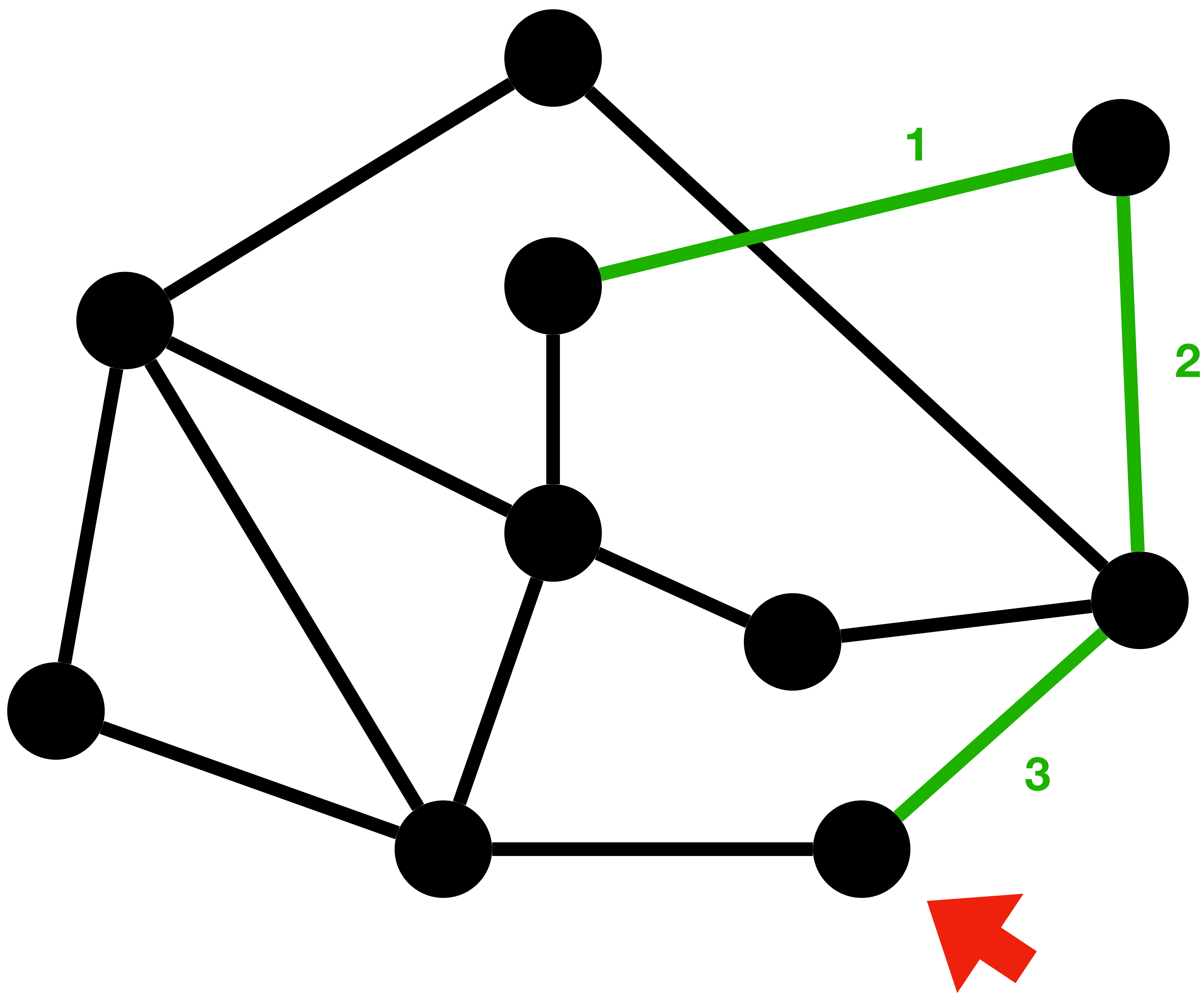

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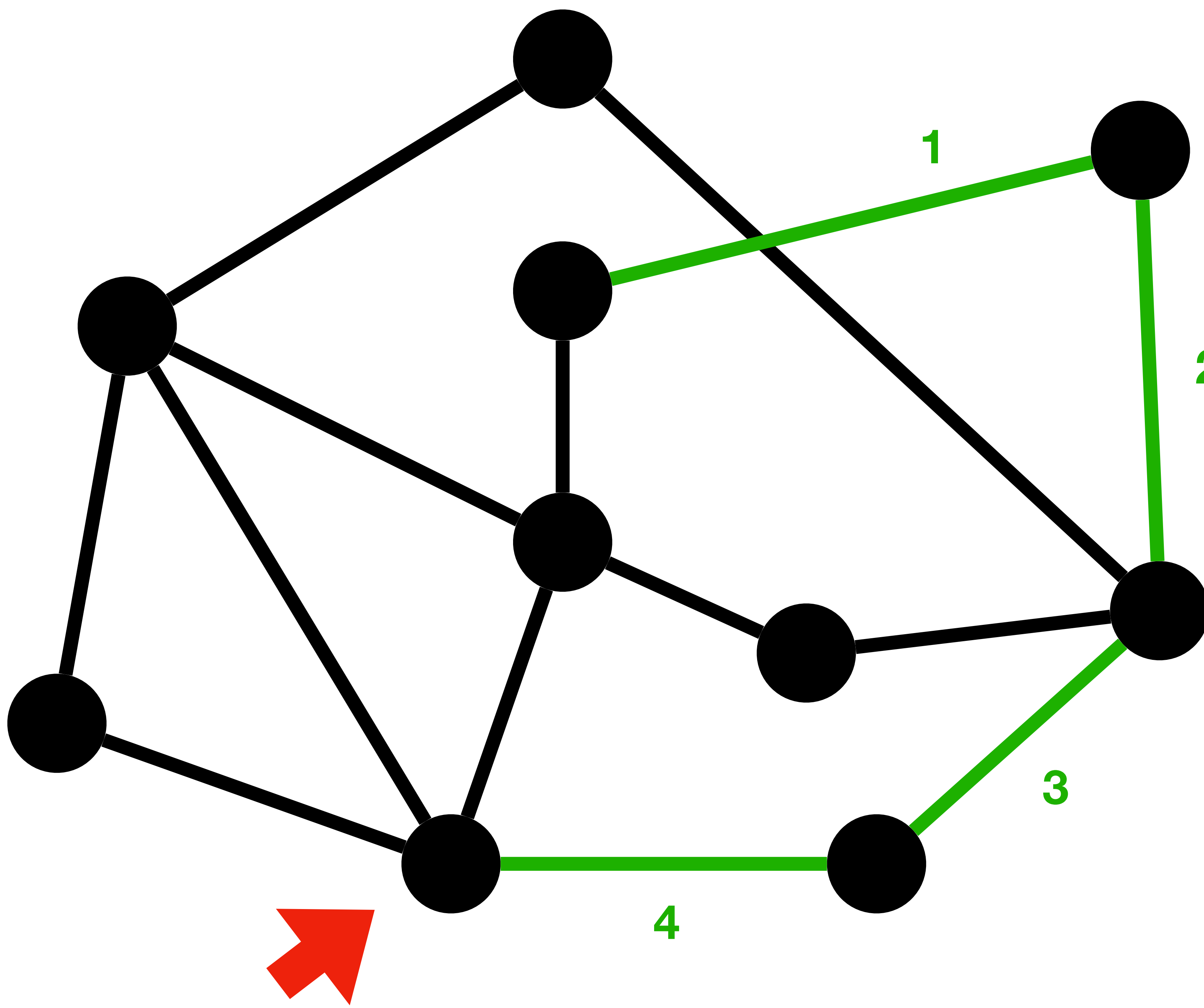

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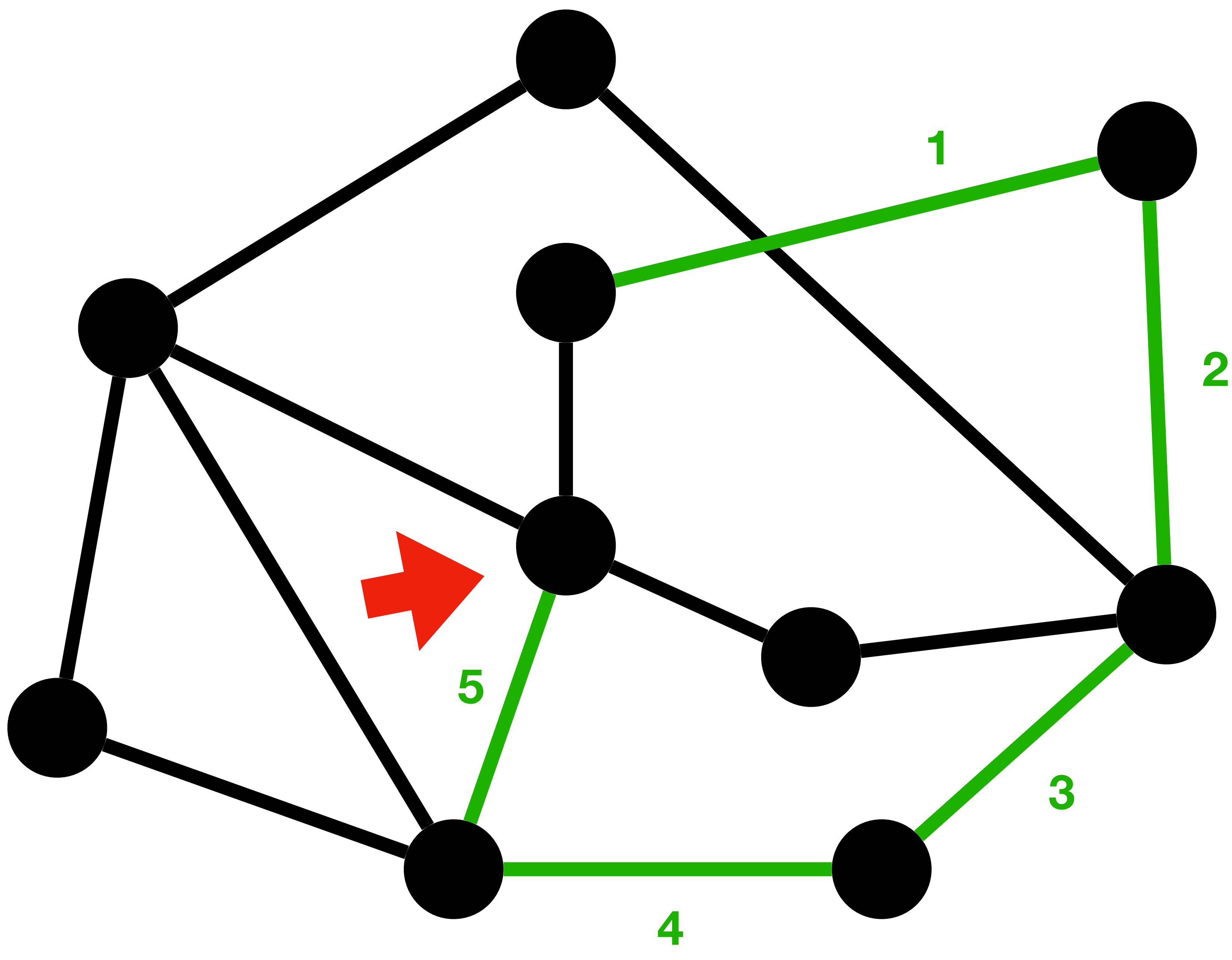

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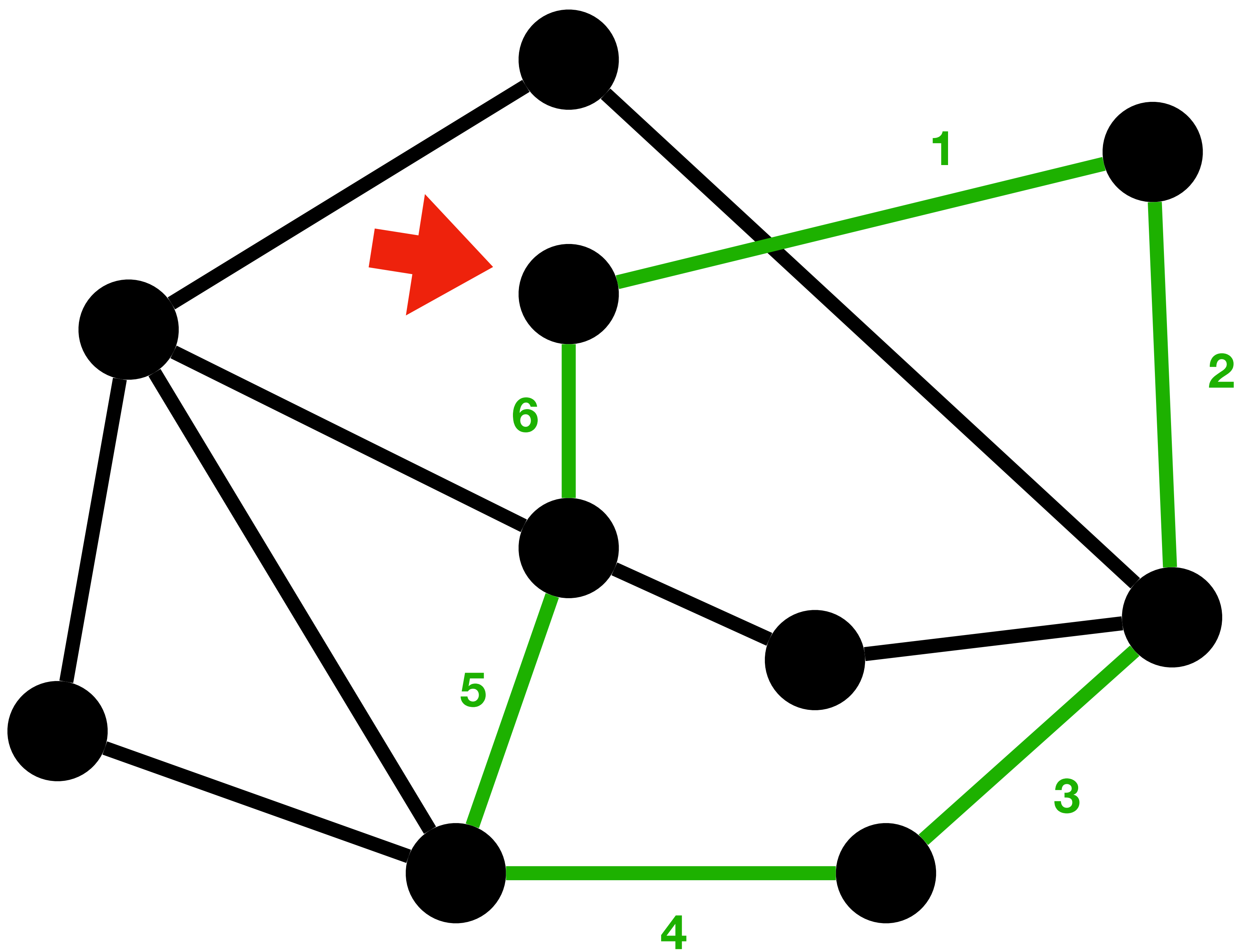

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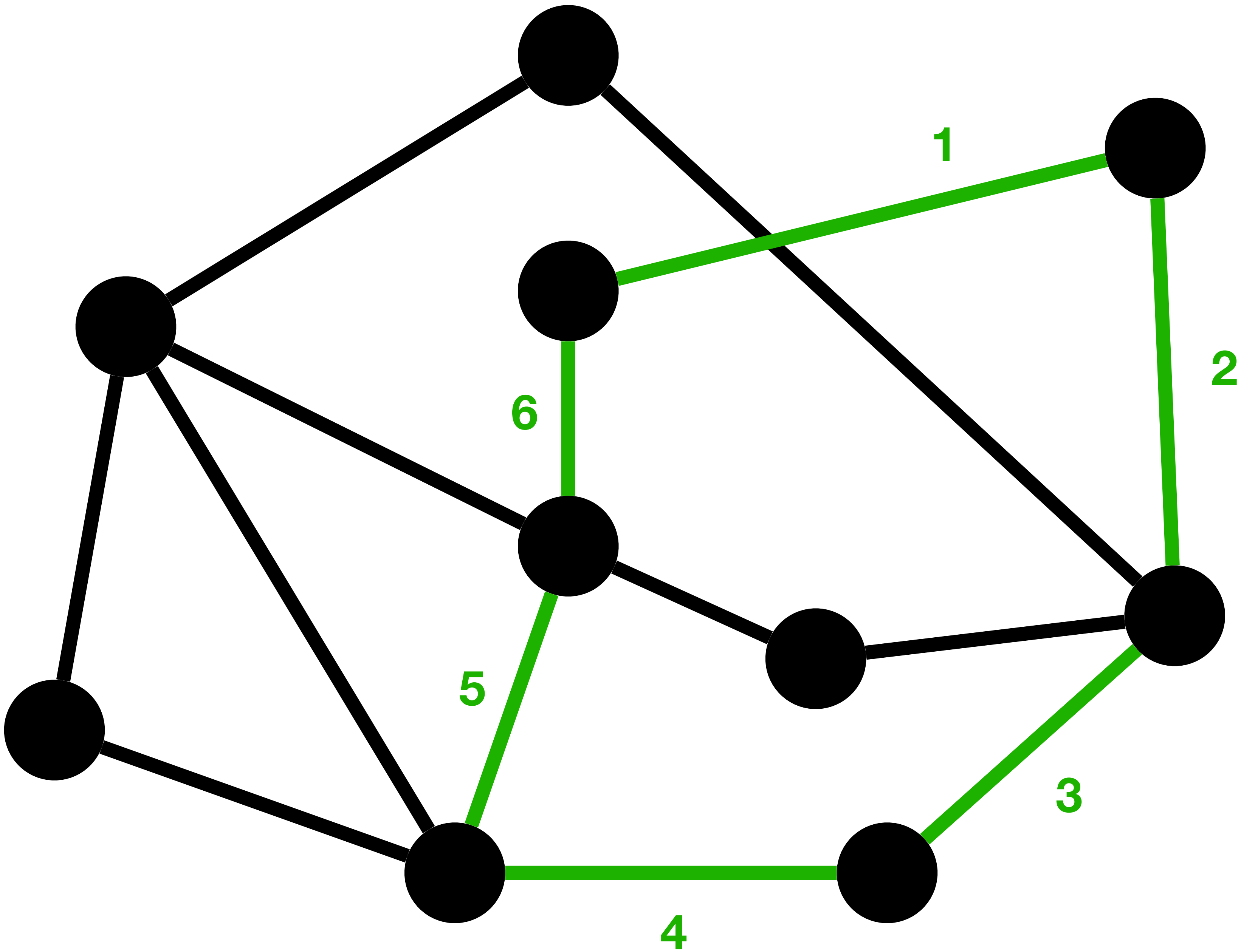

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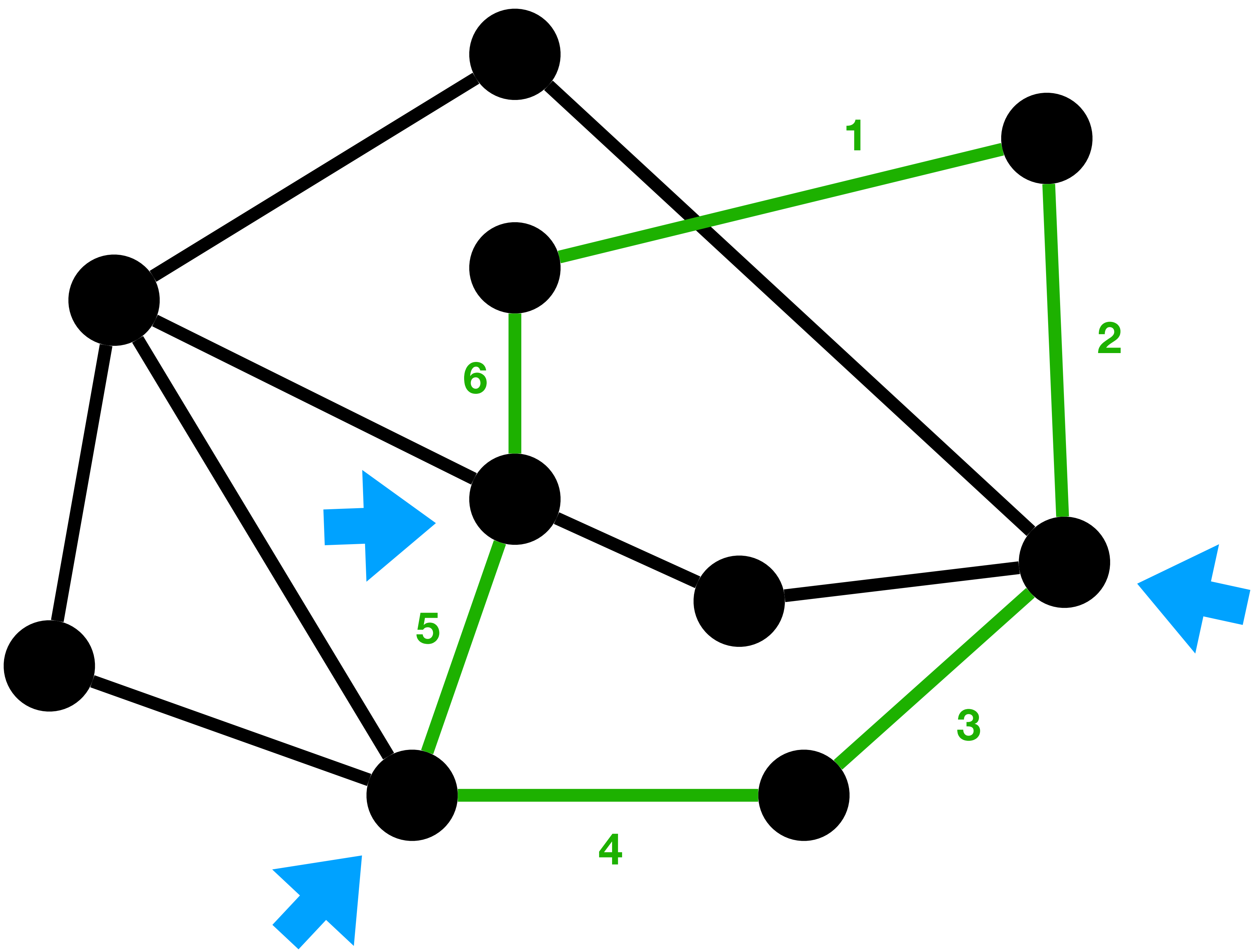

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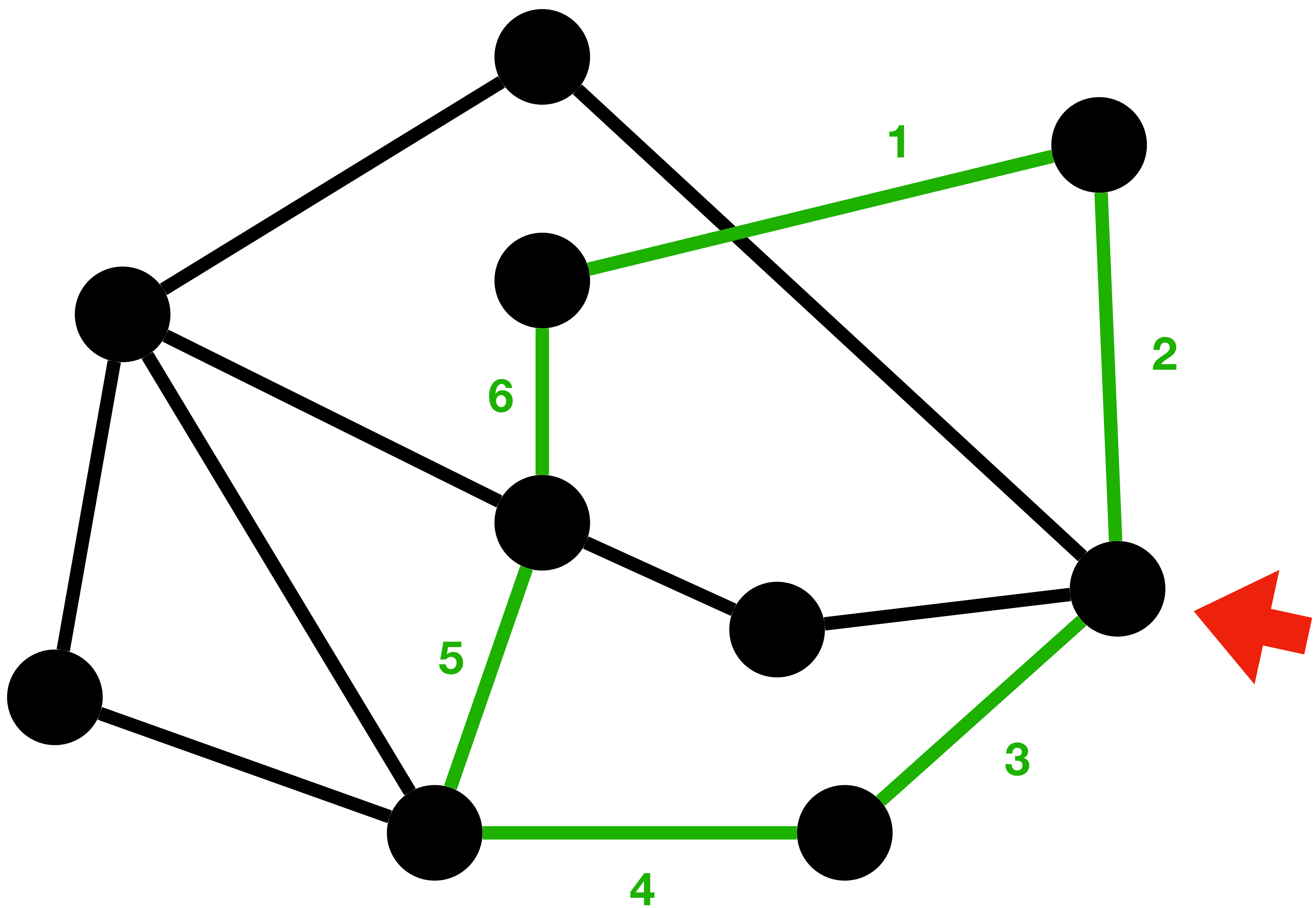

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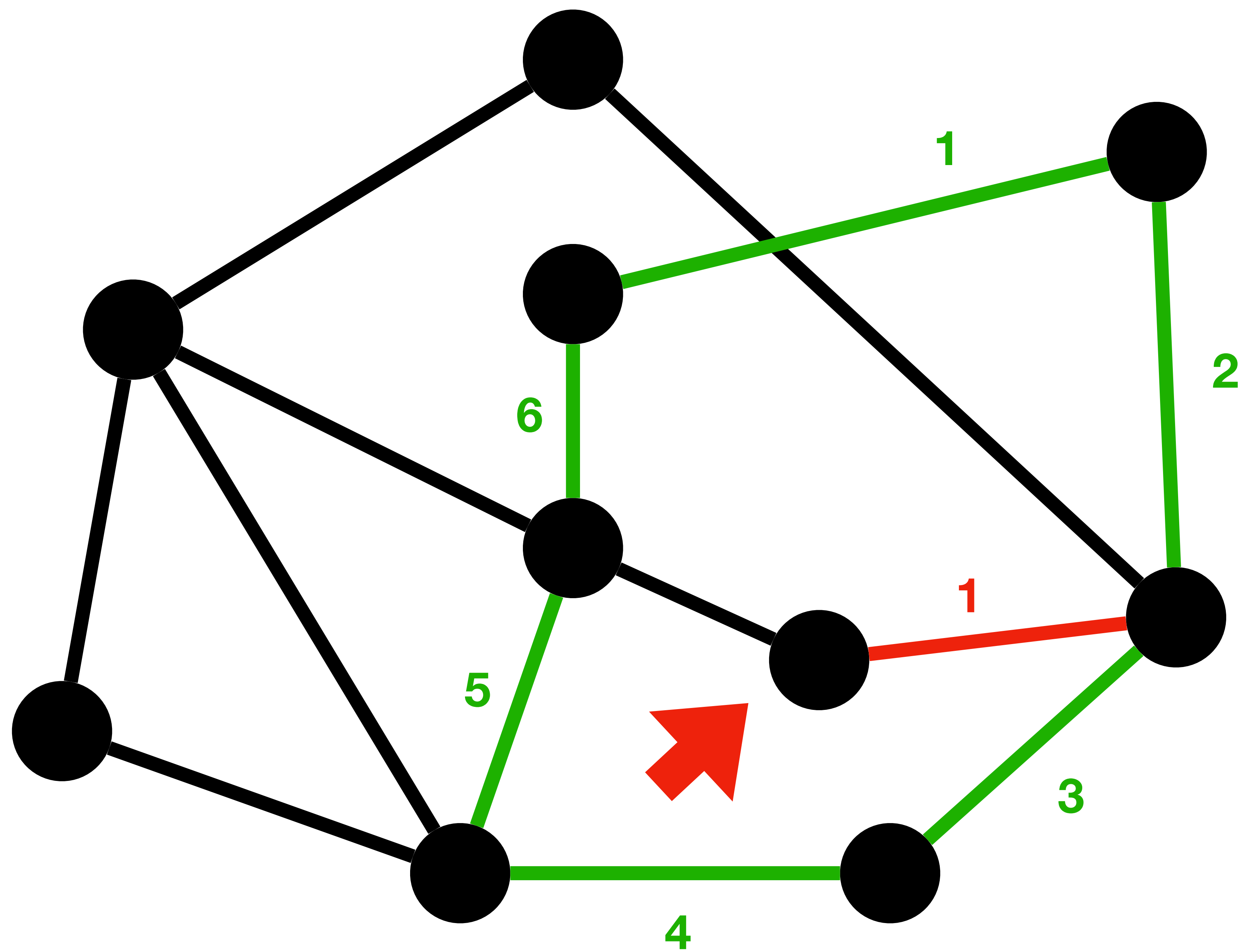

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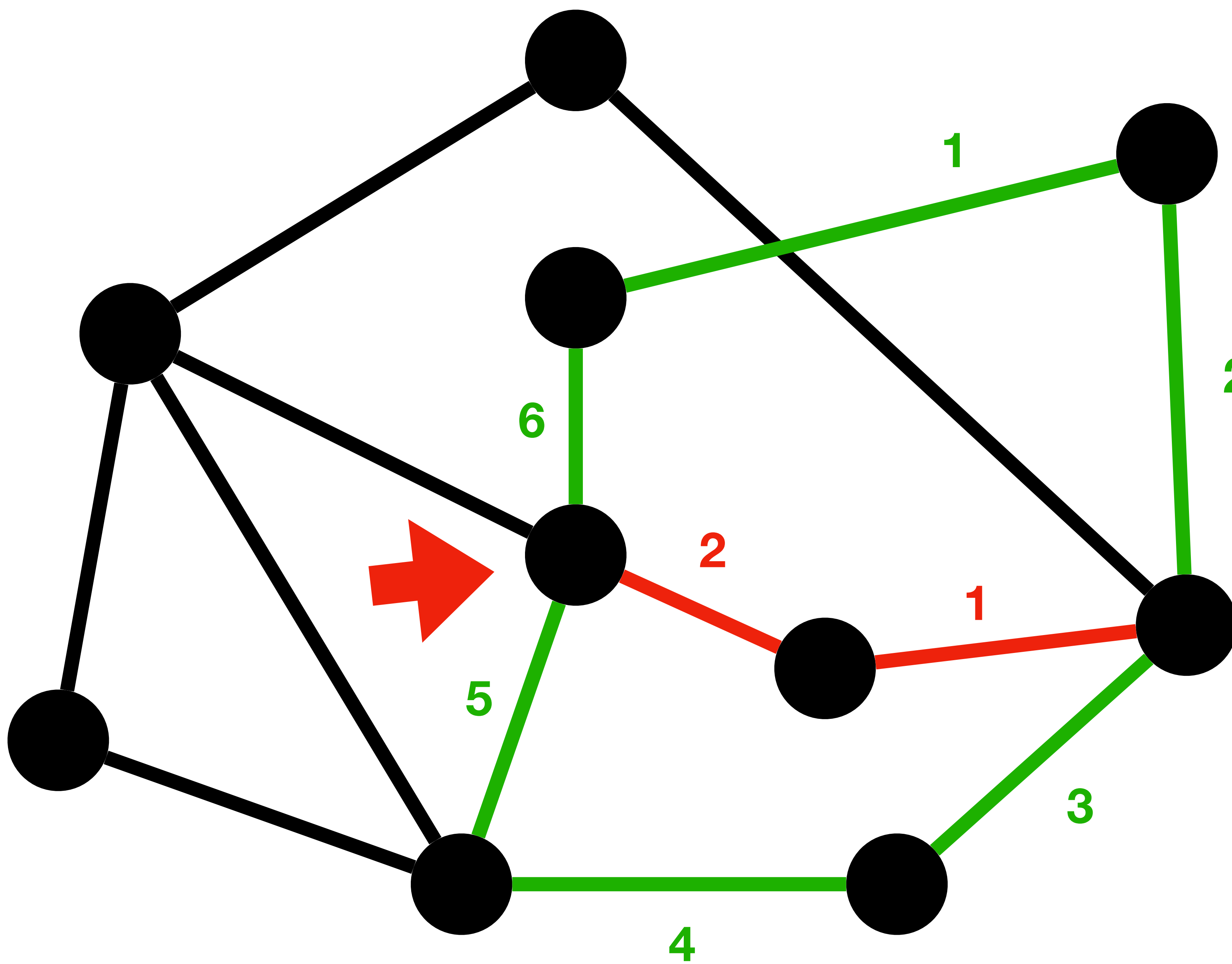

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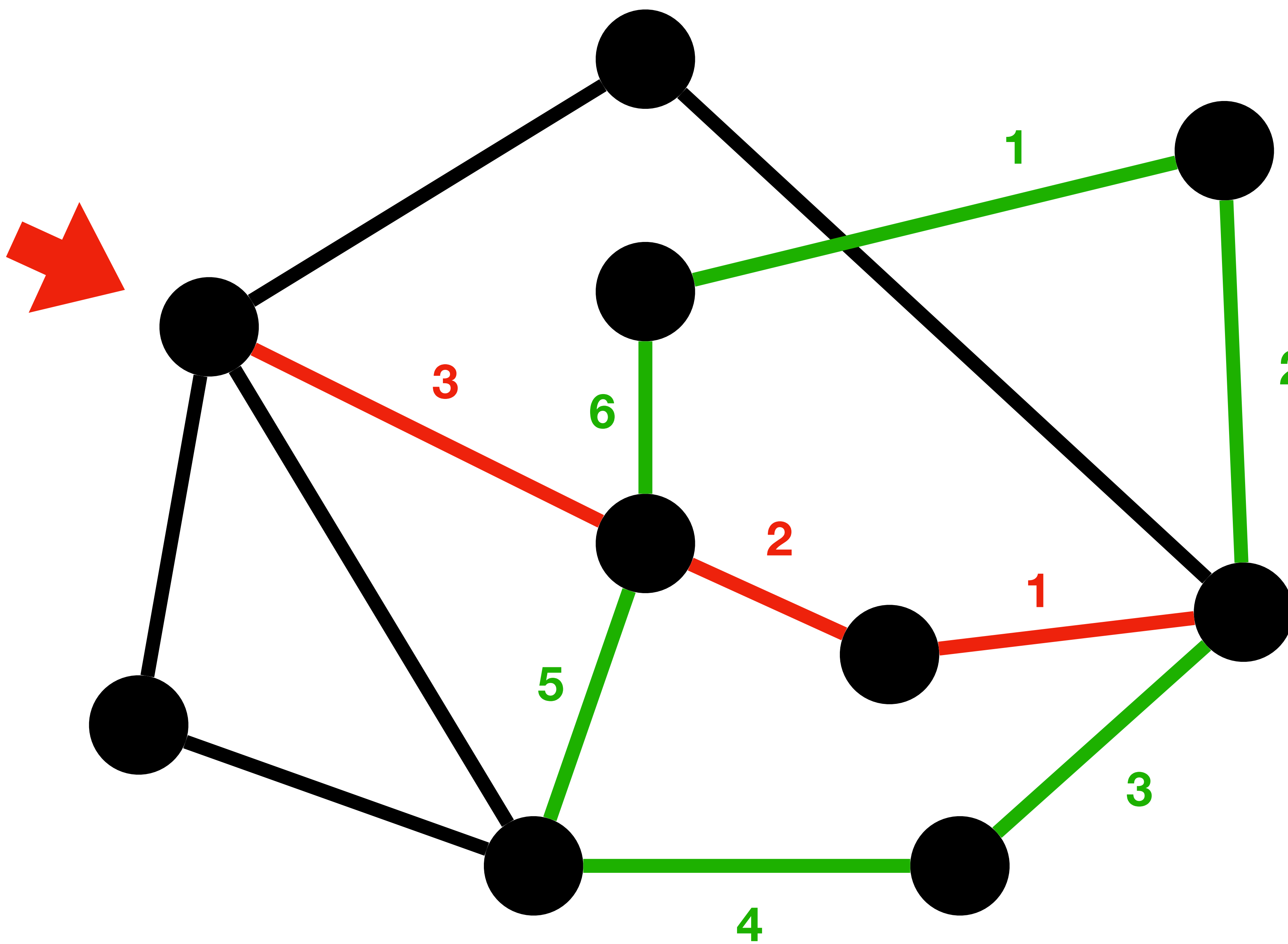

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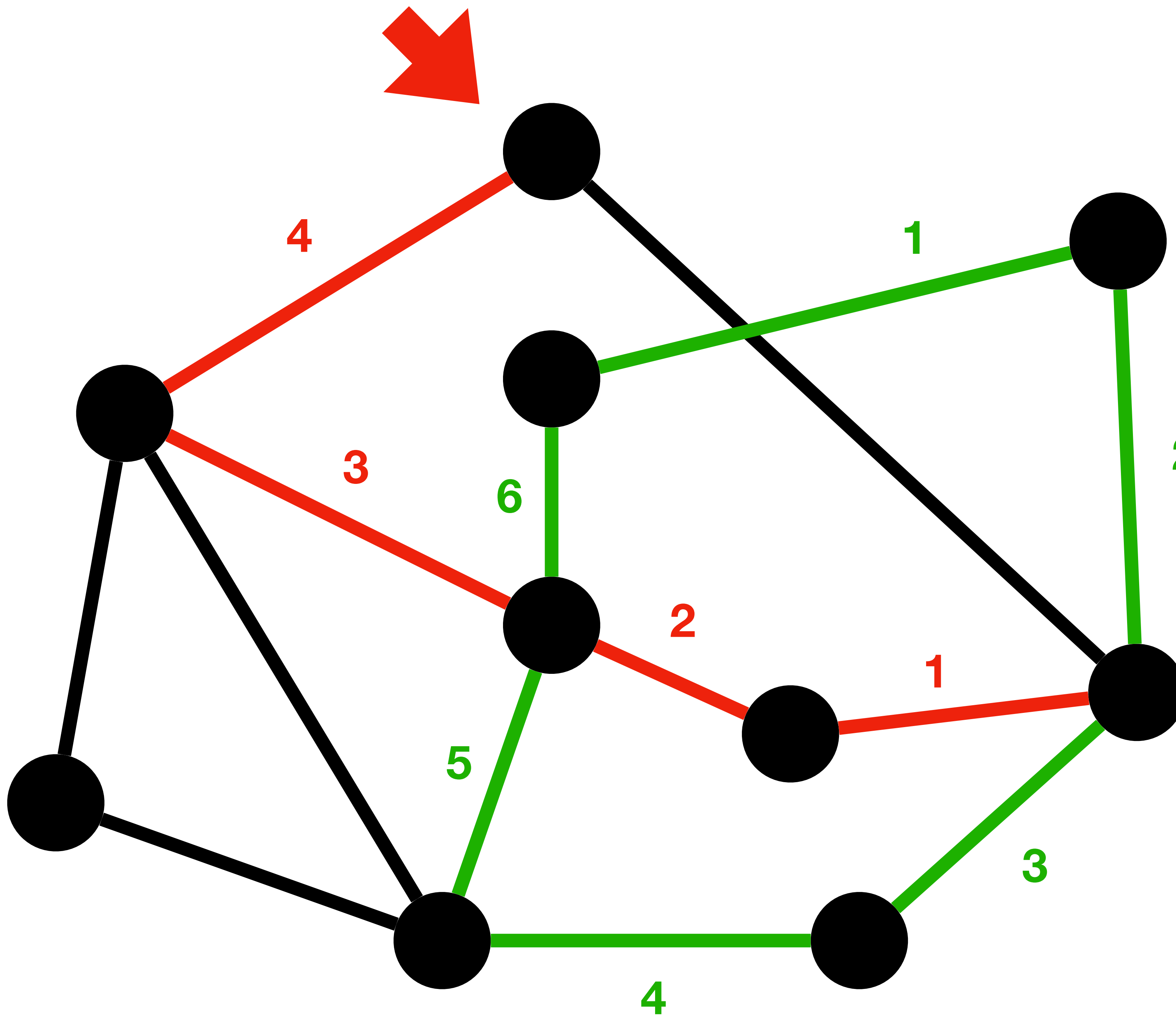

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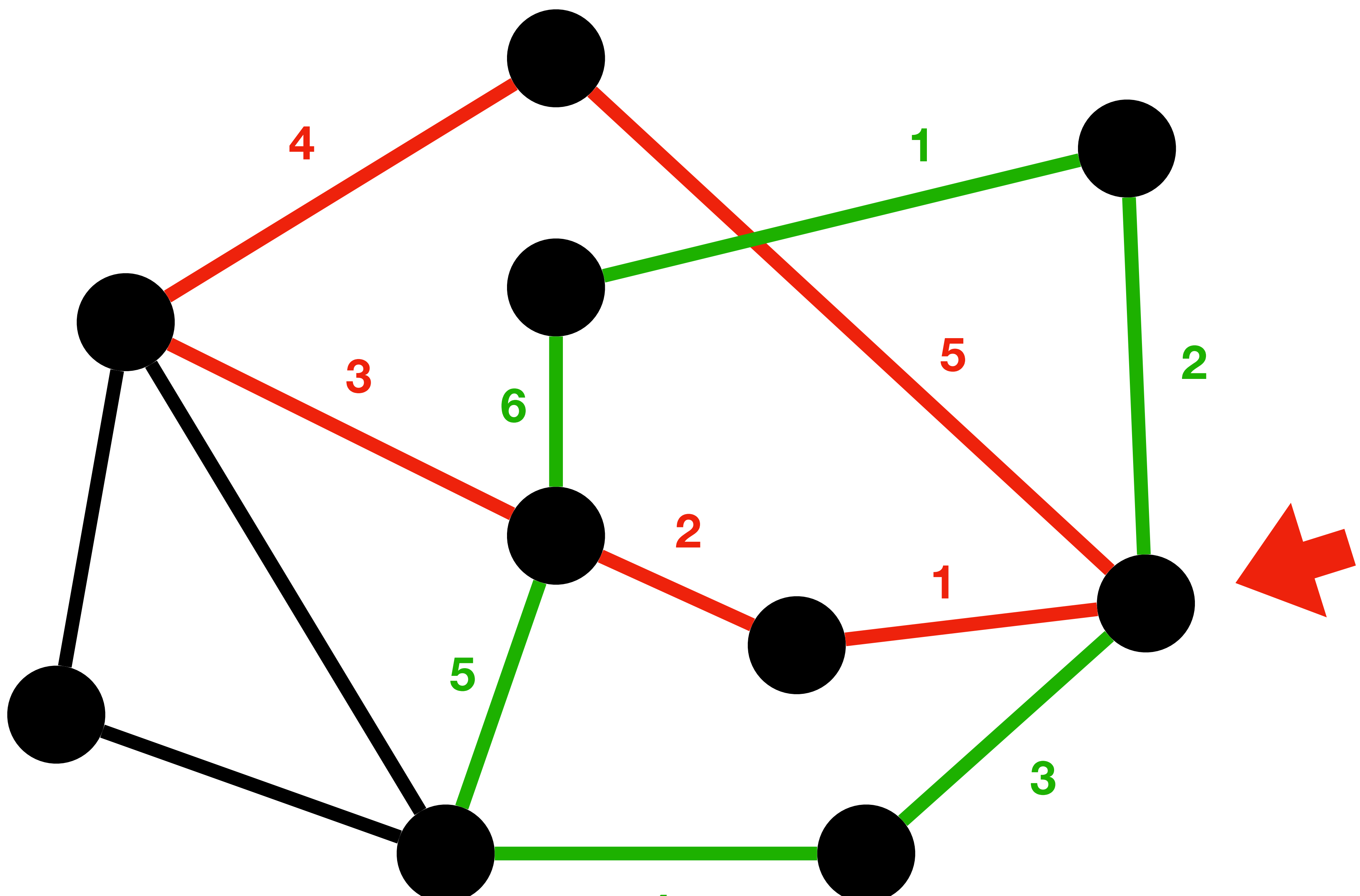

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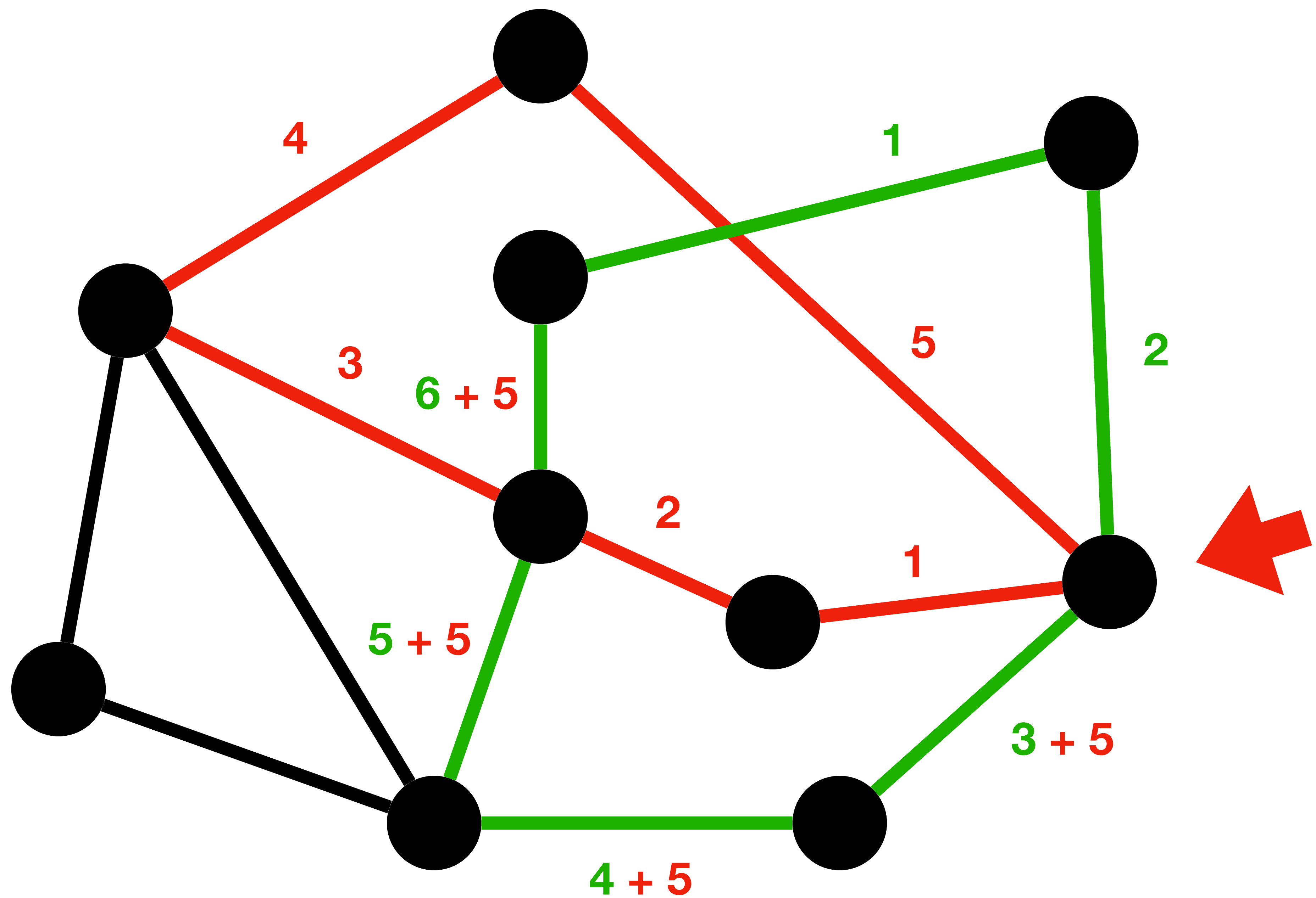

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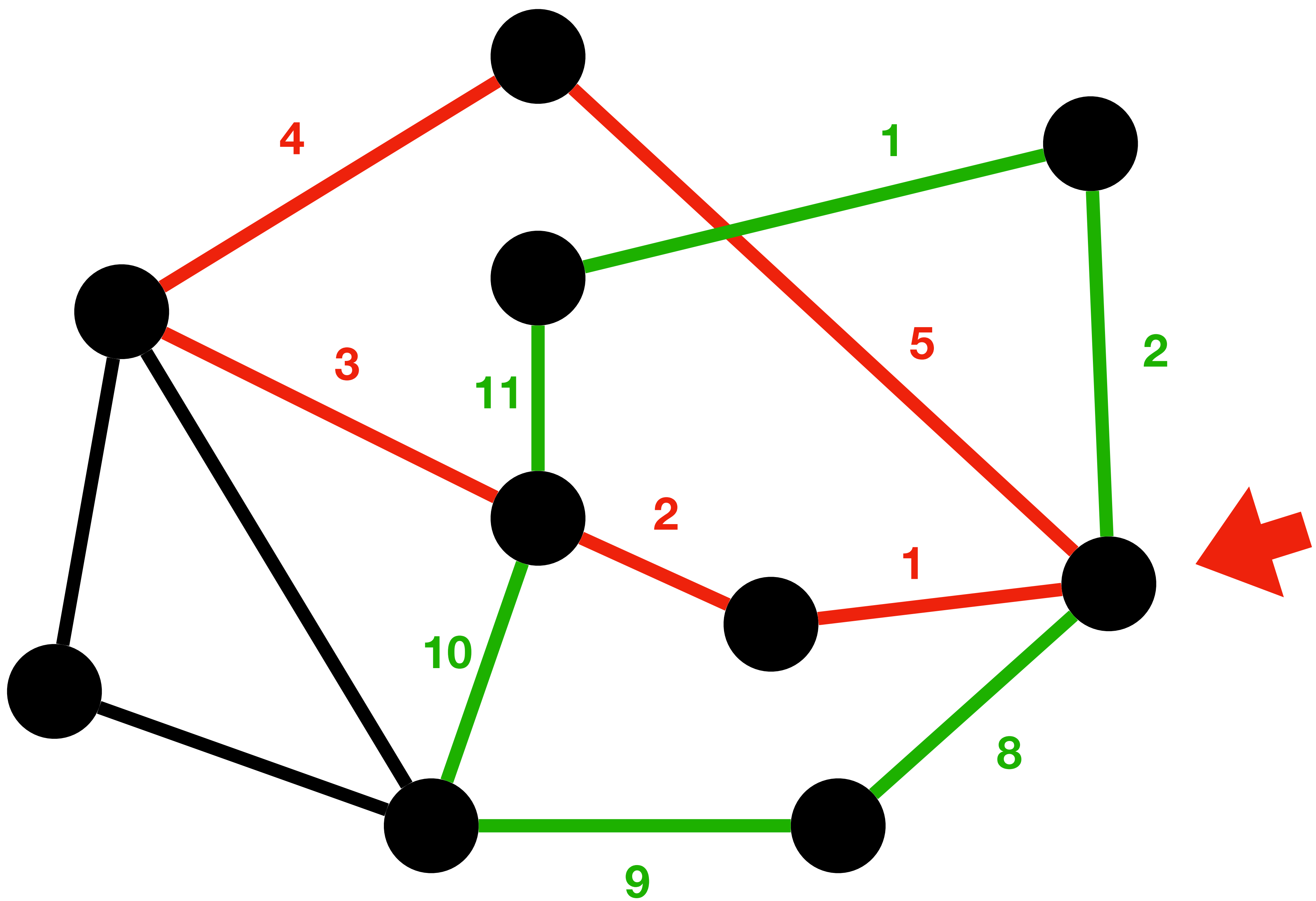

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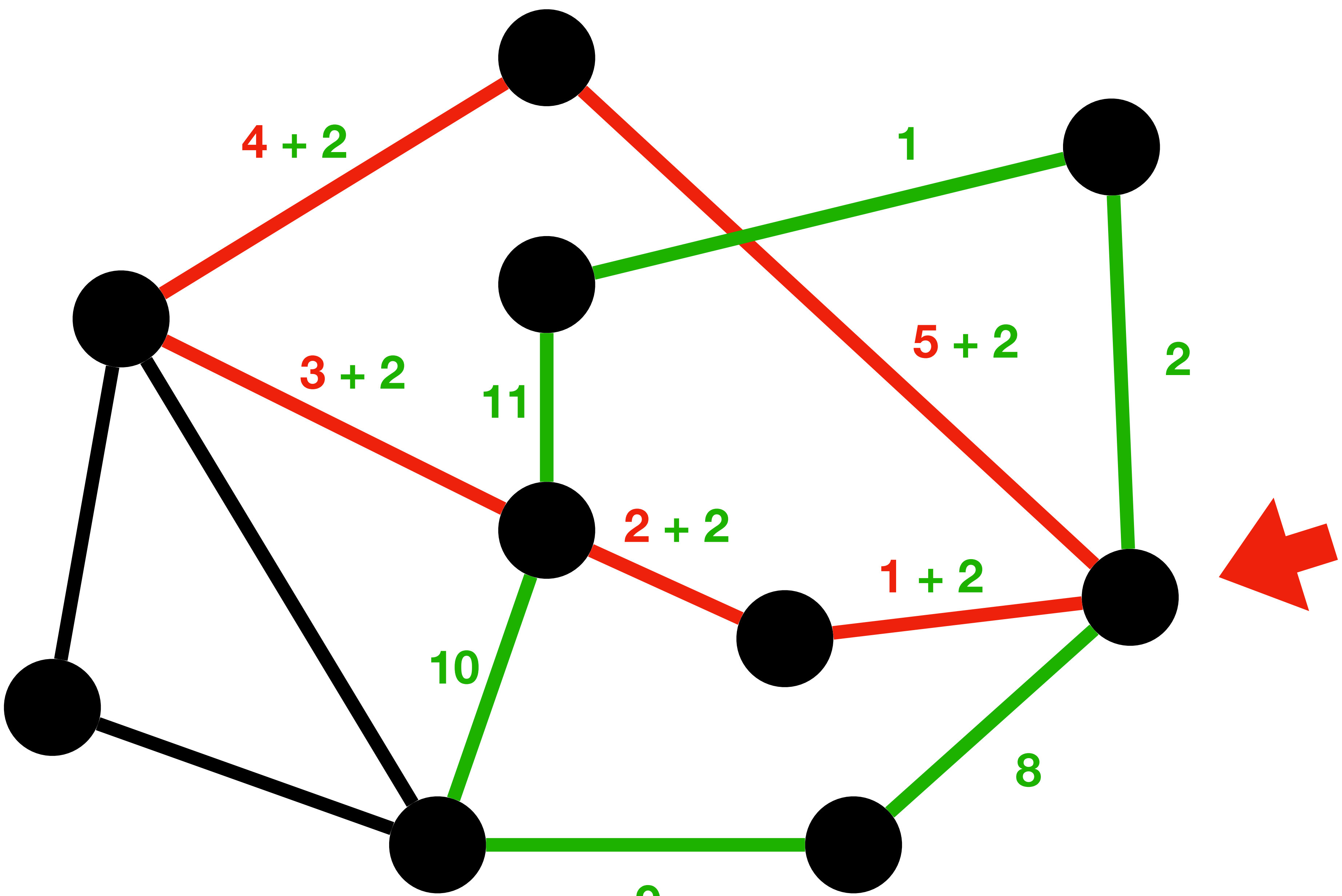

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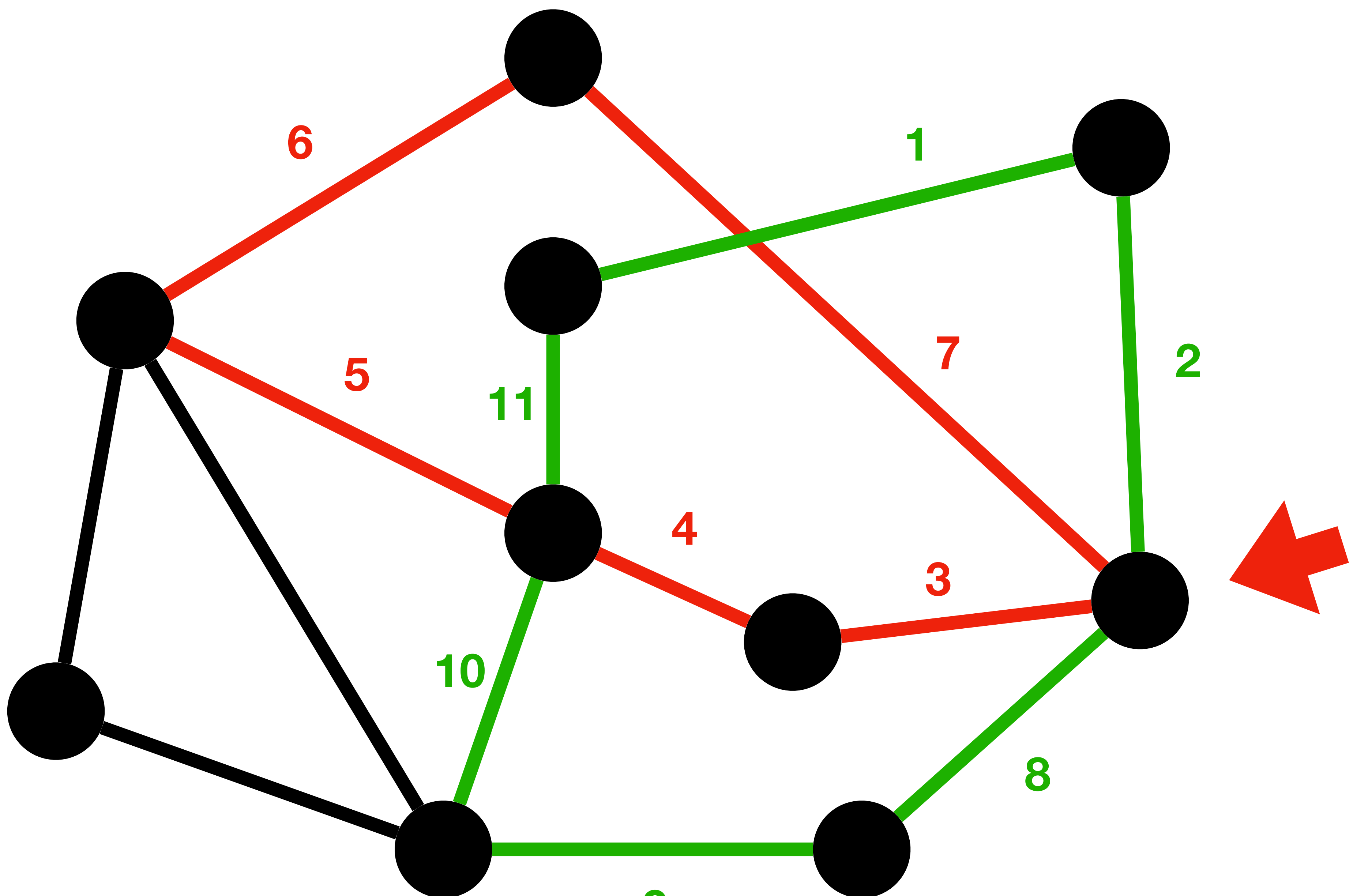

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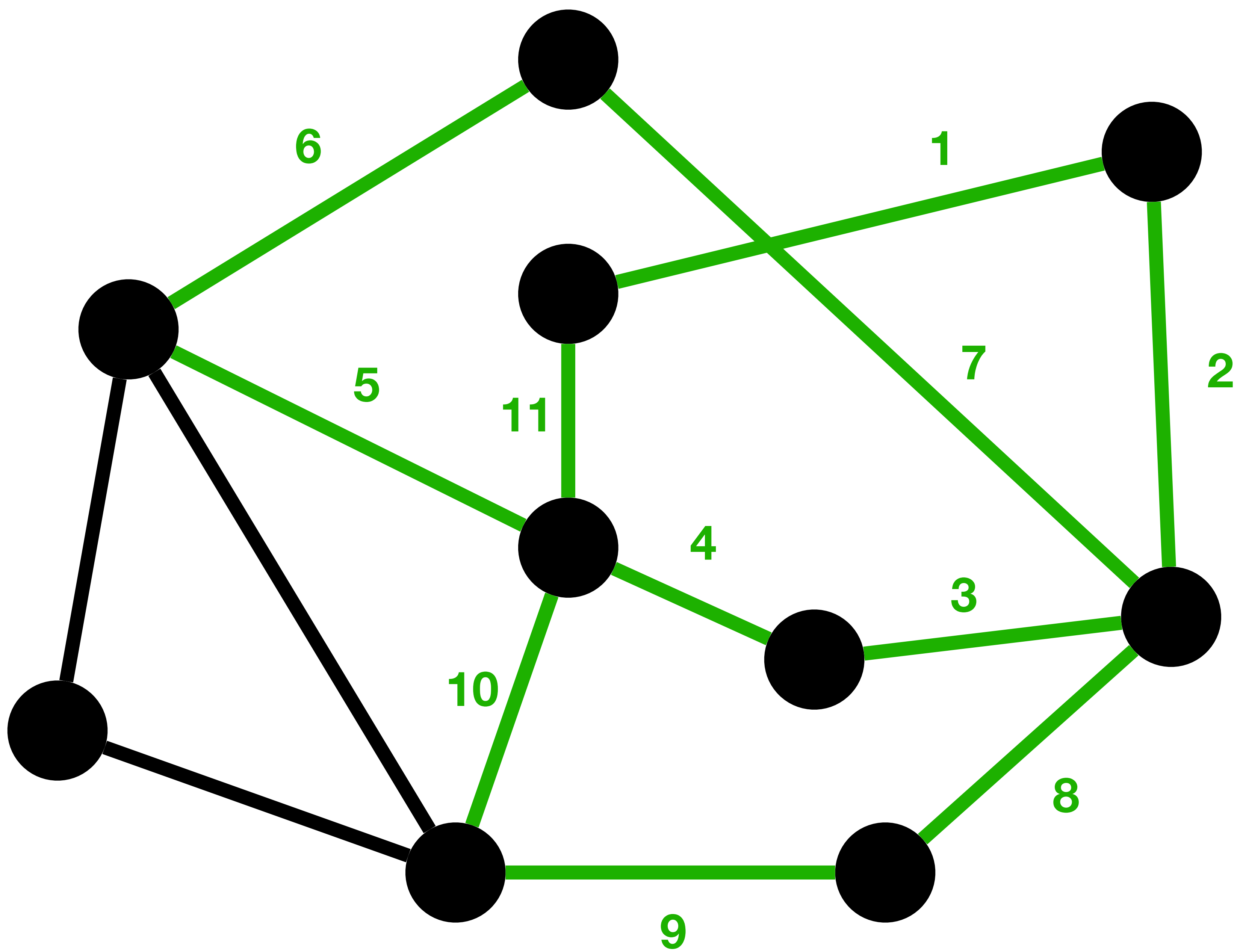

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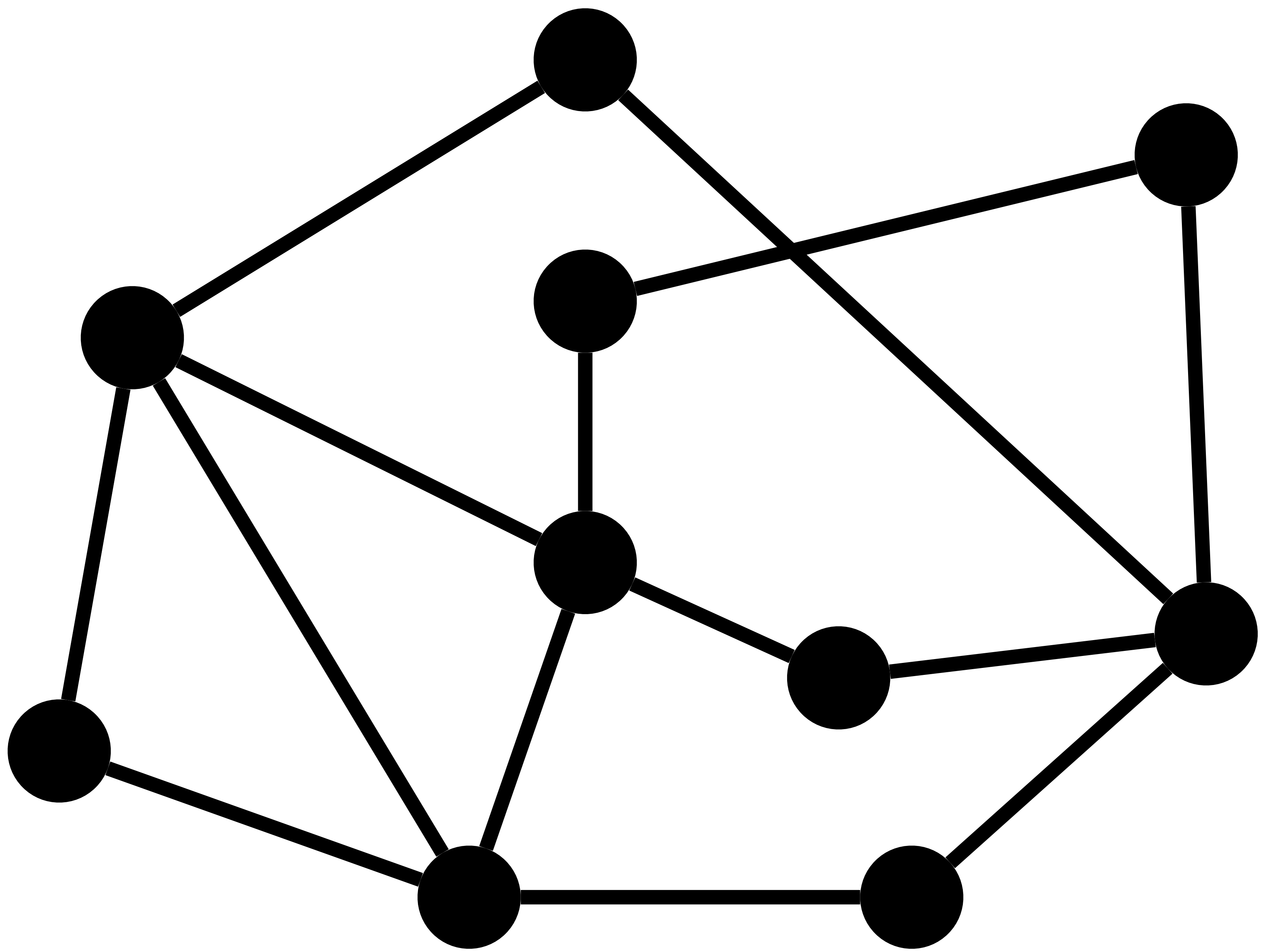

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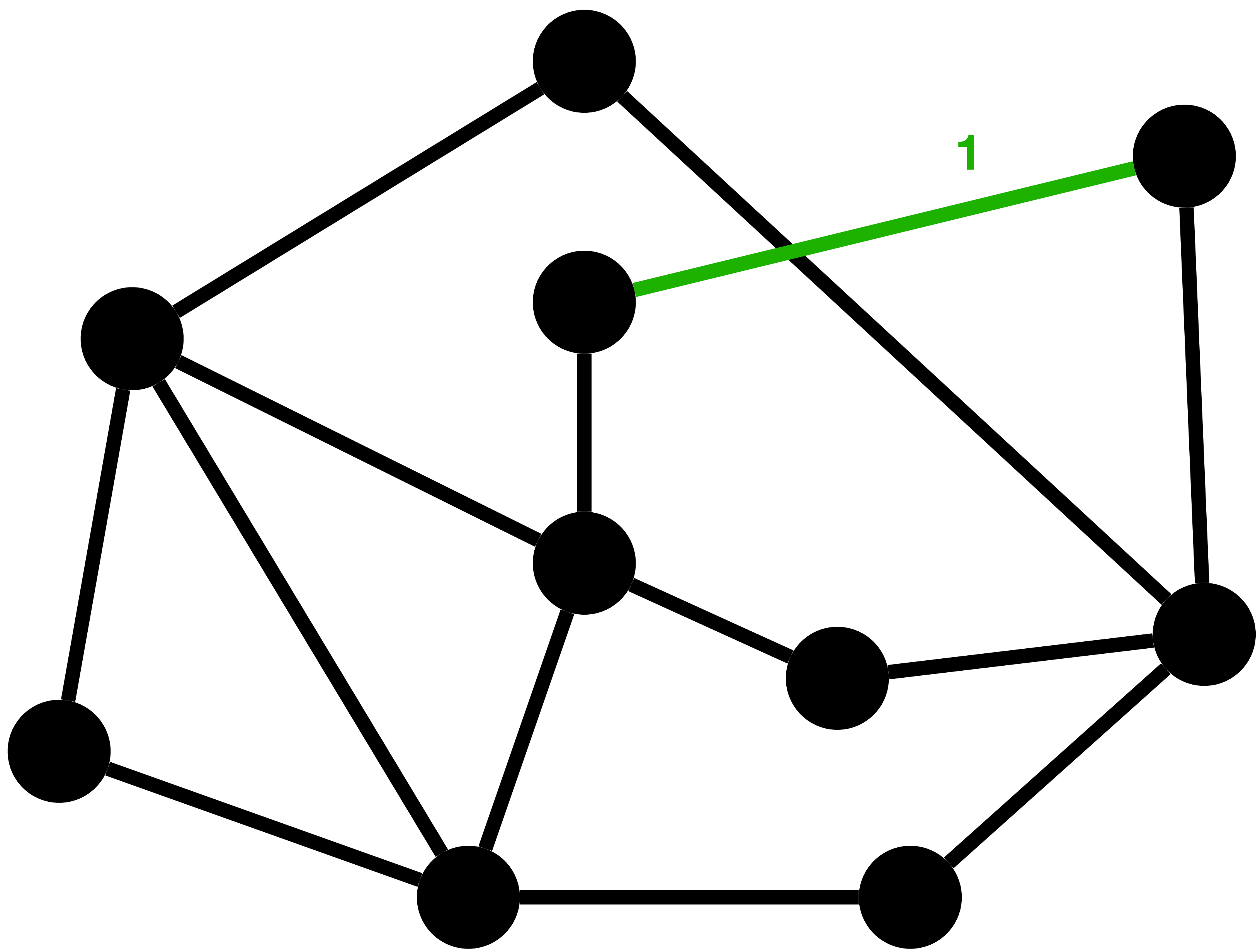

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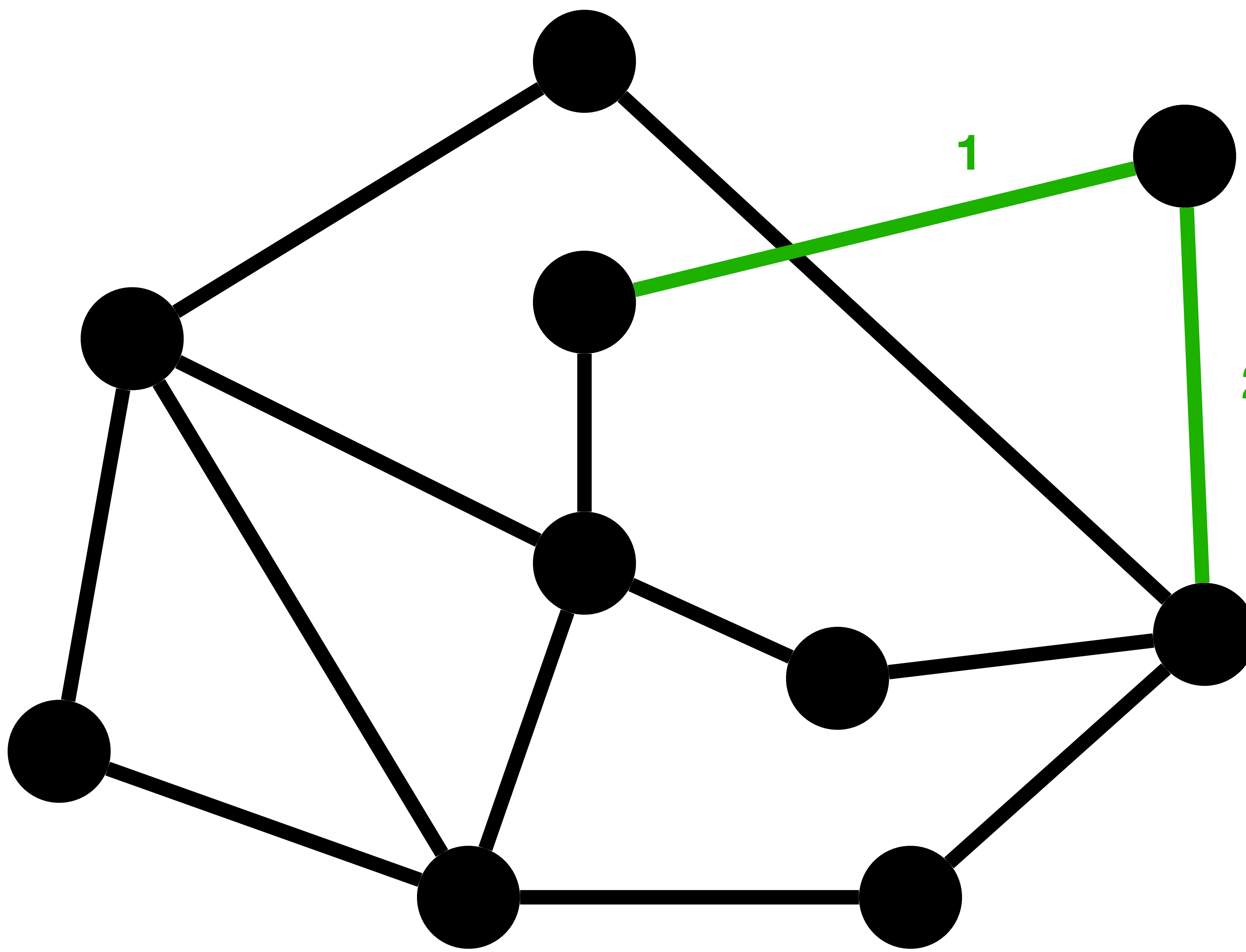

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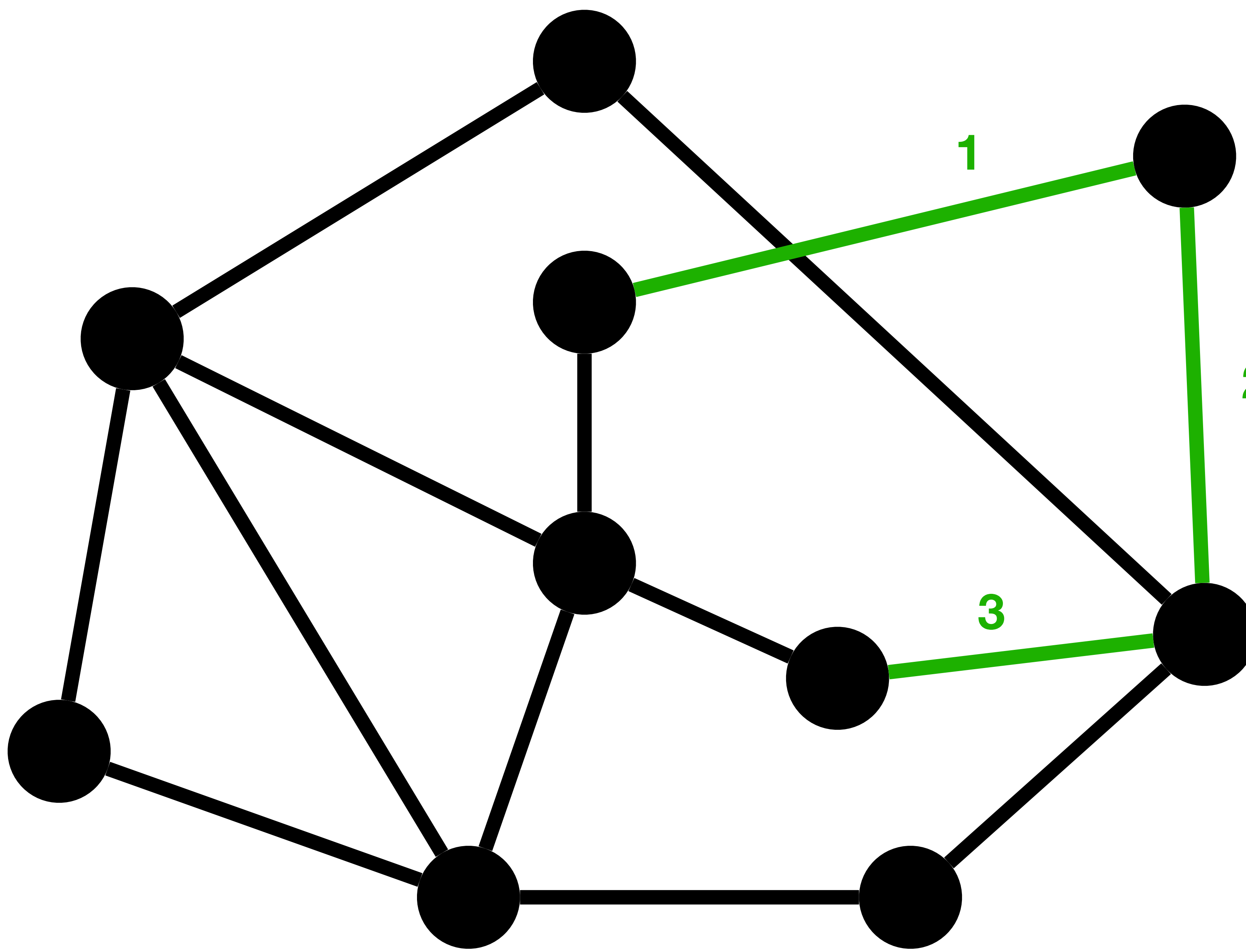

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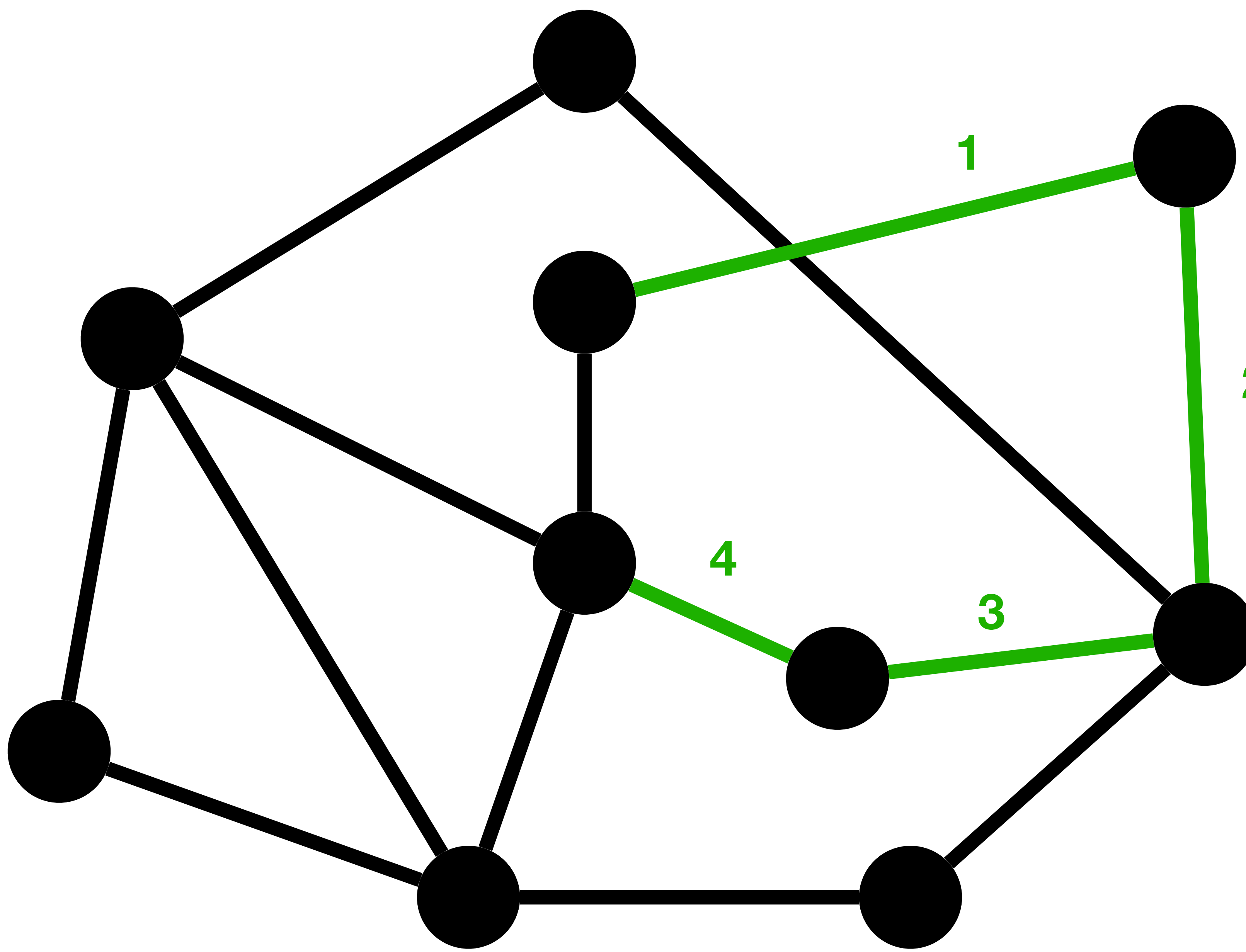

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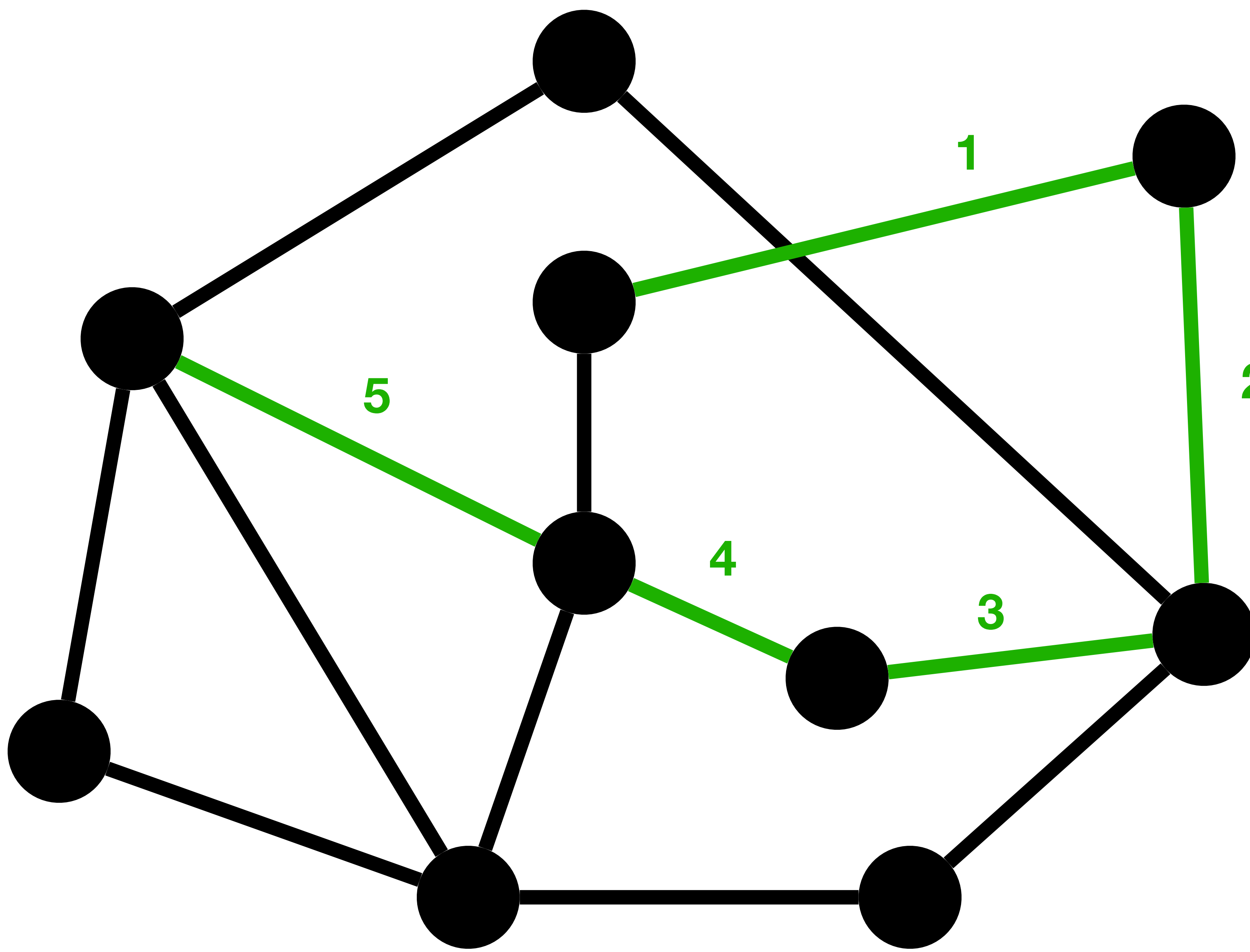

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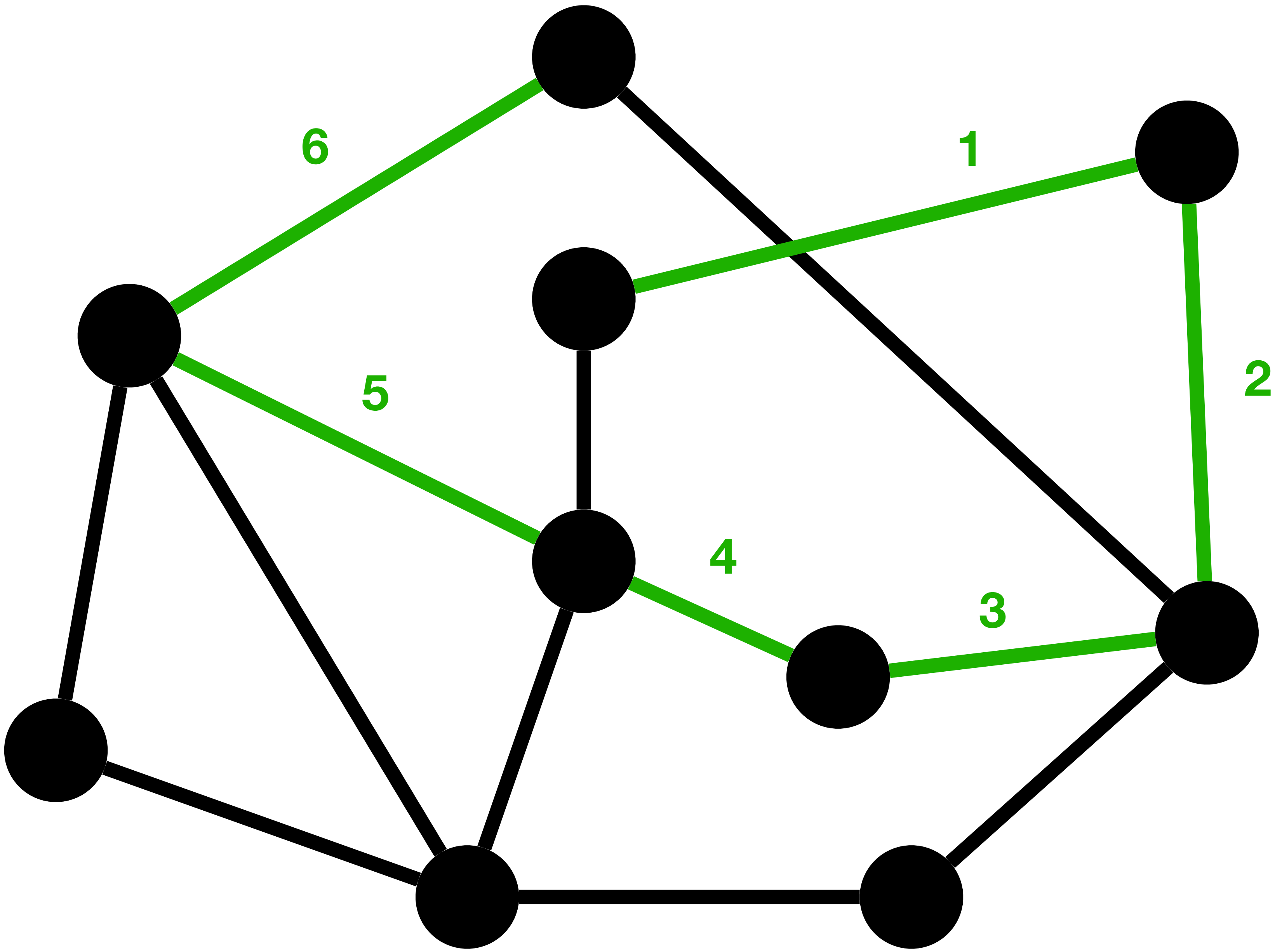

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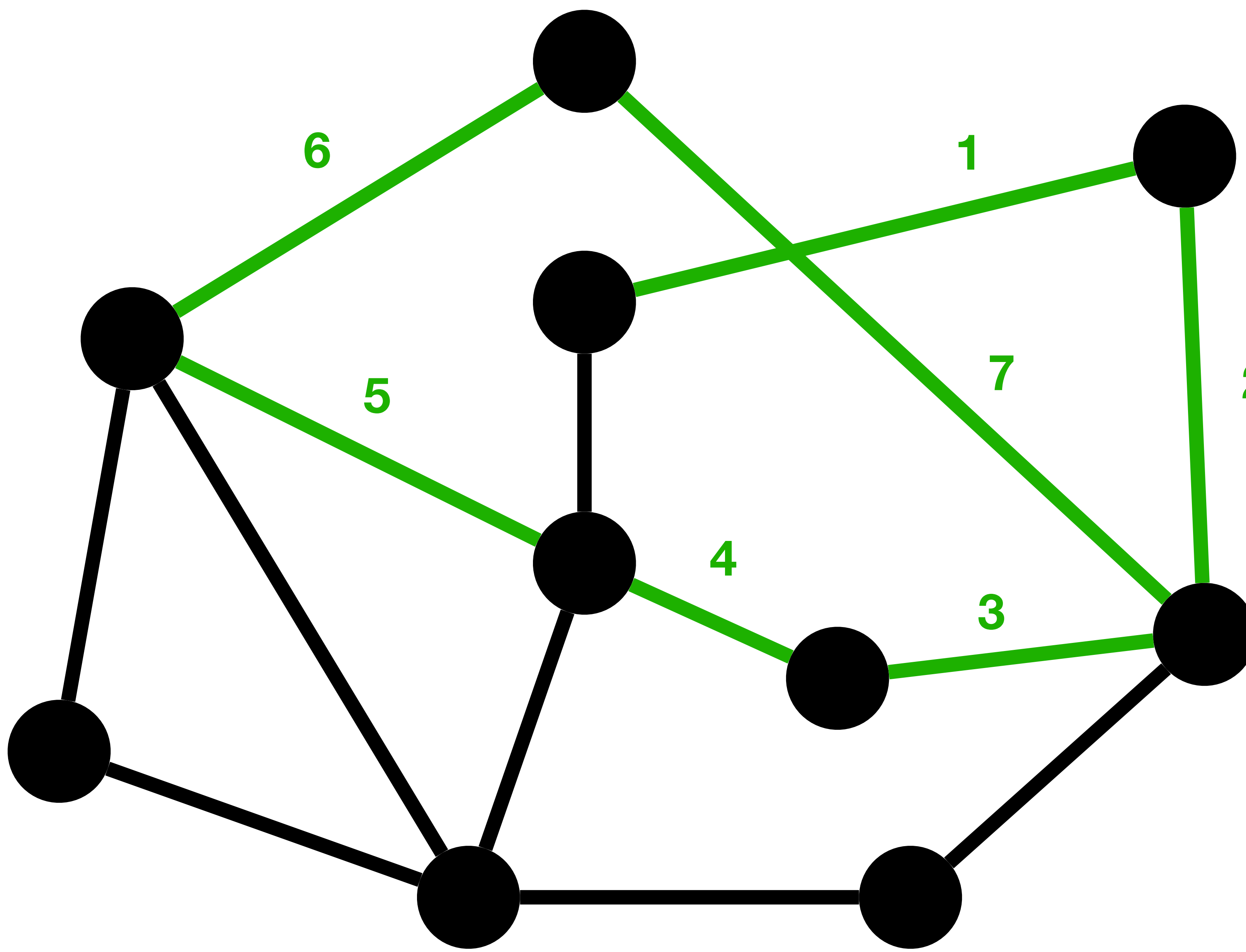

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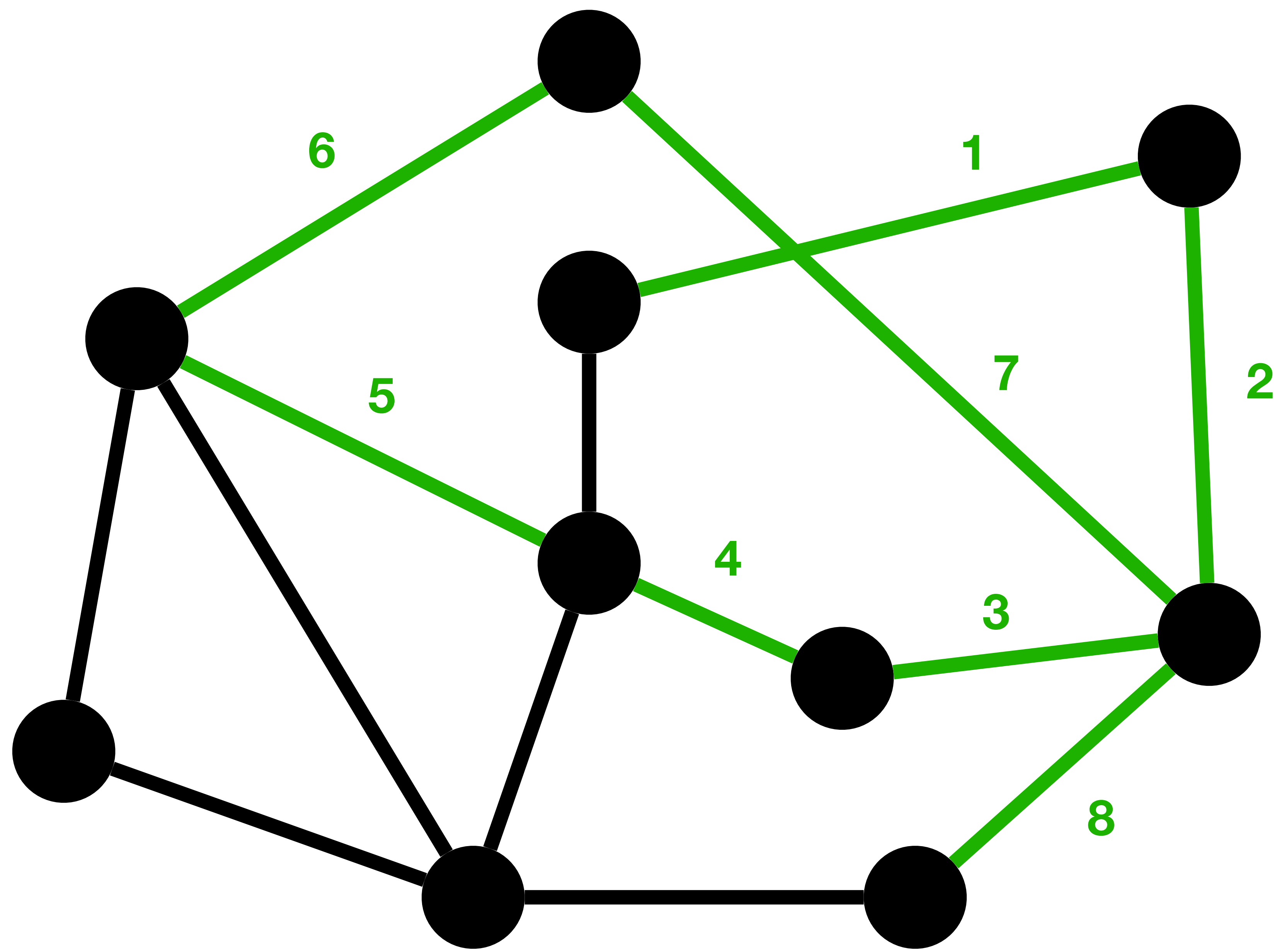

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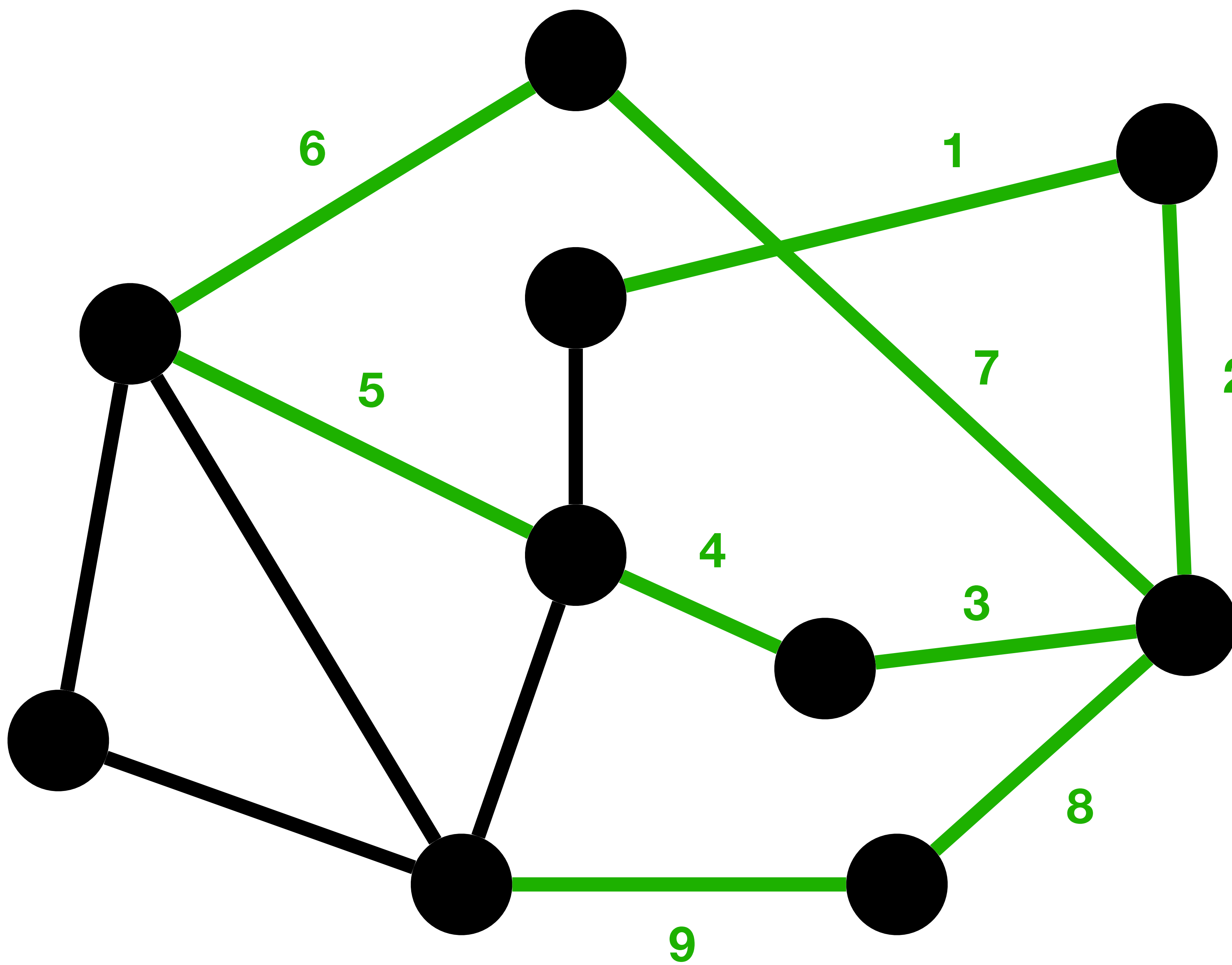

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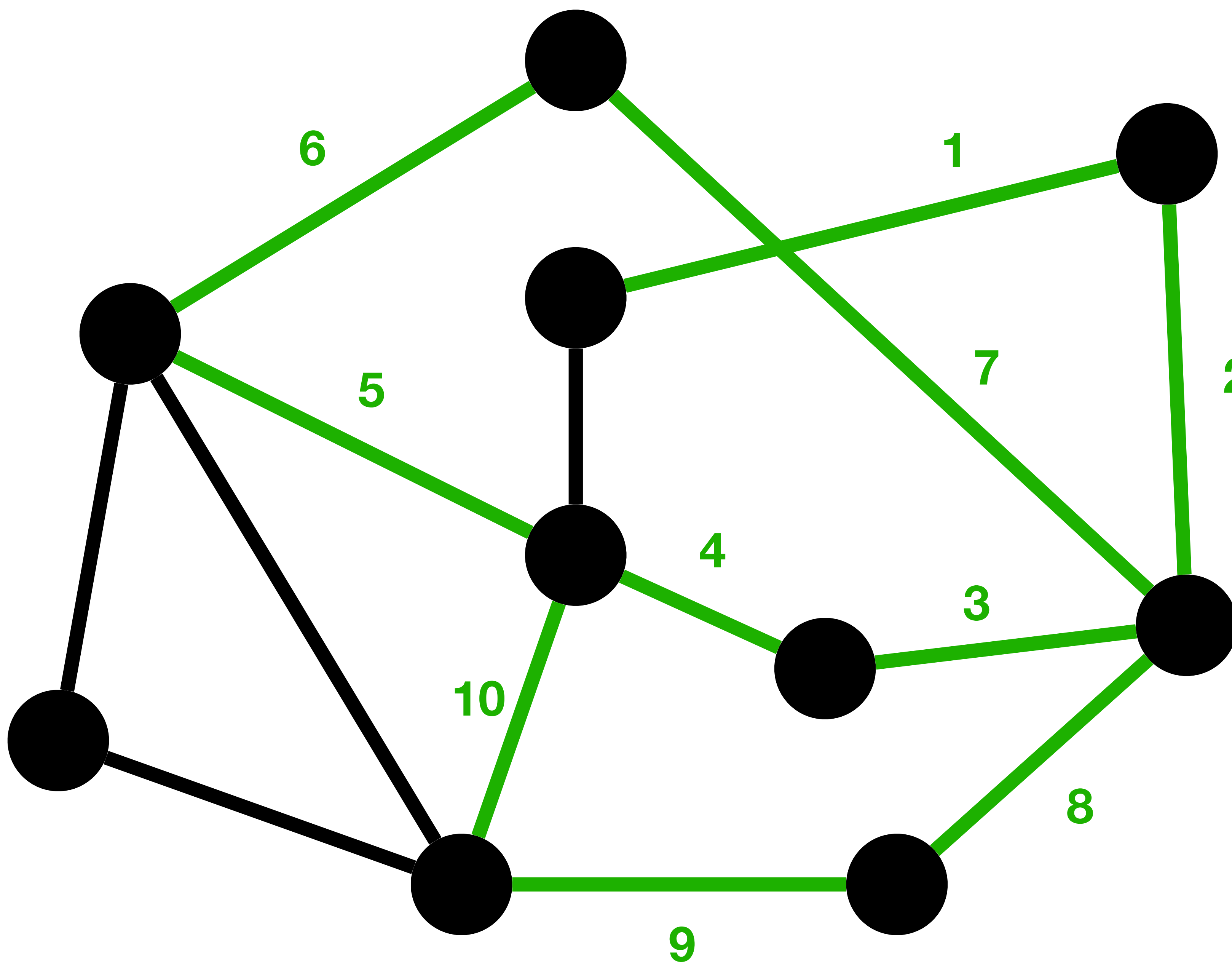

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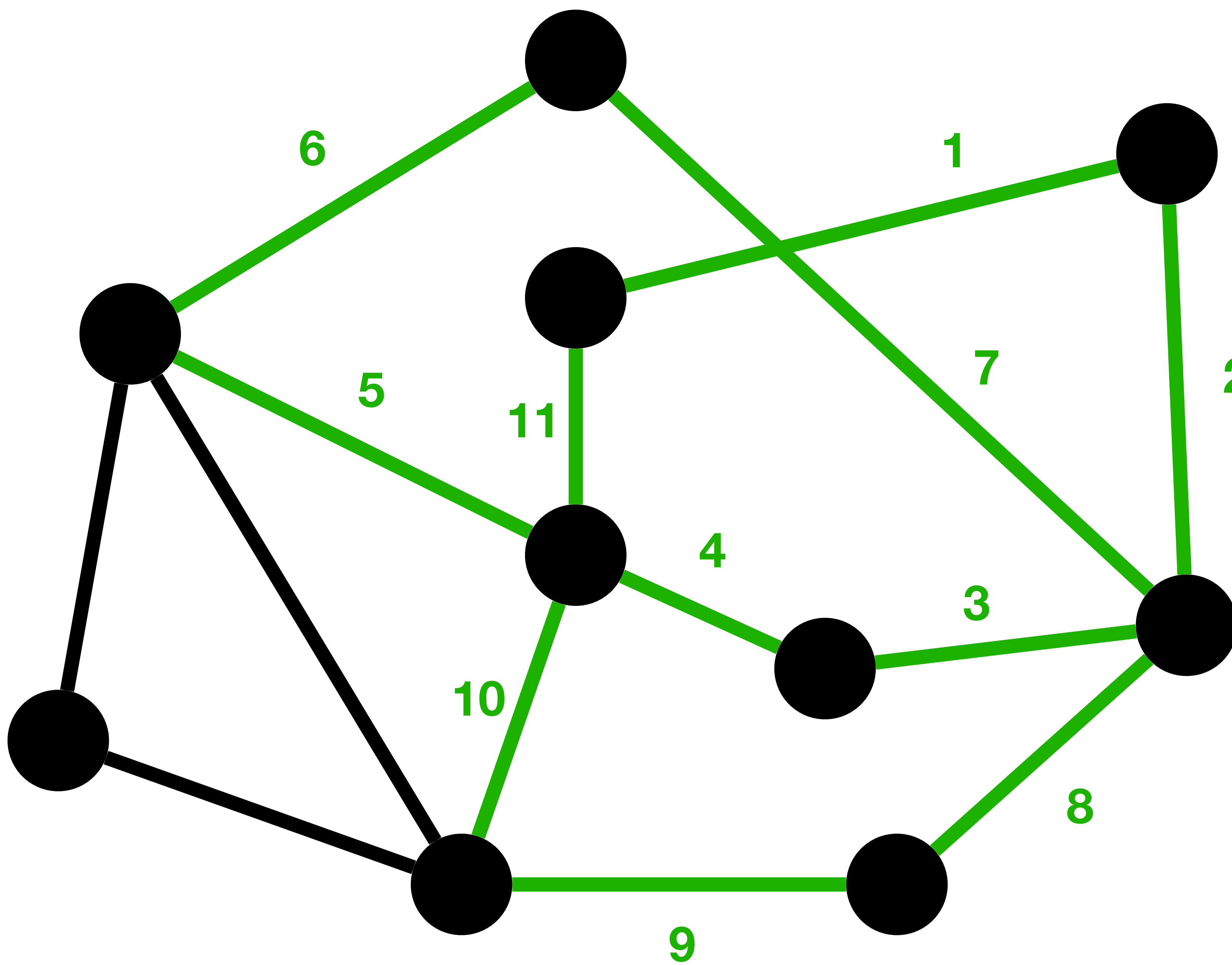

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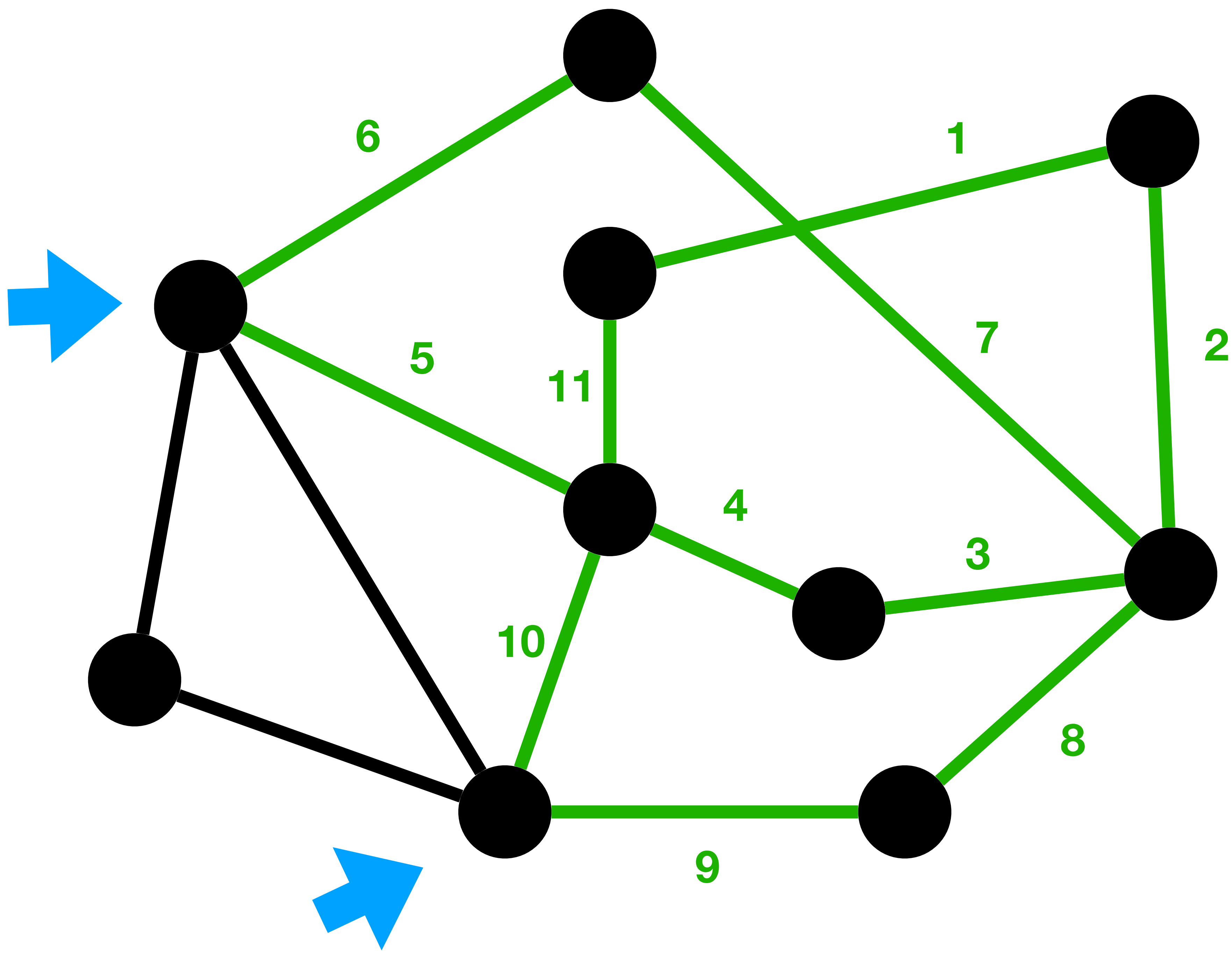

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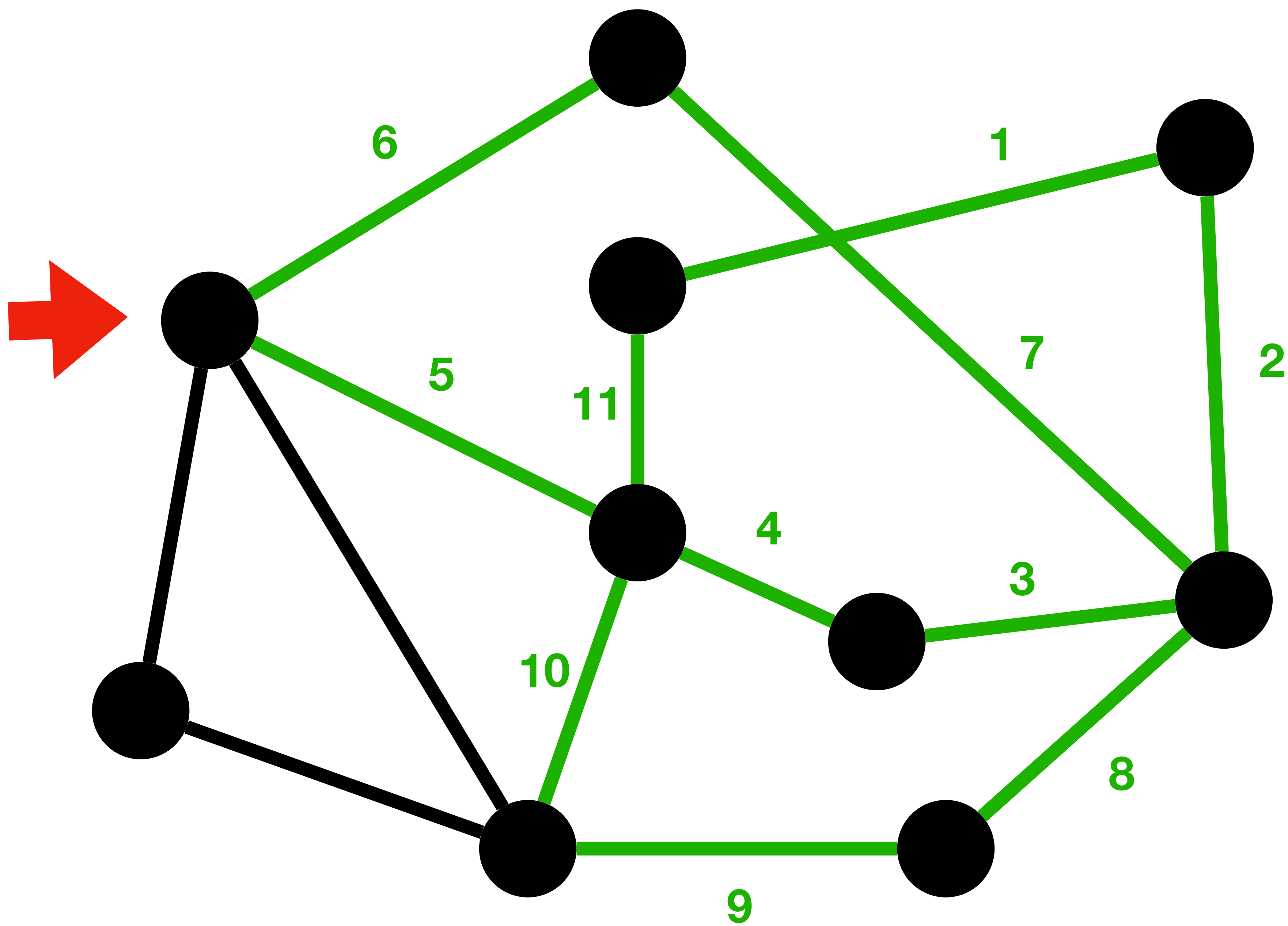

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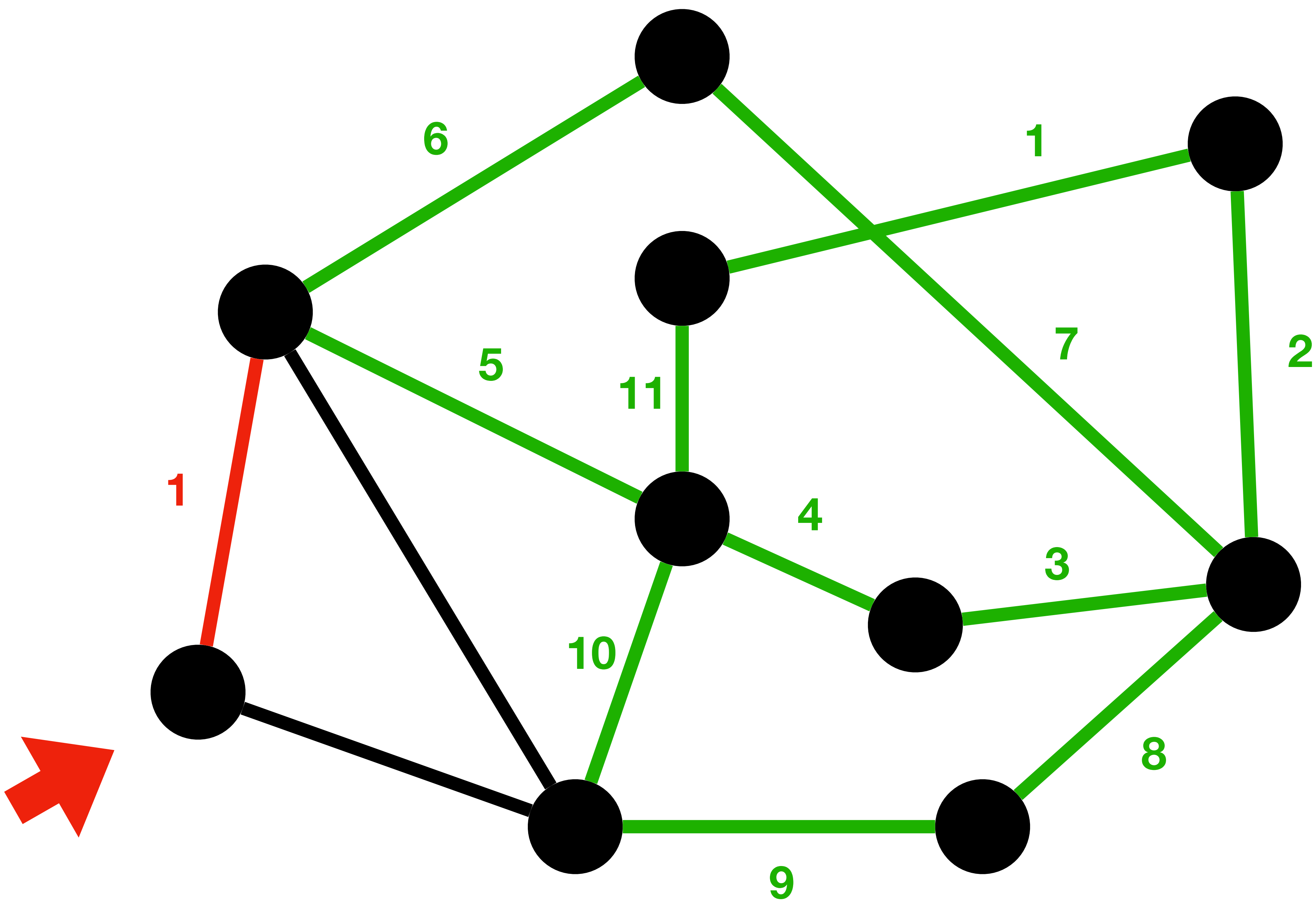

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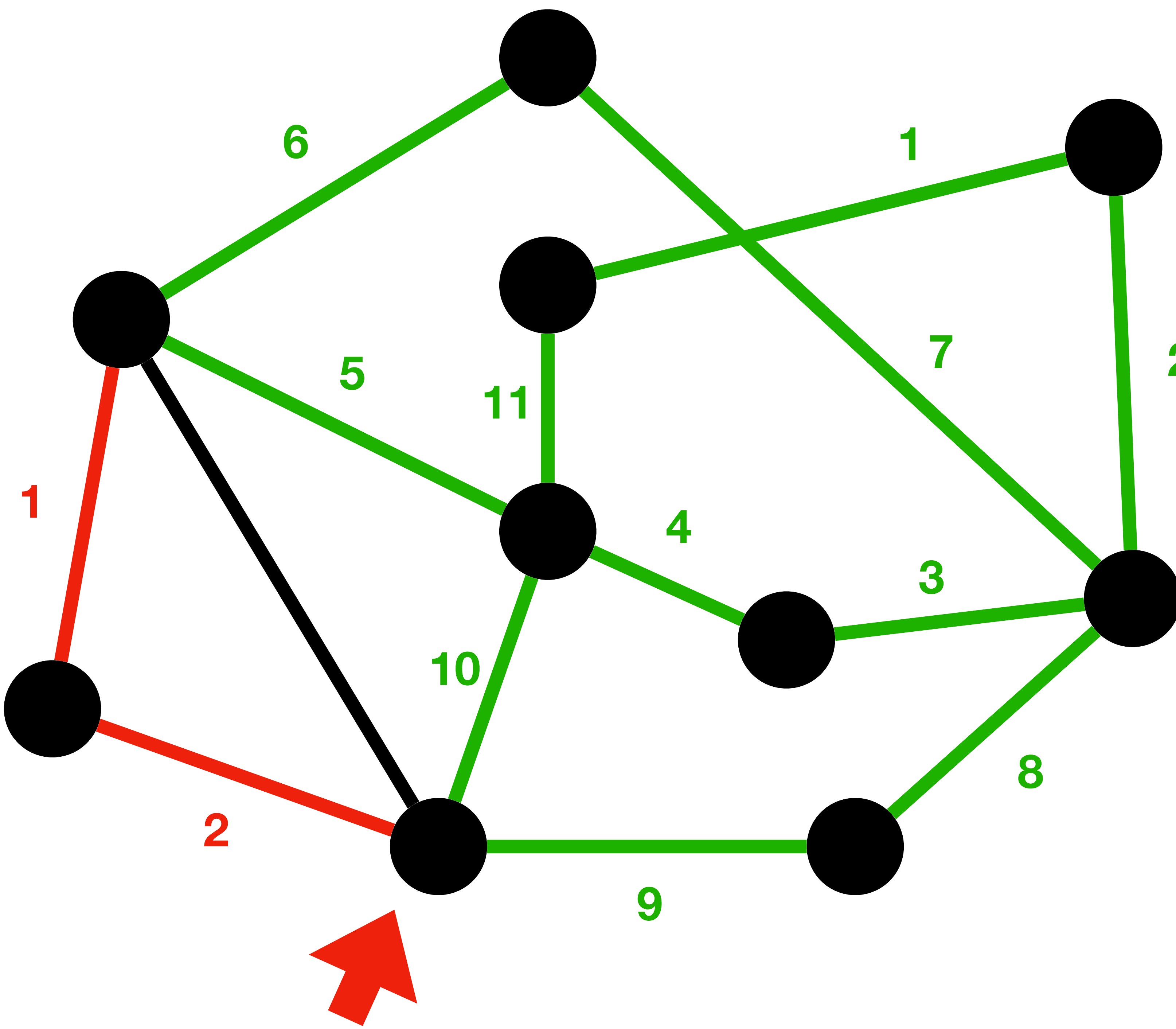

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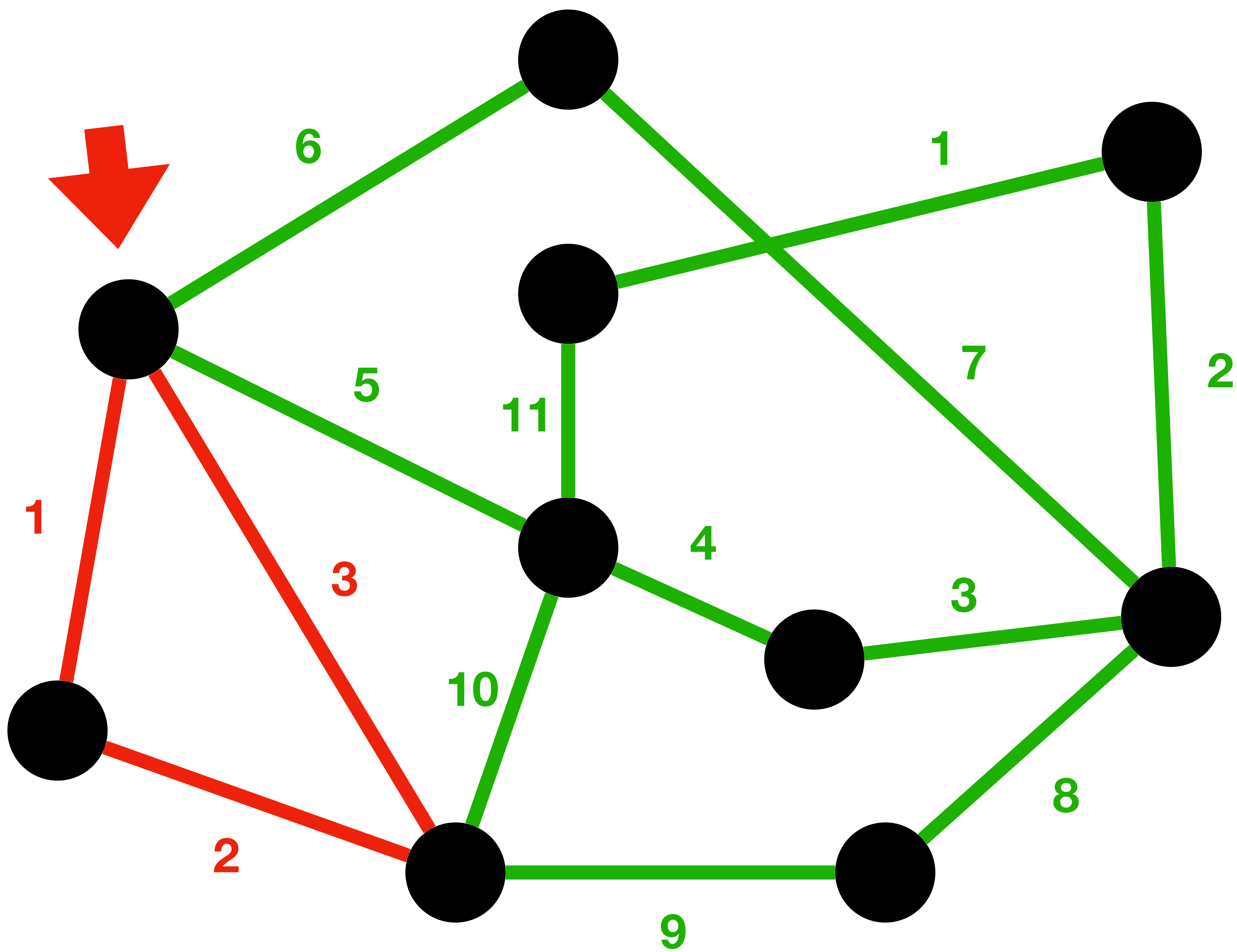

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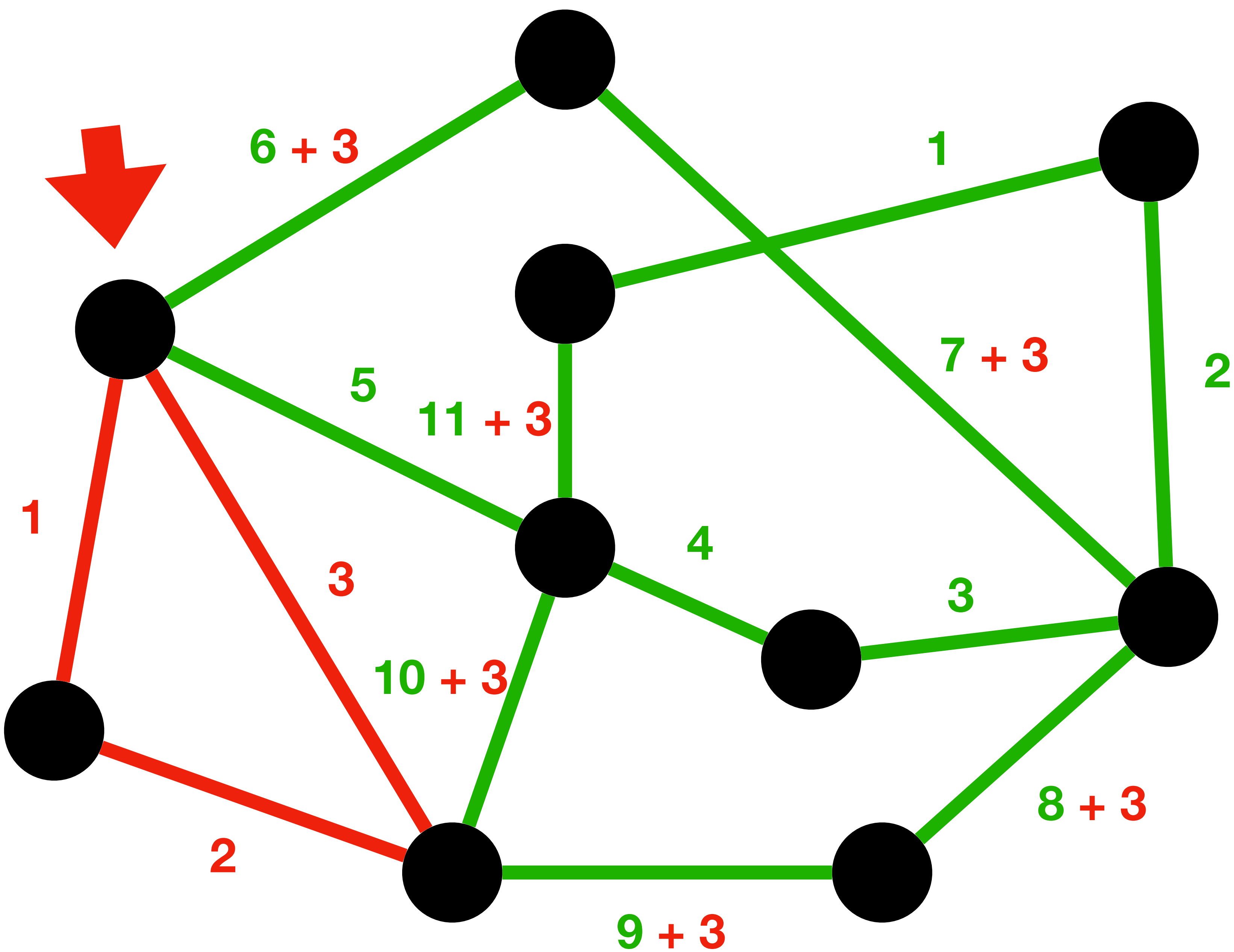

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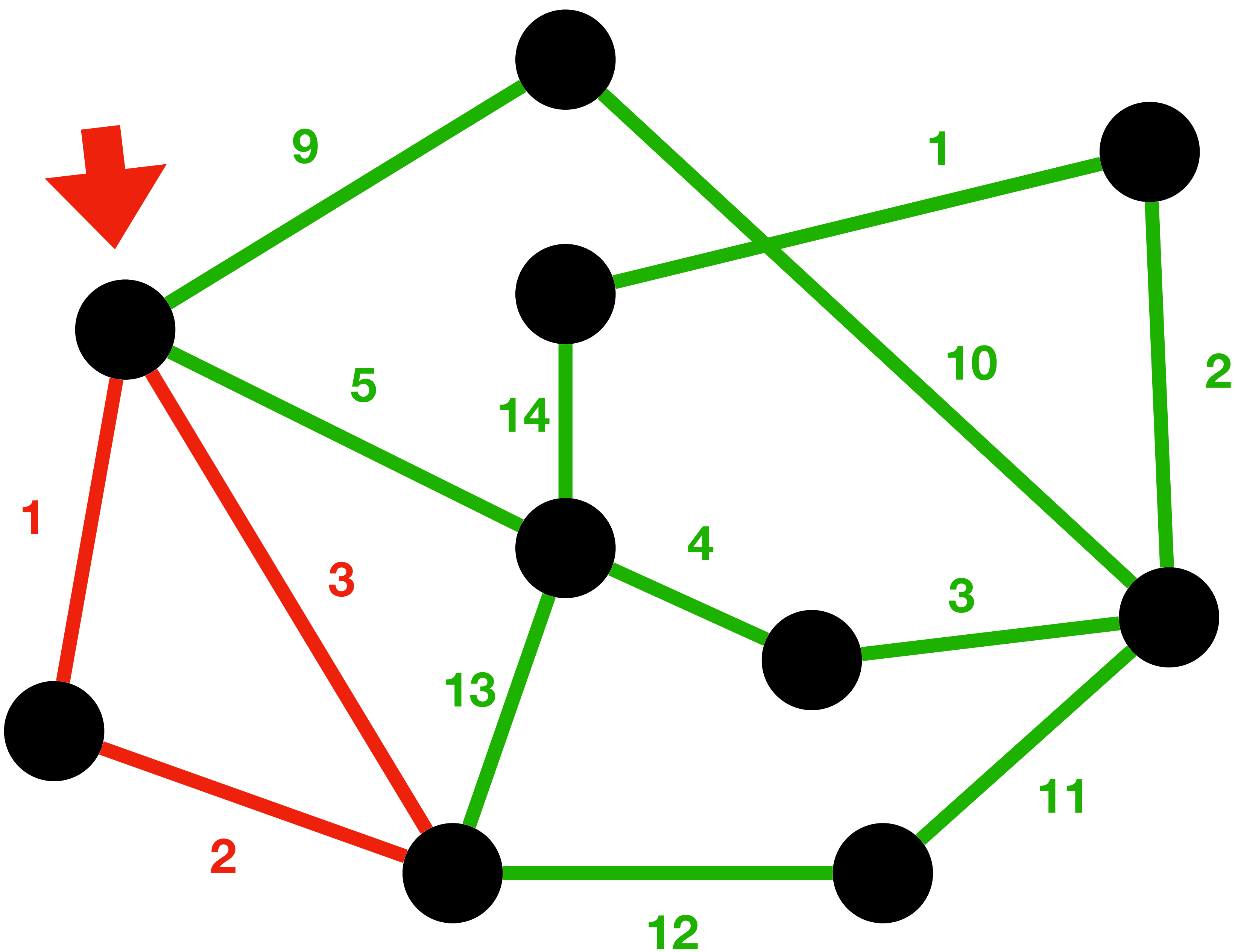

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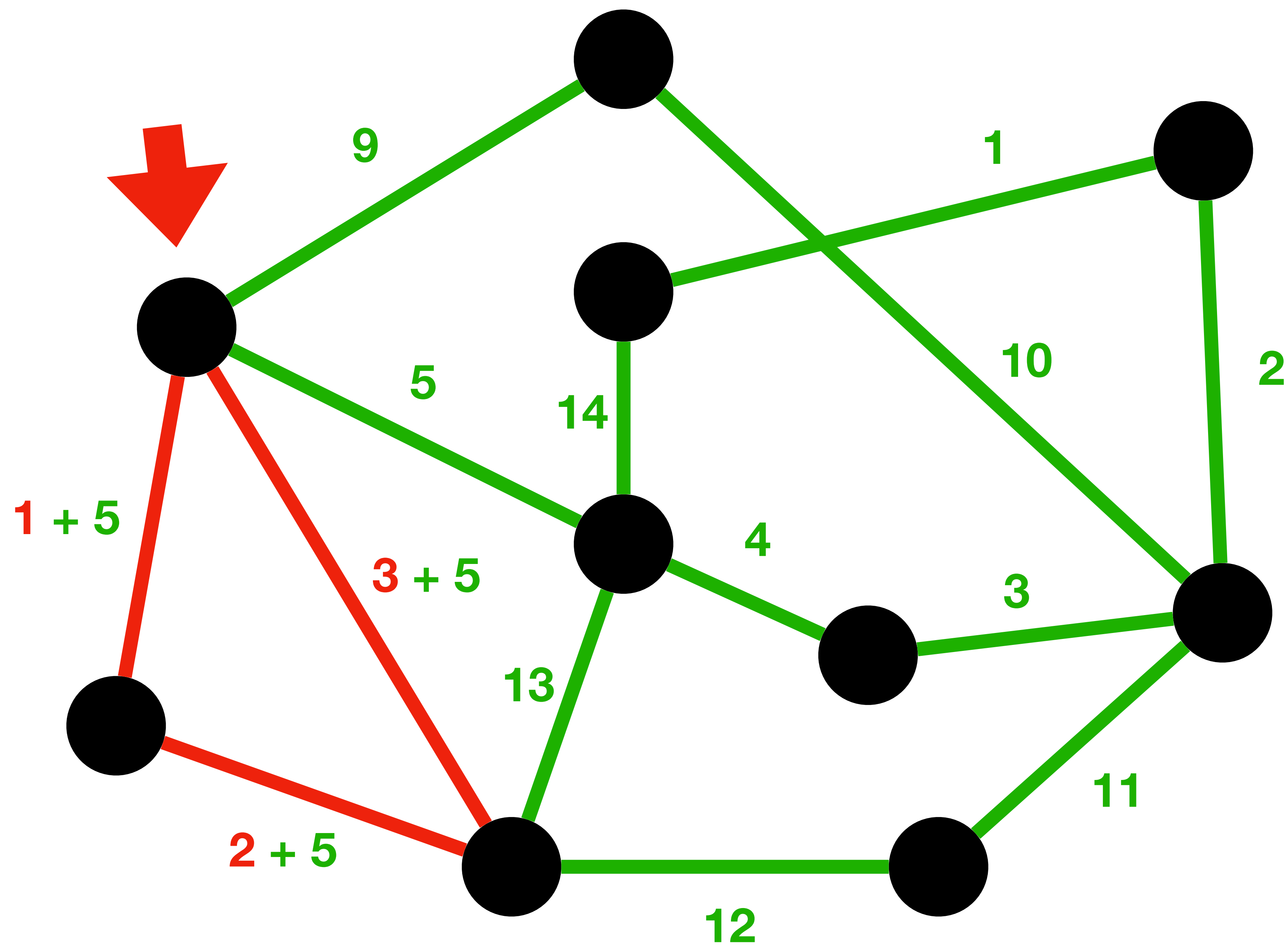

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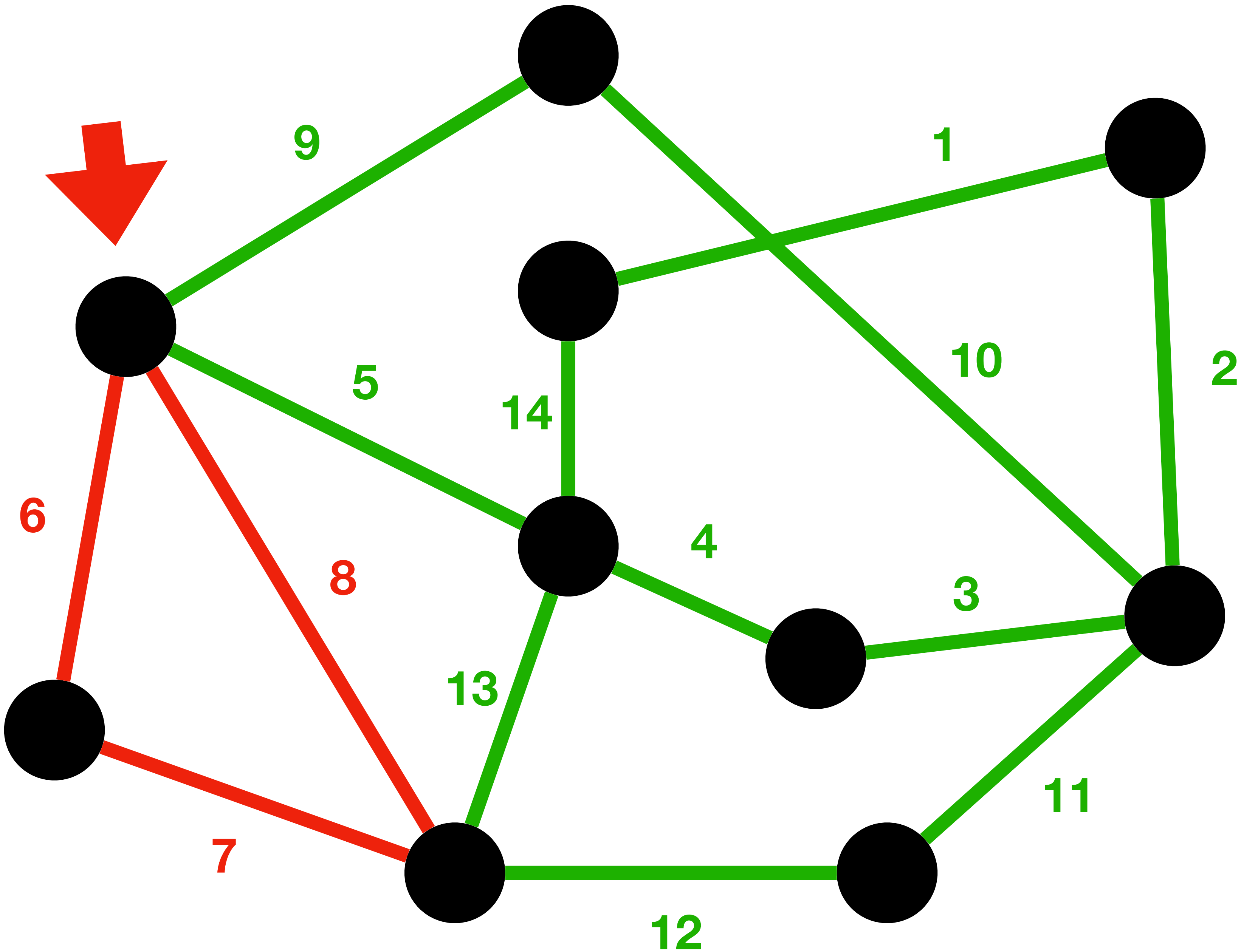

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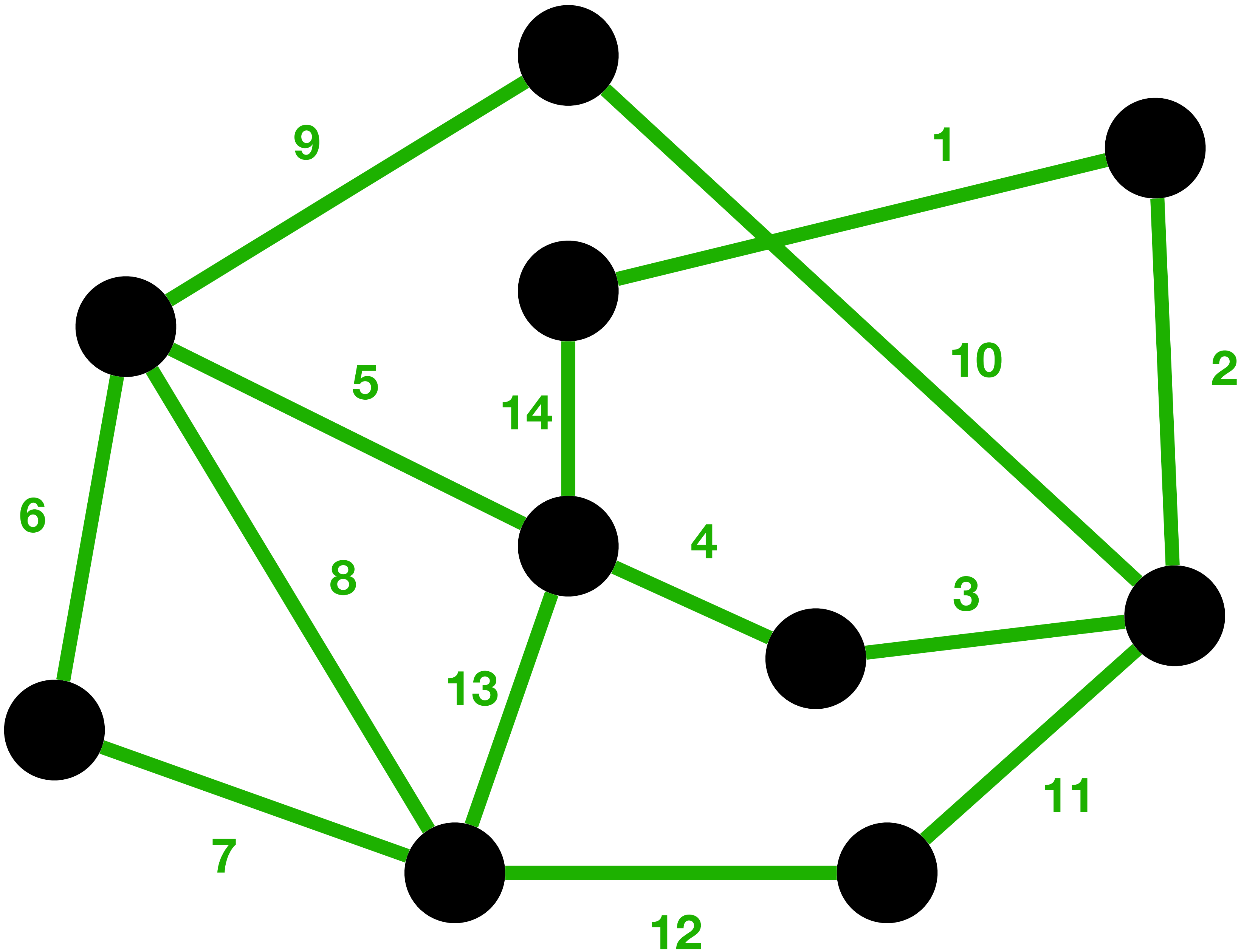

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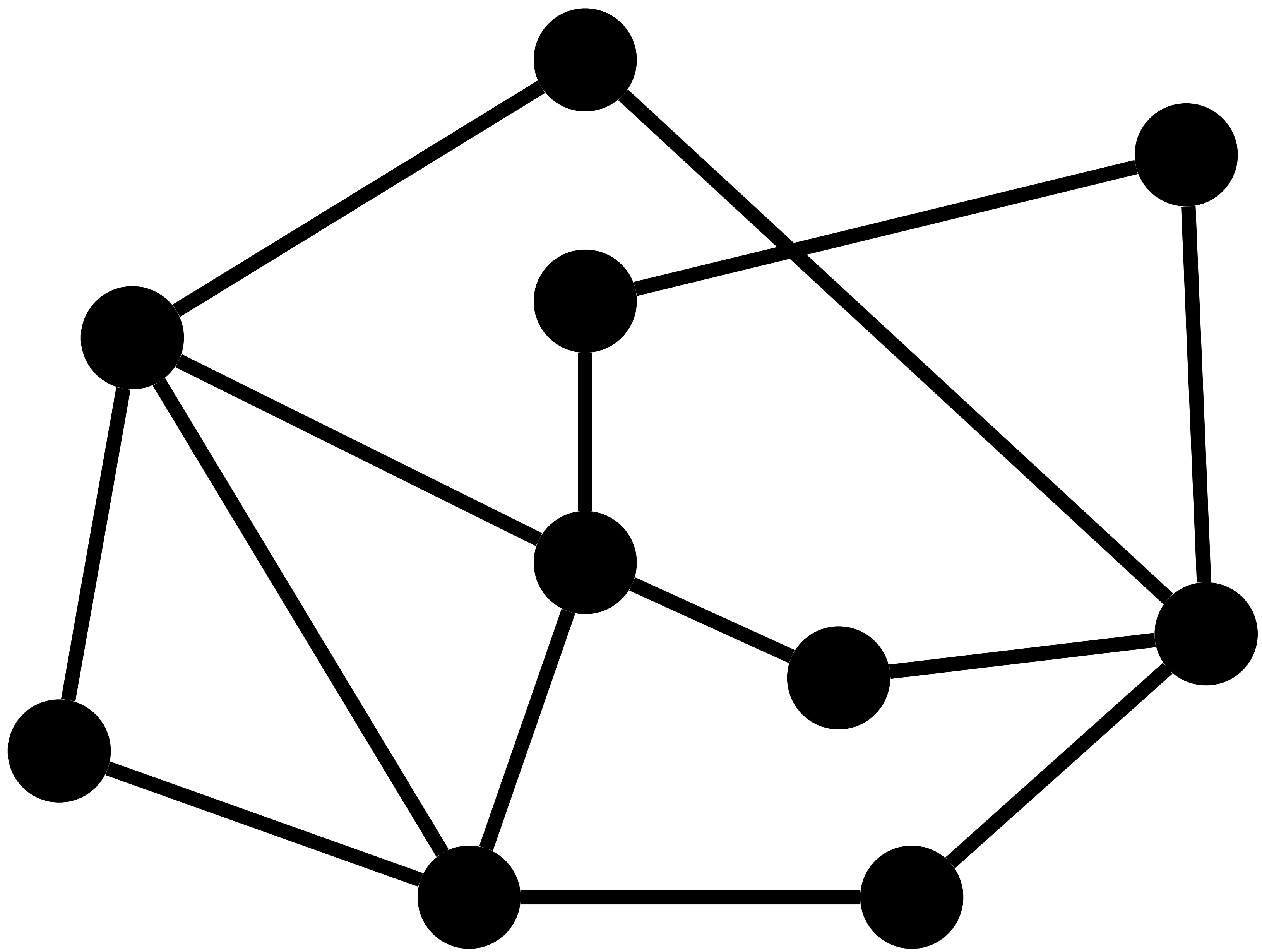

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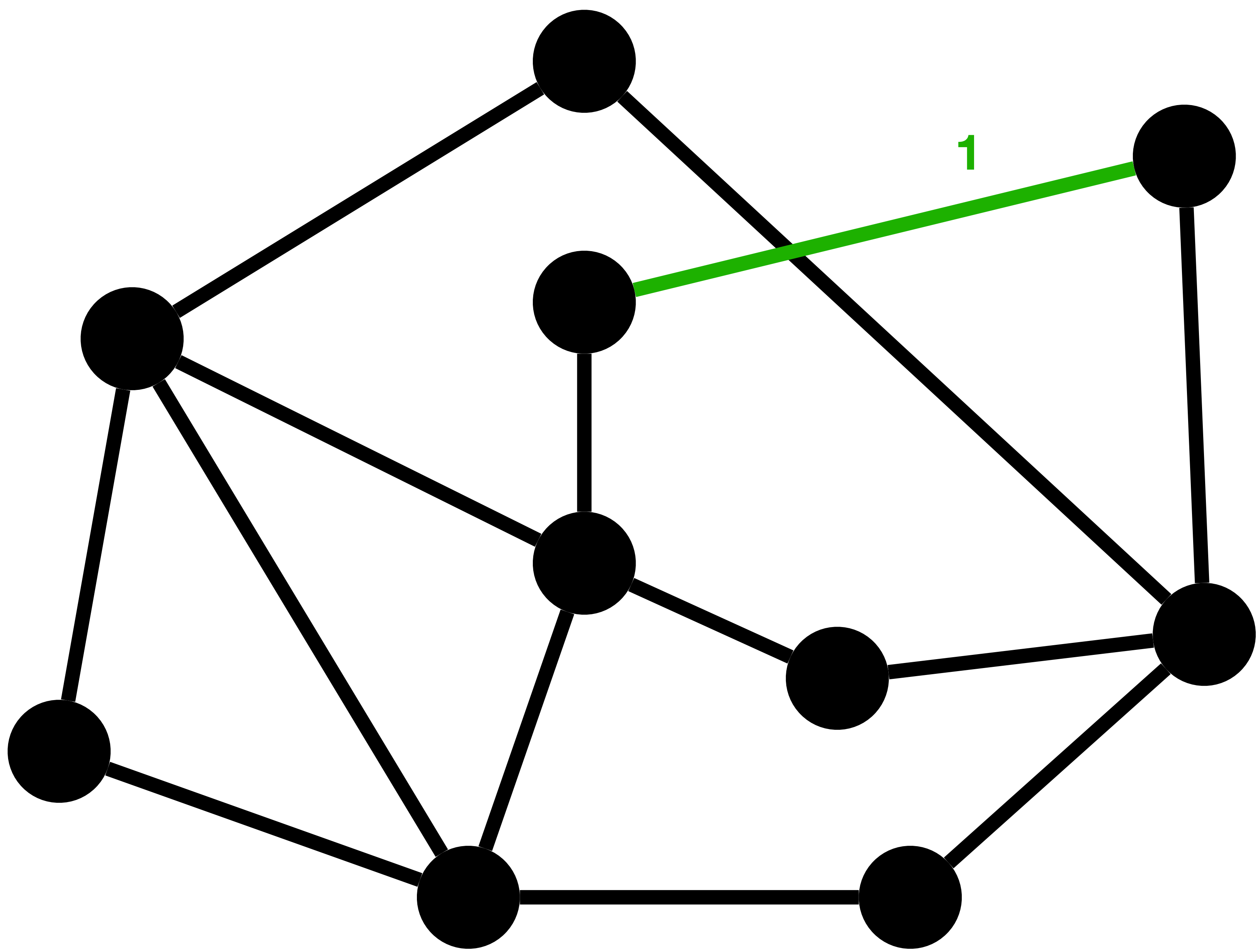

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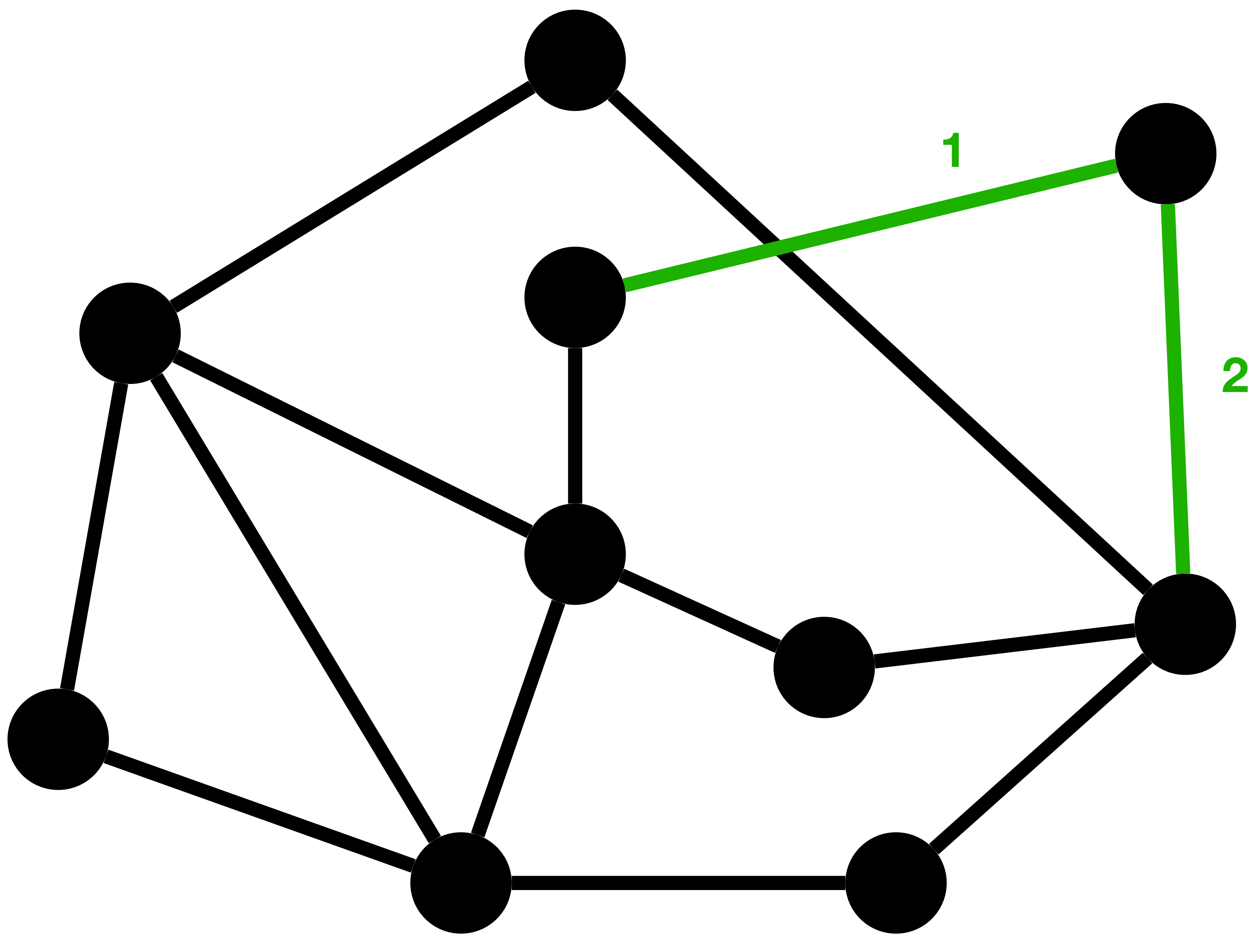

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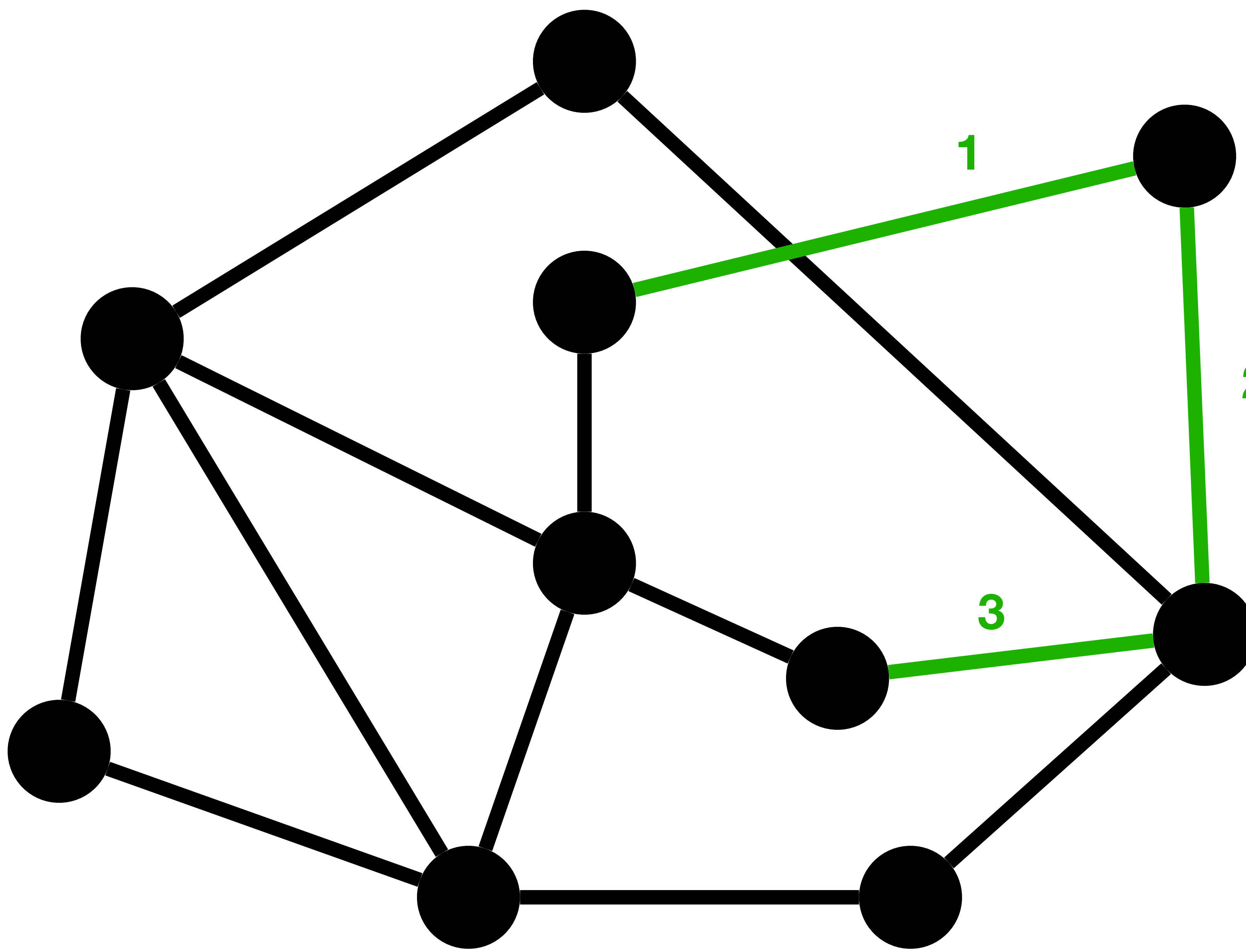

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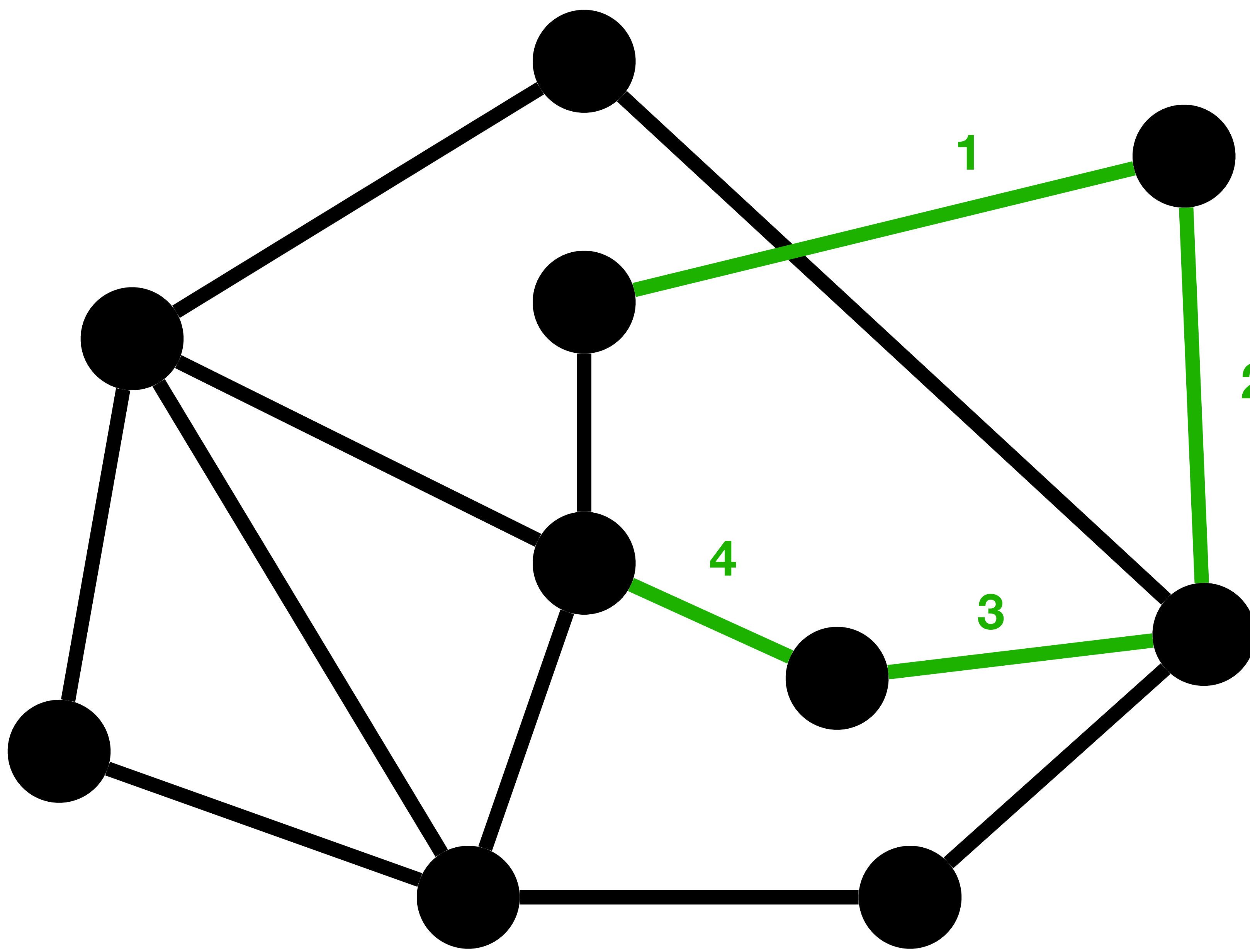

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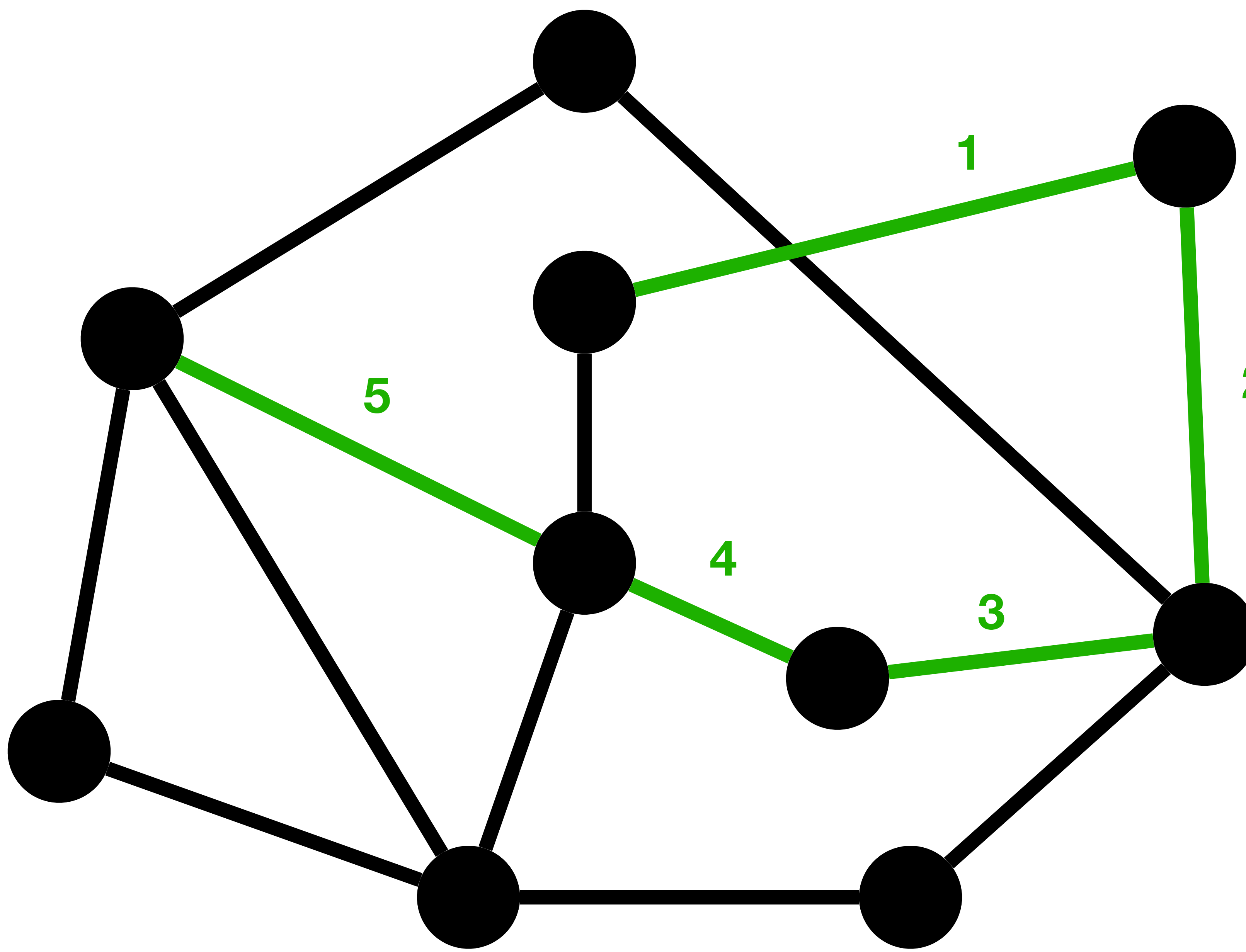

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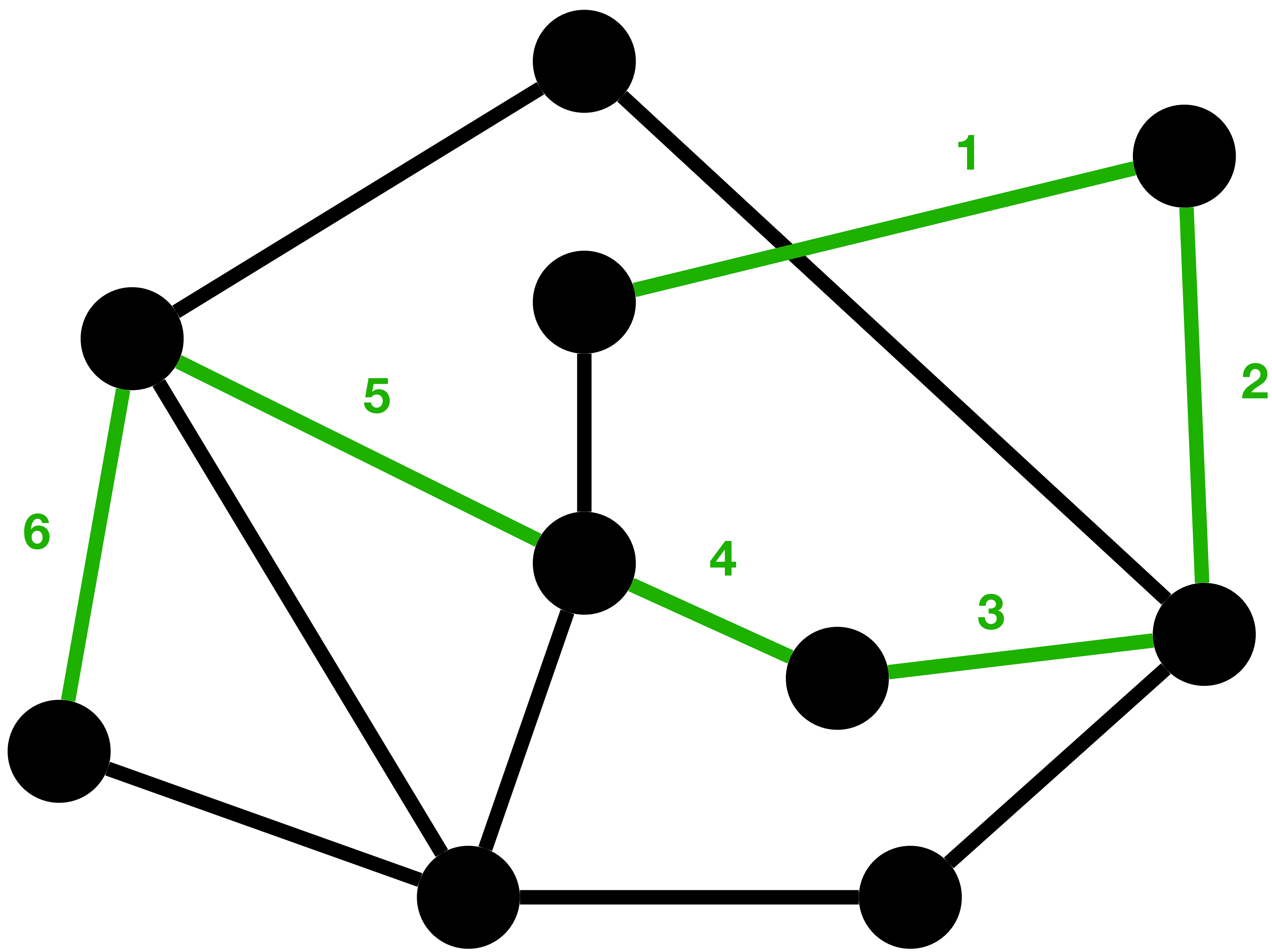

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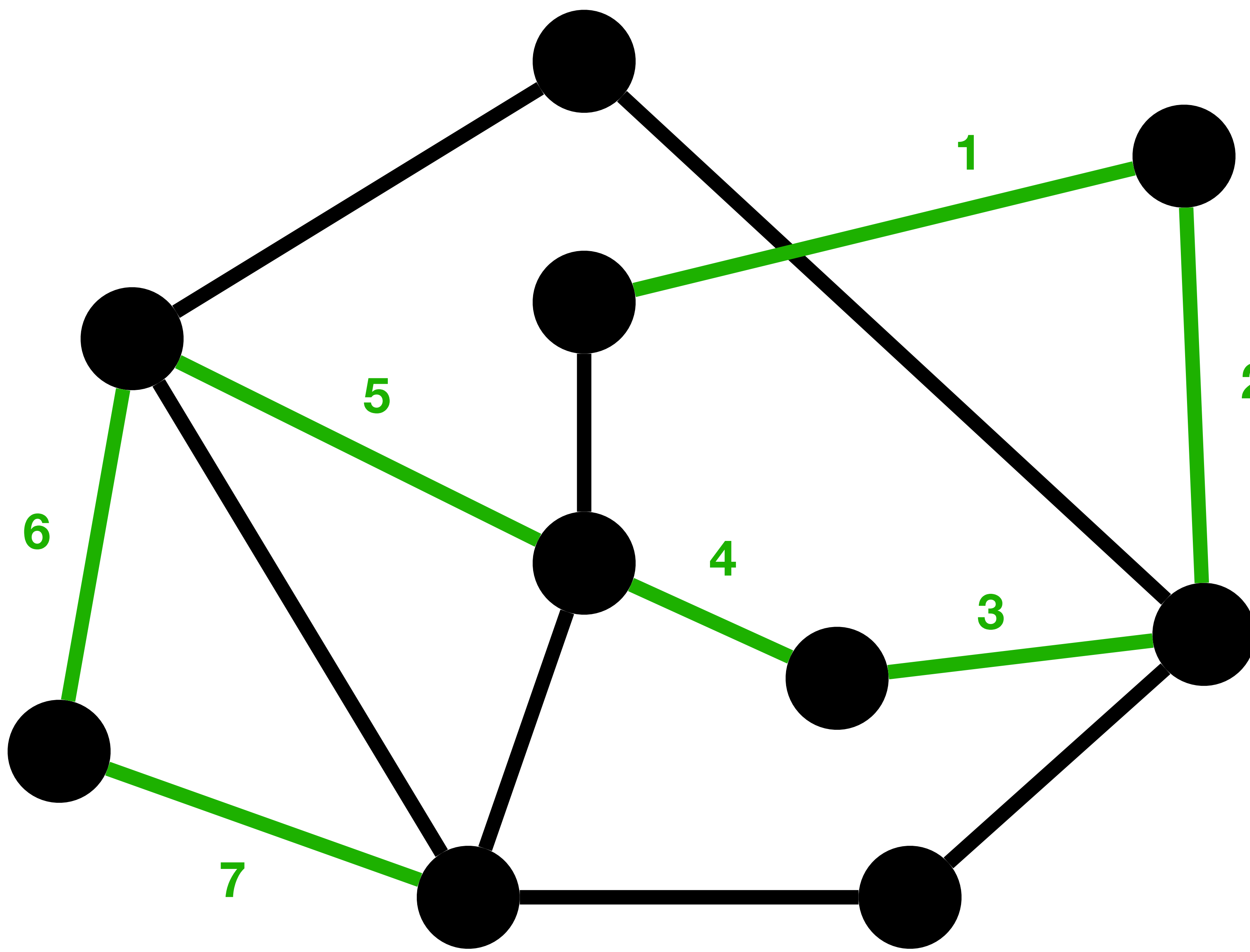

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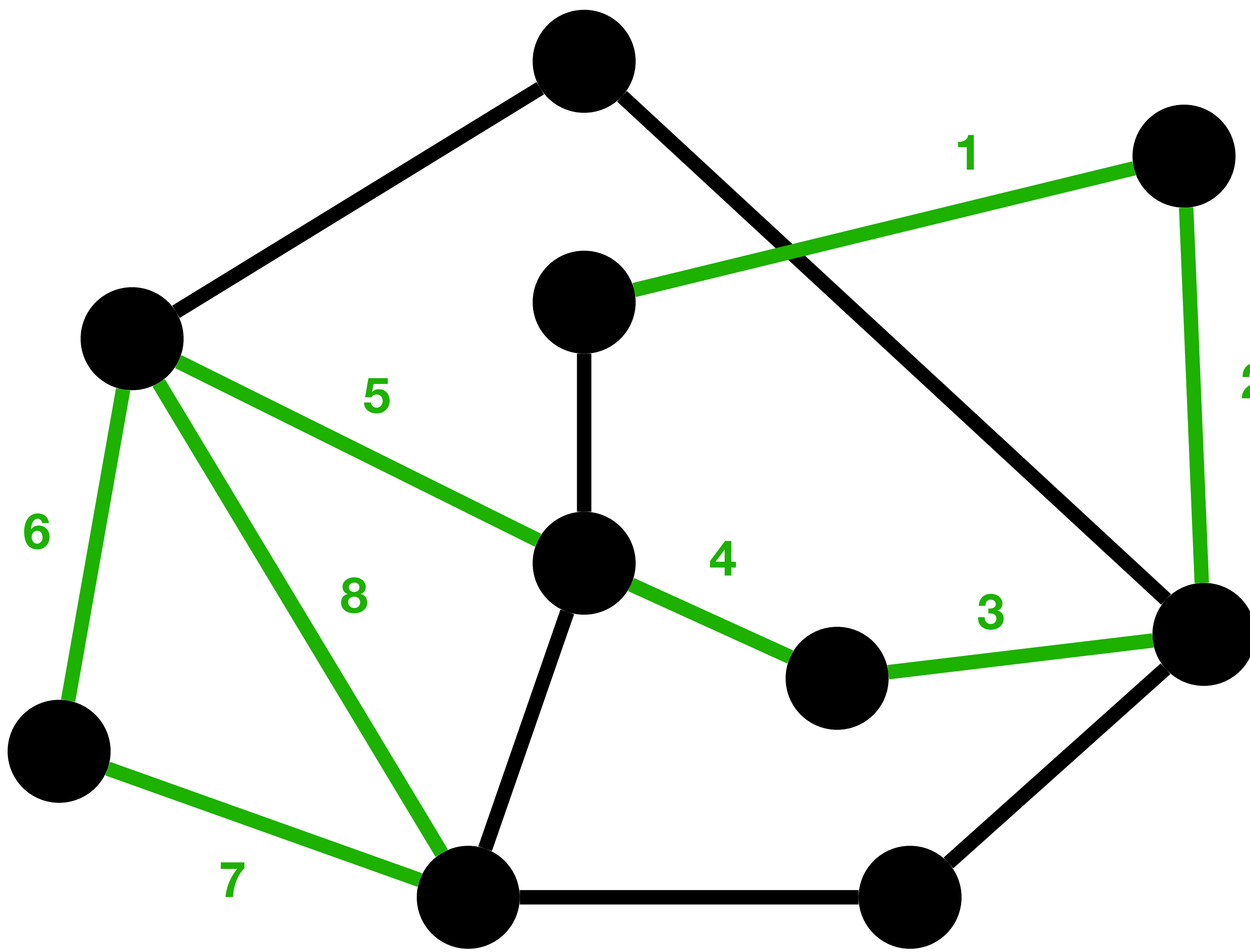

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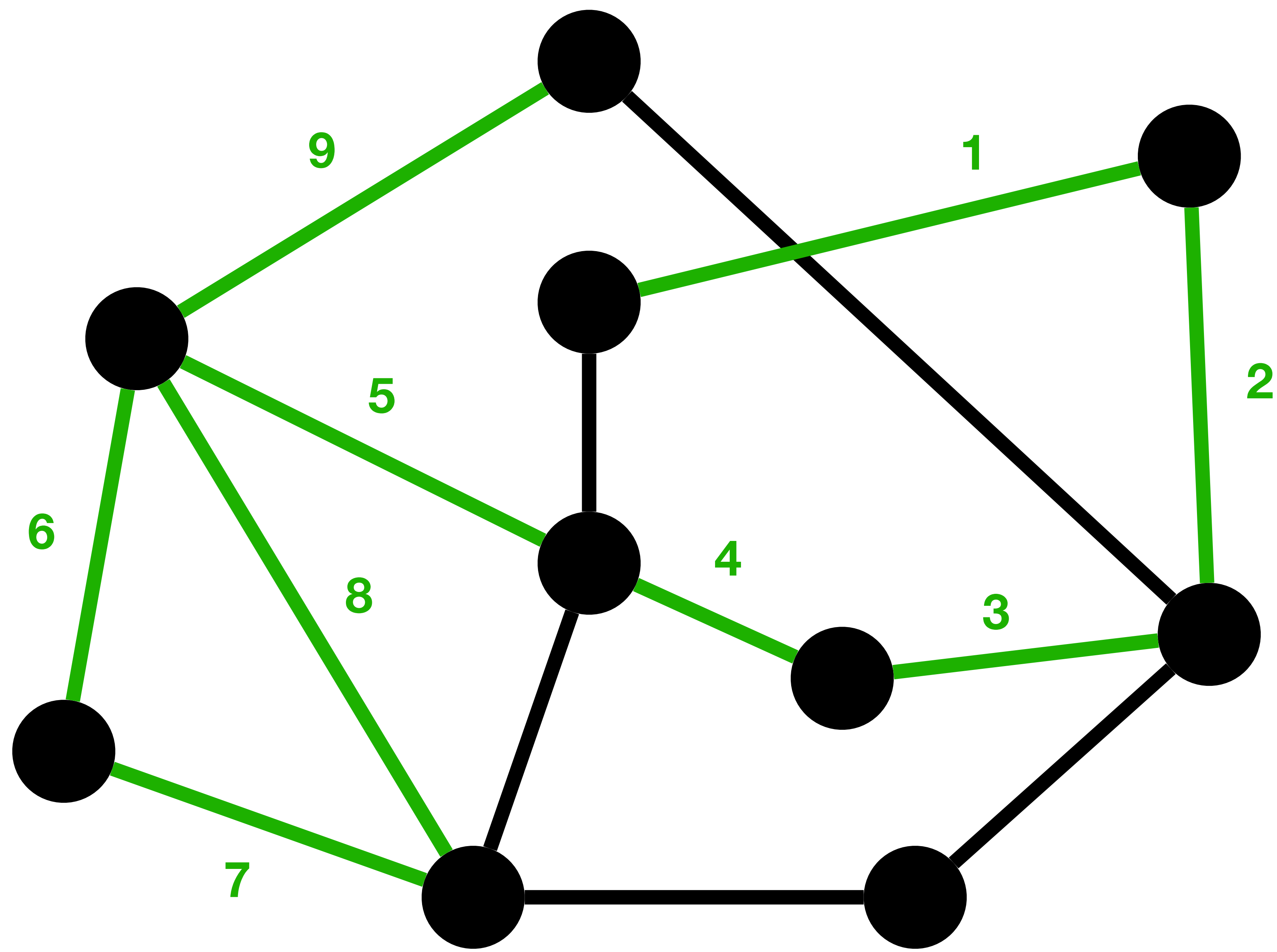

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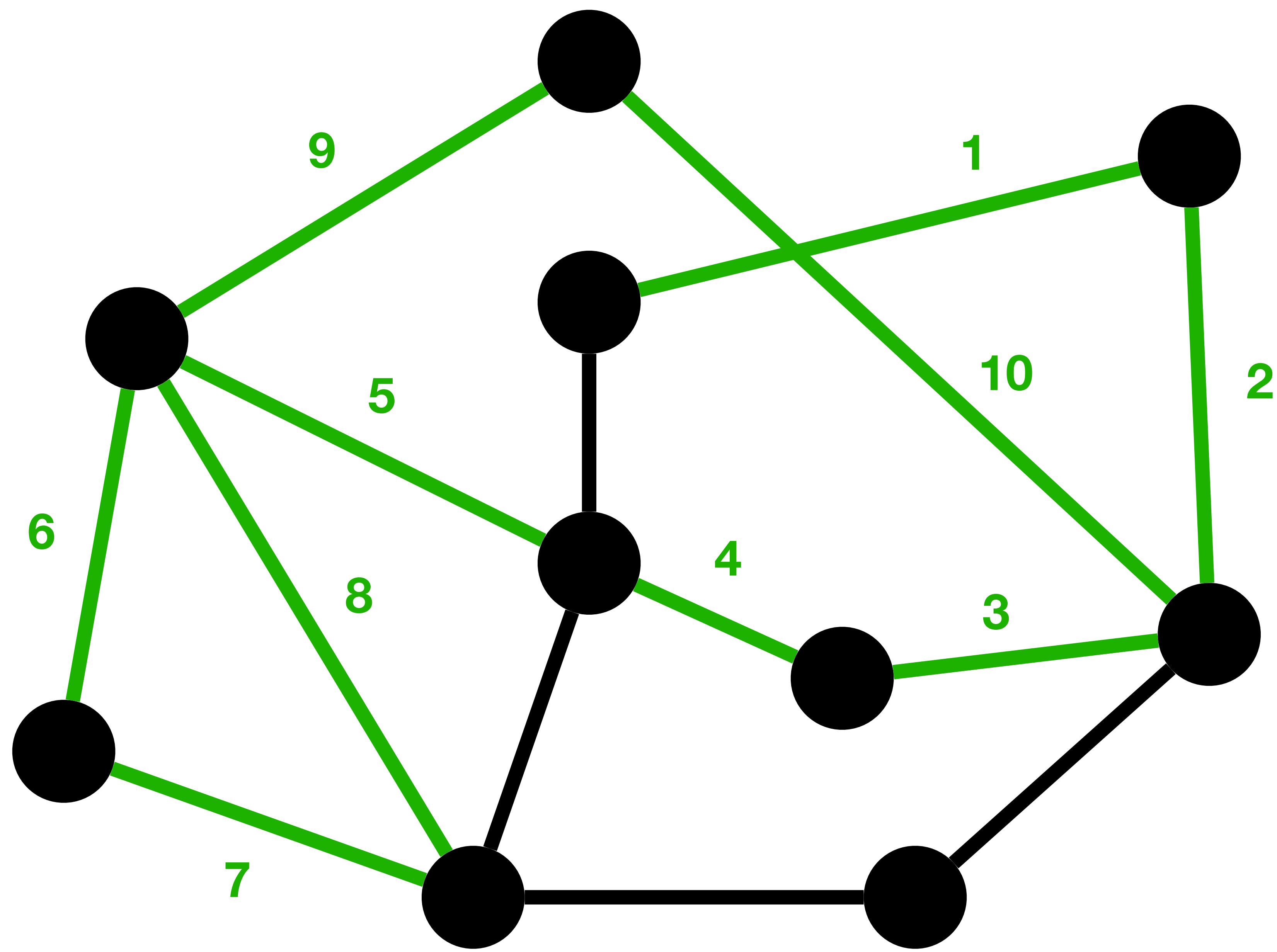
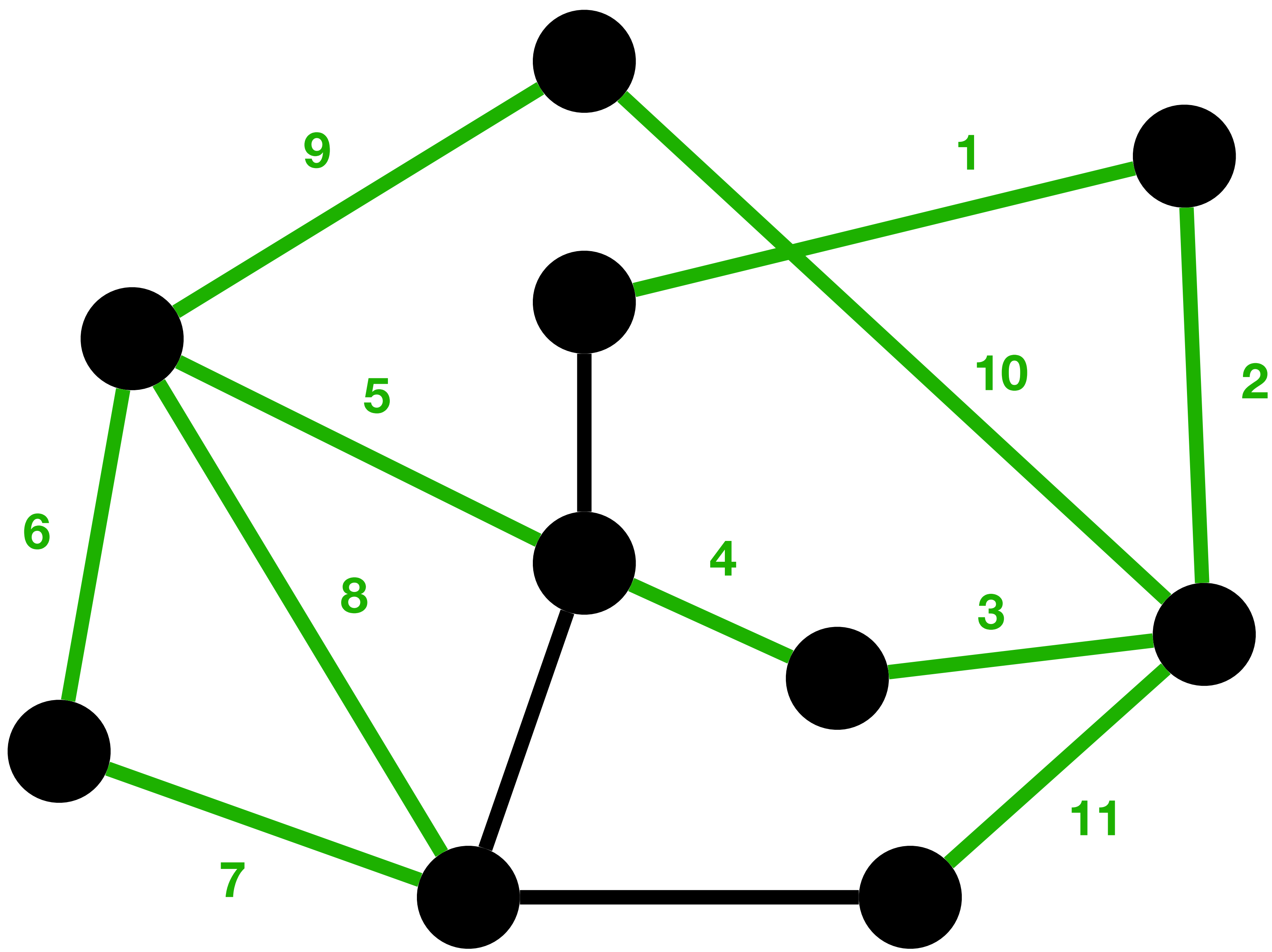
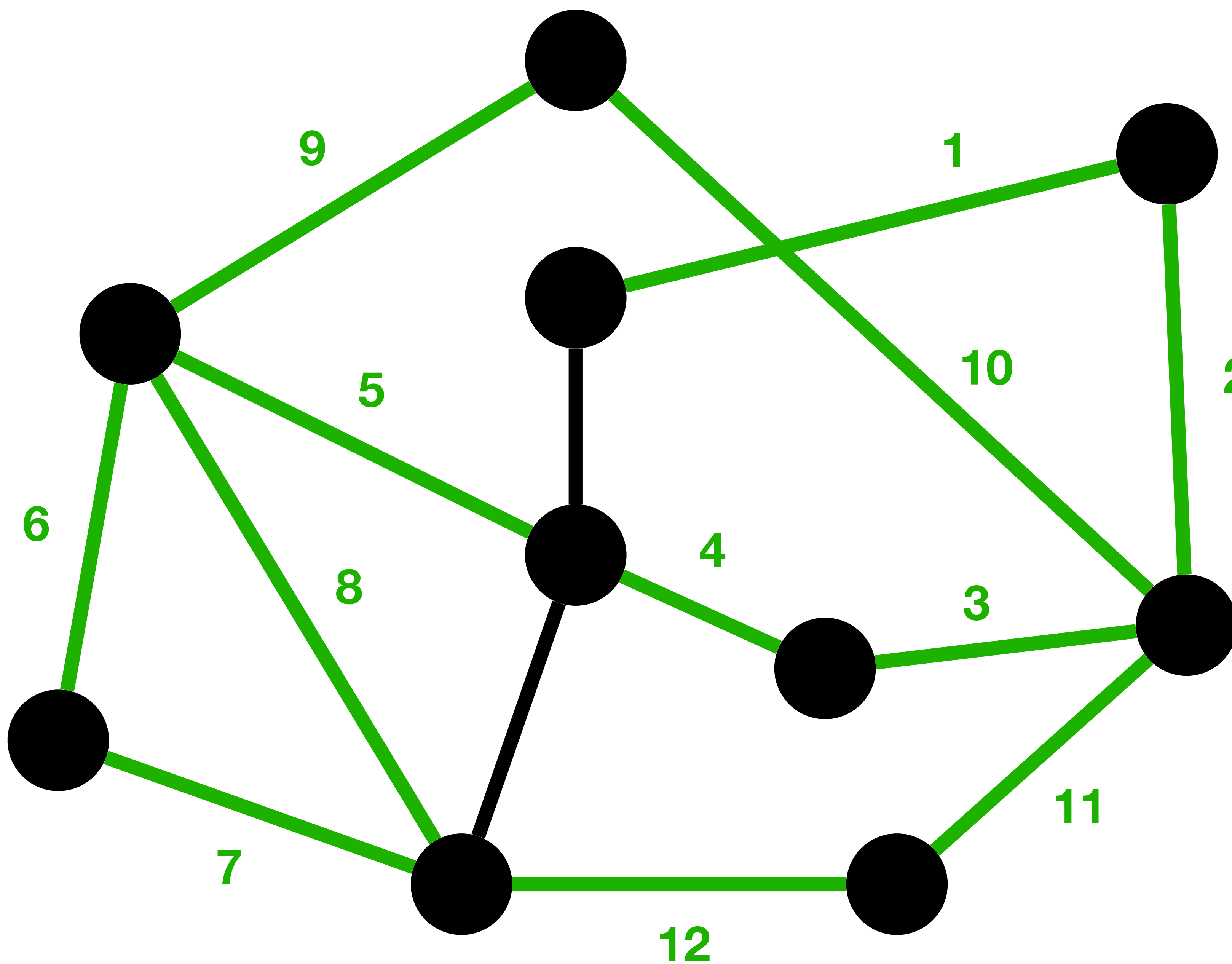


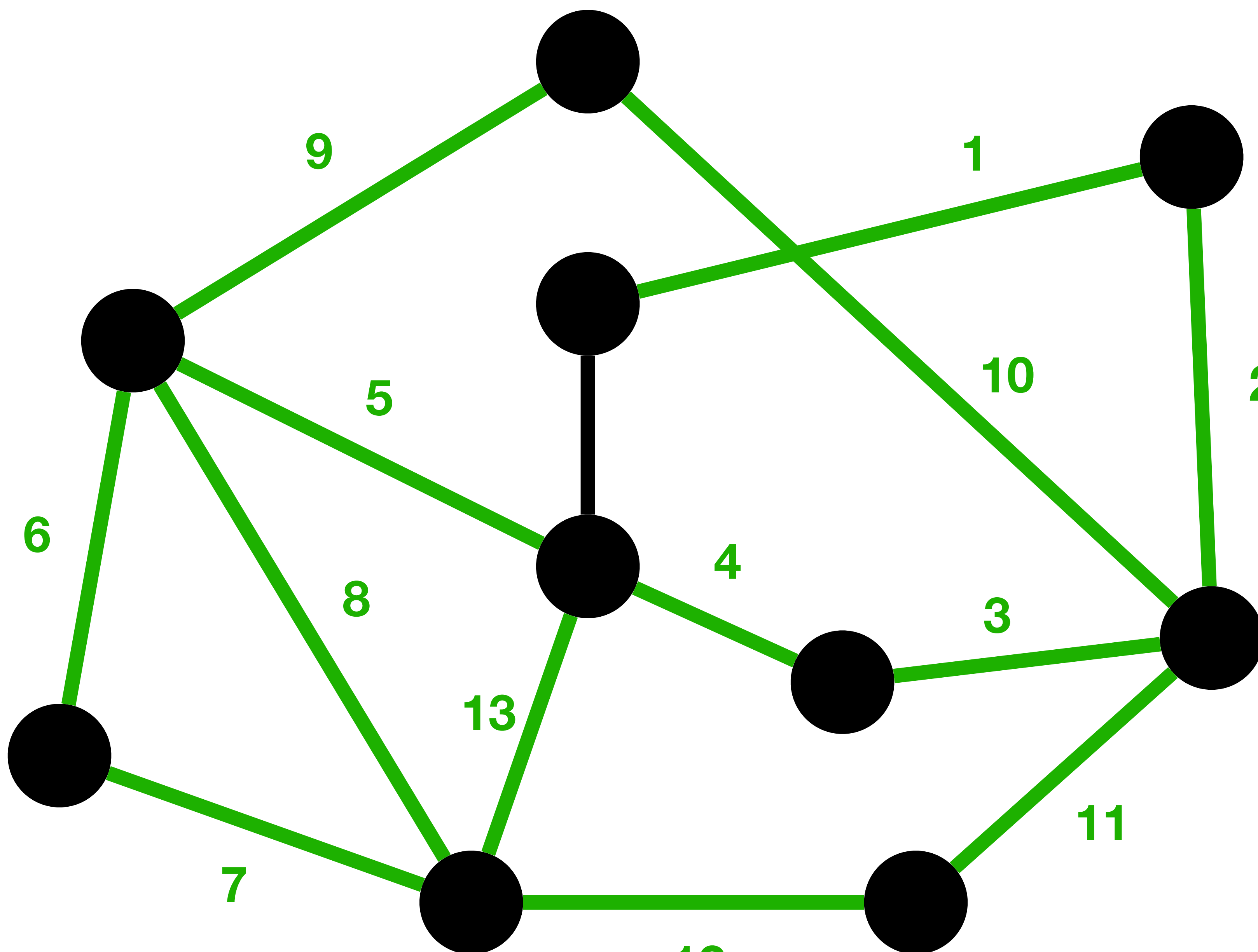
Algorithme de Hierholzer



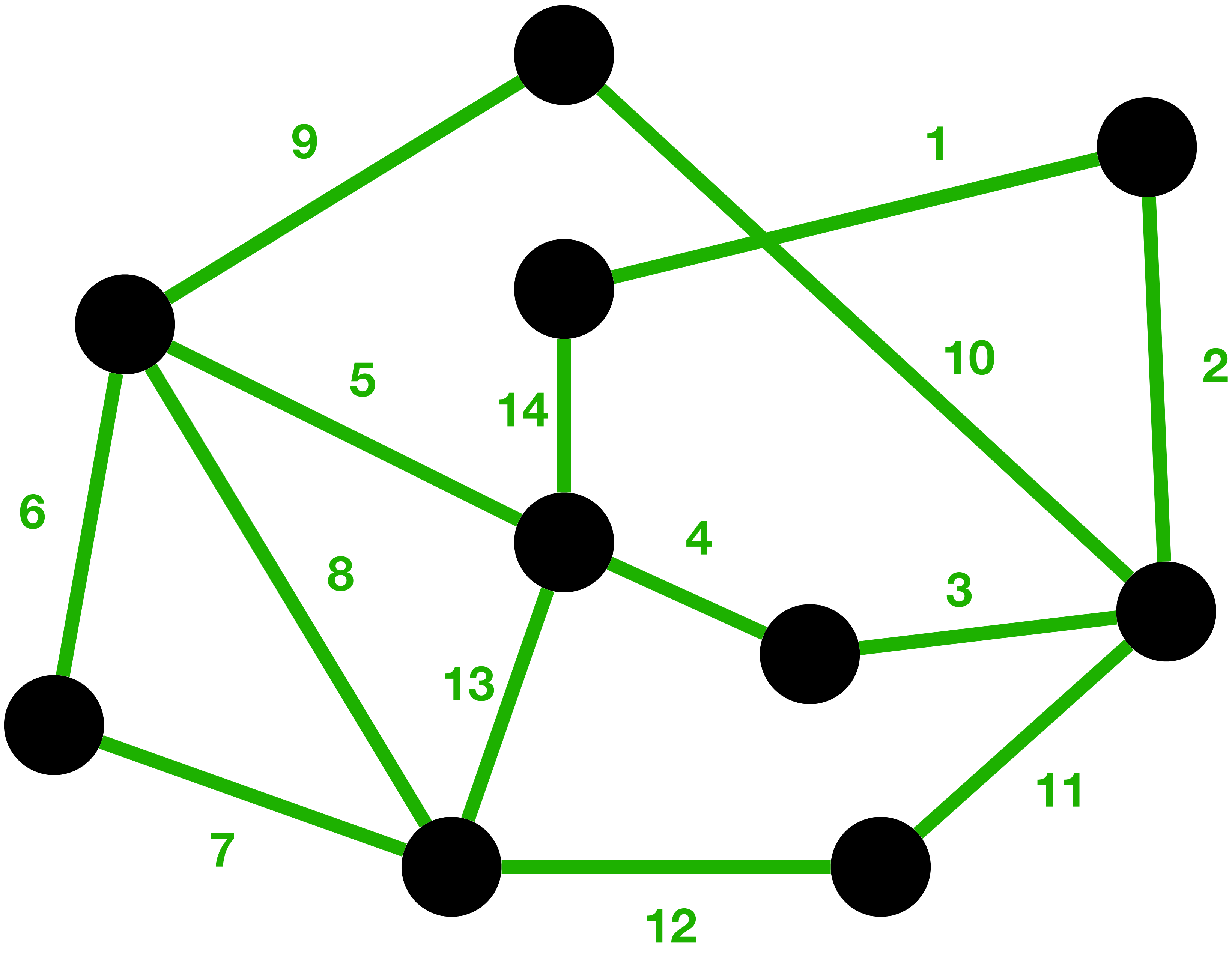
Algorithme de Hierholzer



Algorithme de Hierholzer



Algorithme de Hierholzer



Algorithme de Hierholzer

- Choisir n’importe quel sommet initial *v*
- Suivre un chemin arbitraire d’arêtes jusqu’à retourner à *v*, obtenant ainsi un cycle partiel *c*
- **Tant qu**’il y a des sommets *u* dans le cycle *c* avec des arêtes qu’on n’a pas encore choisit **faire**
  - Suivre un chemin de sommets à partir de *u* jusqu’à retourner à *u*, obtenant un cycle *c’*
  - Prolonger le cycle *c* par *c’*

Terminaison et correction de l'algorithme de Hierholzer

Terminaison et correction de l'algorithme de Hierholzer

- « Choisir n'importe quel sommet initial *v*. Suivre un chemin arbitraire d'arêtes jusqu'à retourner à *v*, obtenant ainsi un cycle partiel *c* »

Terminaison et correction de l’algorithme de Hierholzer

- « Choisir n’importe quel sommet initial *v*. Suivre un chemin arbitraire d’arêtes jusqu’à retourner à *v*, obtenant ainsi un cycle partiel *c* »

- On ne peut pas rester bloqué dans aucun sommet different de *v*, puisque ils ont toujours un nombre pair d’arêtes pas encore explorées (« si on entre dans un sommet, on peut toujours en sortir »)

Terminaison et correction de l’algorithme de Hierholzer

- « Choisir n’importe quel sommet initial *v*. Suivre un chemin arbitraire d’arêtes jusqu’à retourner à *v*, obtenant ainsi un cycle partiel *c* »

- On ne peut pas rester bloqué dans aucun sommet different de *v*, puisque ils ont toujours un nombre pair d’arêtes pas encore explorées (« si on entre dans un sommet, on peut toujours en sortir »)

- Et, tôt ou tard, on retourne à *v*, puisque dans le pire des cas on épuise l’ensemble des autres arêtes, et *v* a un nombre **impair** d’arêtes inexplorées (donc il reste une arête pour y entrer)

Terminaison et correction de l'algorithme de Hierholzer

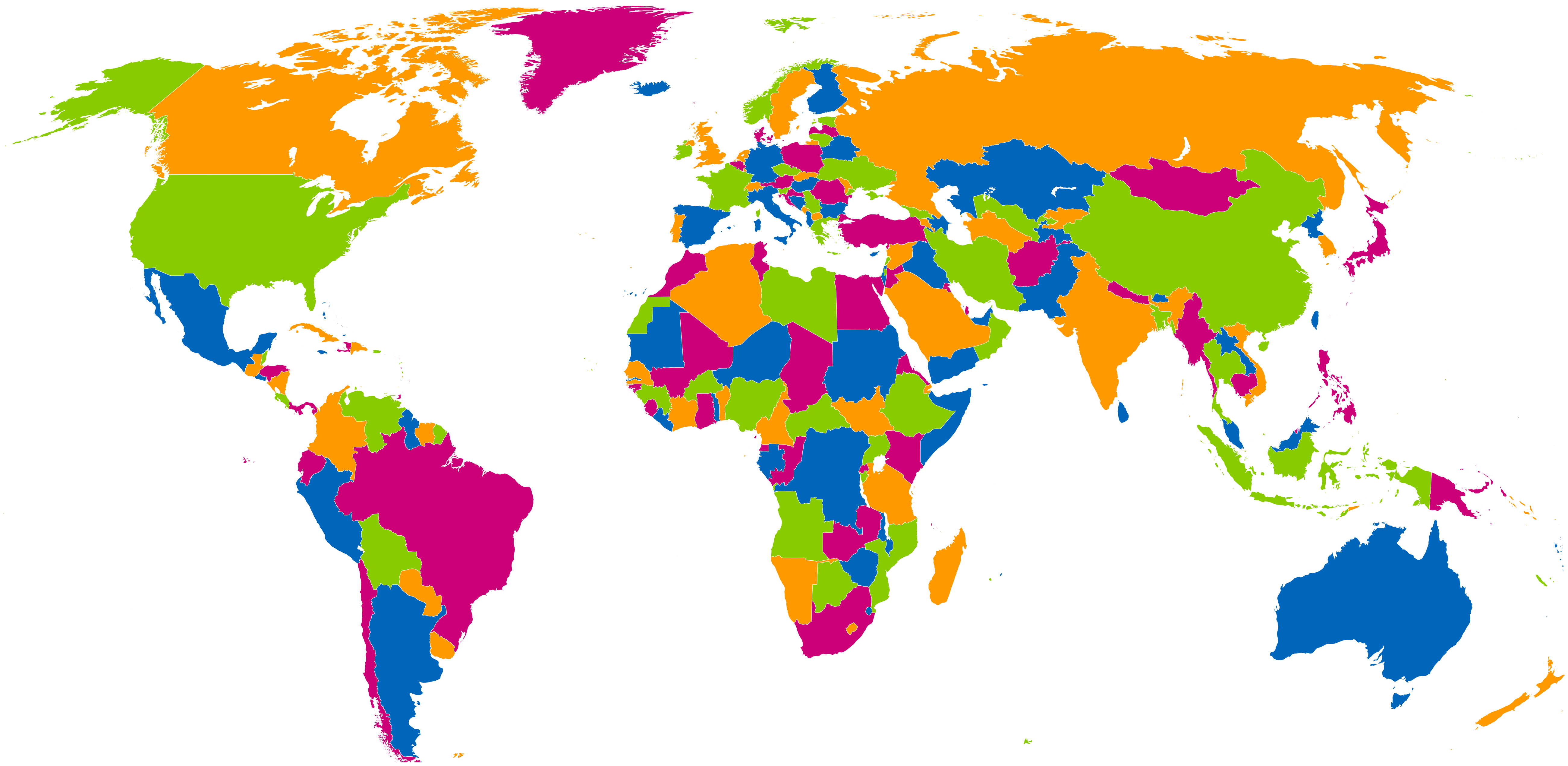
- « Choisir n'importe quel sommet initial *v*. Suivre un chemin arbitraire d'arêtes jusqu'à retourner à *v*, obtenant ainsi un cycle partiel *c* »

- On ne peut pas rester bloqué dans aucun sommet different de *v*, puisque ils ont toujours un nombre pair d'arêtes pas encore explorées (« si on entre dans un sommet, on peut toujours en sortir »)

- Et, tôt ou tard, on retourne à *v*, puisque dans le pire des cas on épuise l'ensemble des autres arêtes, et *v* a un nombre **impair** d'arêtes inexplorées (donc il reste une arête pour y entrer)

- Ça est également vrais pour chaque sommet *u* choisi après *v*

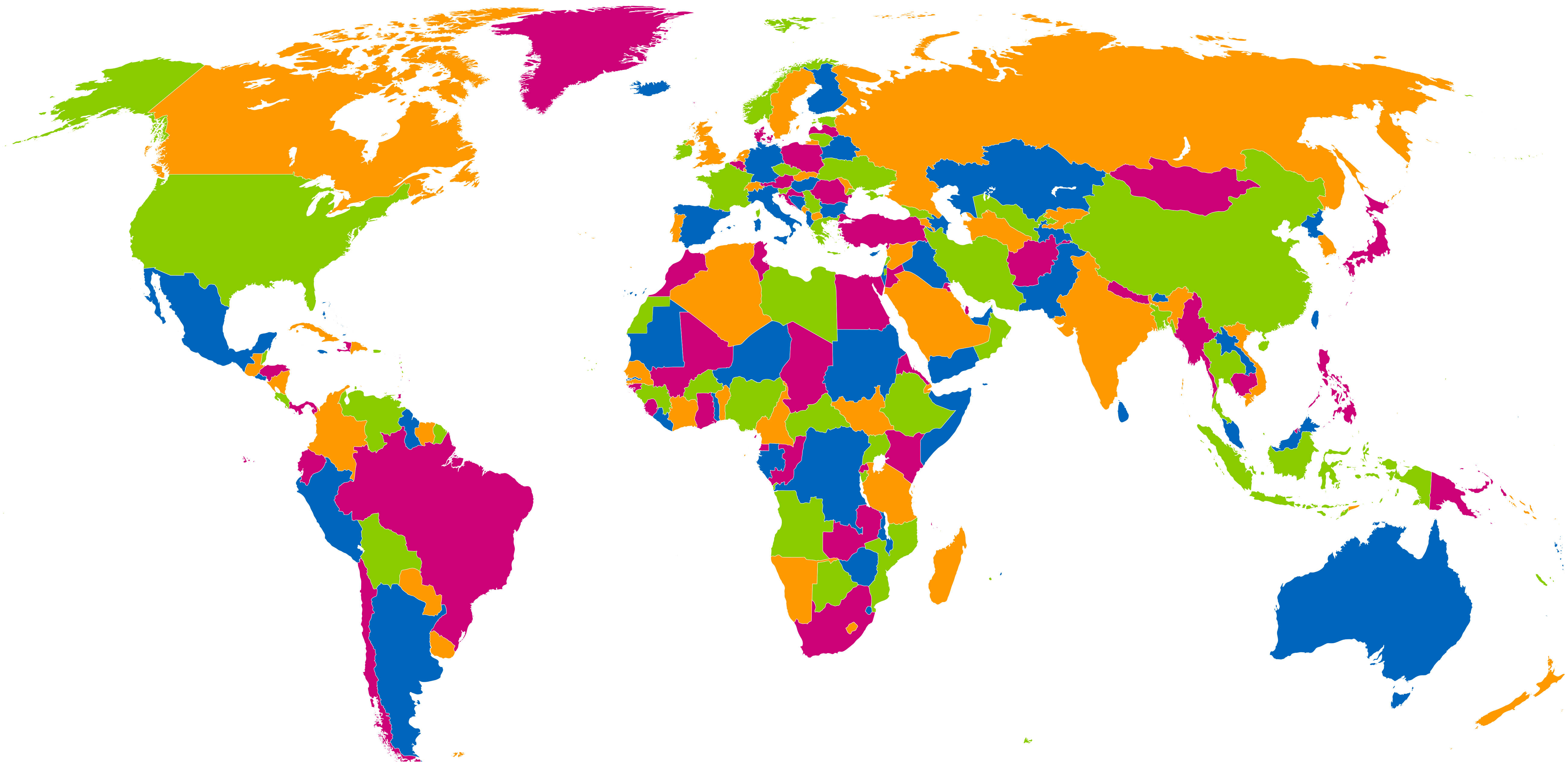
Coloration de cartes



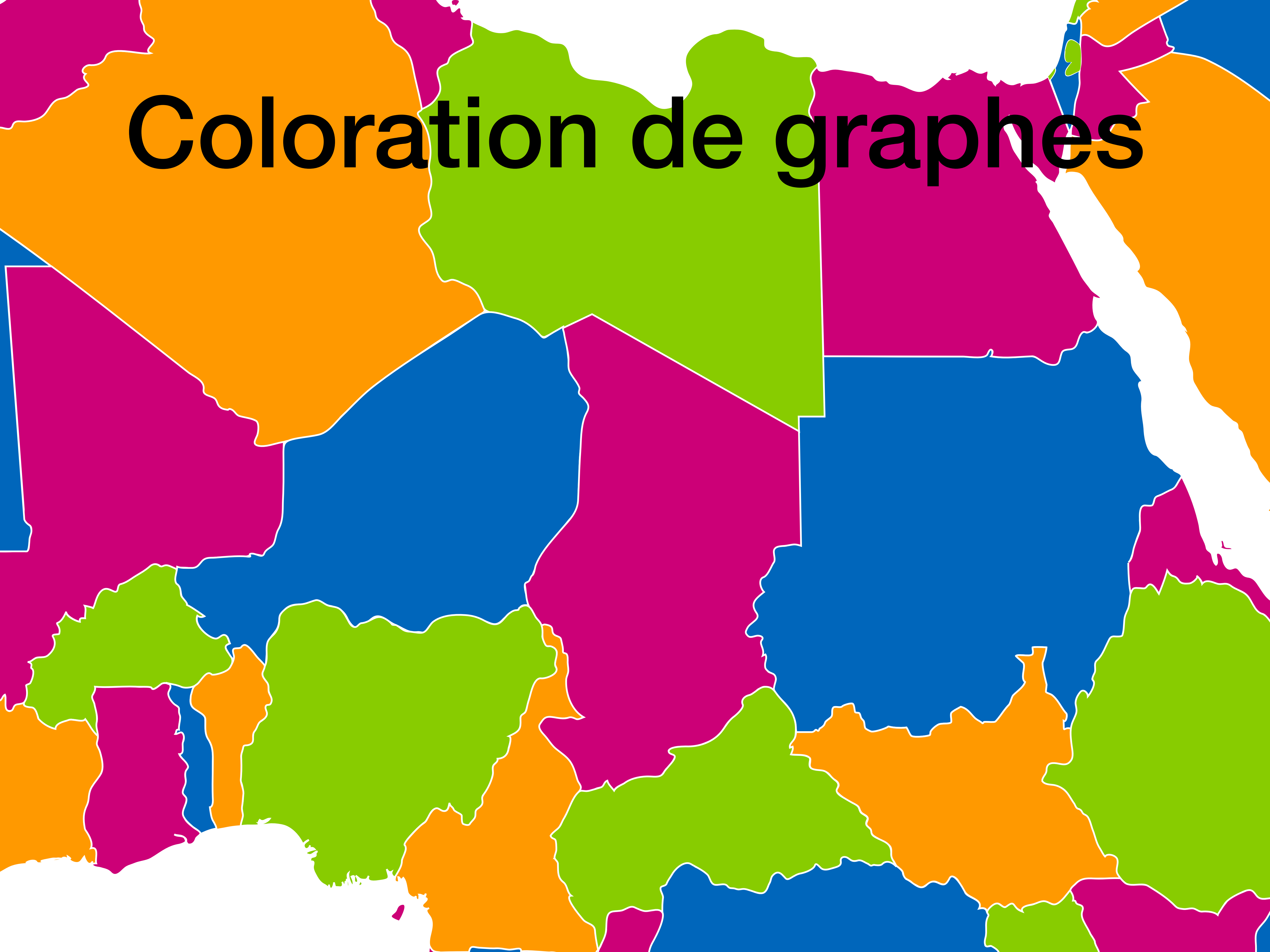
Théorème des quatre couleurs

On peut colorer n’importe quelle carte
avec un maximum de 4 couleurs
de sorte que les pays (régions, etc.) adjacents
reçoivent toujours deux couleurs distinctes

Coloration de graphes



Coloration de graphes



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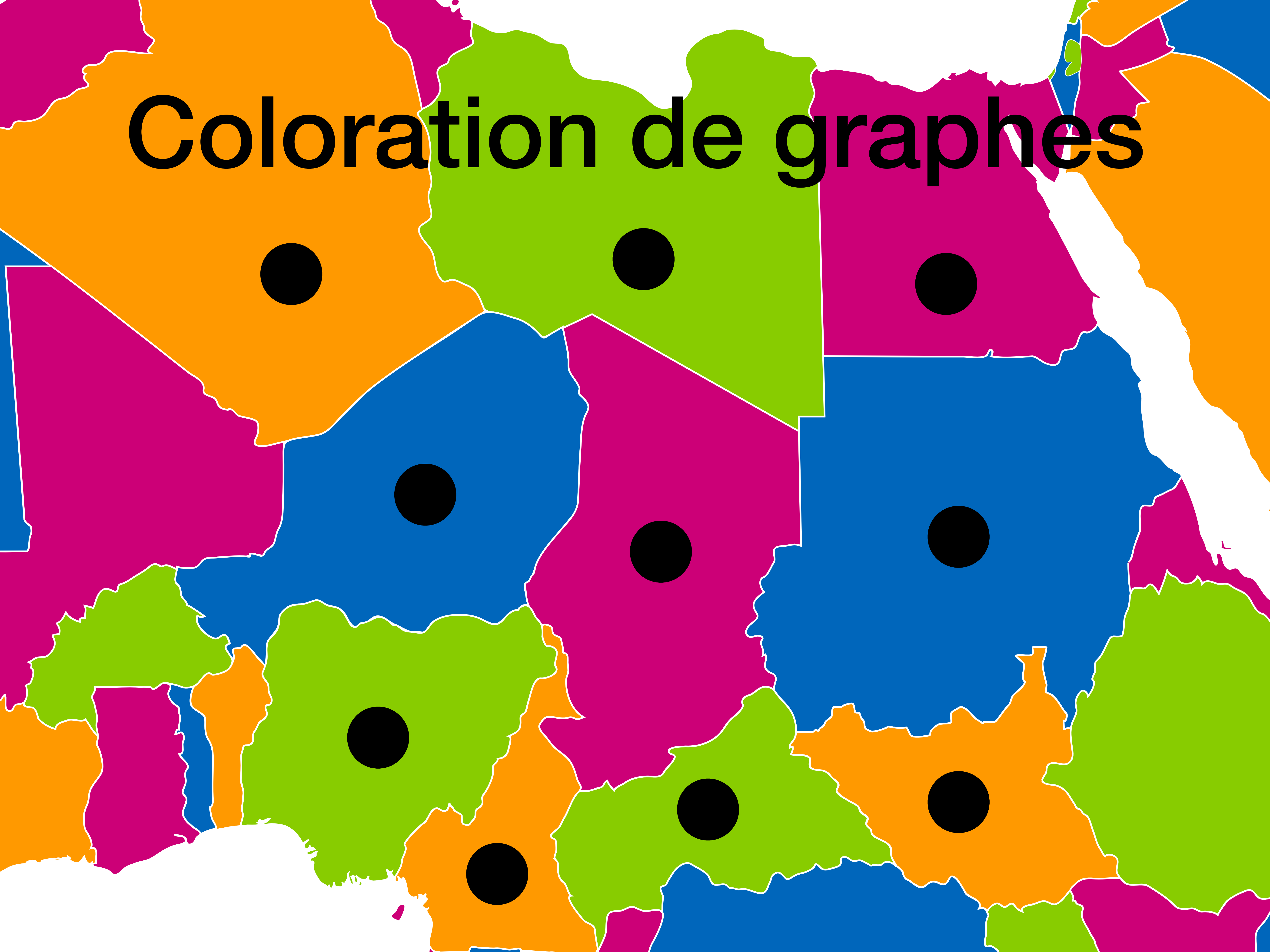
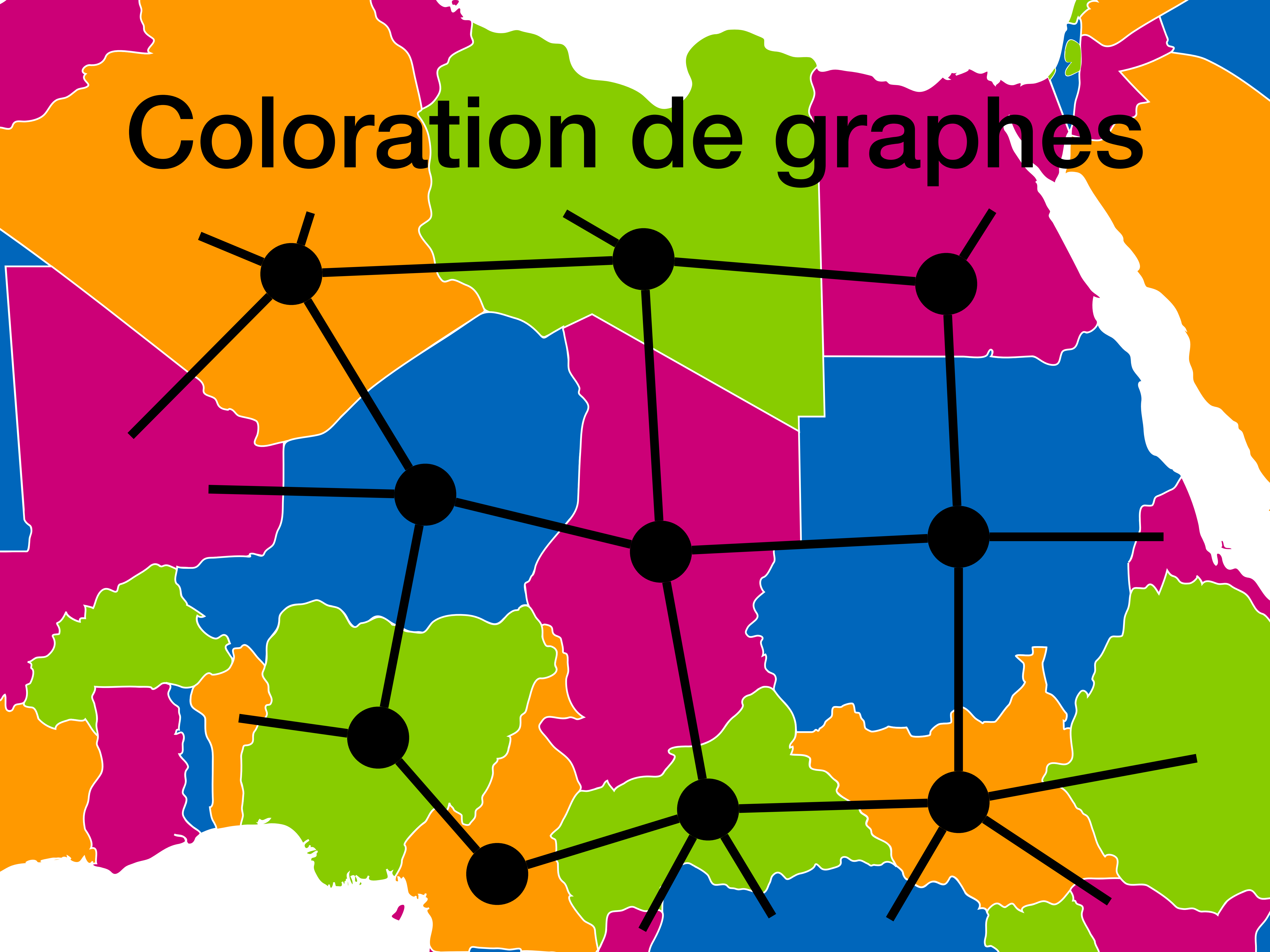
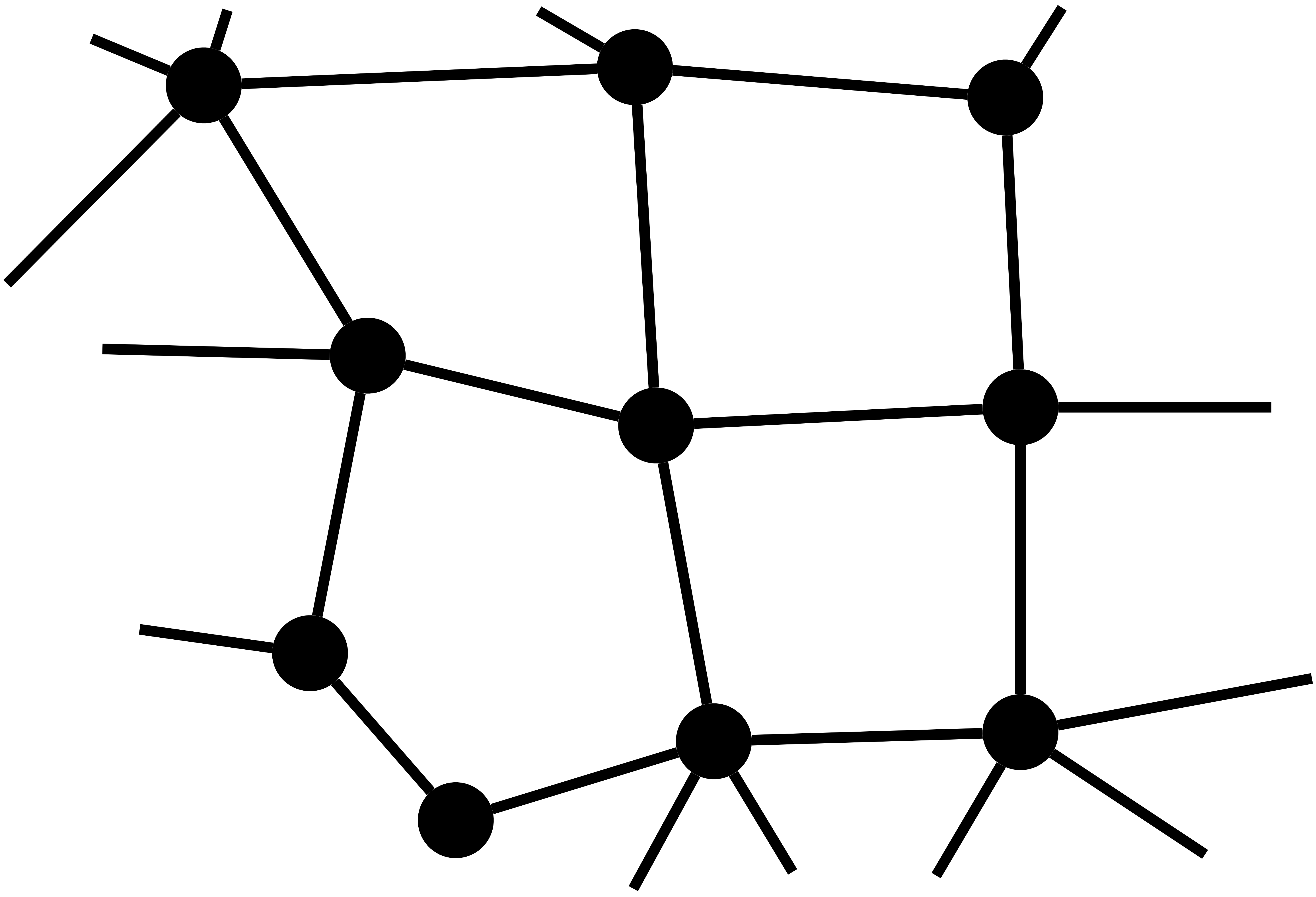


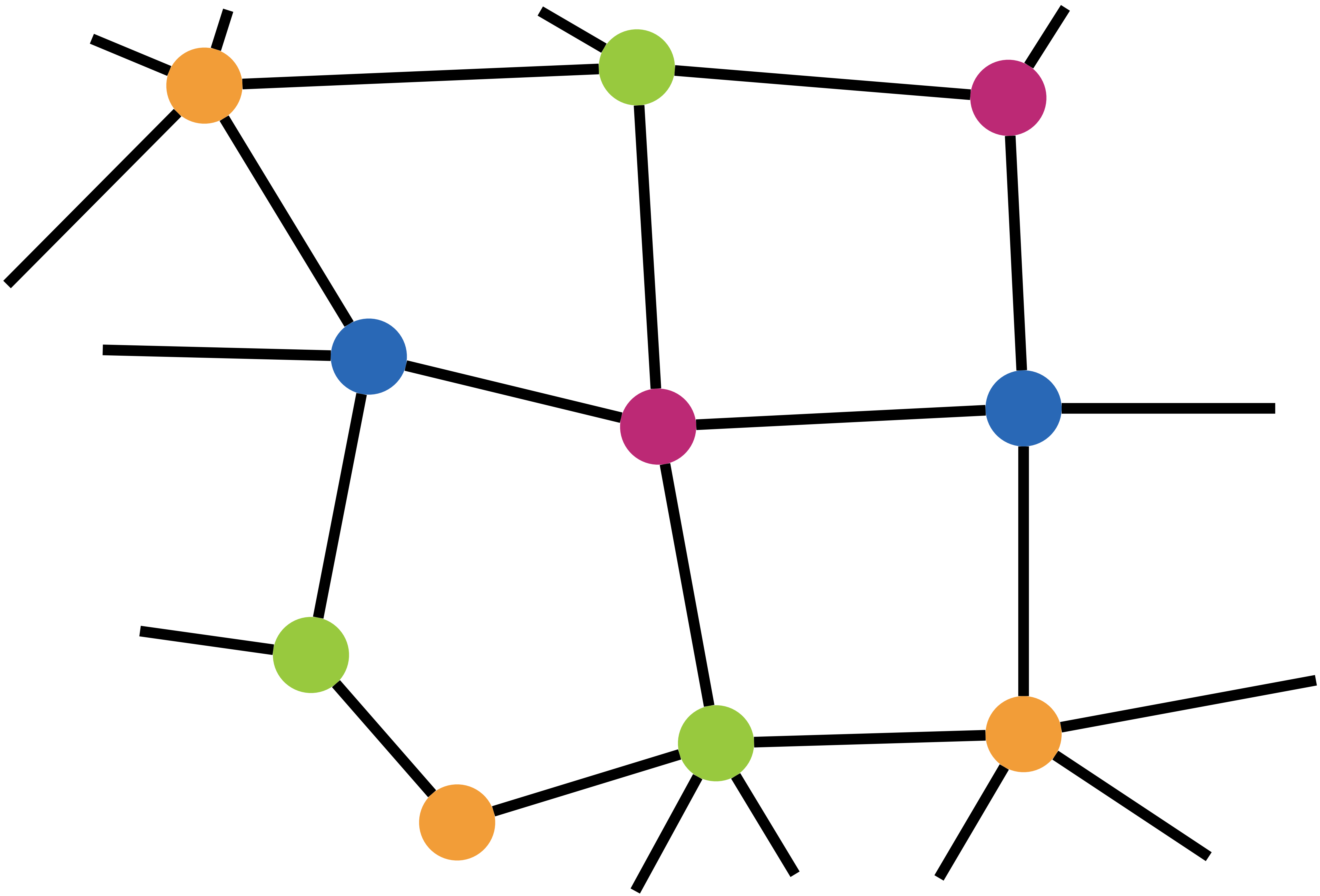
Coloration de graphes



Coloration de graphes



Coloration de graphes

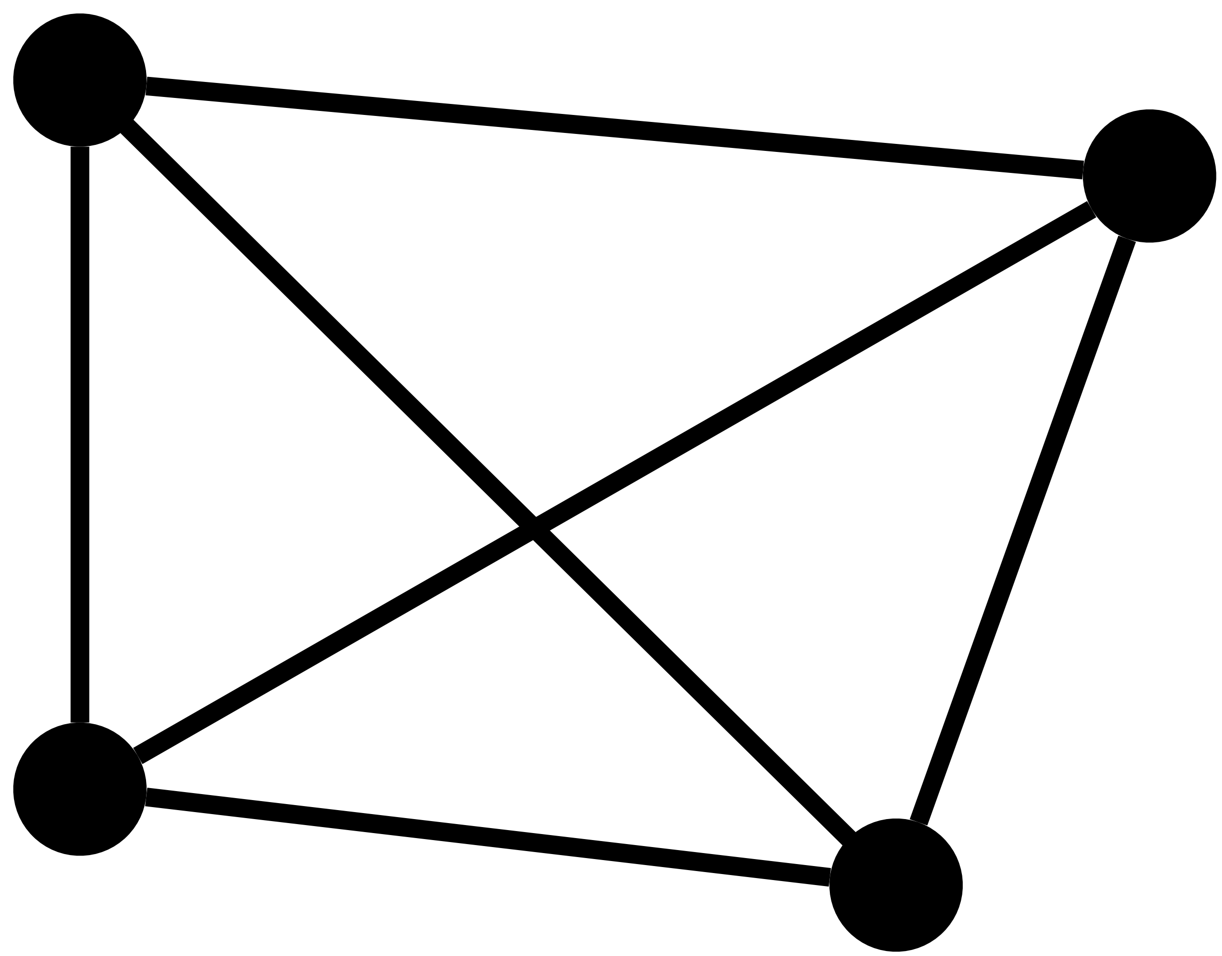


Graphes planaires

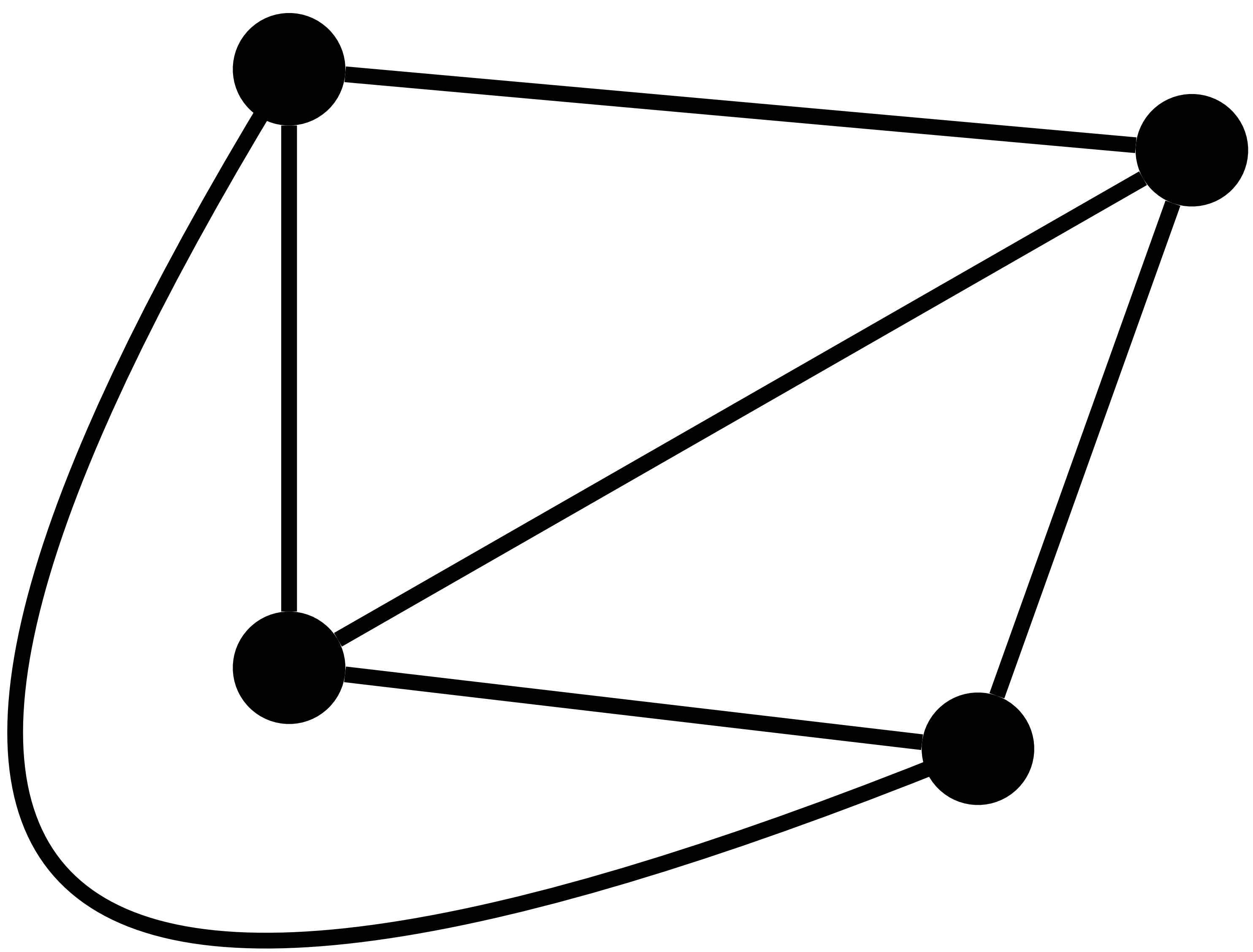
- Un graphe (non orienté) planaire est un graphe qu’on peut dessiner sur un plan (une feuille de papier) sans qu’aucune arête n’en croise une autre

- Un graphe non orienté G est planaire si et seulement si il existe une carte ayant G comme graphe associé

Graphes qui ne semblent pas planaires



Graphes qui ne semblent pas planaires



Théorème des quatre couleurs

On peut colorer les sommets de n’importe
quel graphe non orienté planaire
avec un maximum de 4 couleurs
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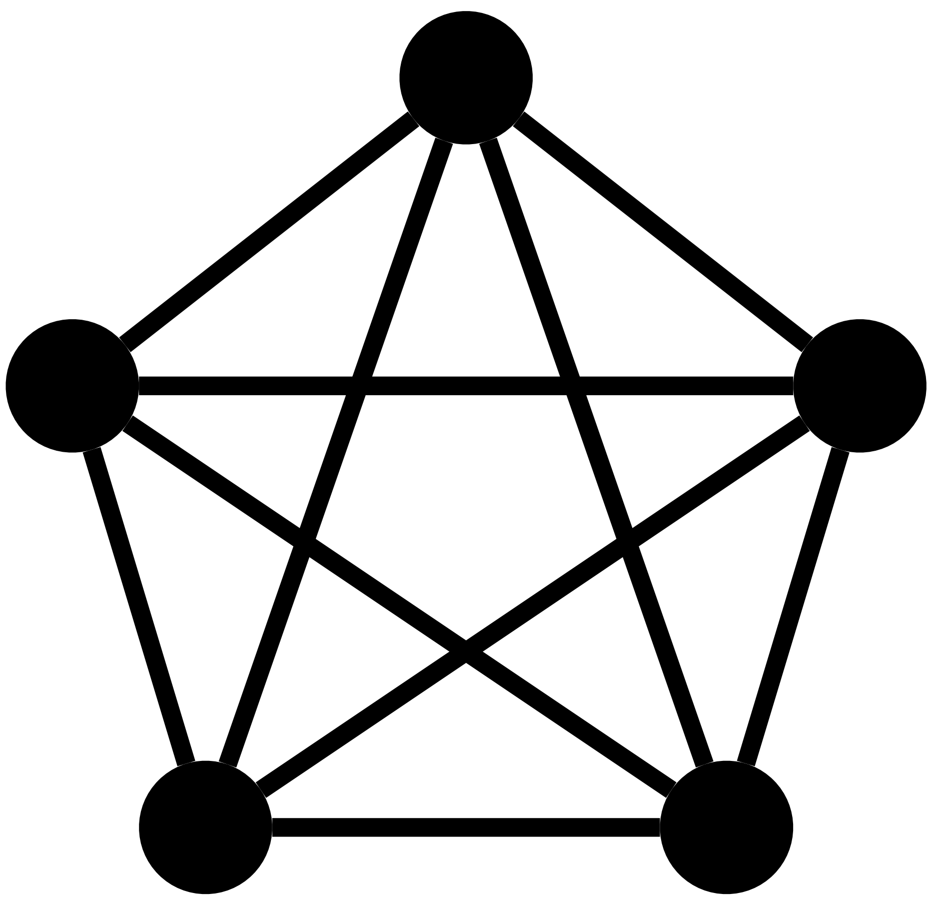
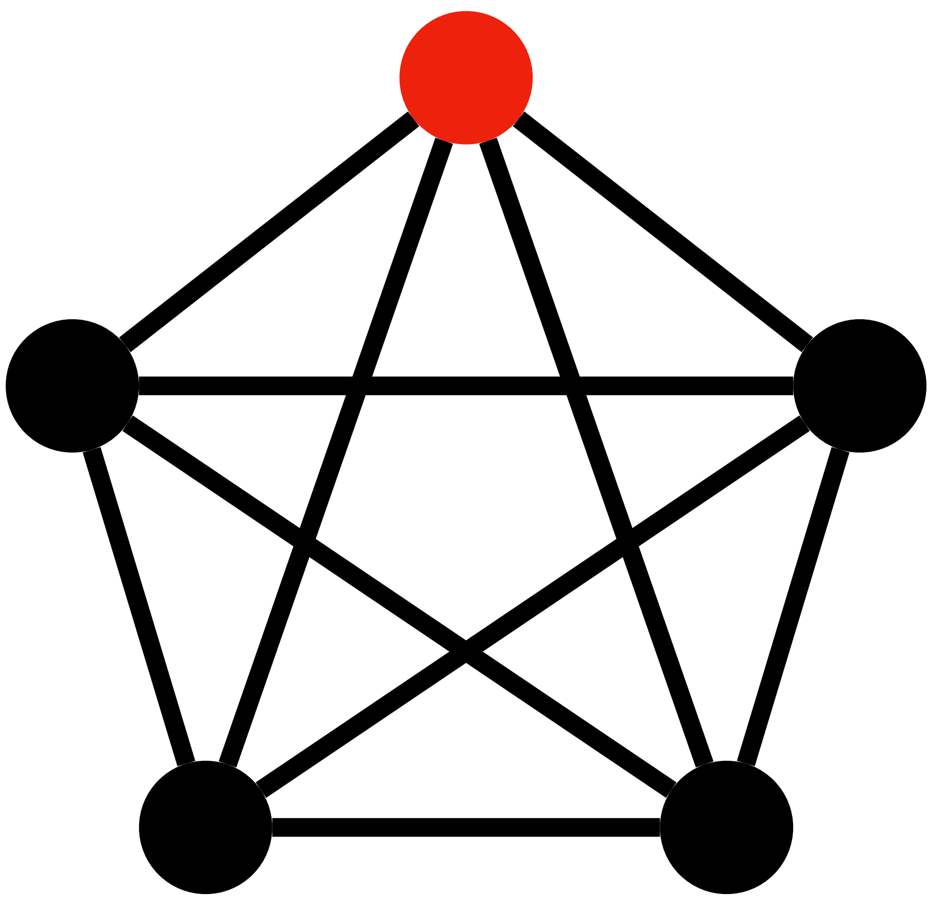
reçoivent toujours deux couleurs distinctes

Graphes non planaires



Graphes non planaires



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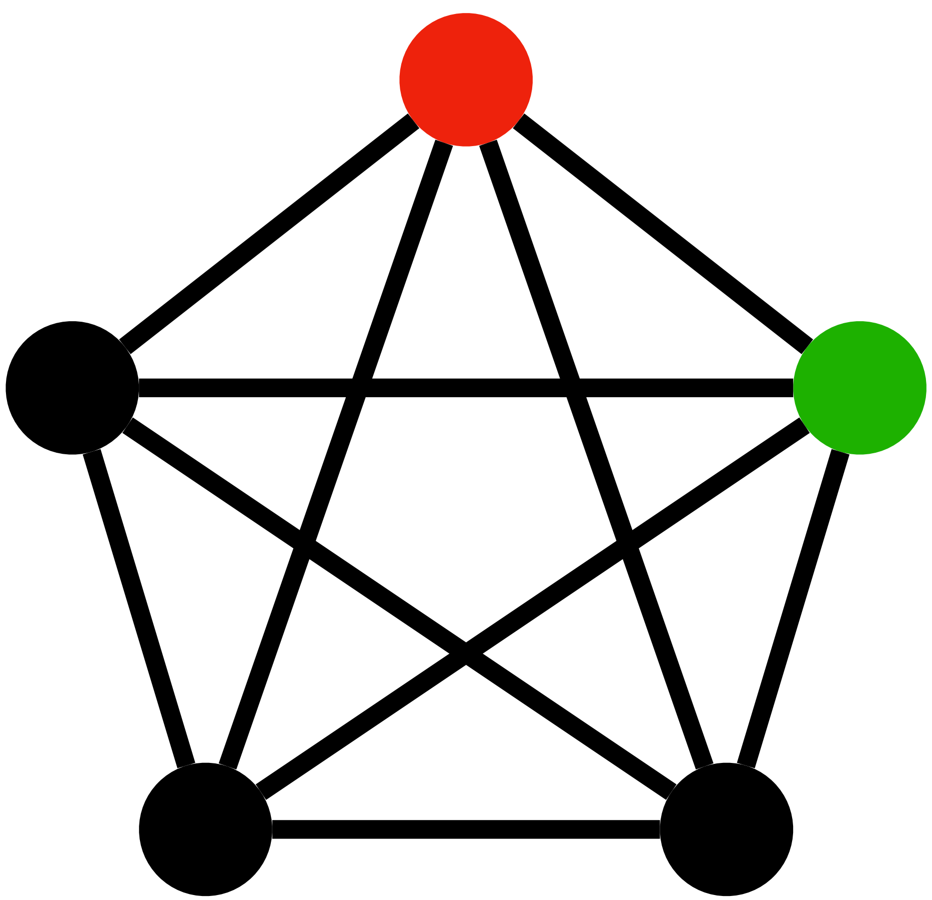

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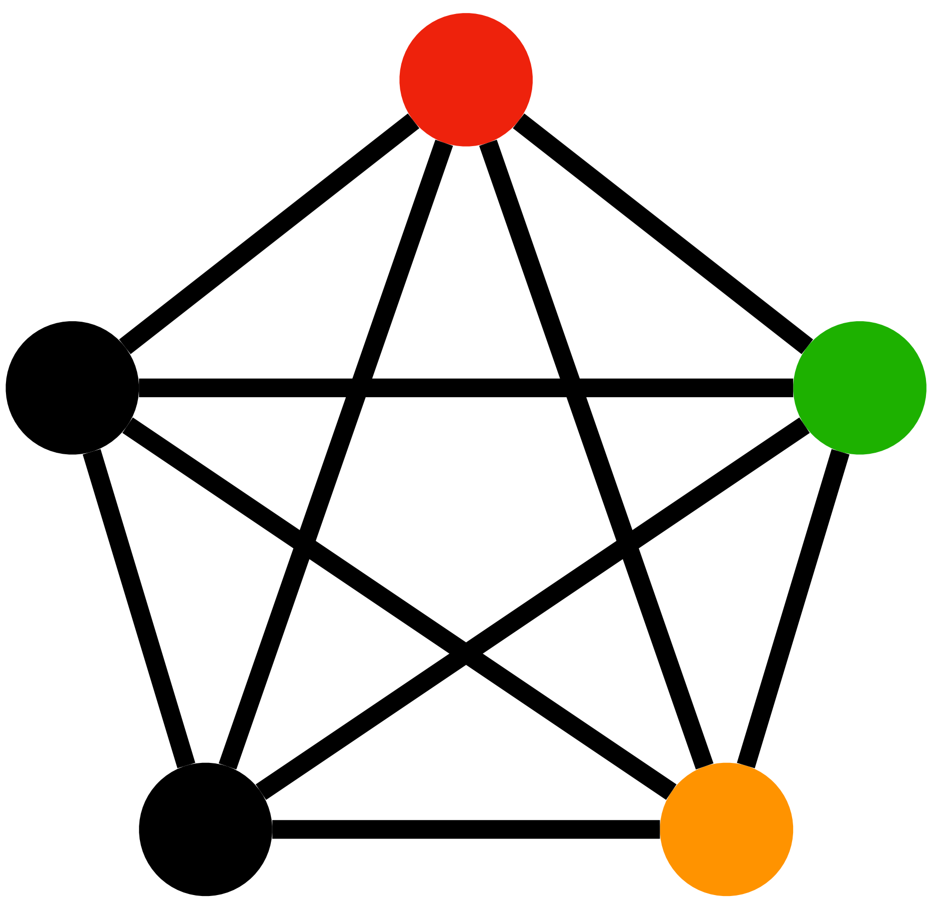

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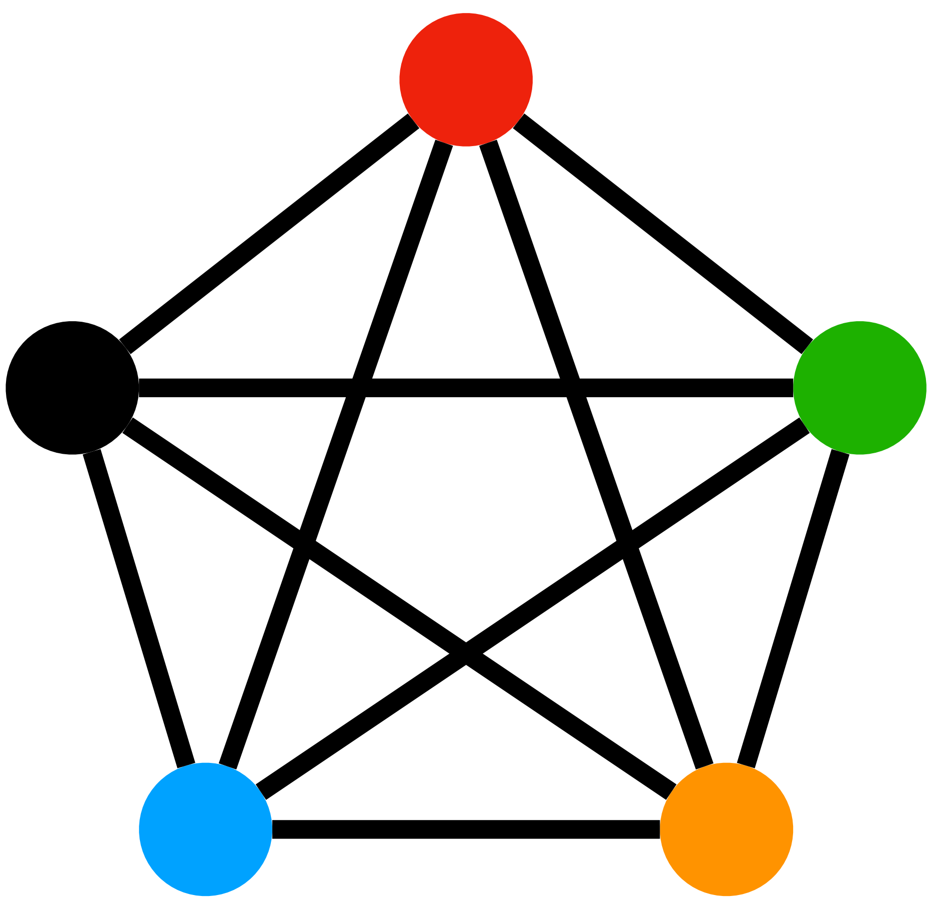

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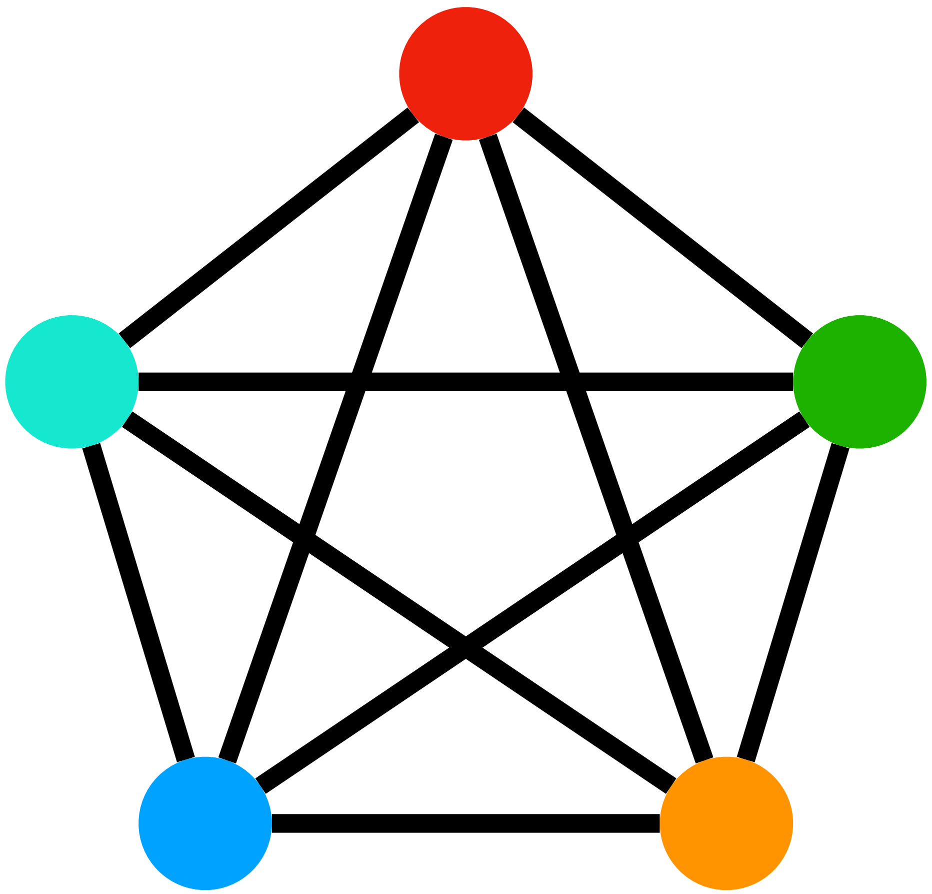

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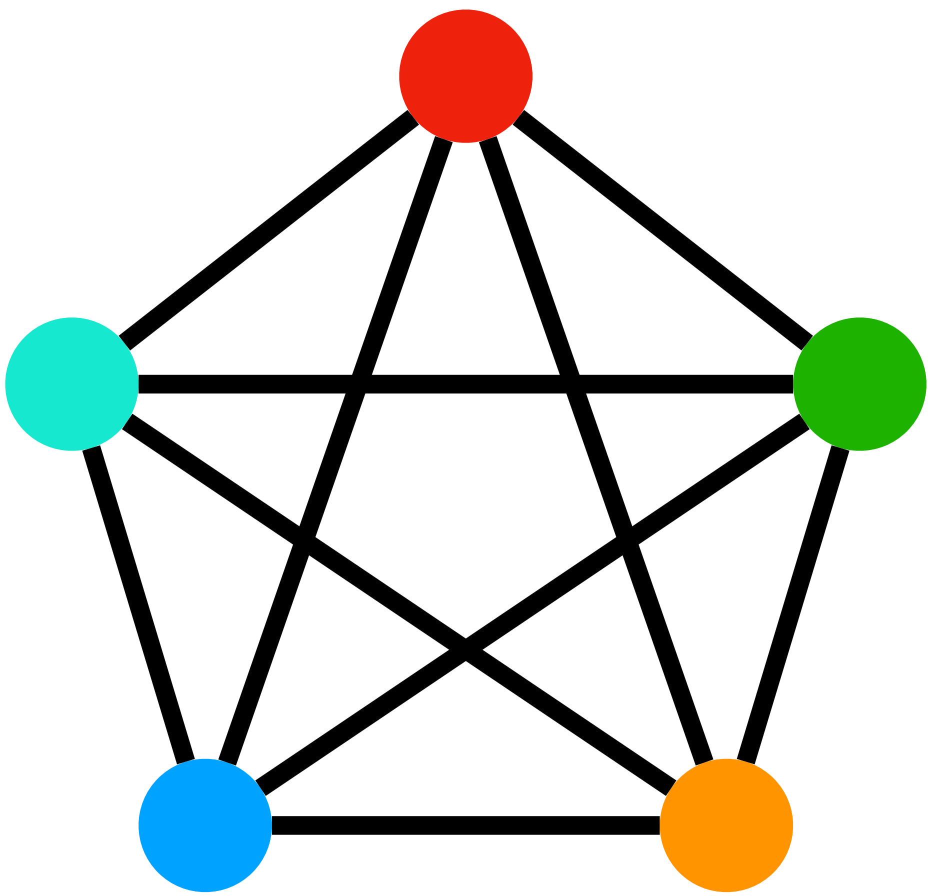

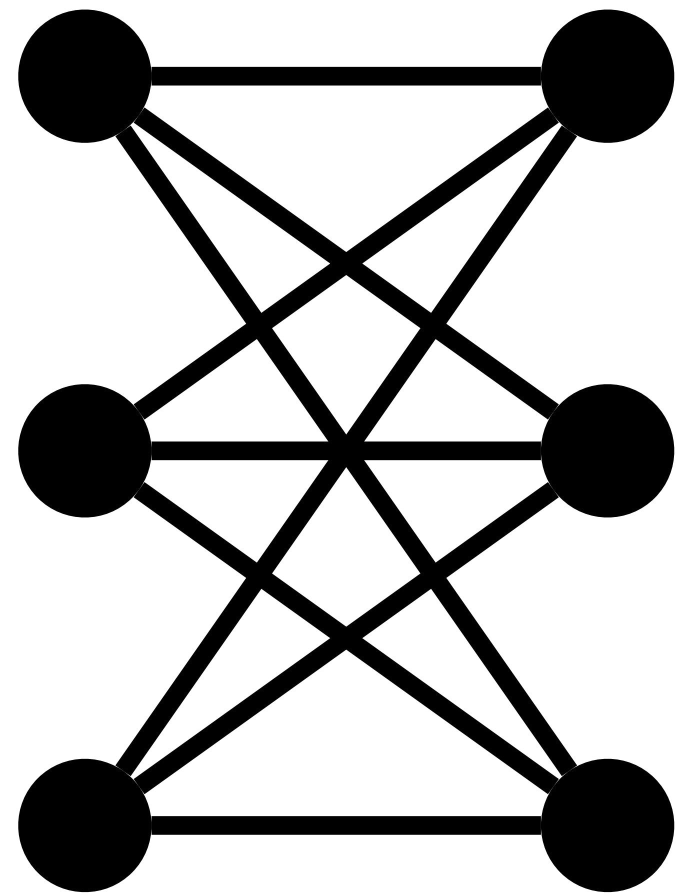

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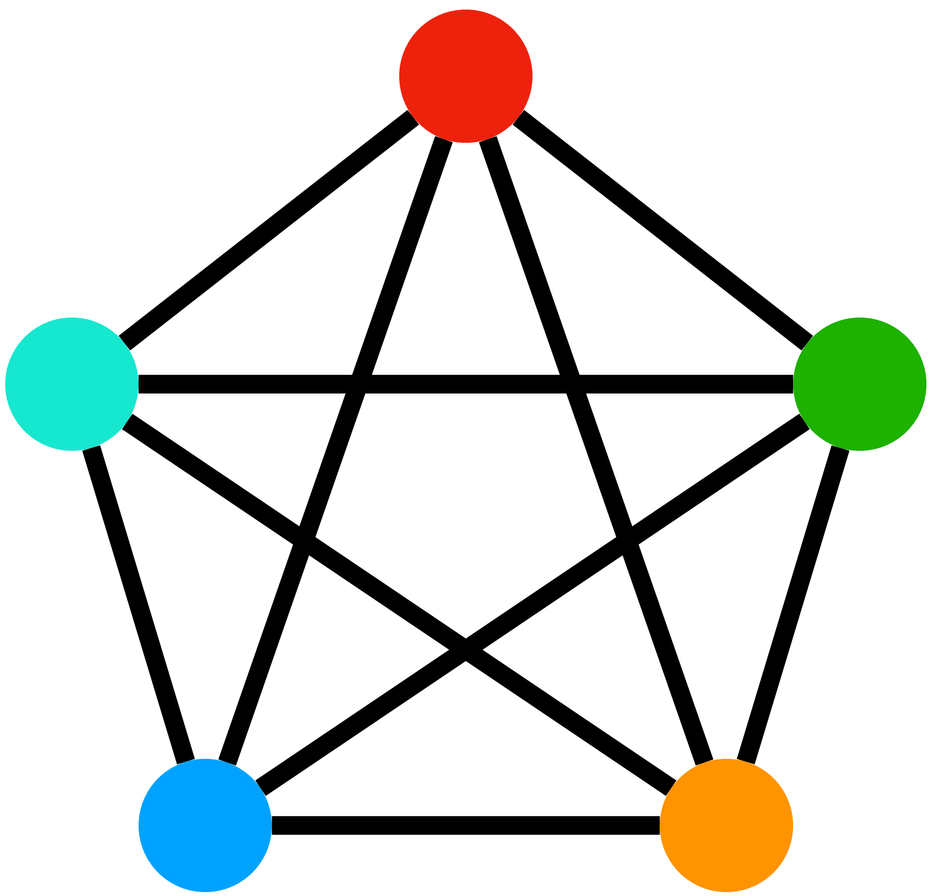

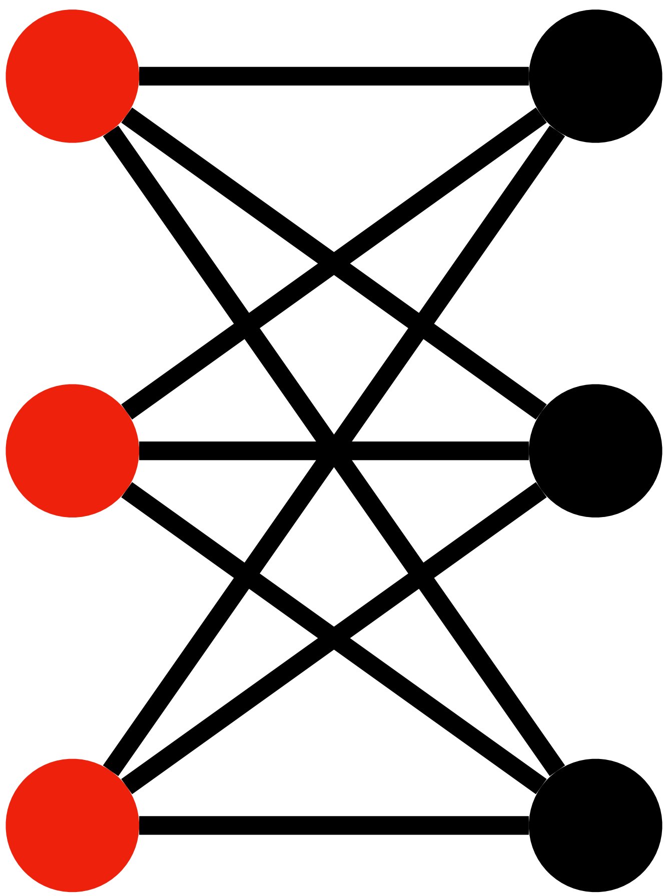

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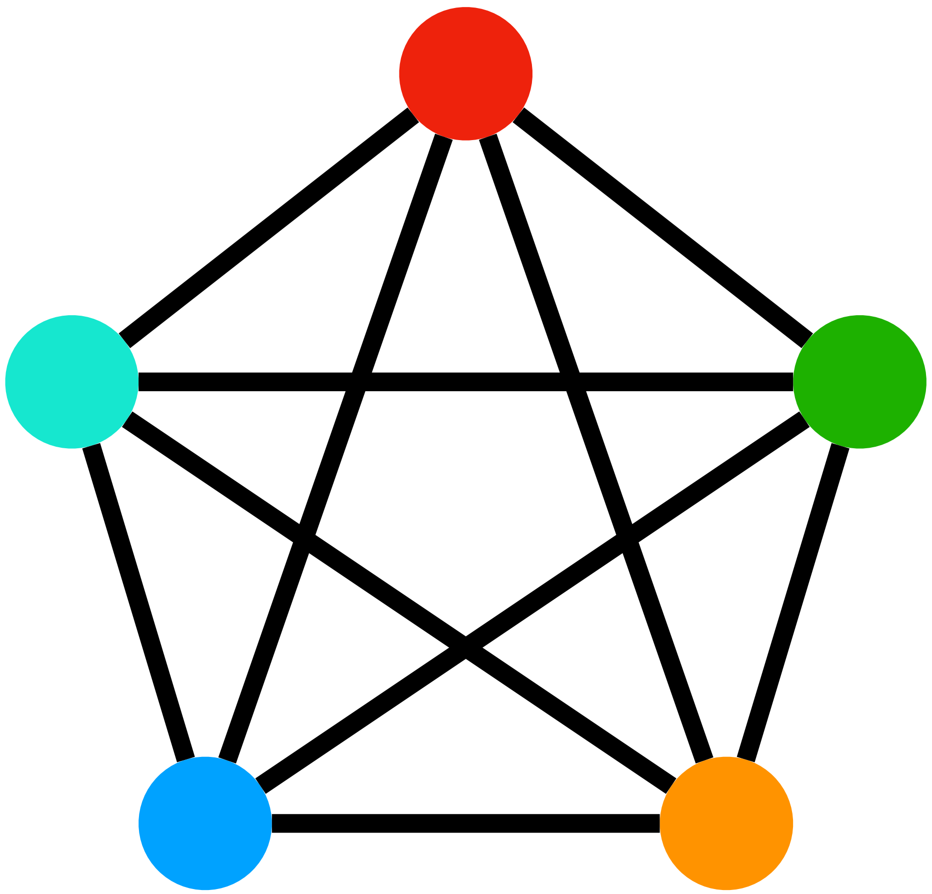
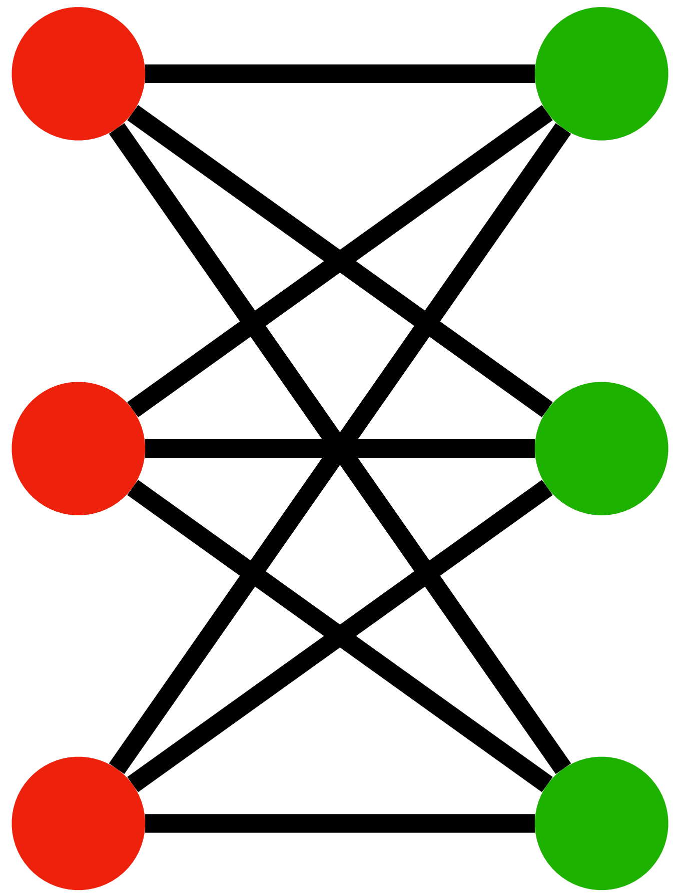
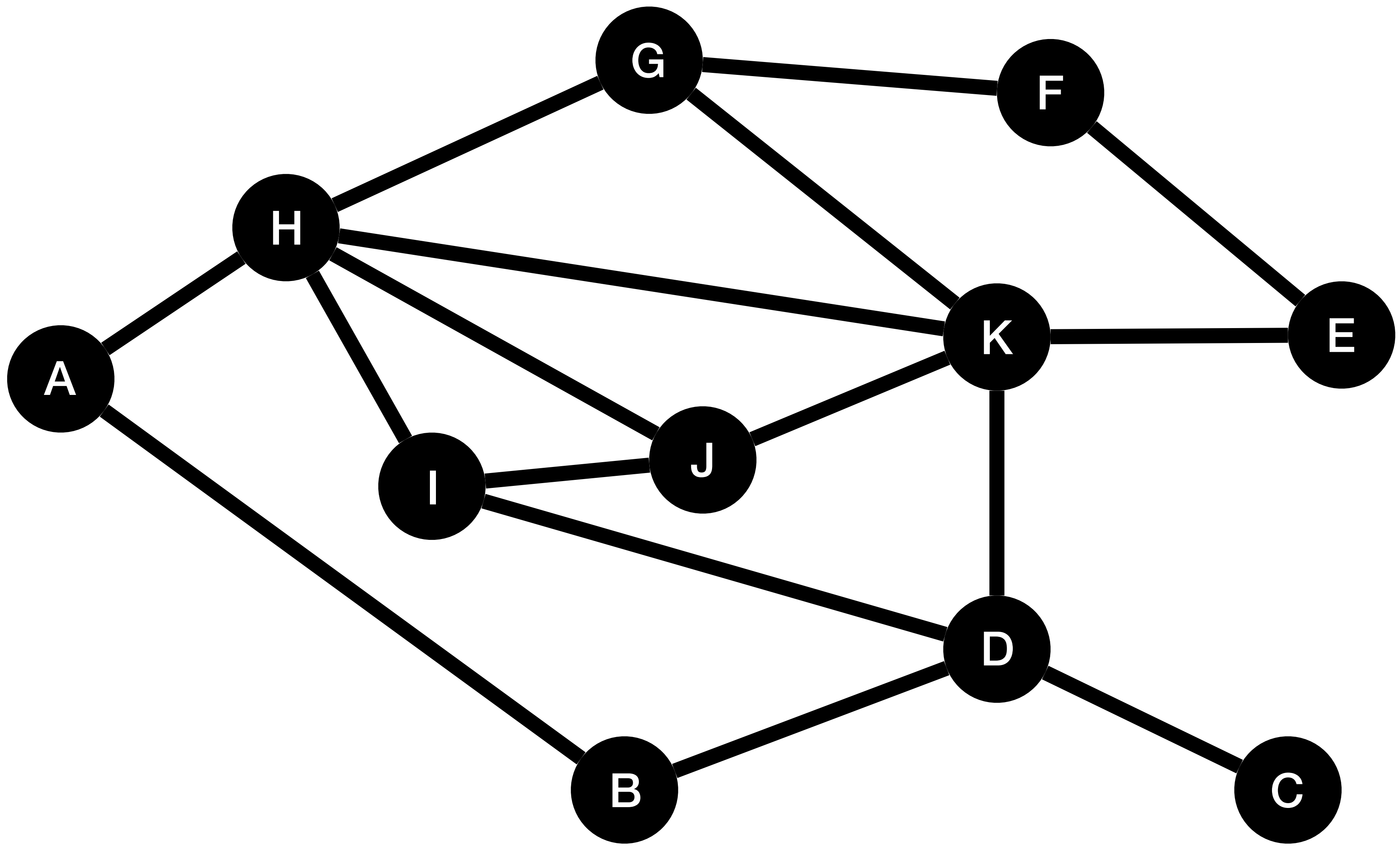
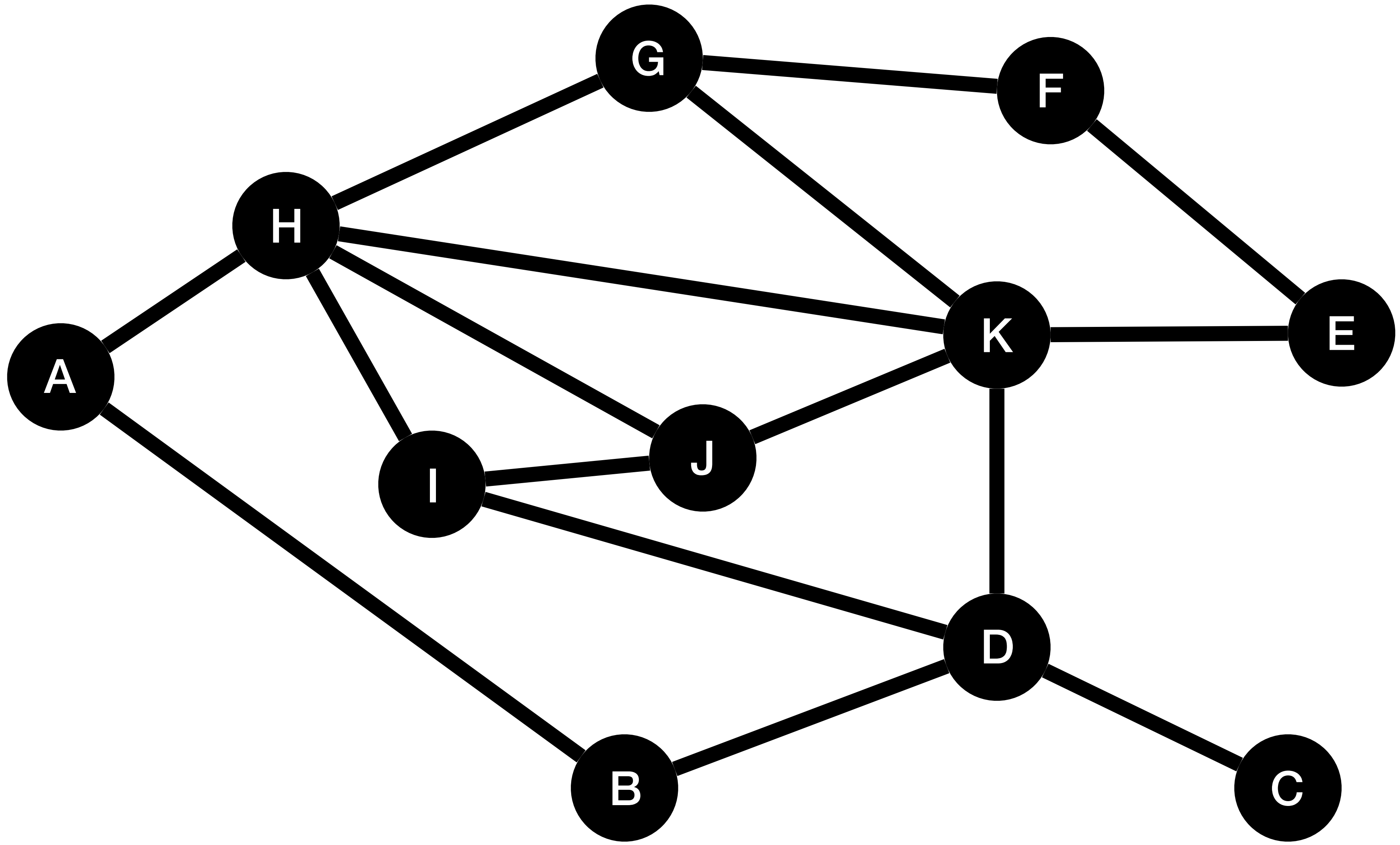


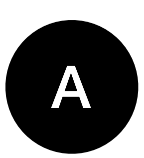


Algorithme de Welsh-Powell



Algorithme de Welsh-Powell





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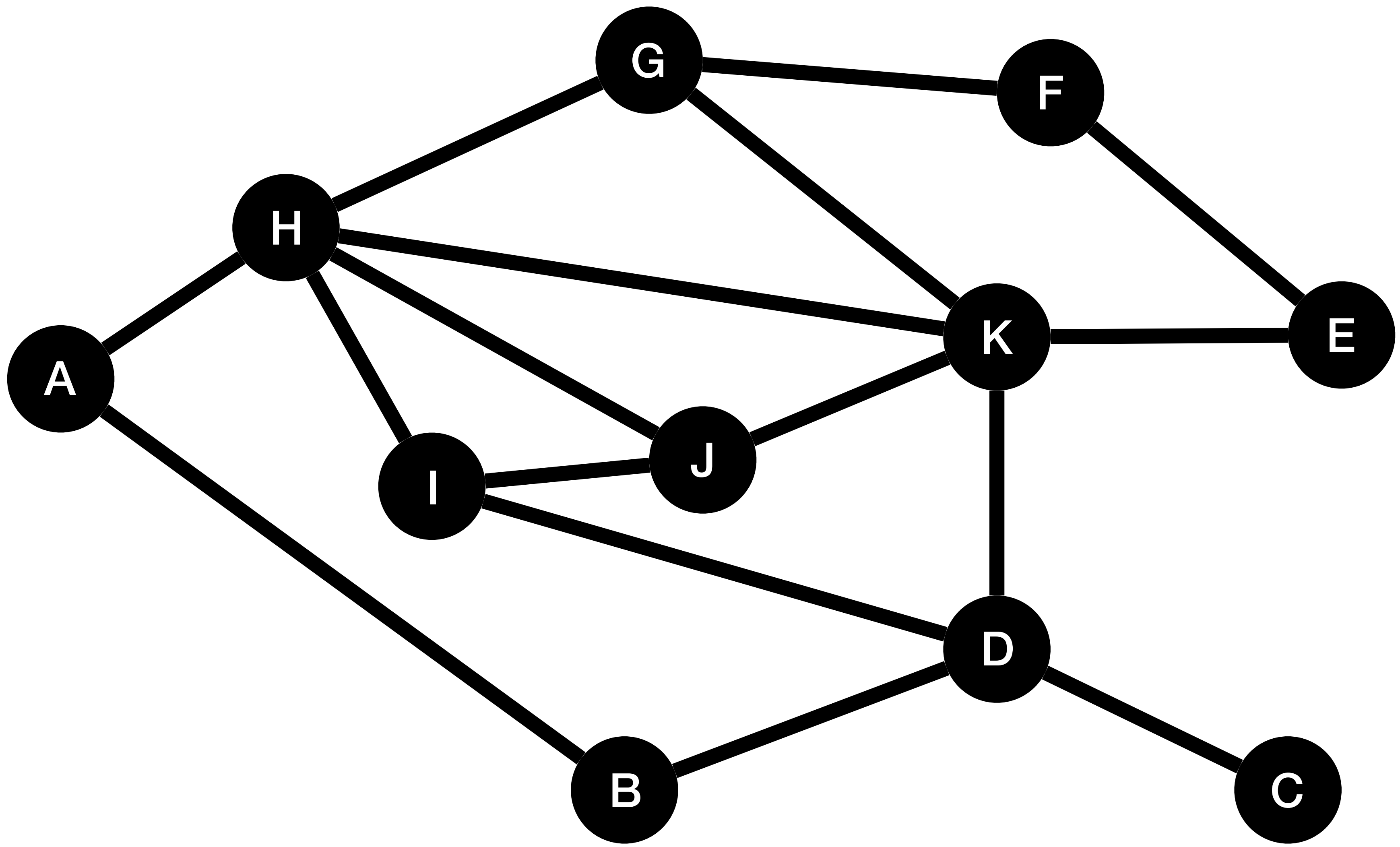
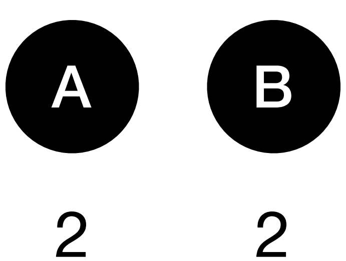
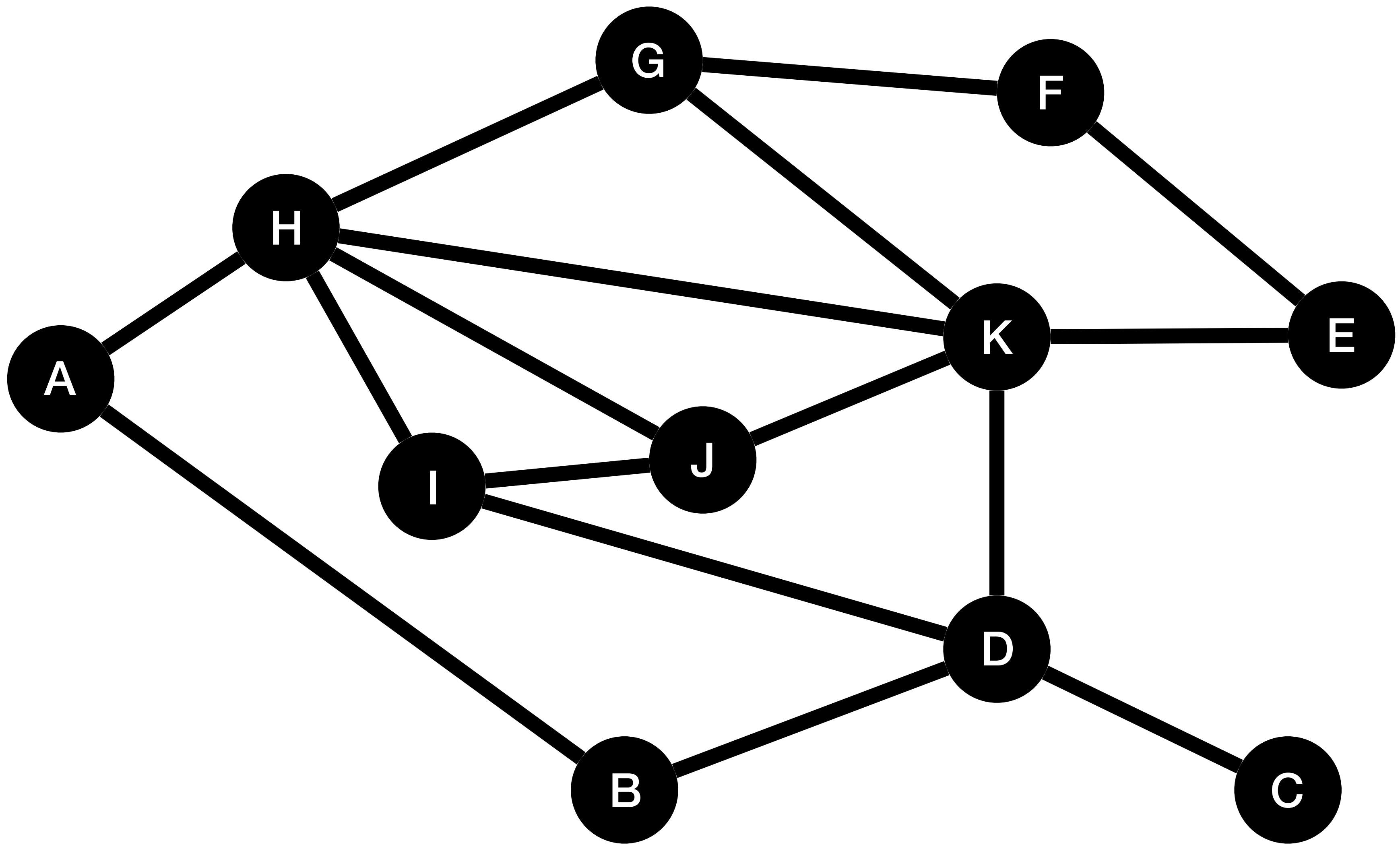
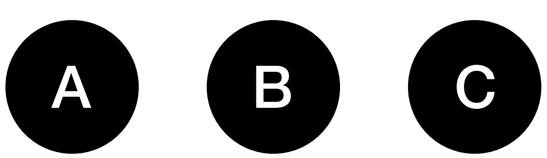
Algorithme de Welsh-Powell





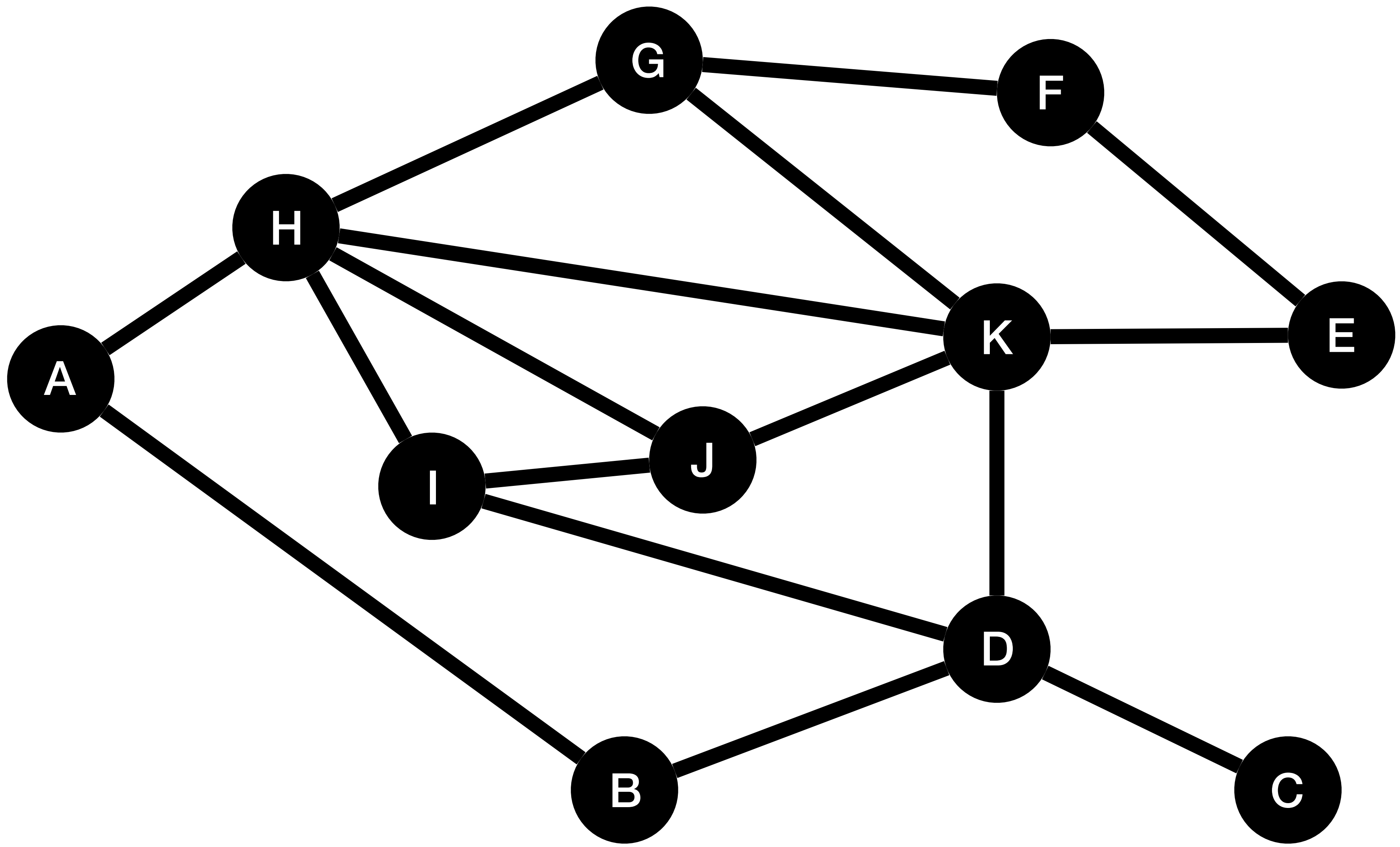
Algorithme de Welsh-Powell

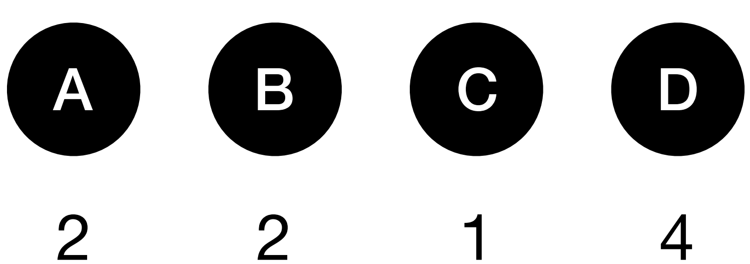




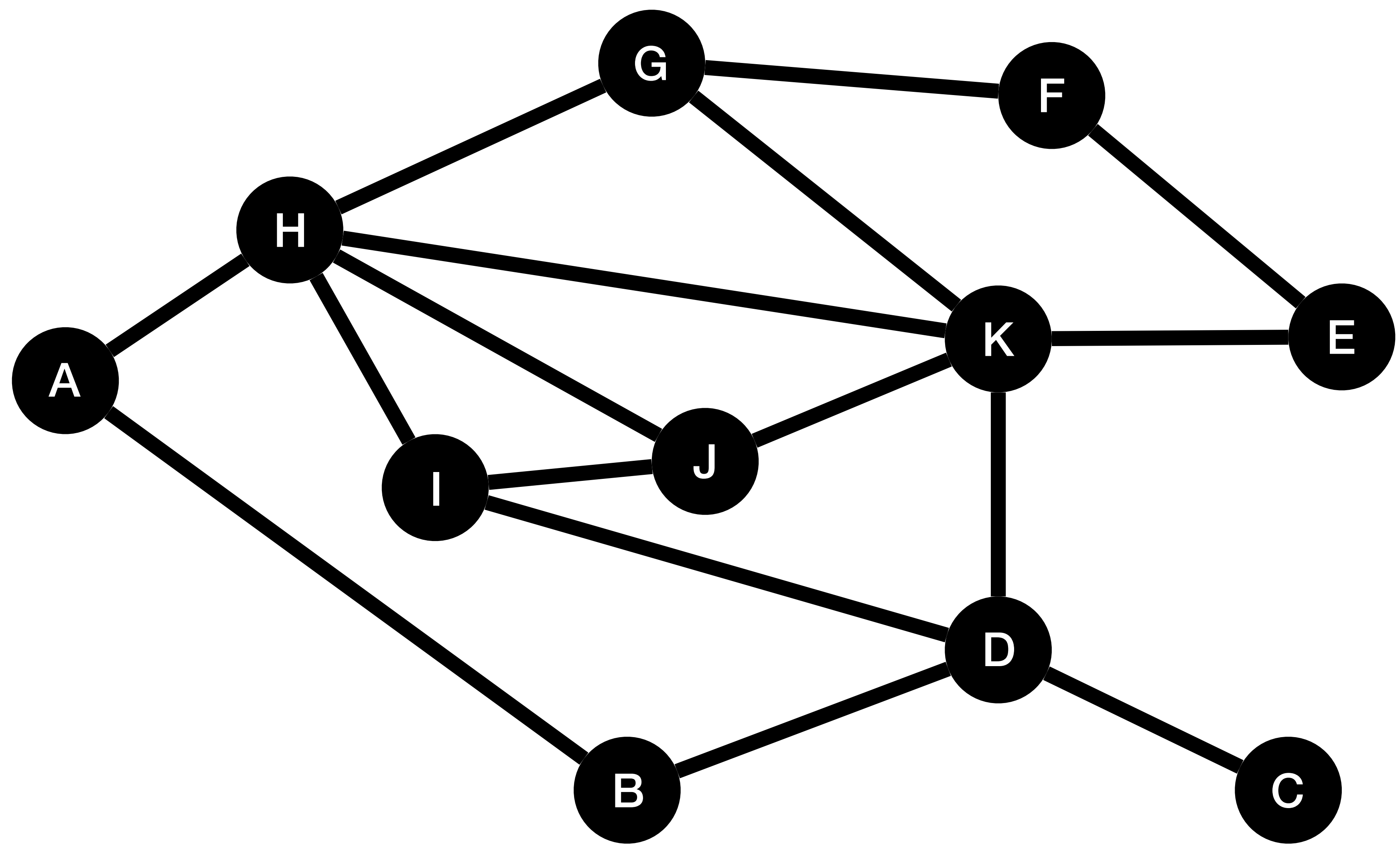
2	2	1

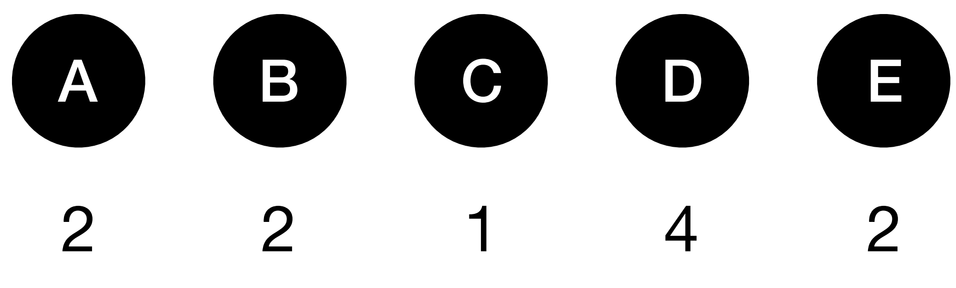
Algorithme de Welsh-Powell



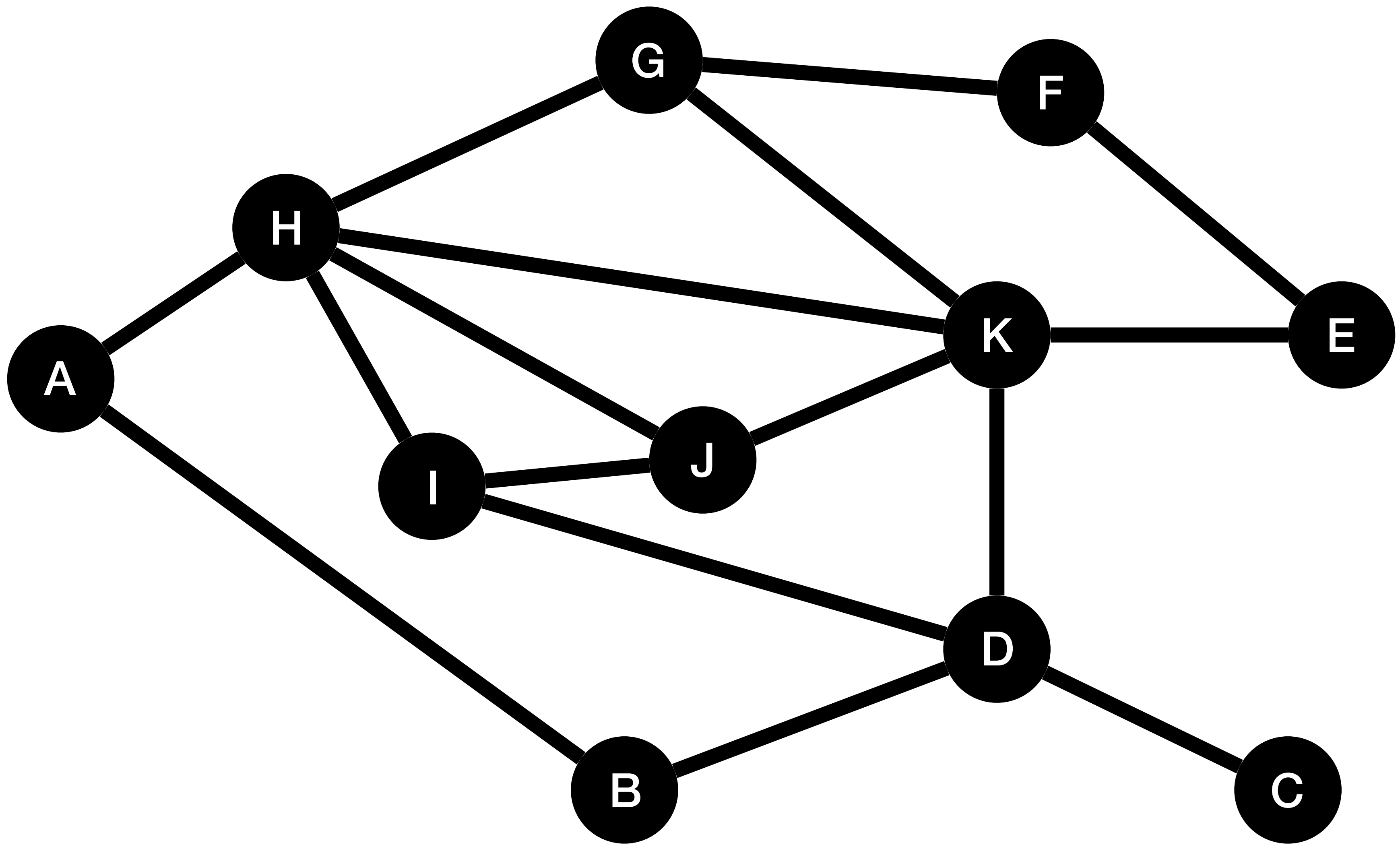


Algorithme de Welsh-Powell



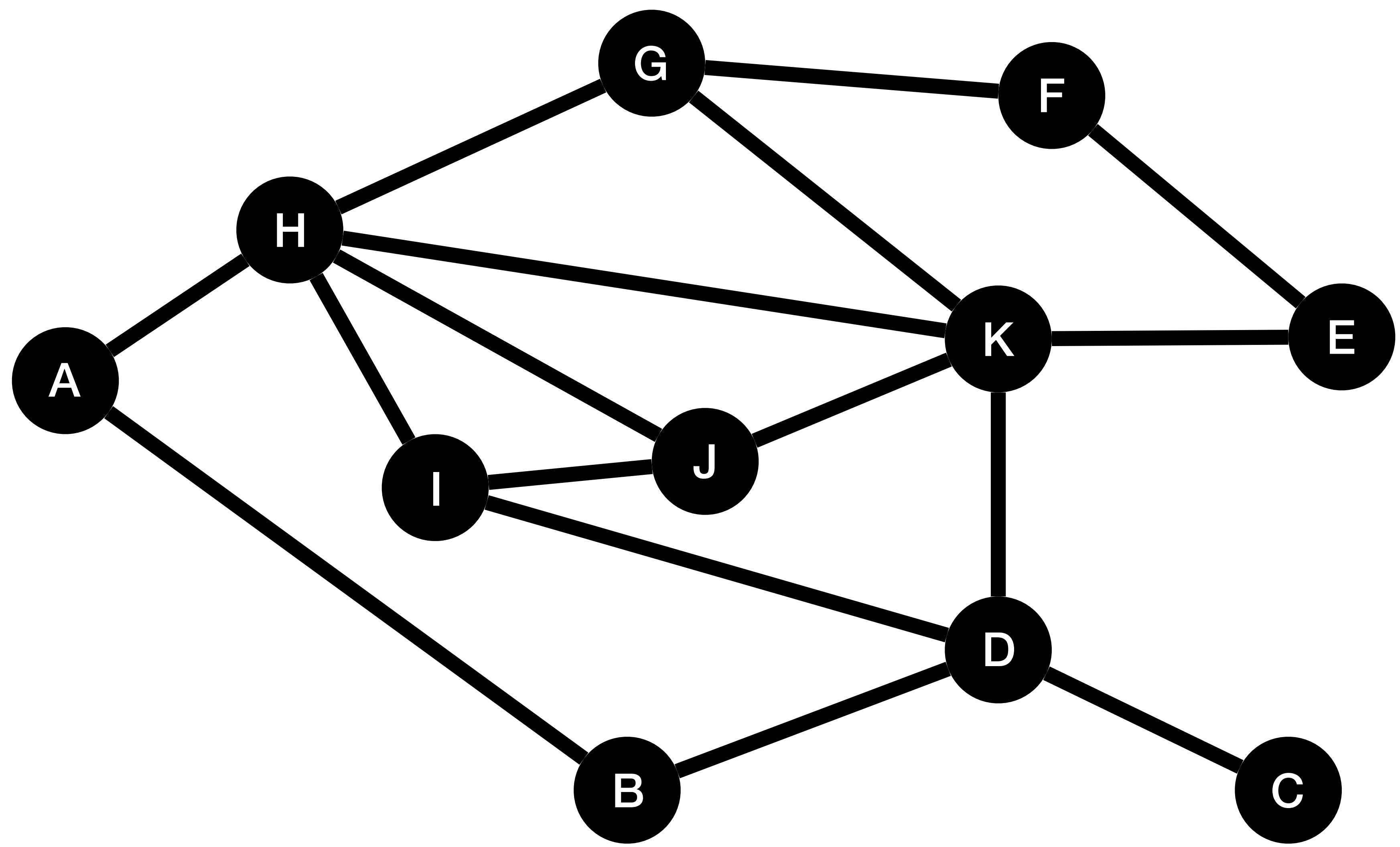


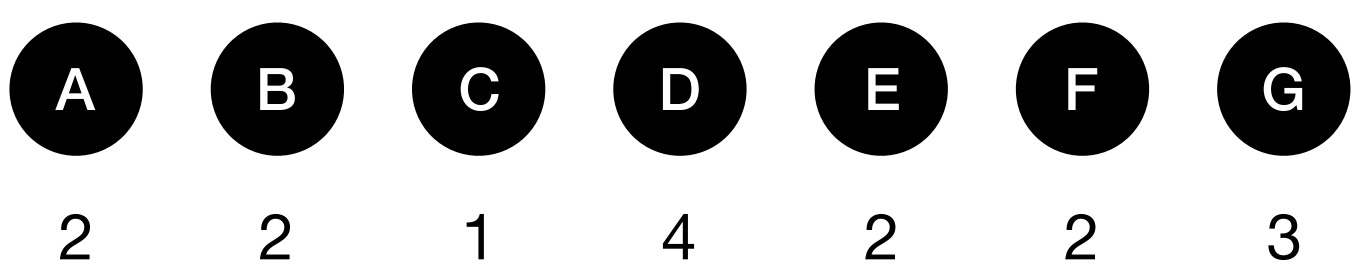
Algorithme de Welsh-Powell



A	B	C	D	E	F
2	2	1	4	2	2

Algorithme de Welsh-Powell





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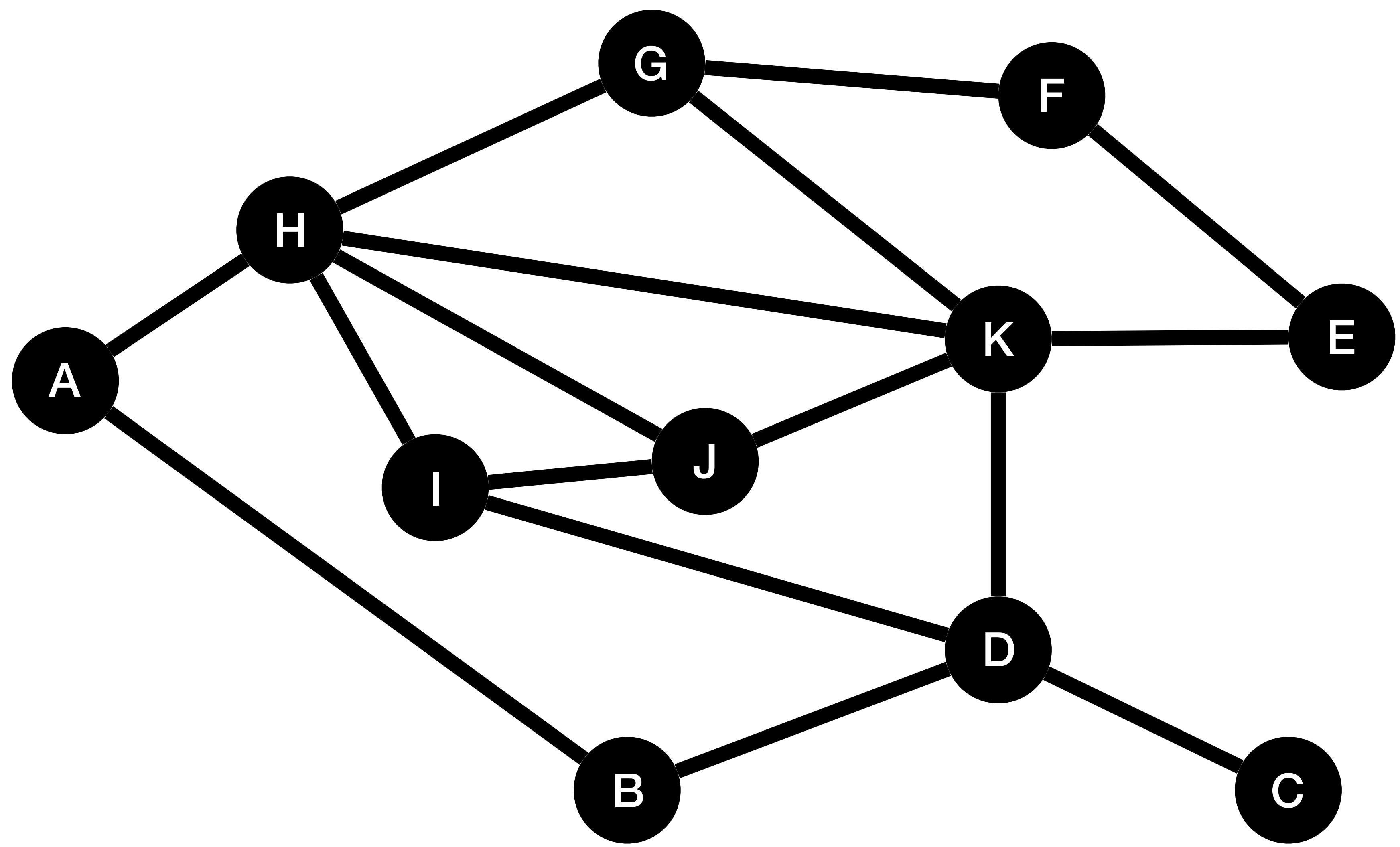

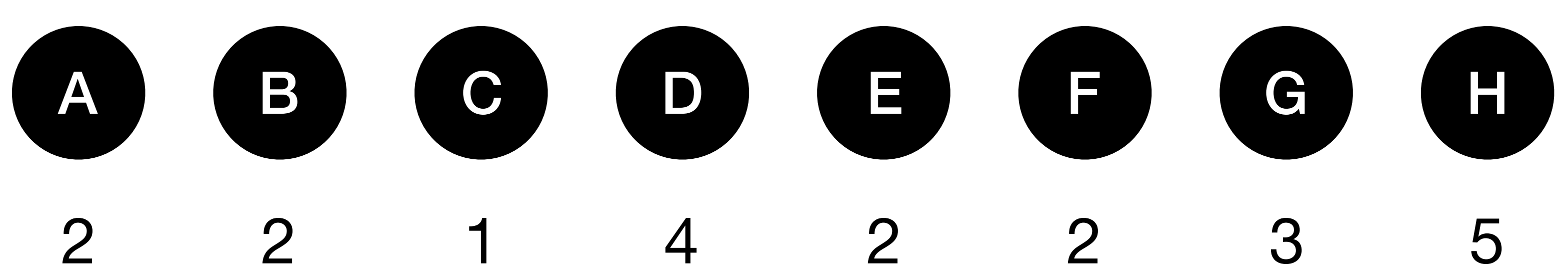

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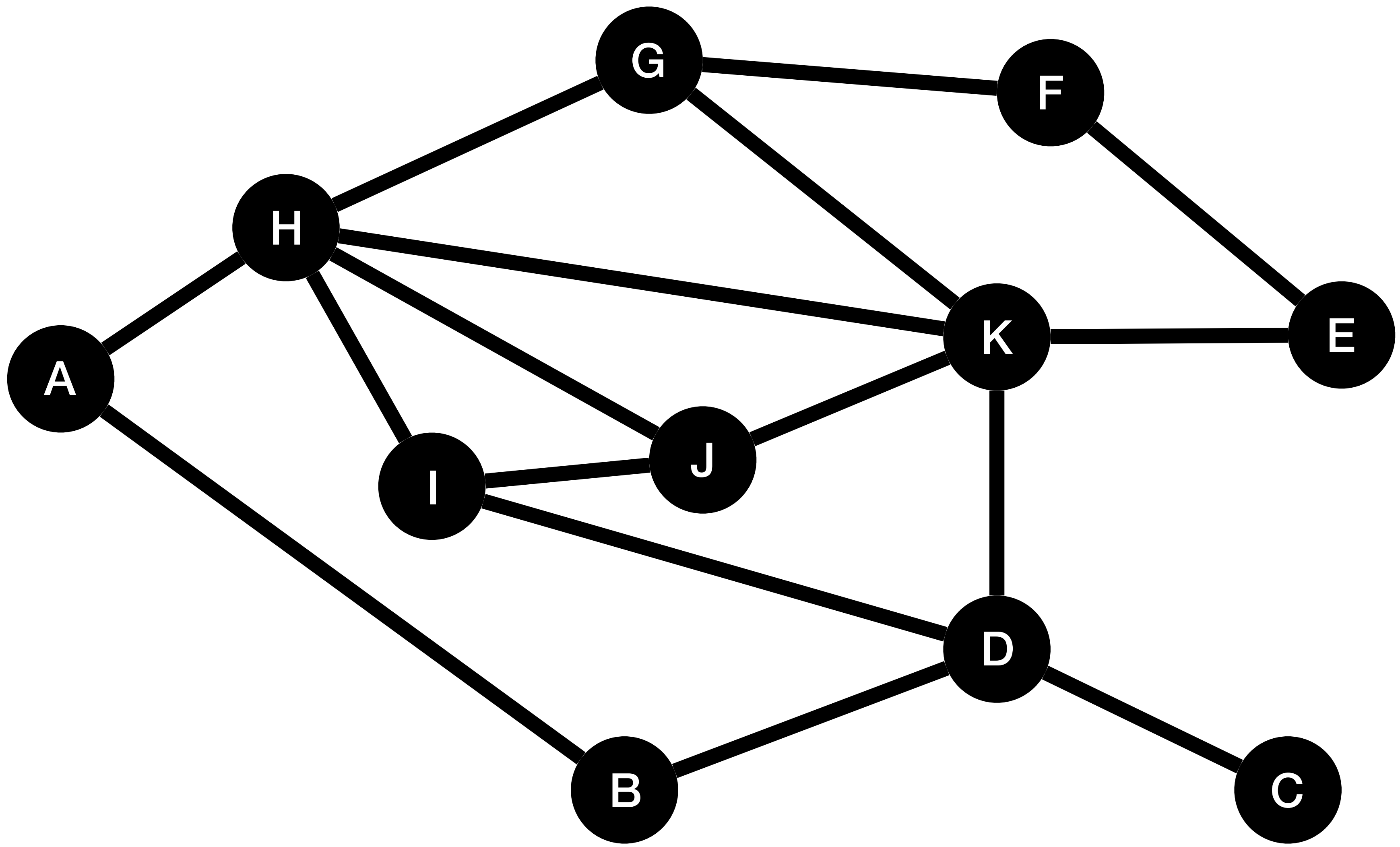
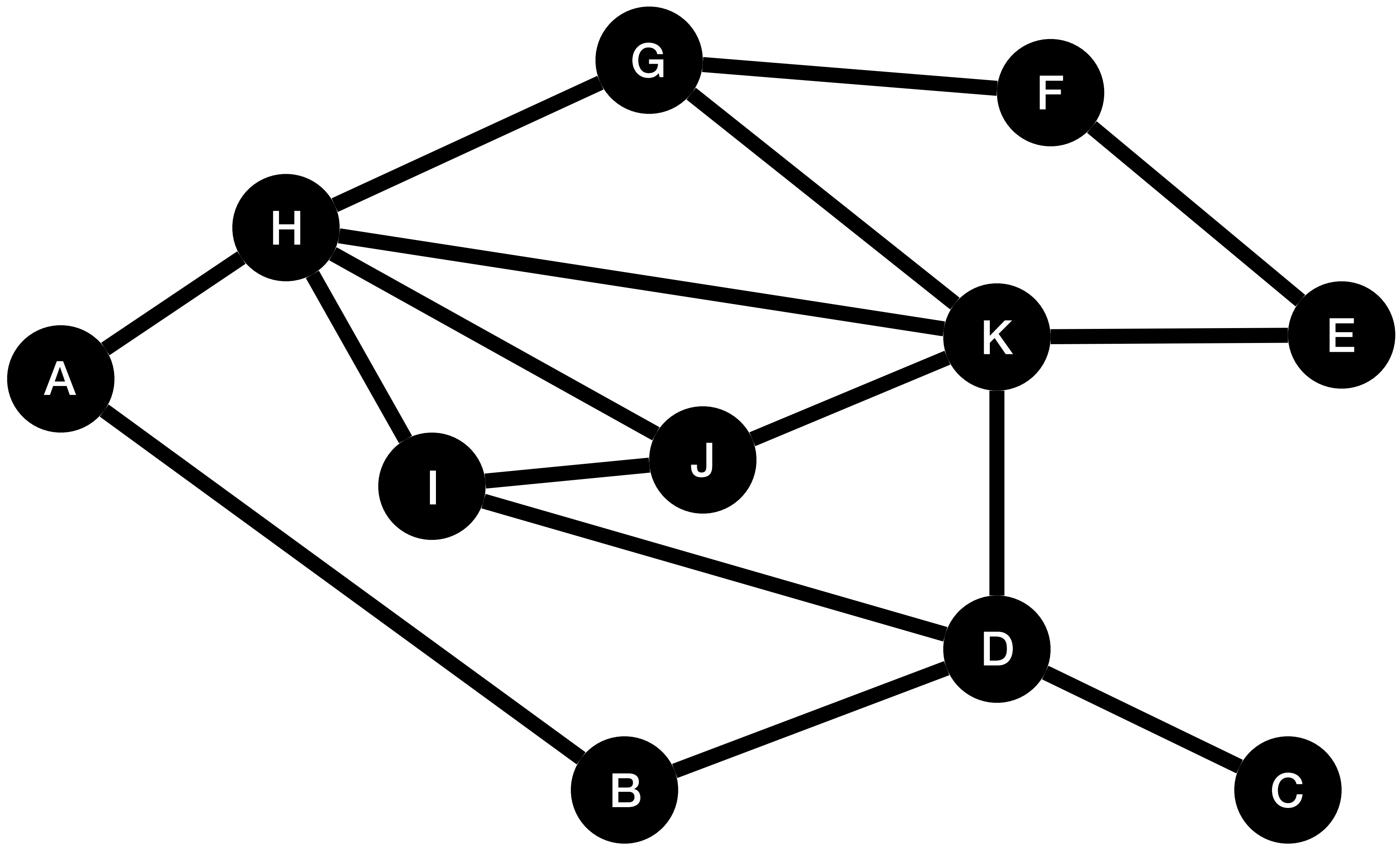


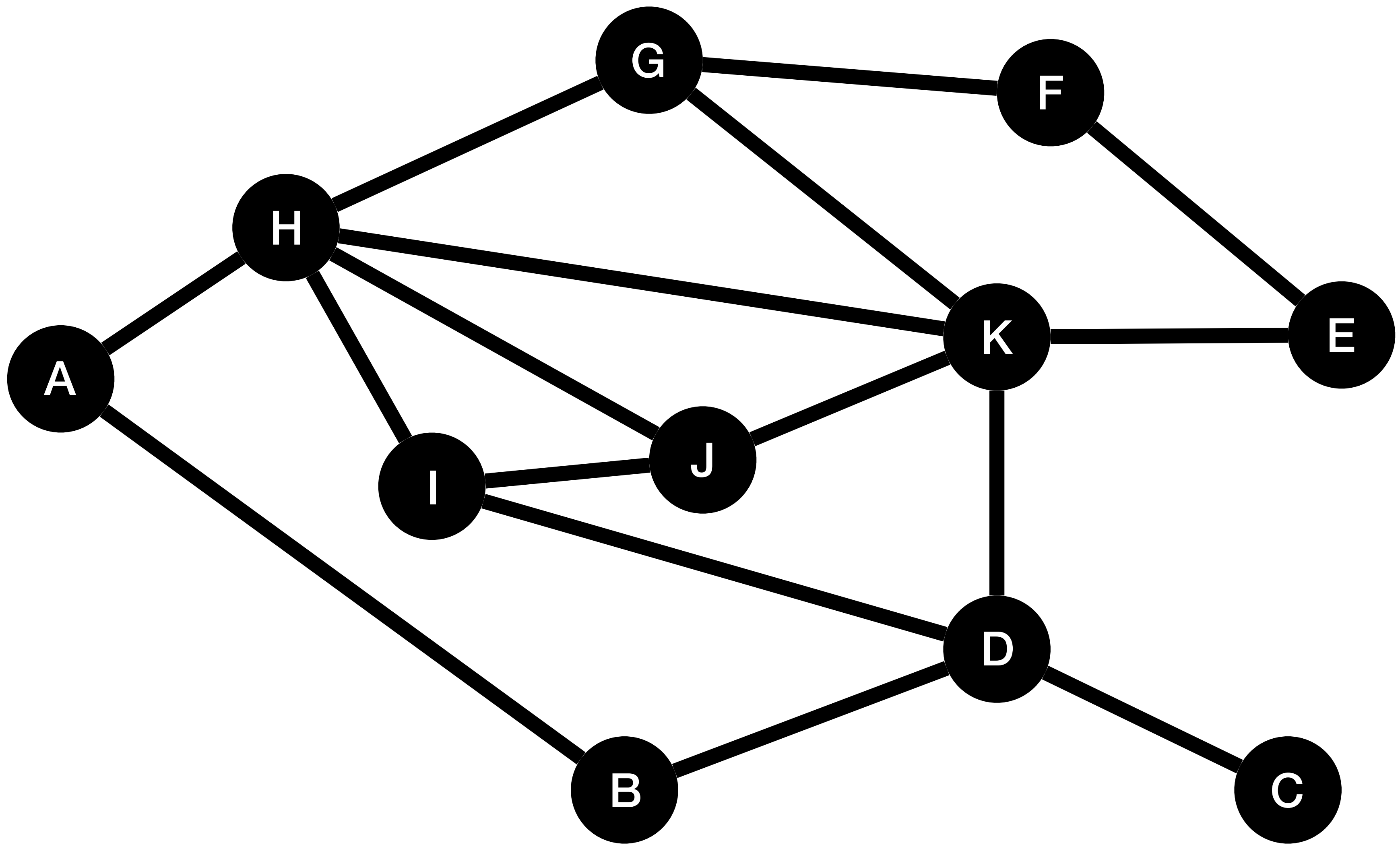
A	B	C	D	E	F	G	H	I
2	2	1	4	2	2	3	5	3

Algorithme de Welsh-Powell



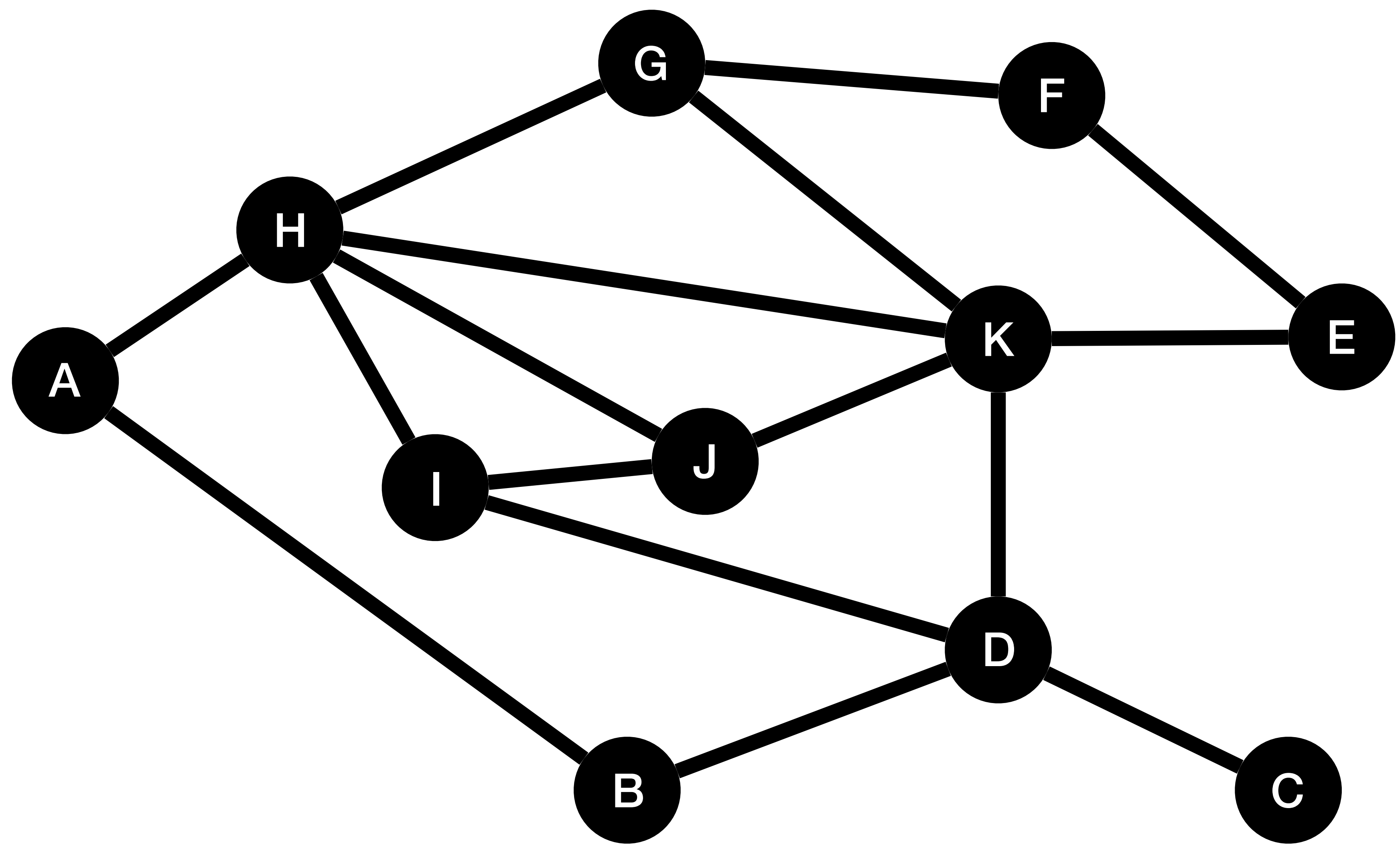
A	B	C	D	E	F	G	H	I	J
2	2	1	4	2	2	3	5	3	3

Algorithme de Welsh-Powell



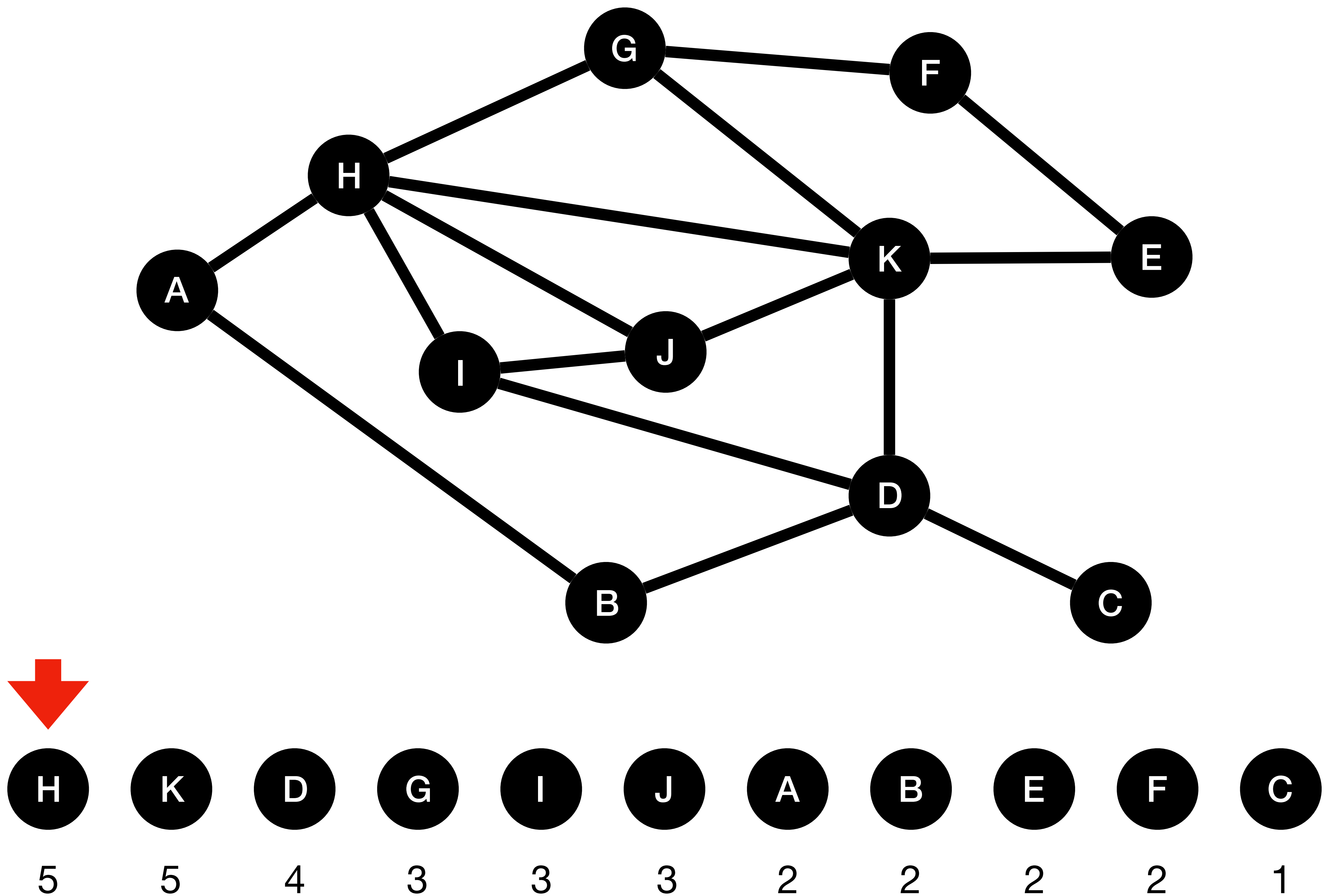
A	B	C	D	E	F	G	H	I	J	K
2	2	1	4	2	2	3	5	3	3	5

Algorithme de Welsh-Powell



H	K	D	G	I	J	A	B	E	F	C
5	5	4	3	3	3	2	2	2	2	1

Algorithme de Welsh-Powell



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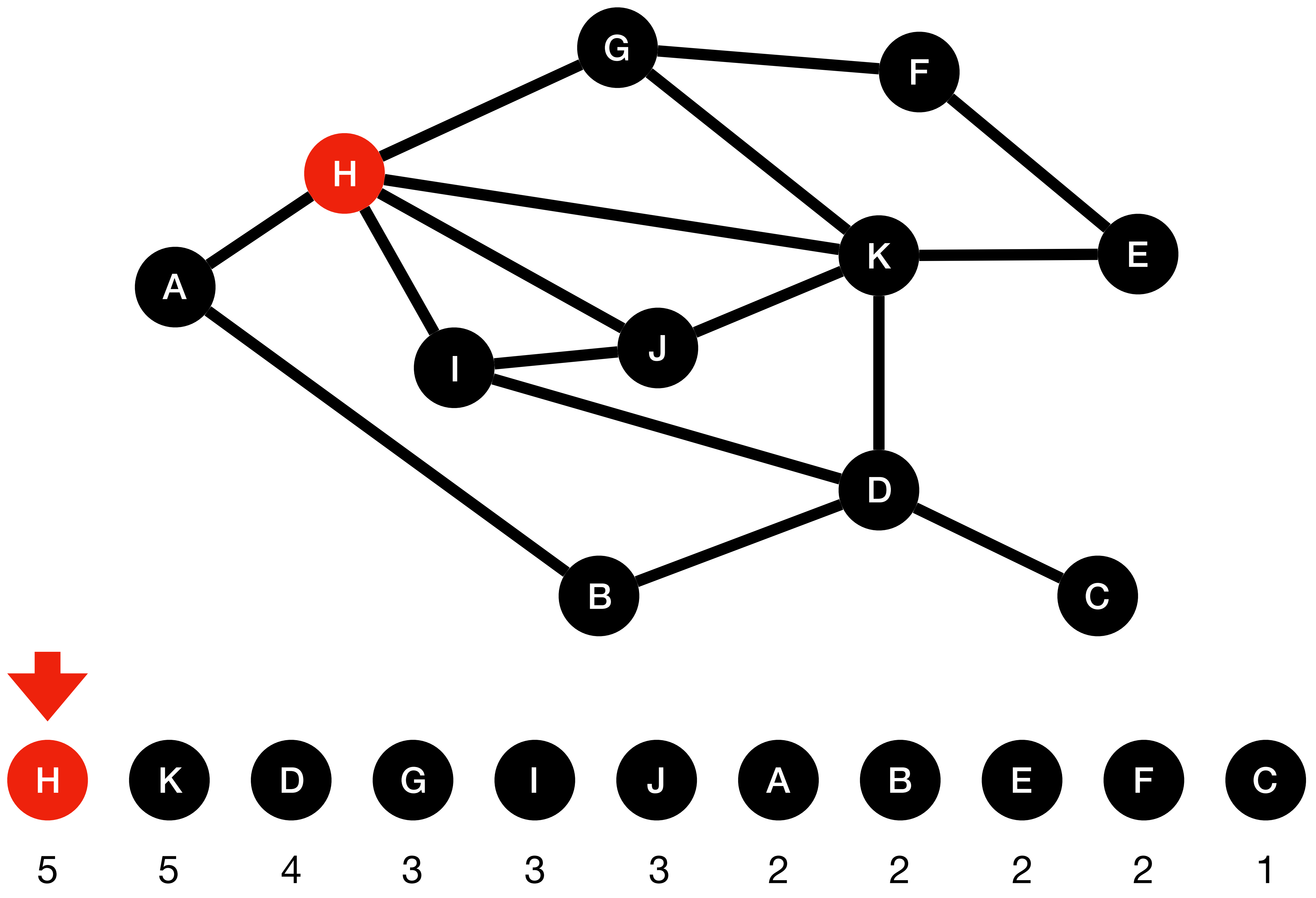

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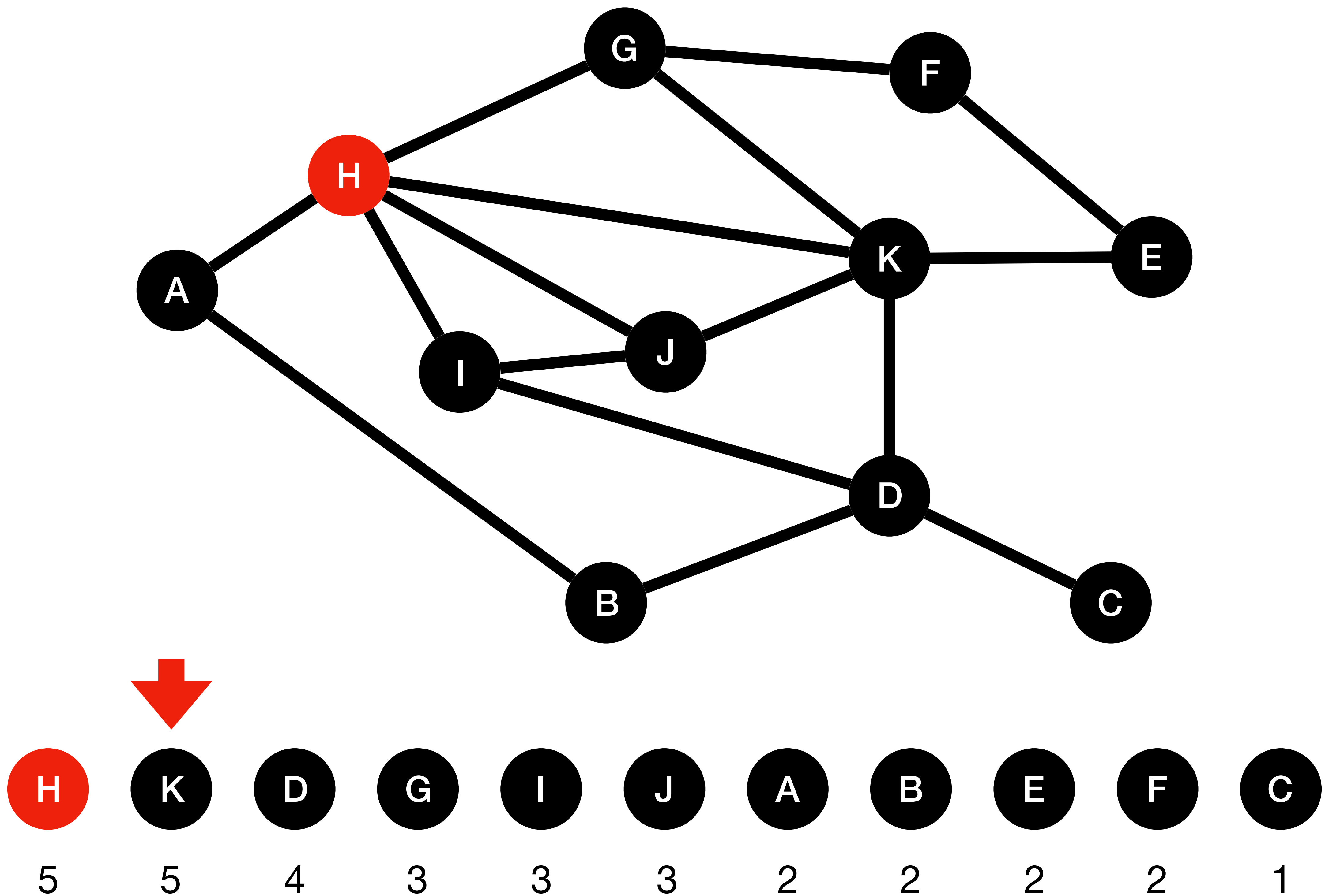

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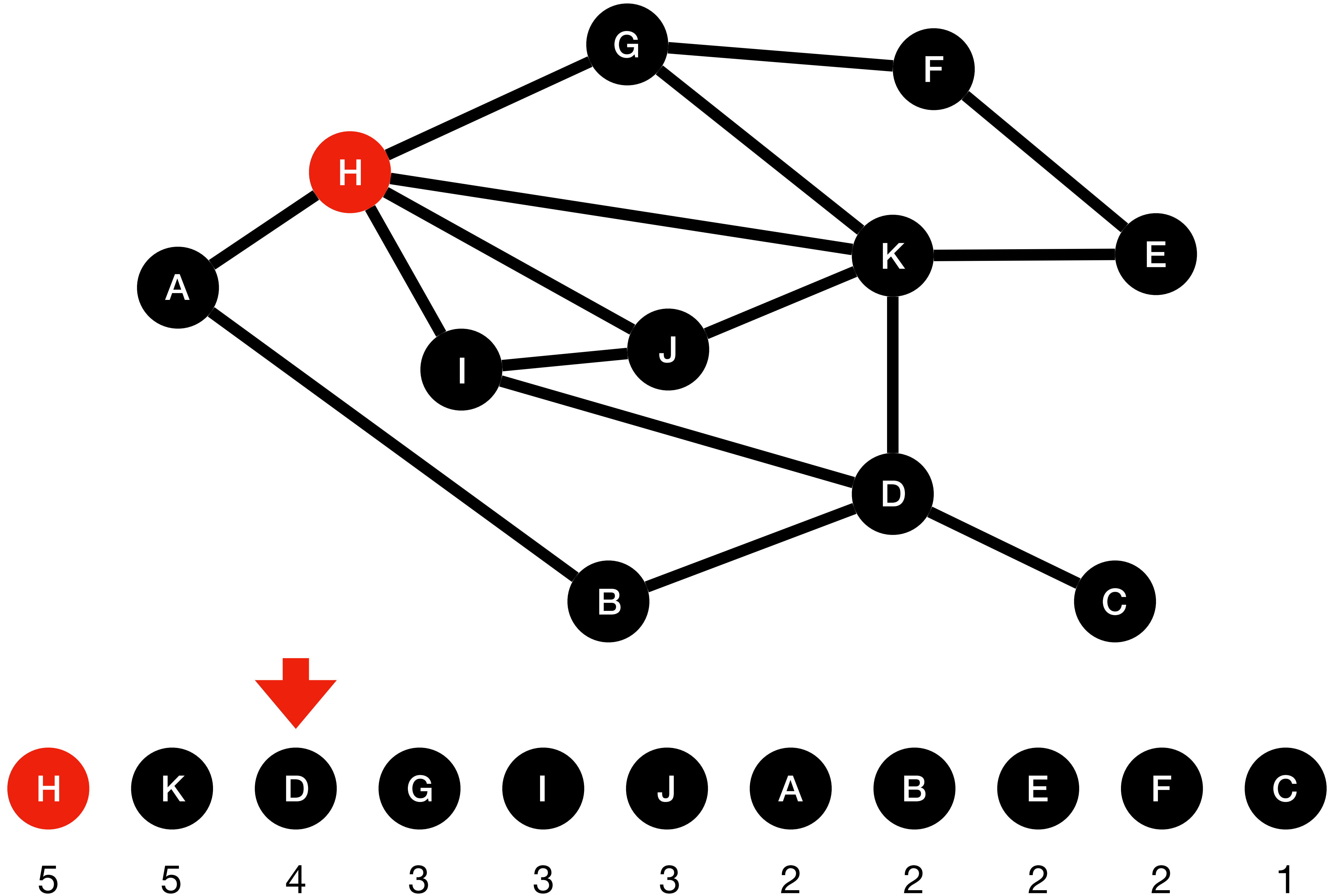

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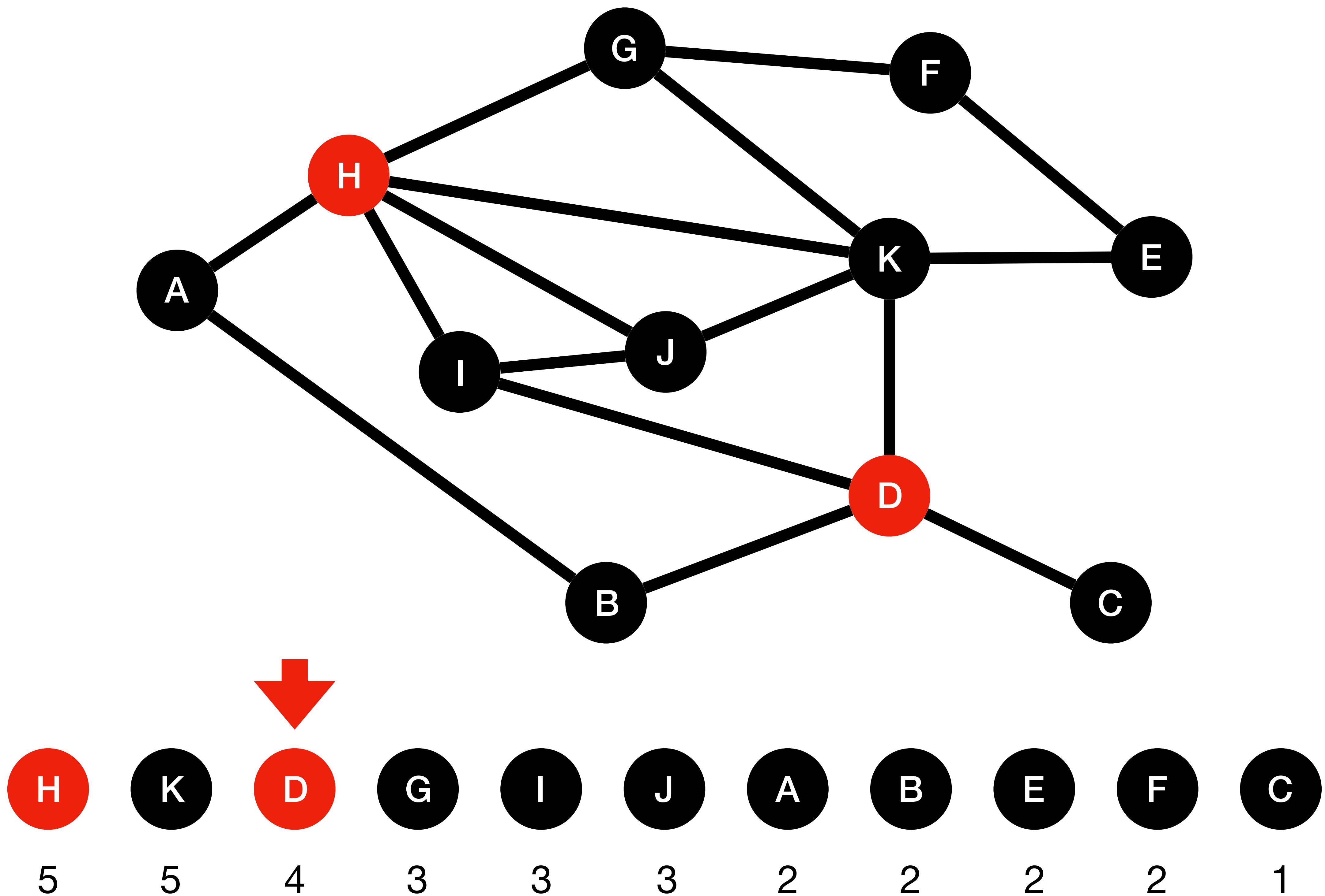
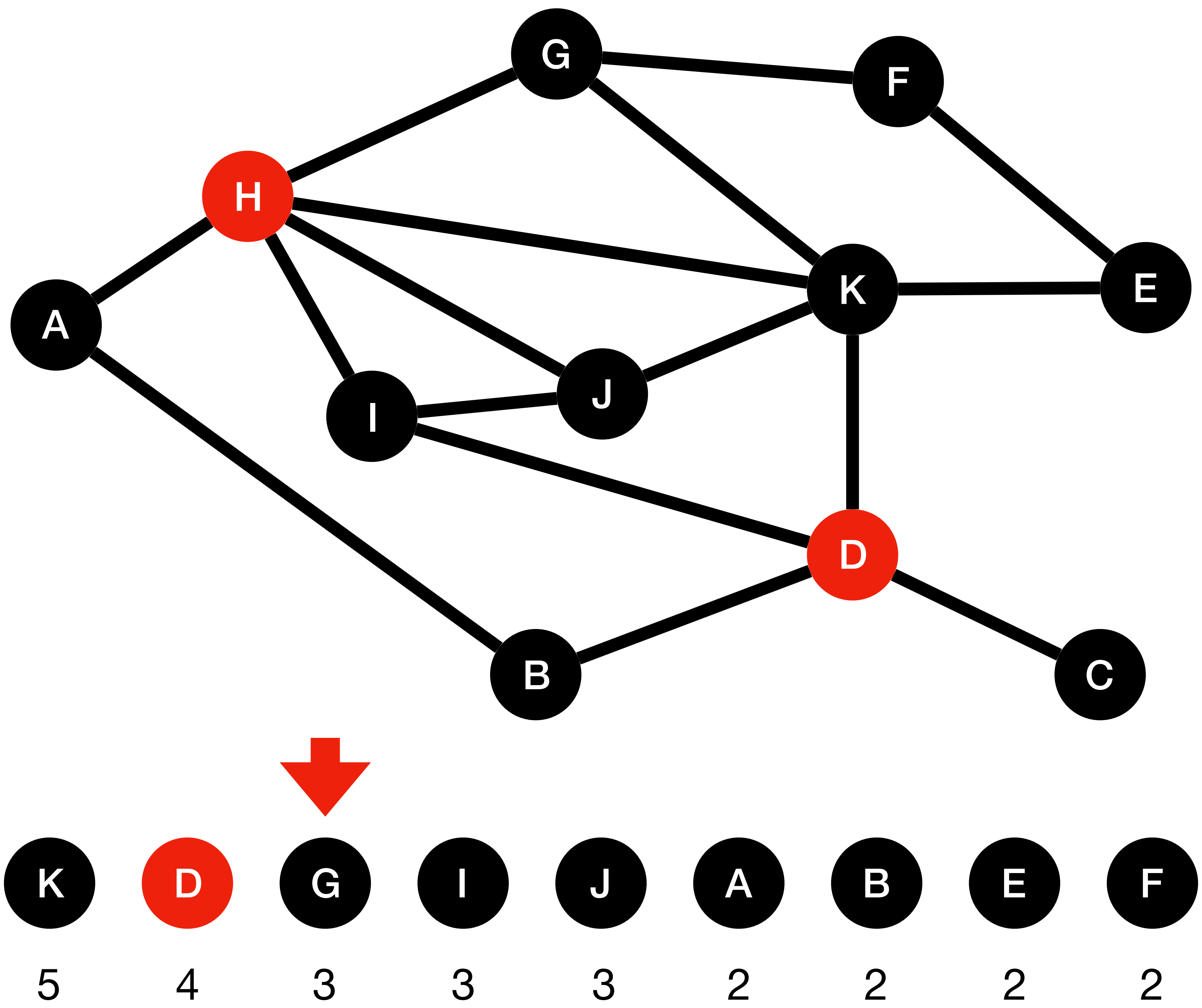


Algorithme de Welsh-Powell



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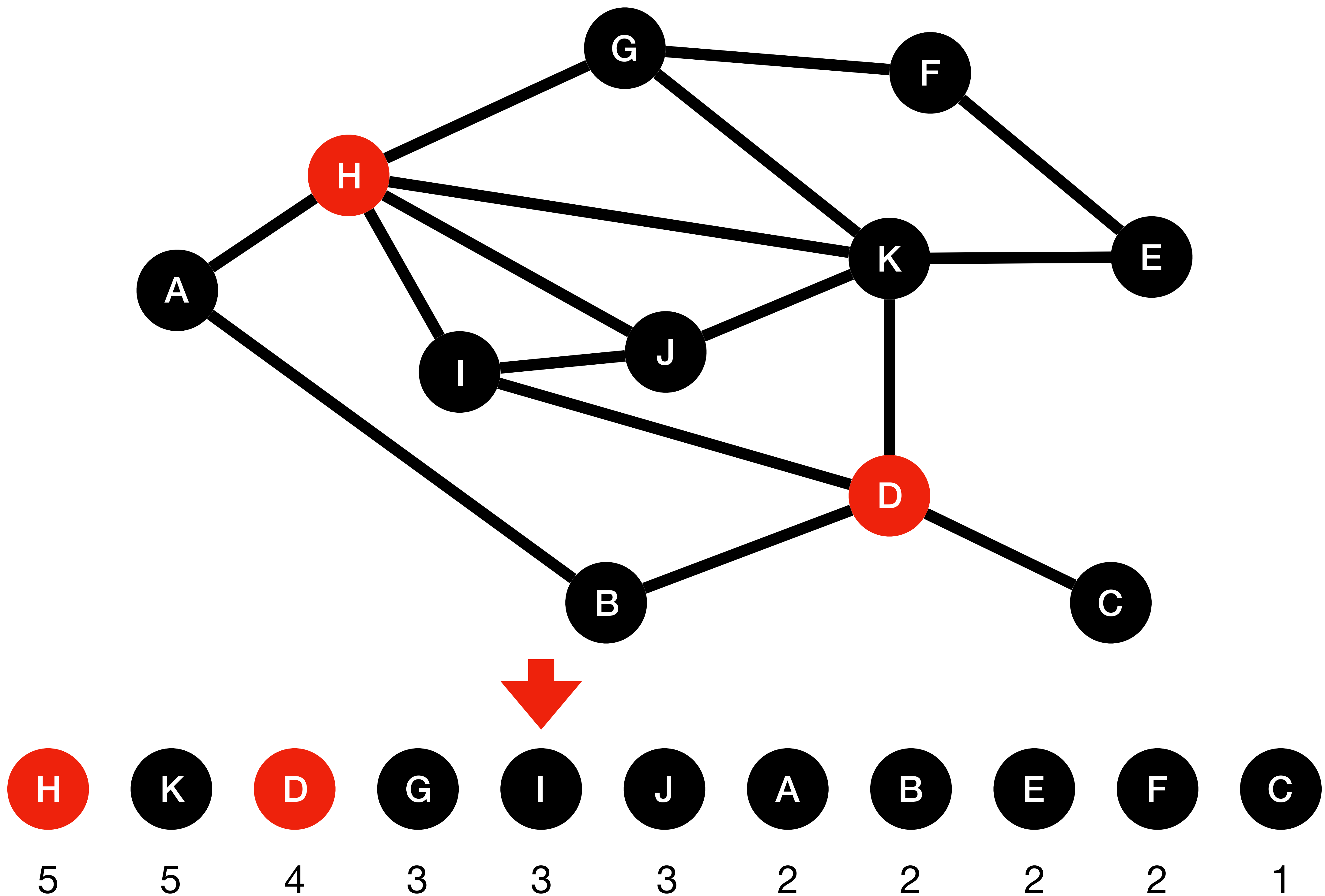
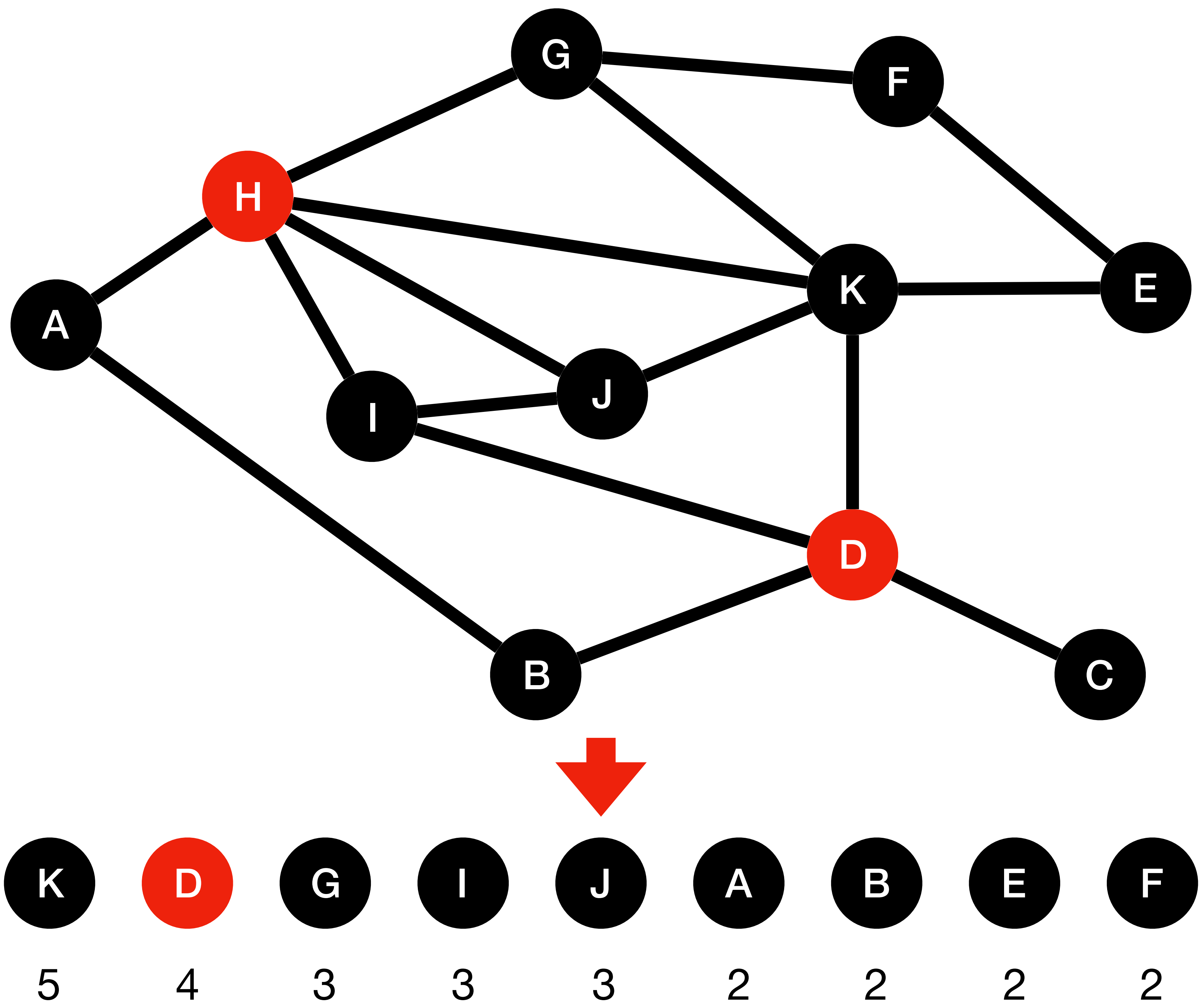
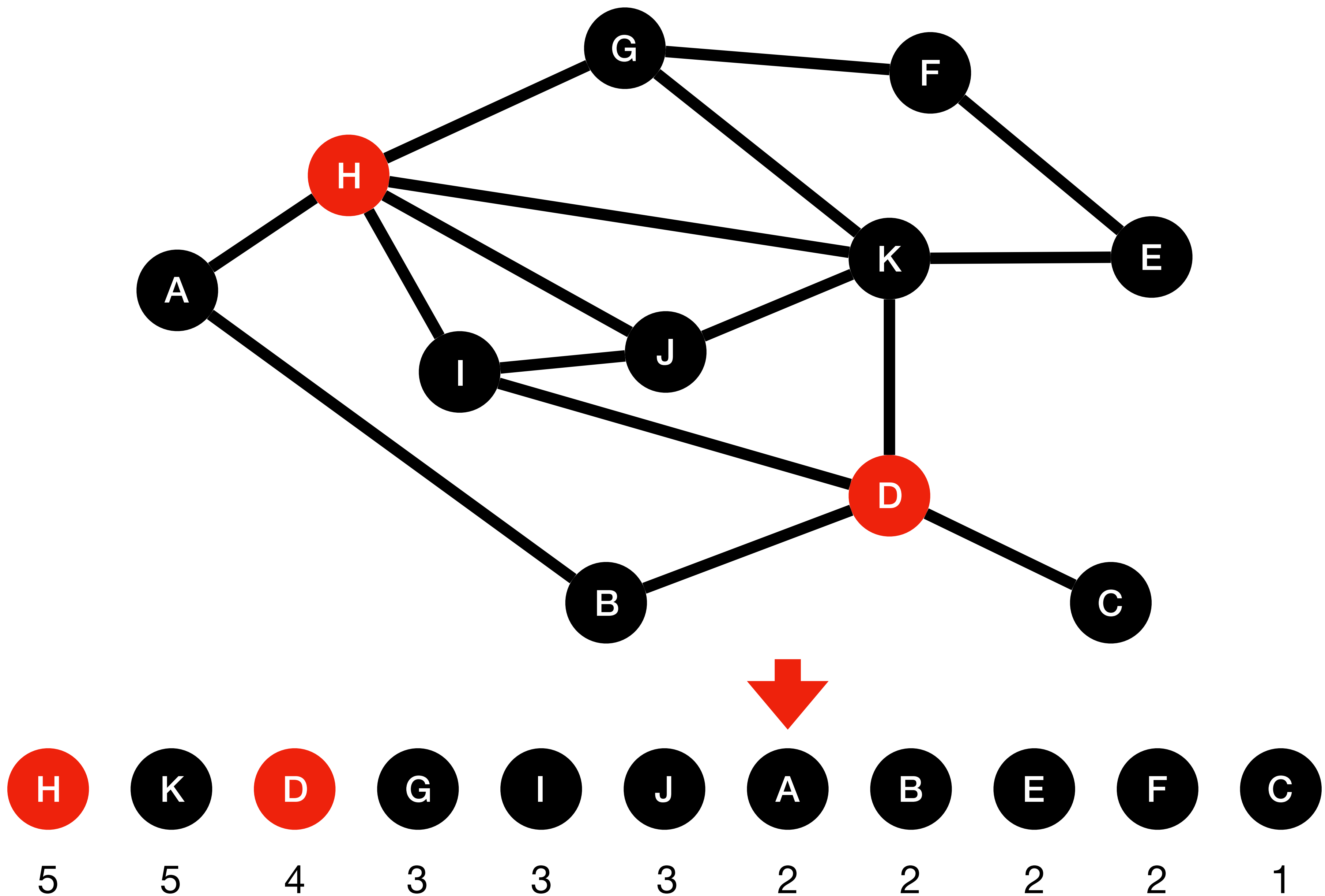


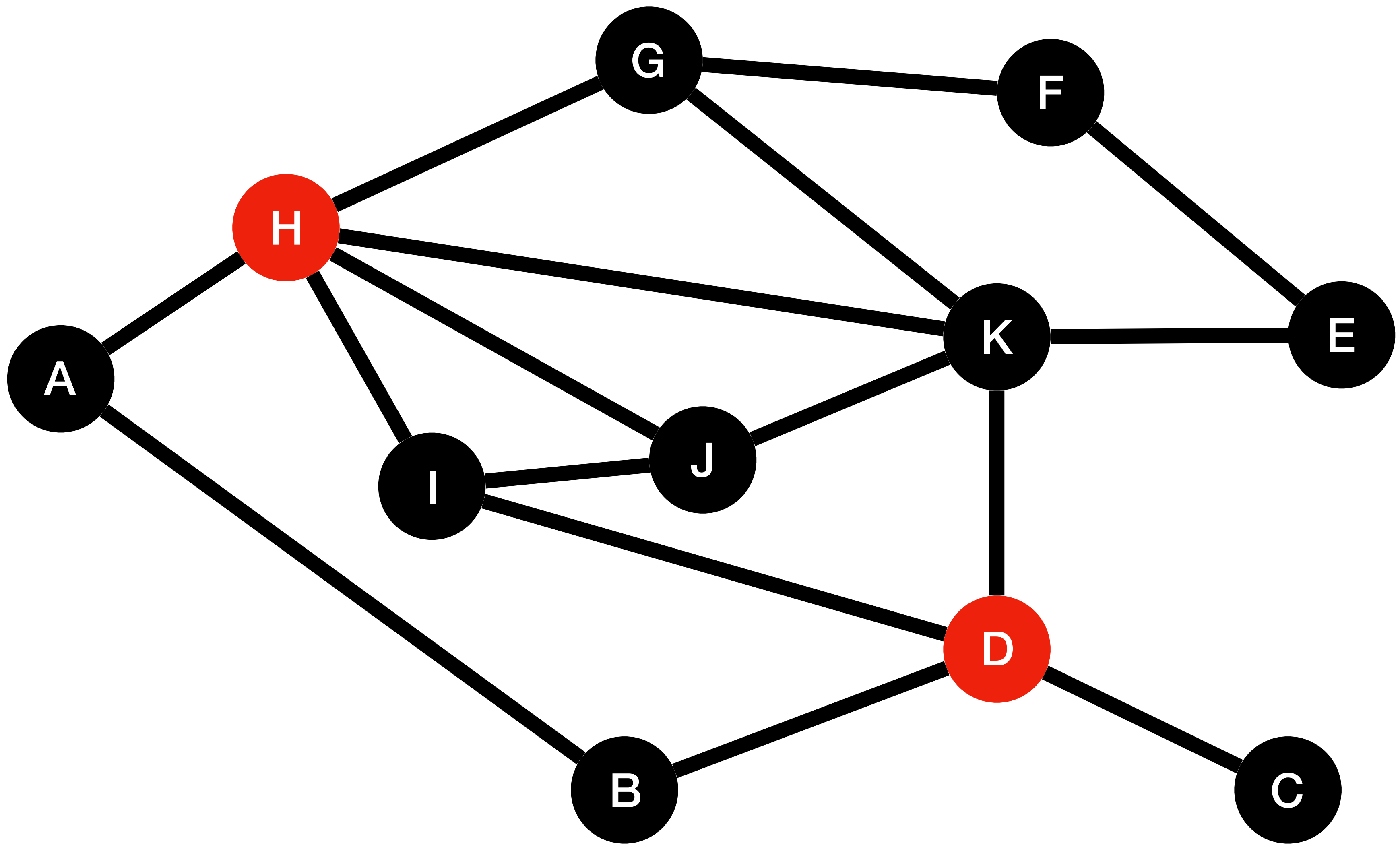
Algorithme de Welsh-Powell

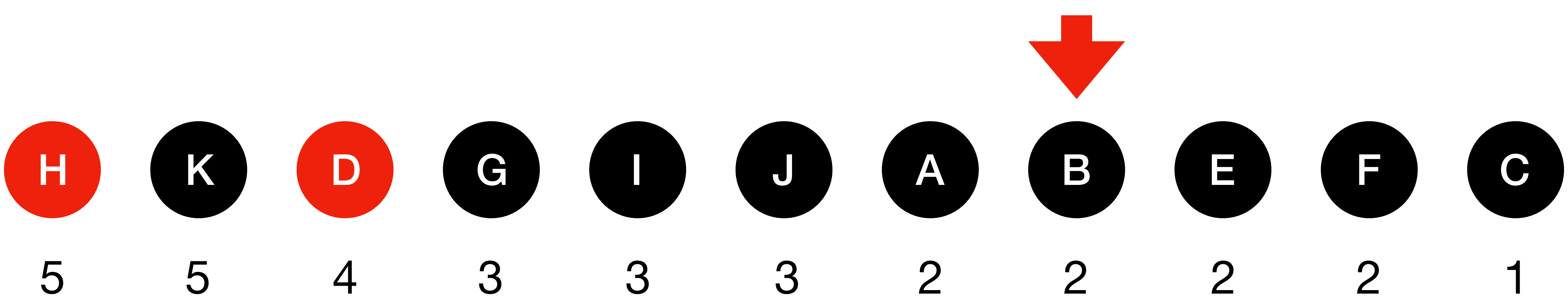


Algorithme de Welsh-Powell

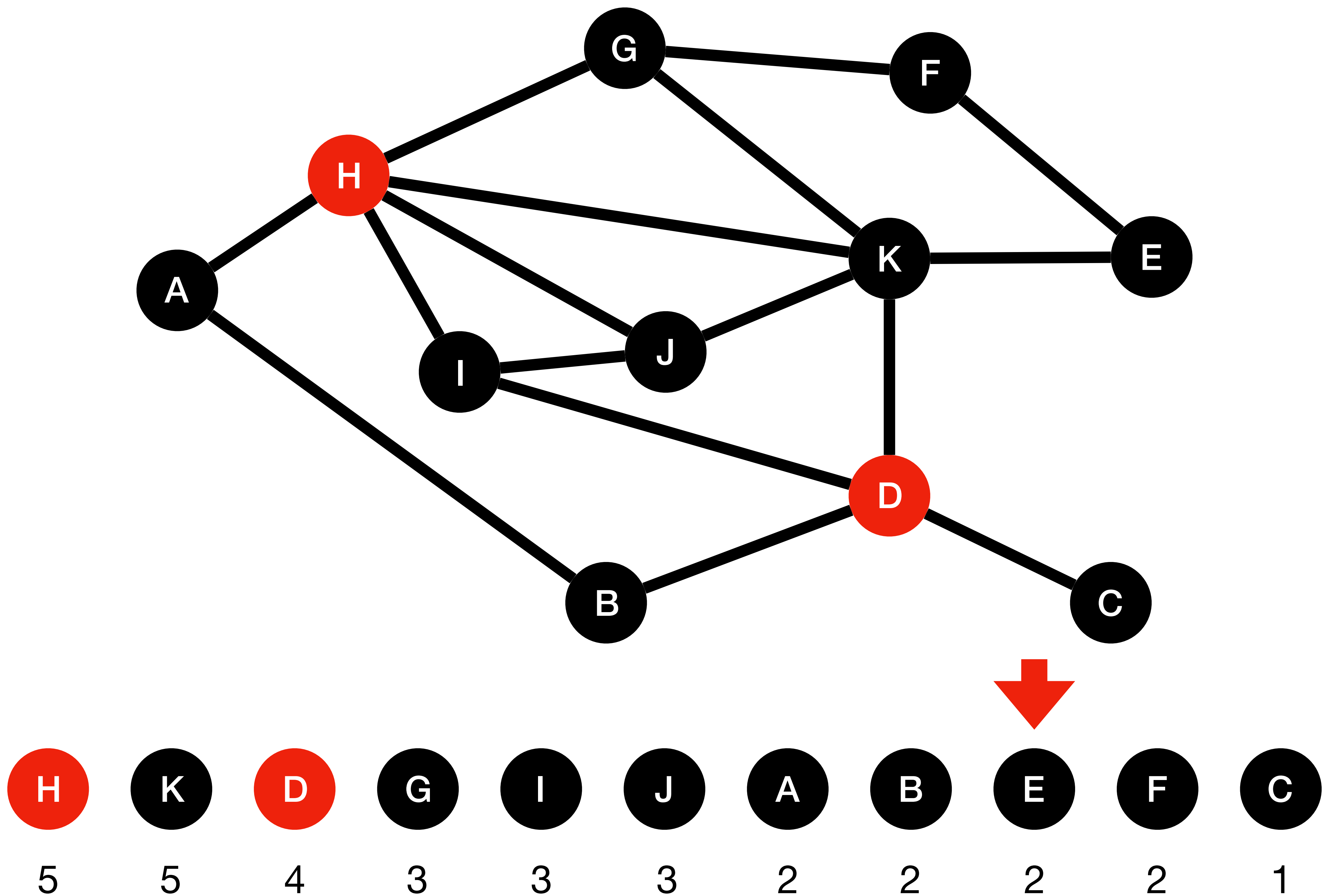


Algorithme de Welsh-Powell

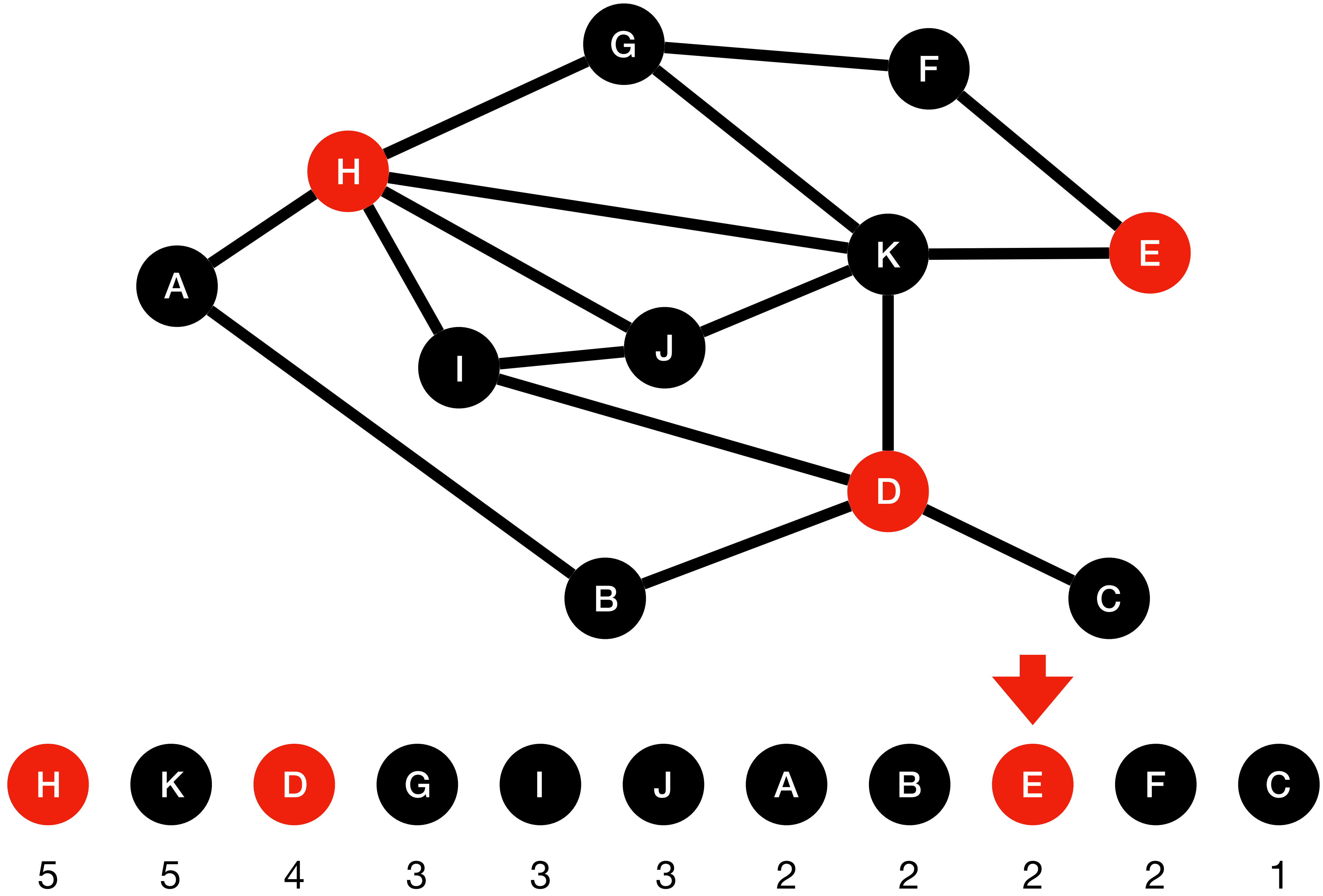




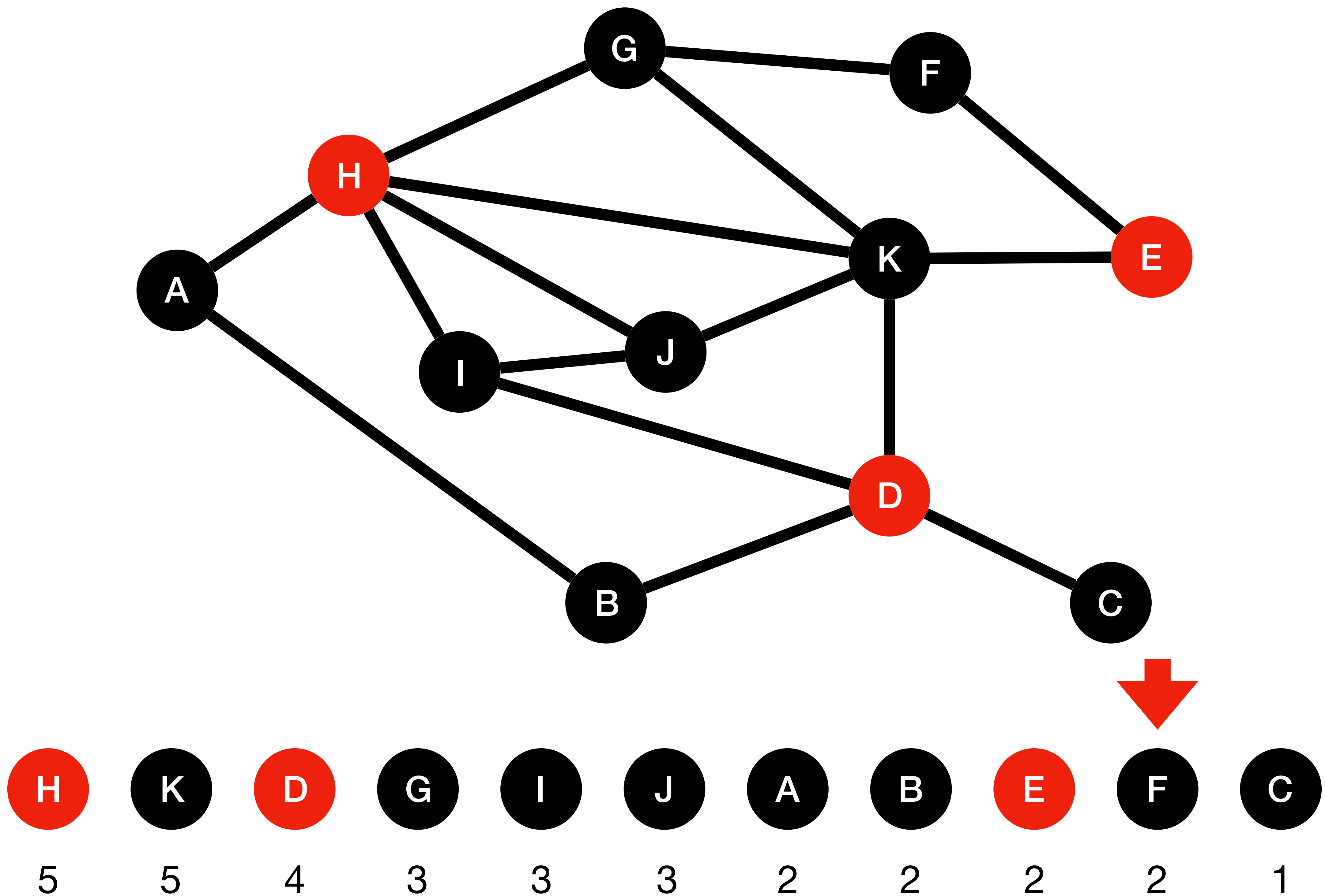
Algorithme de Welsh-Powell



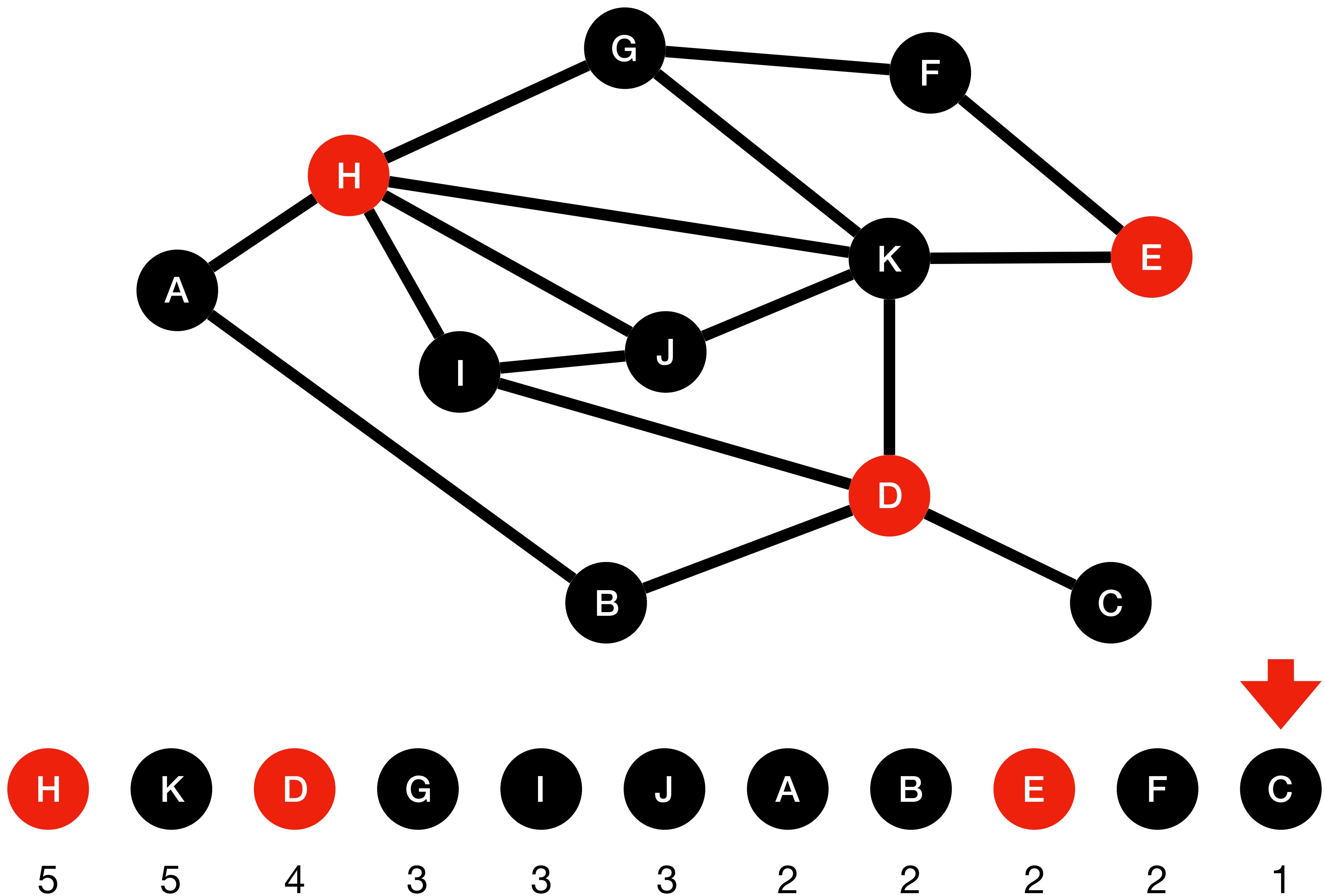
Algorithme de Welsh-Powell



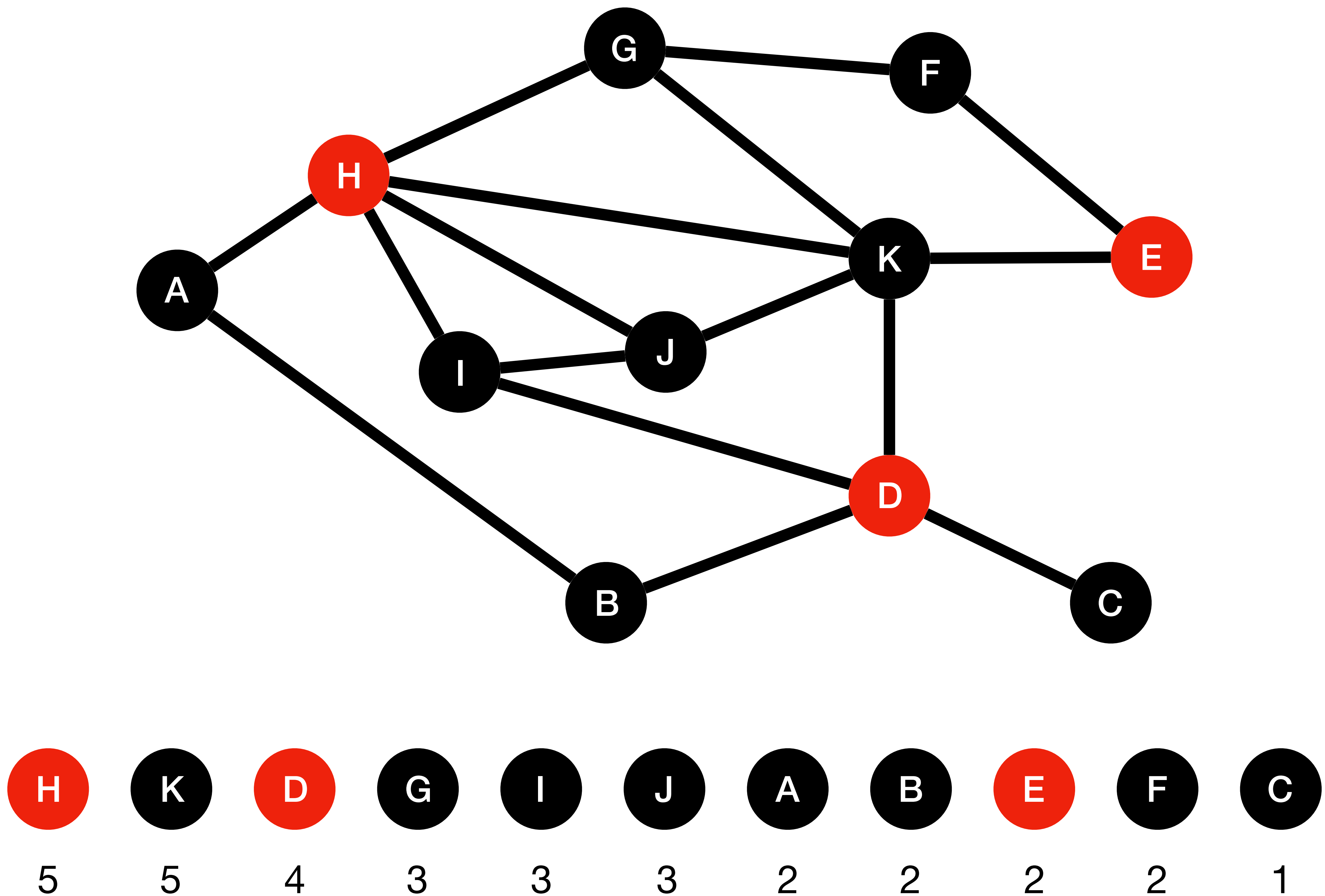
Algorithme de Welsh-Powell



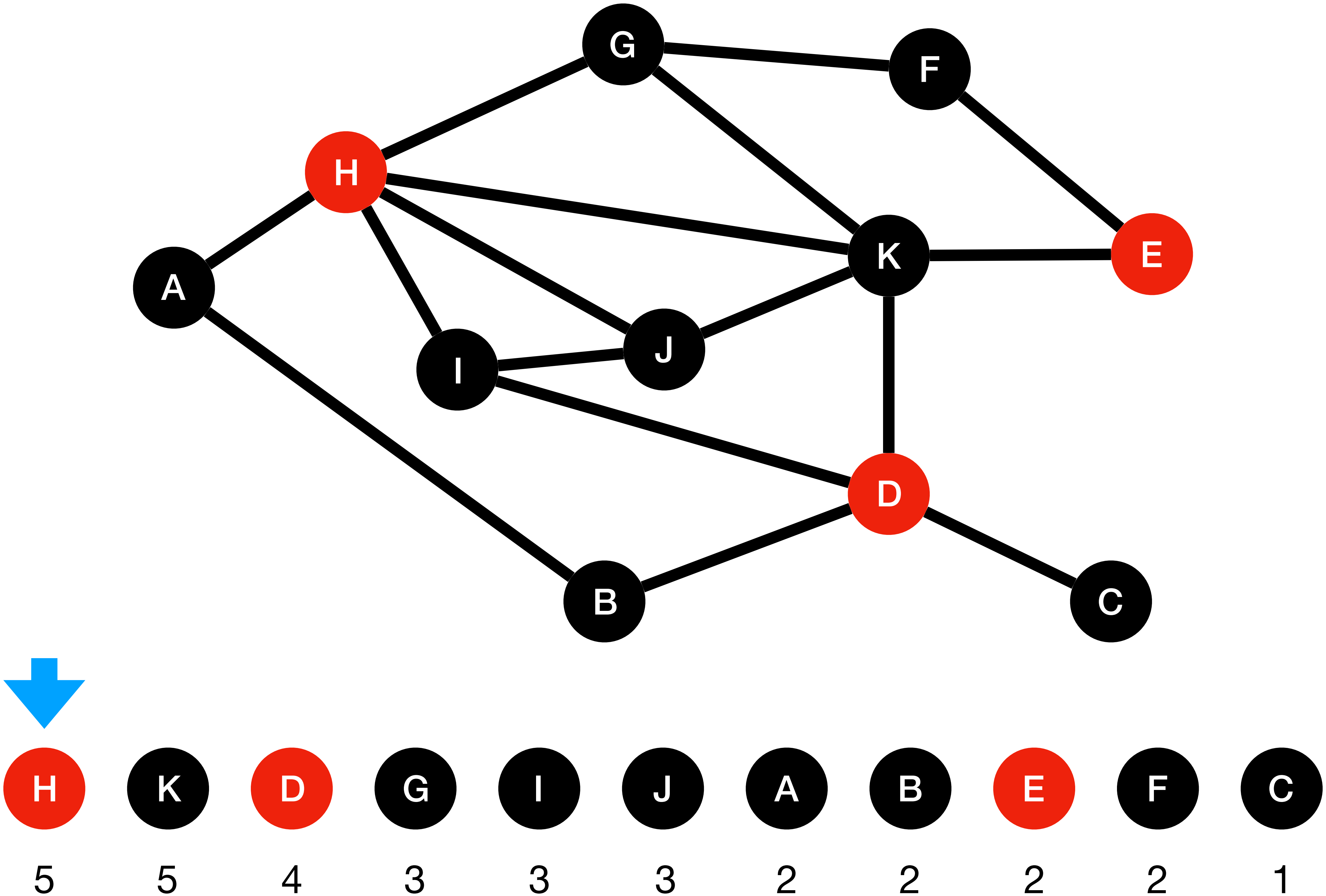
Algorithme de Welsh-Powell



Algorithme de Welsh-Powell



Algorithme de Welsh-Powell



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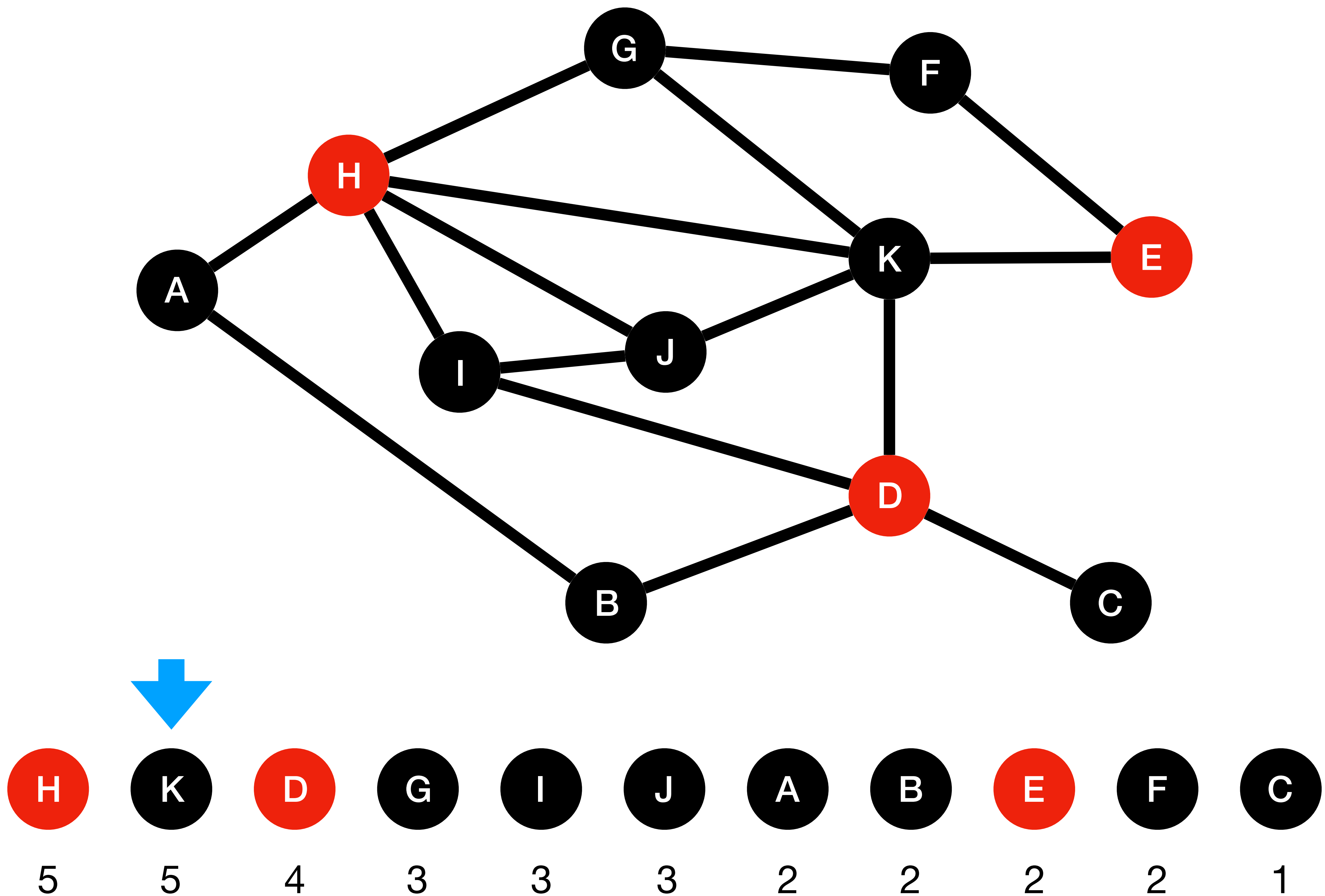

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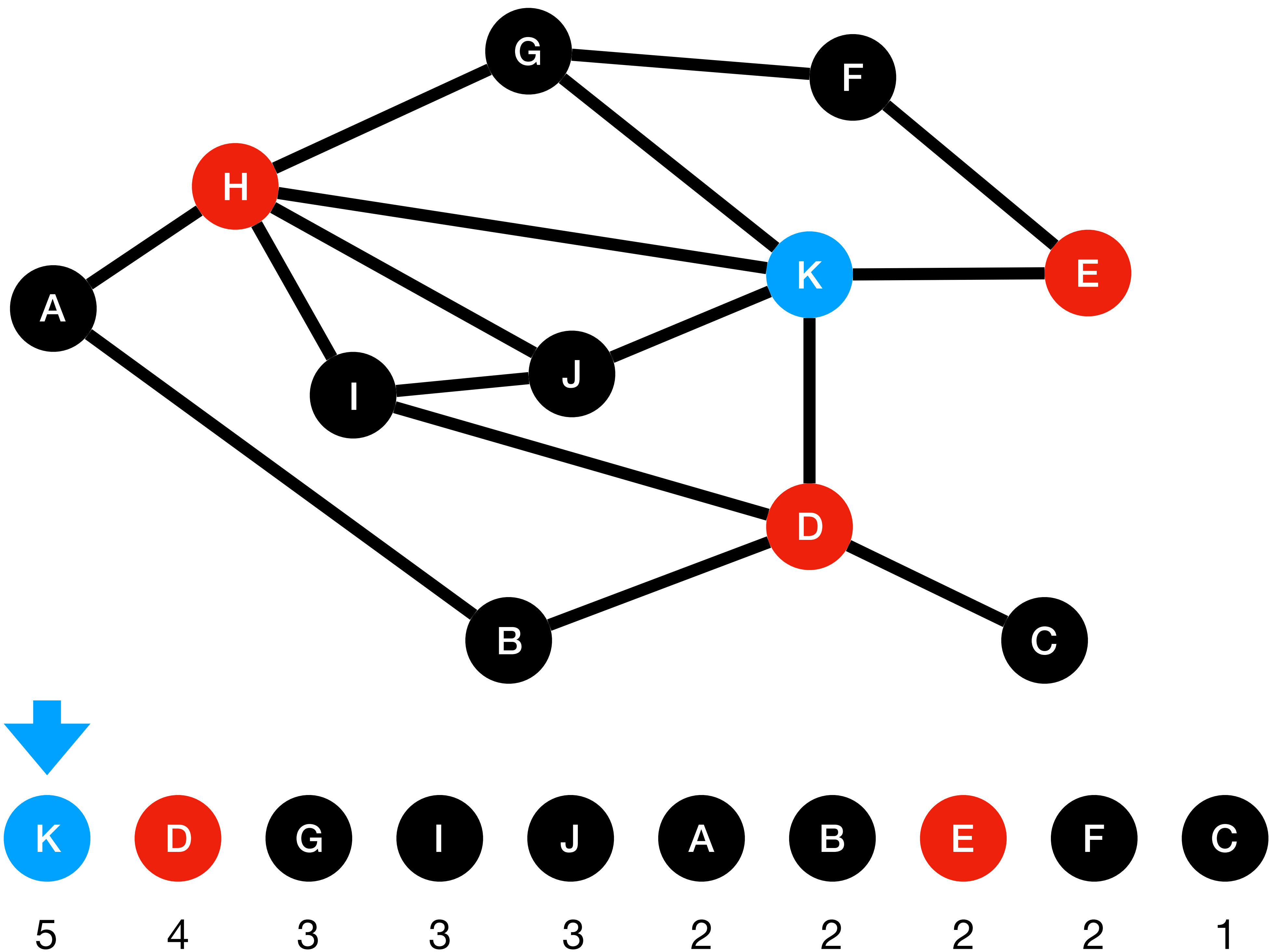

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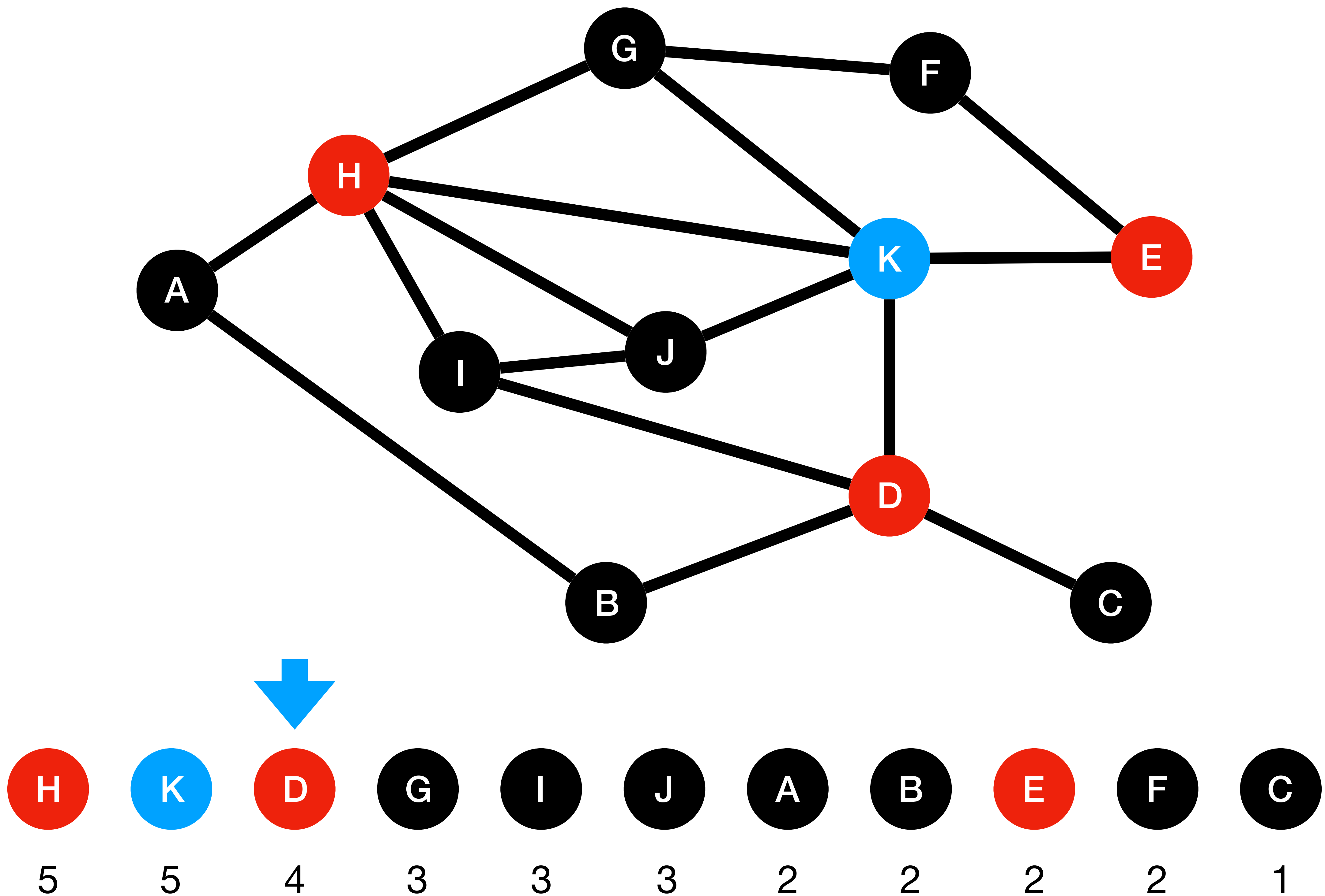

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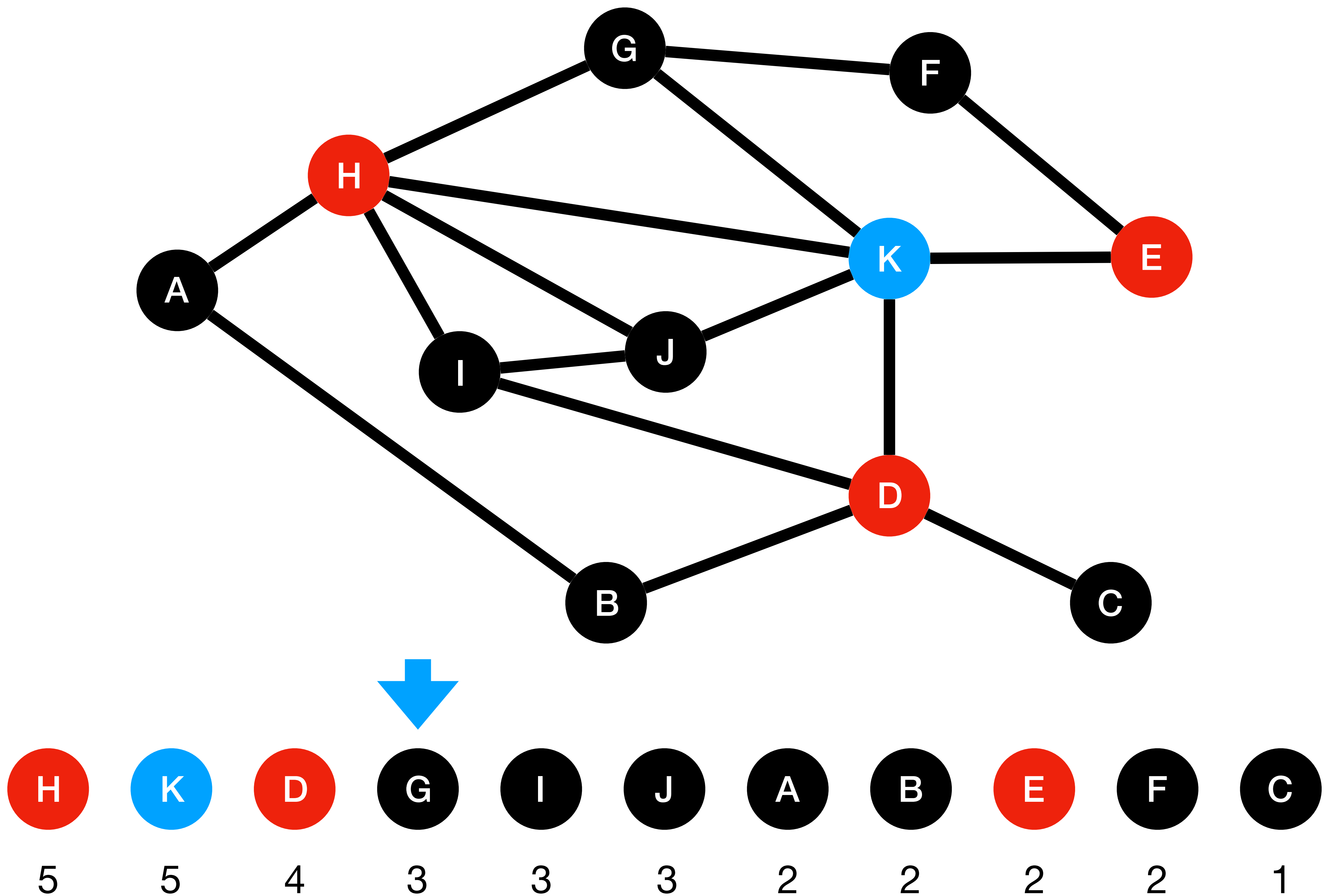

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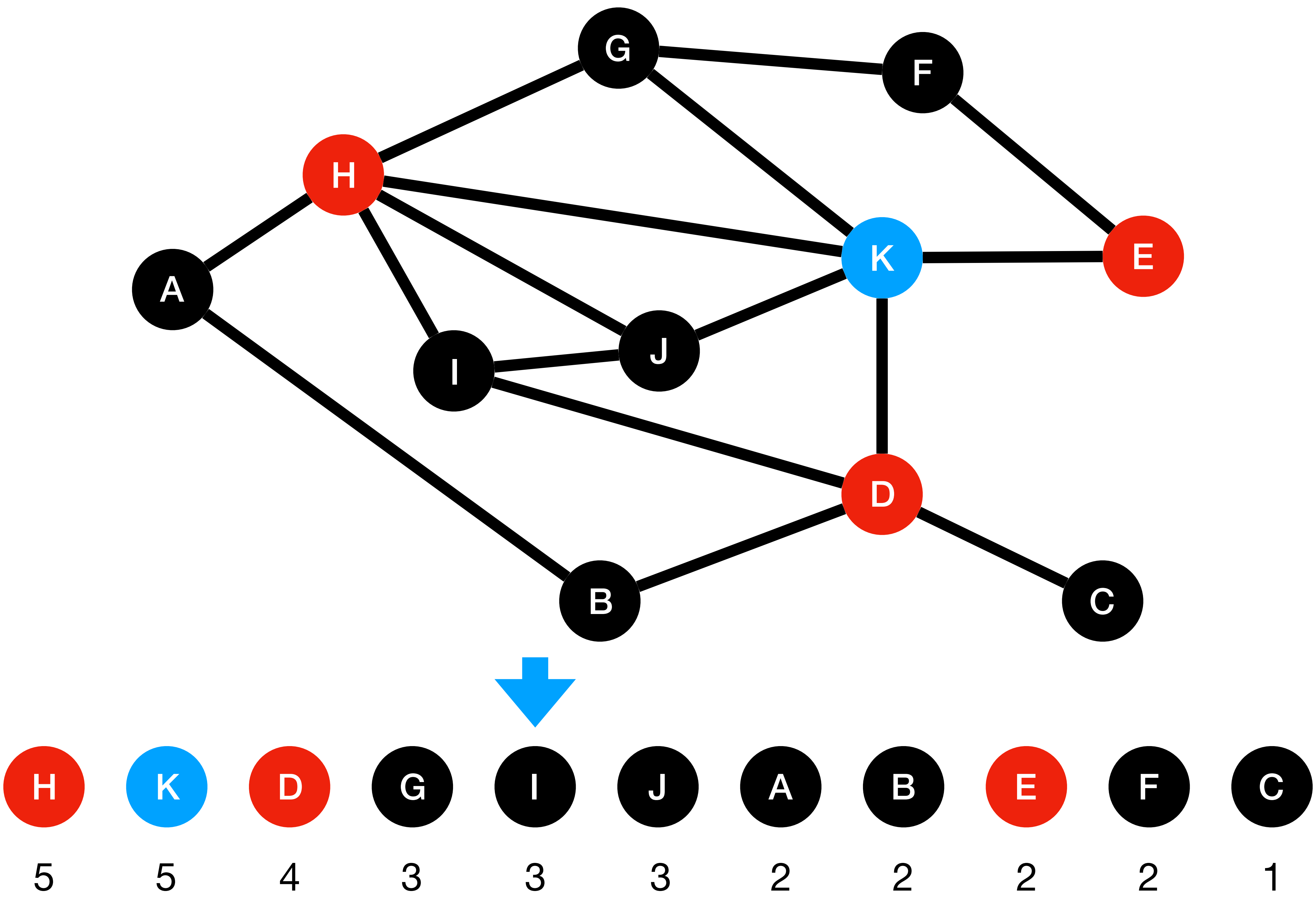

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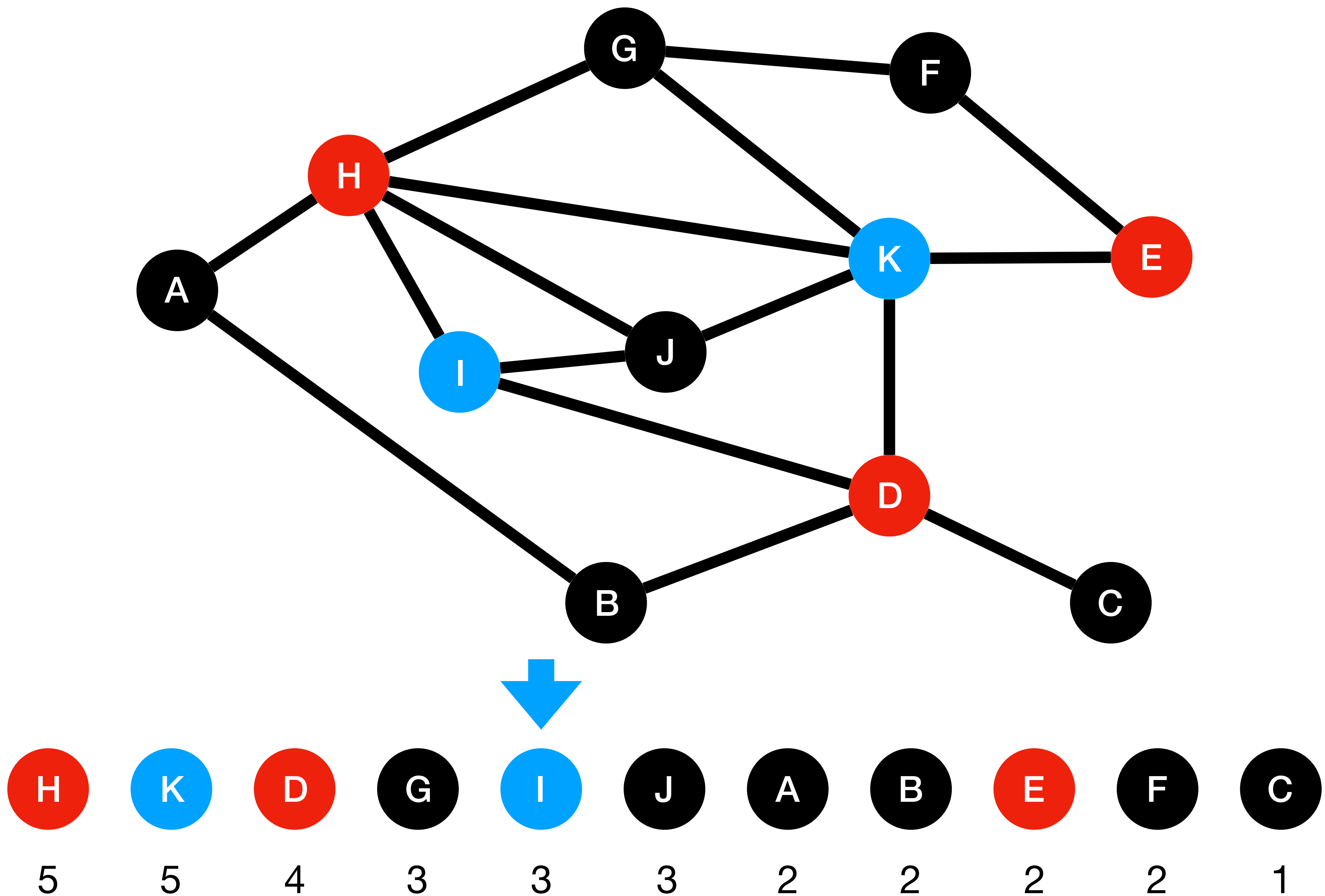

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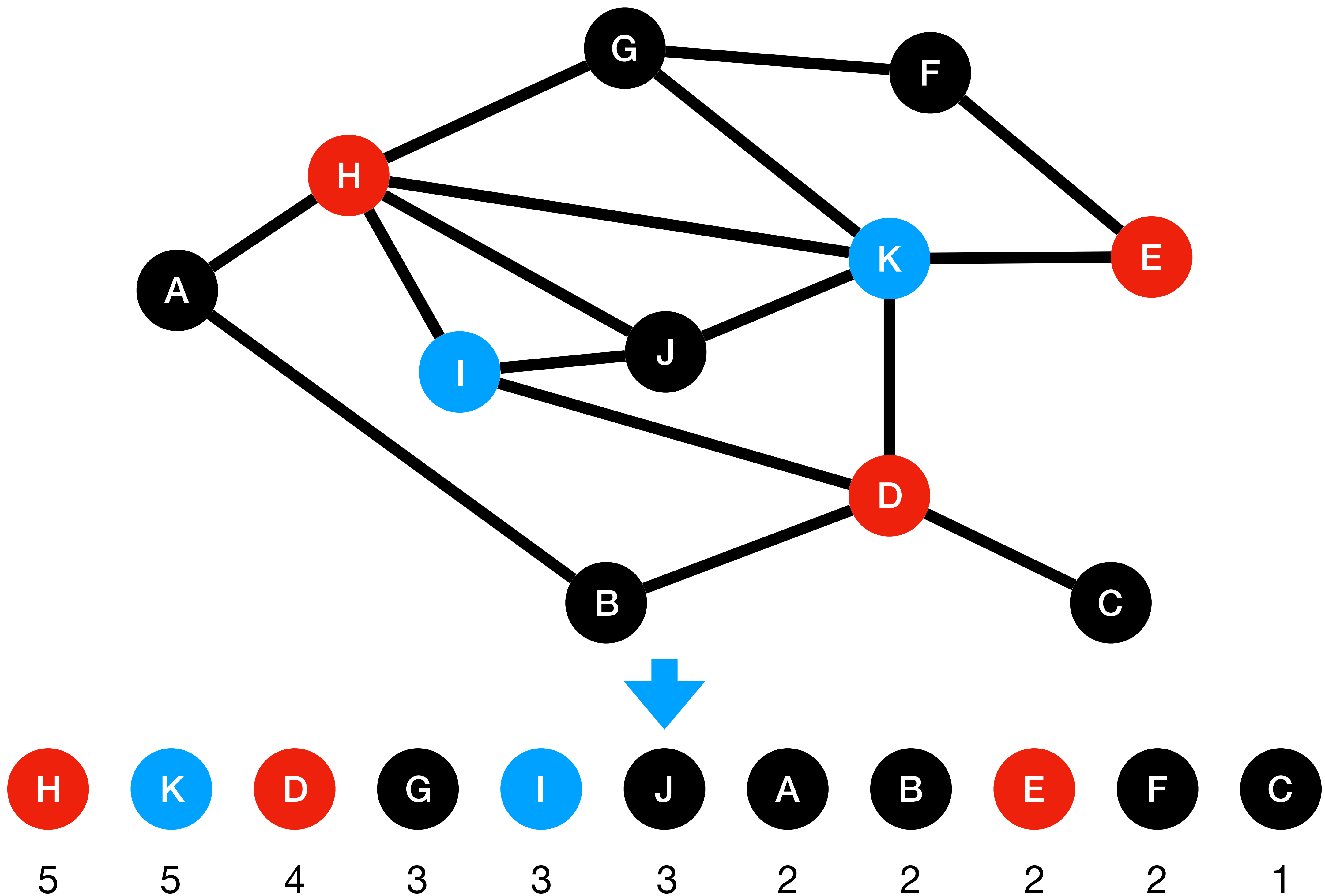

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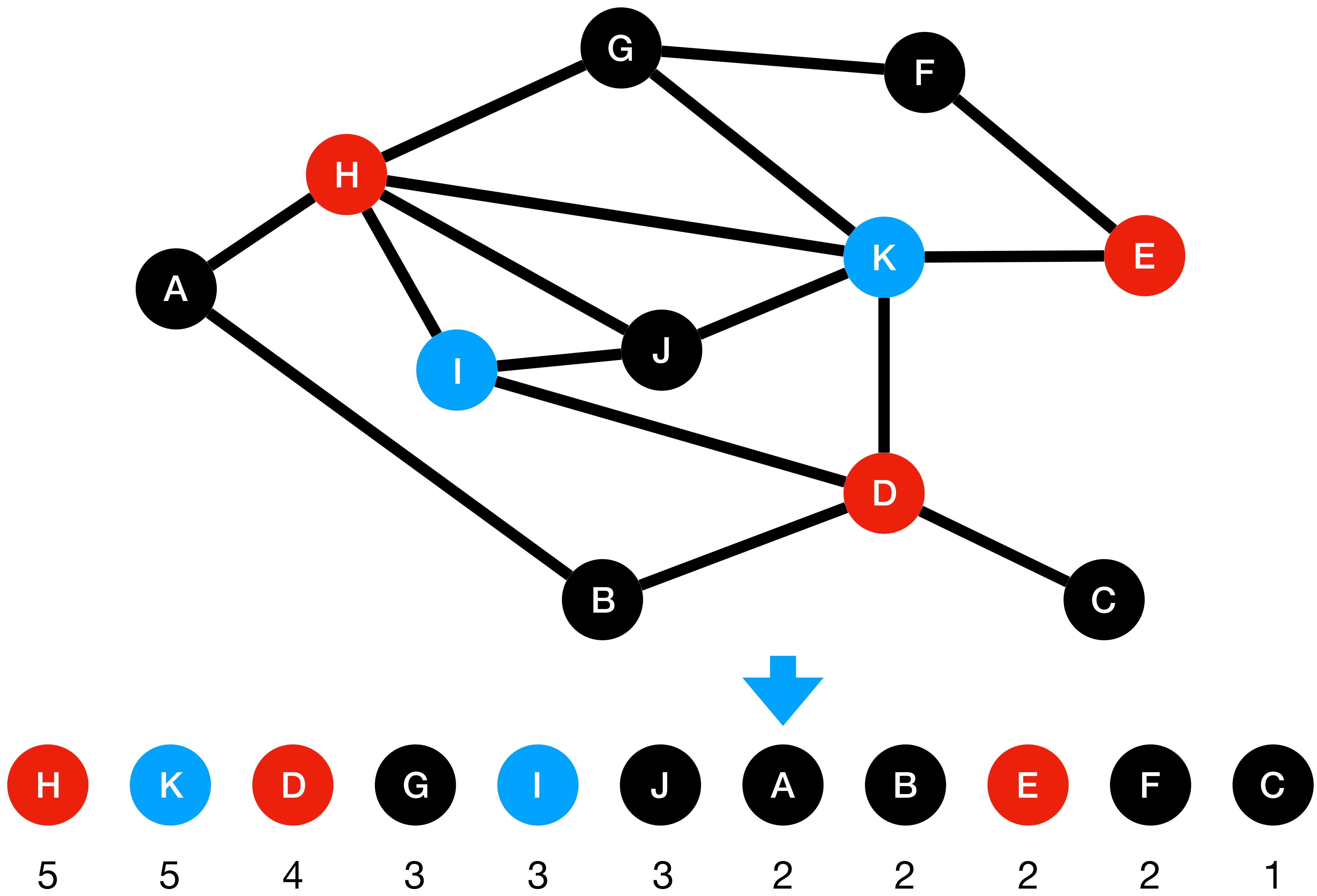

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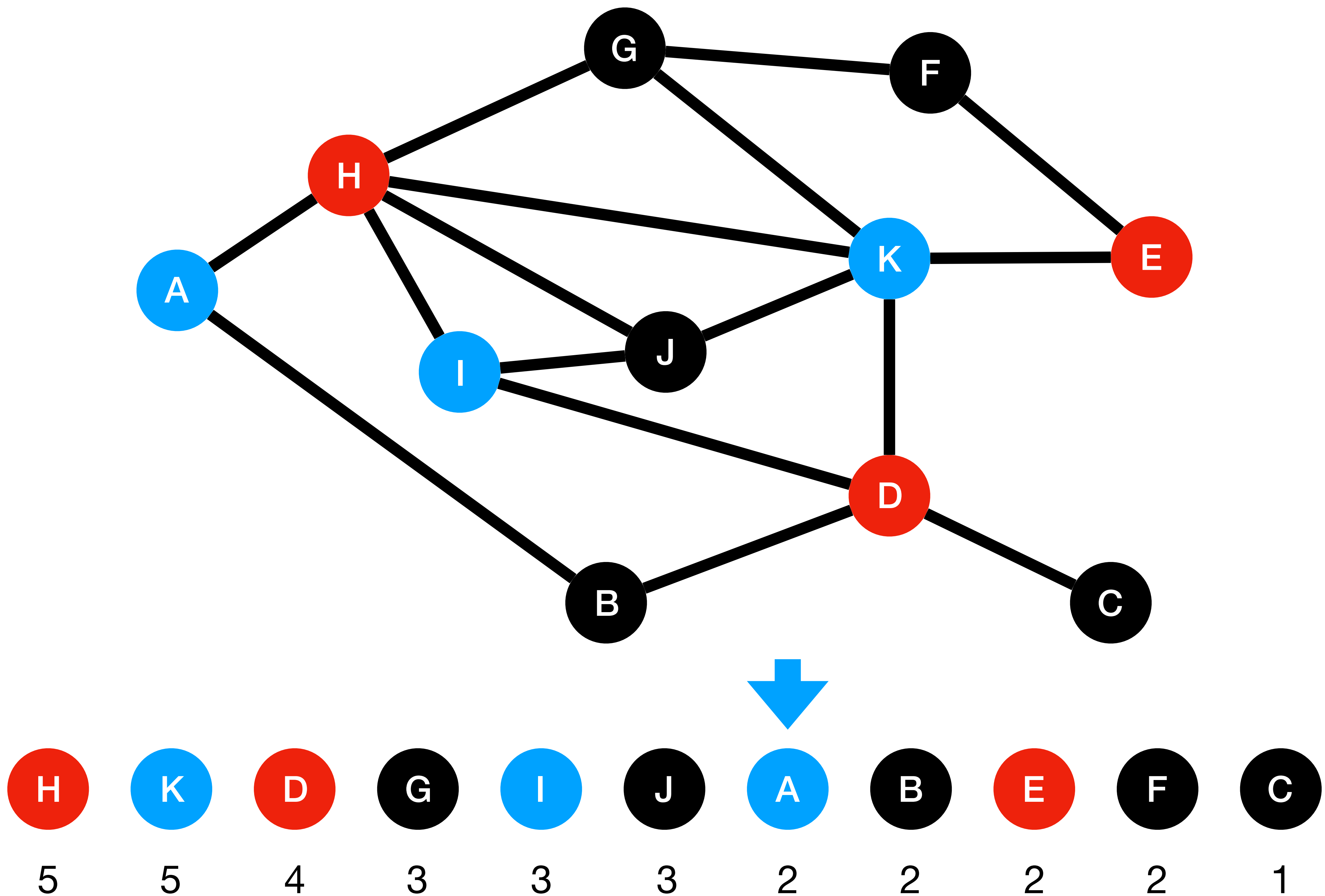

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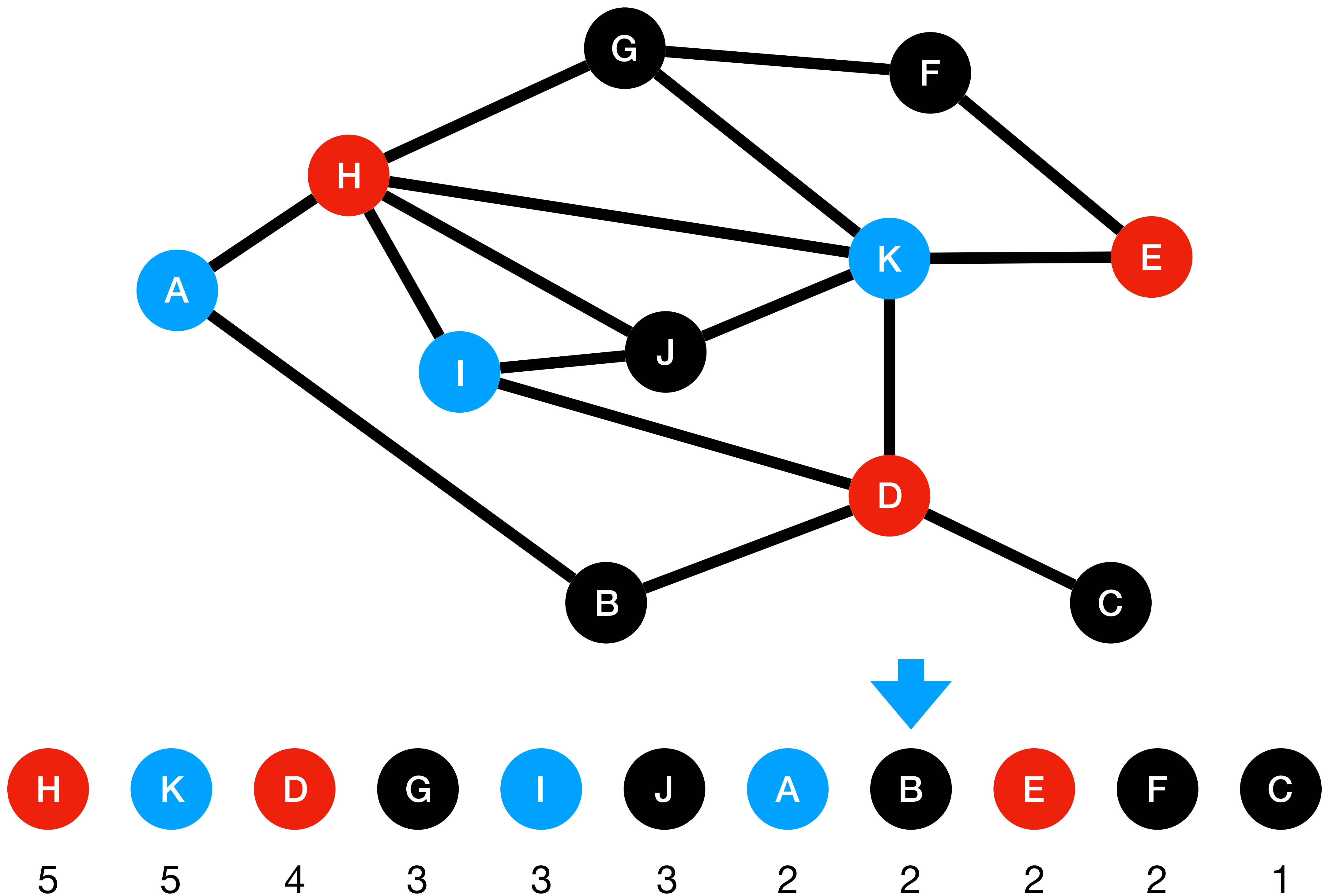

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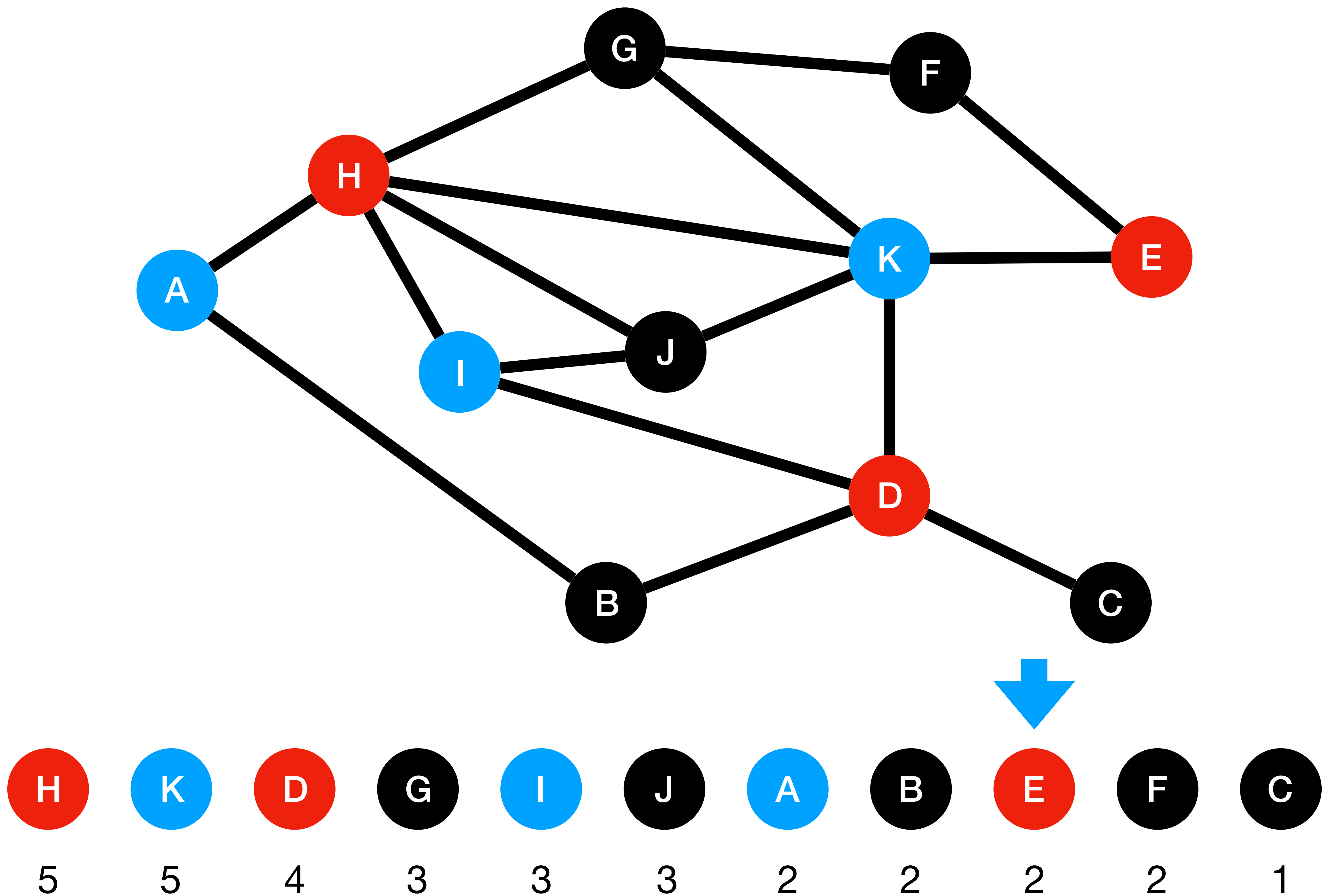

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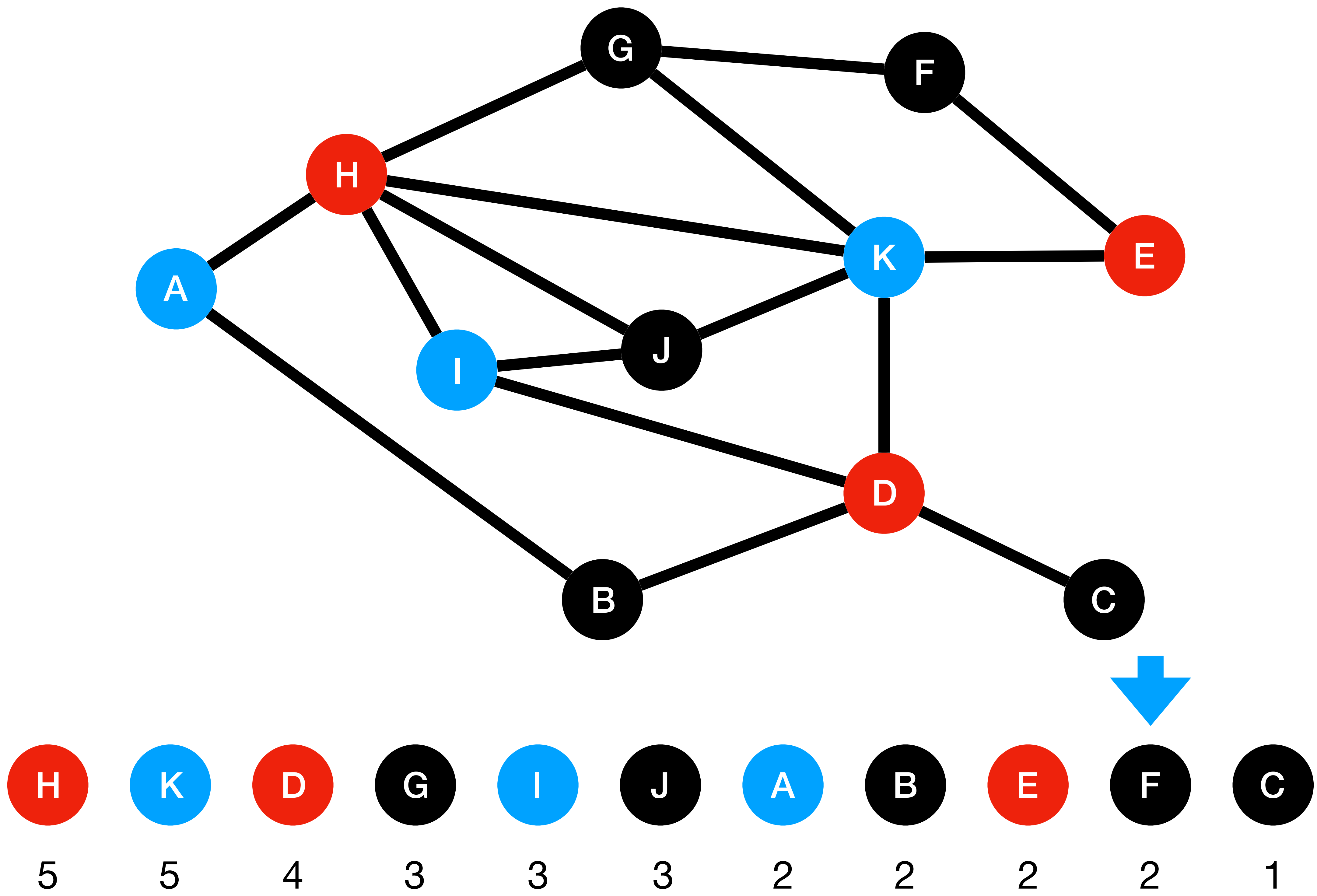

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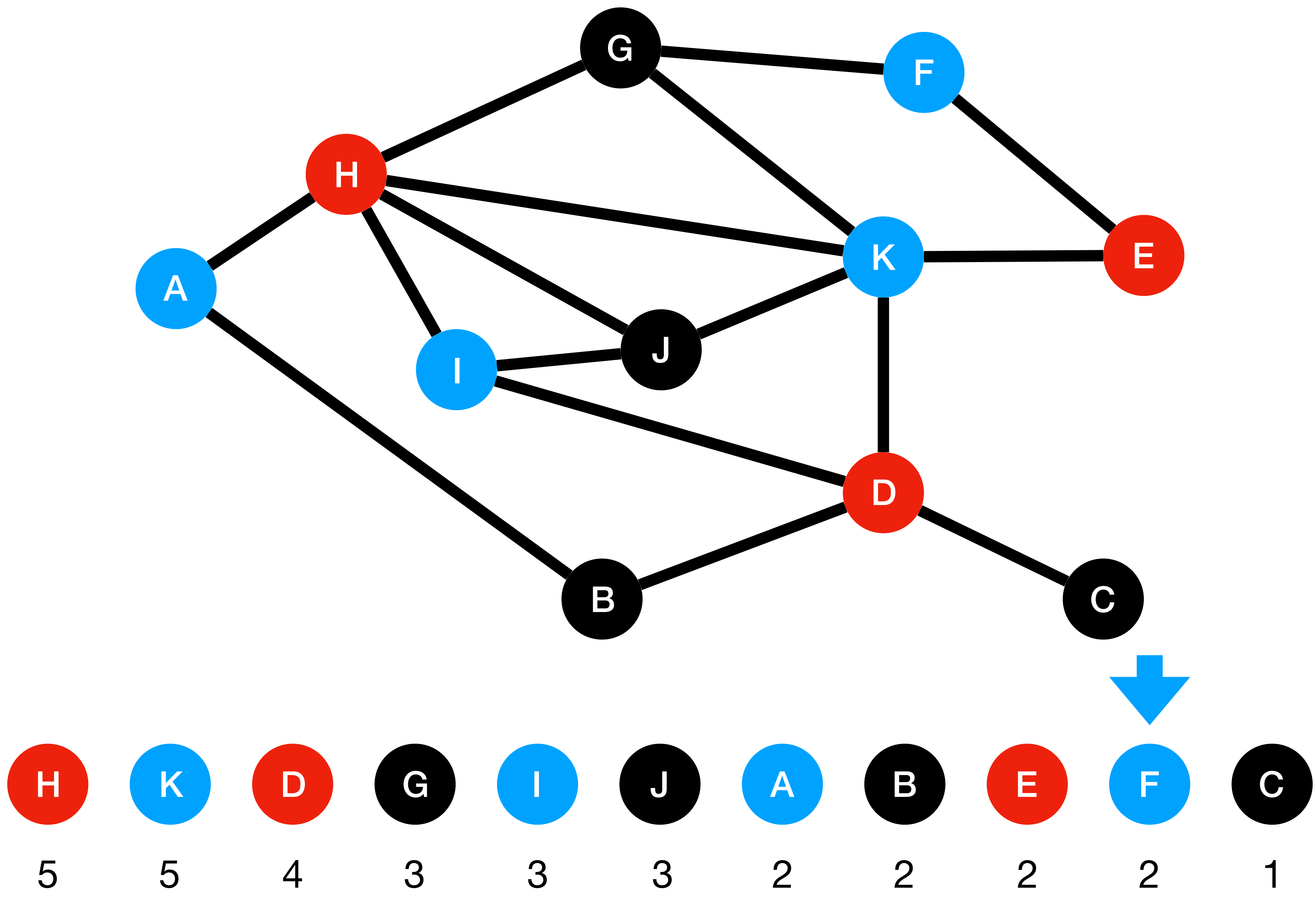

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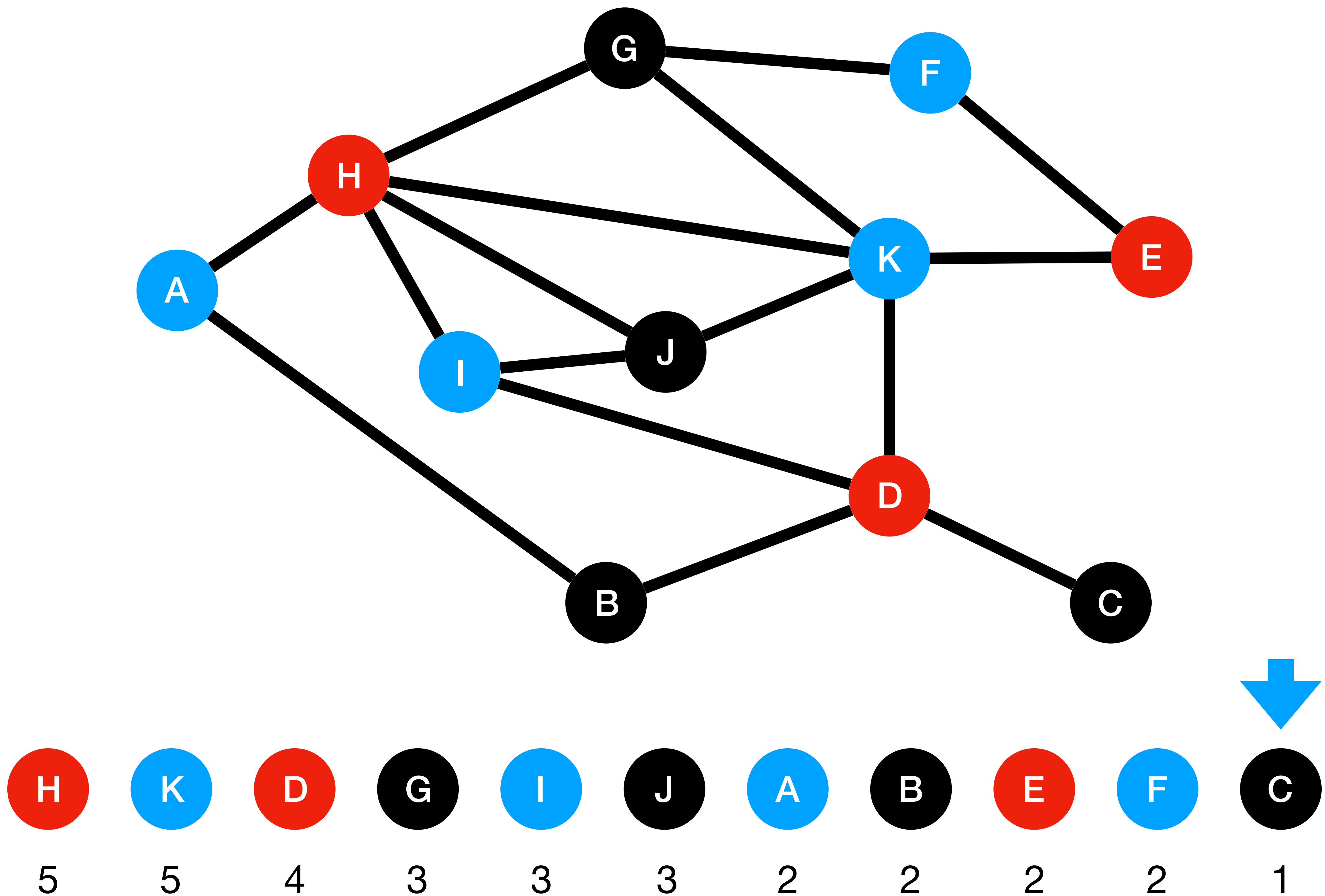

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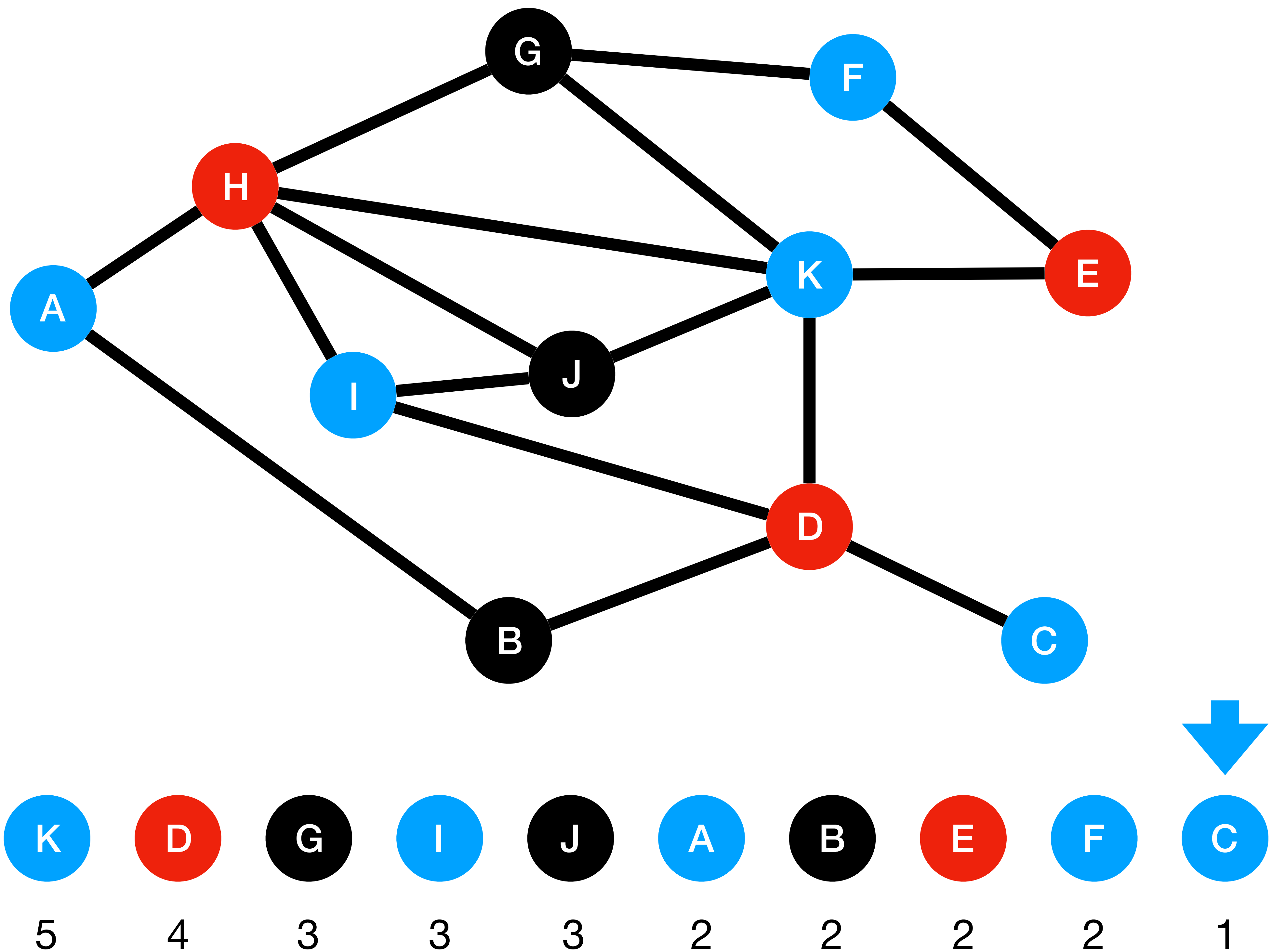

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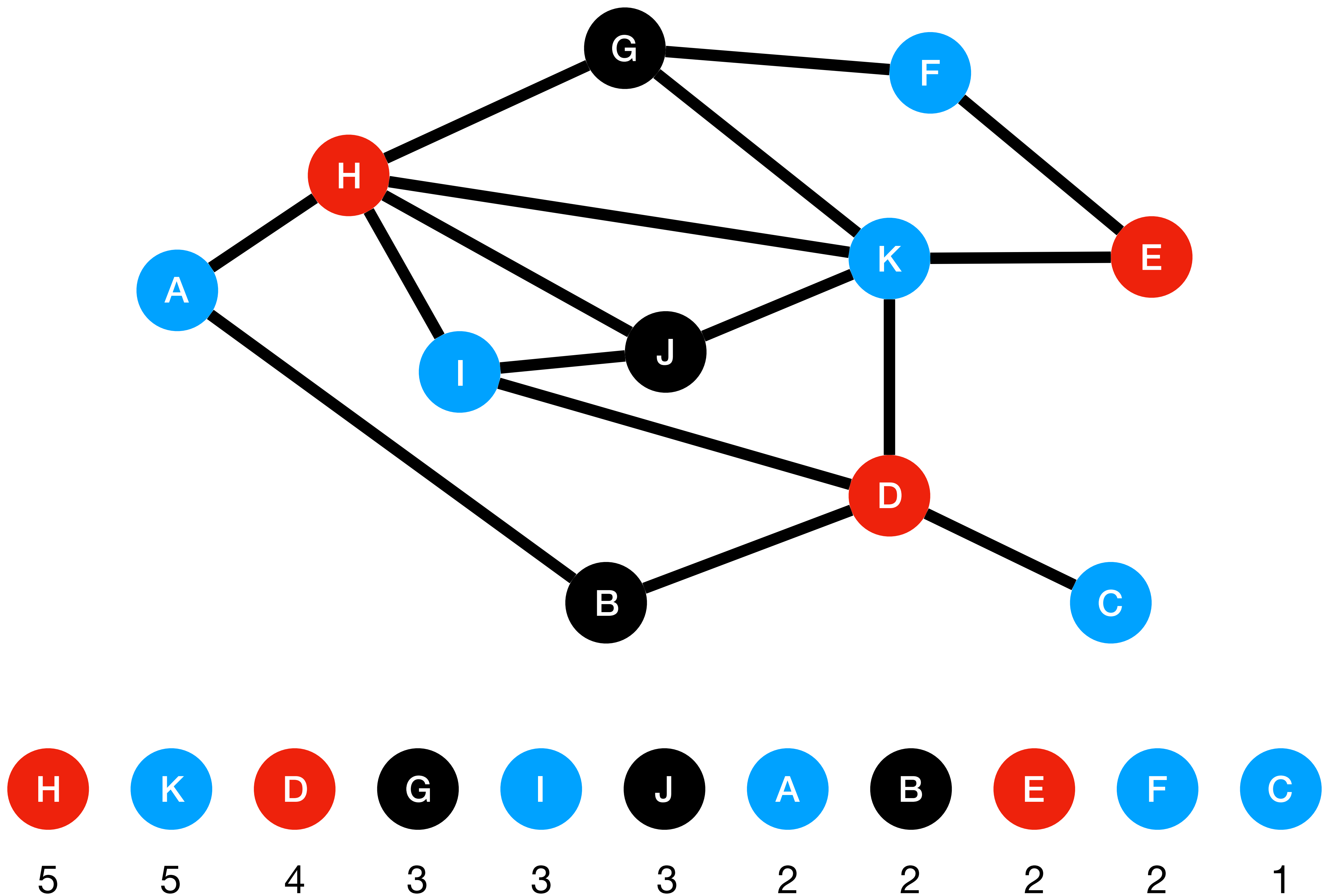

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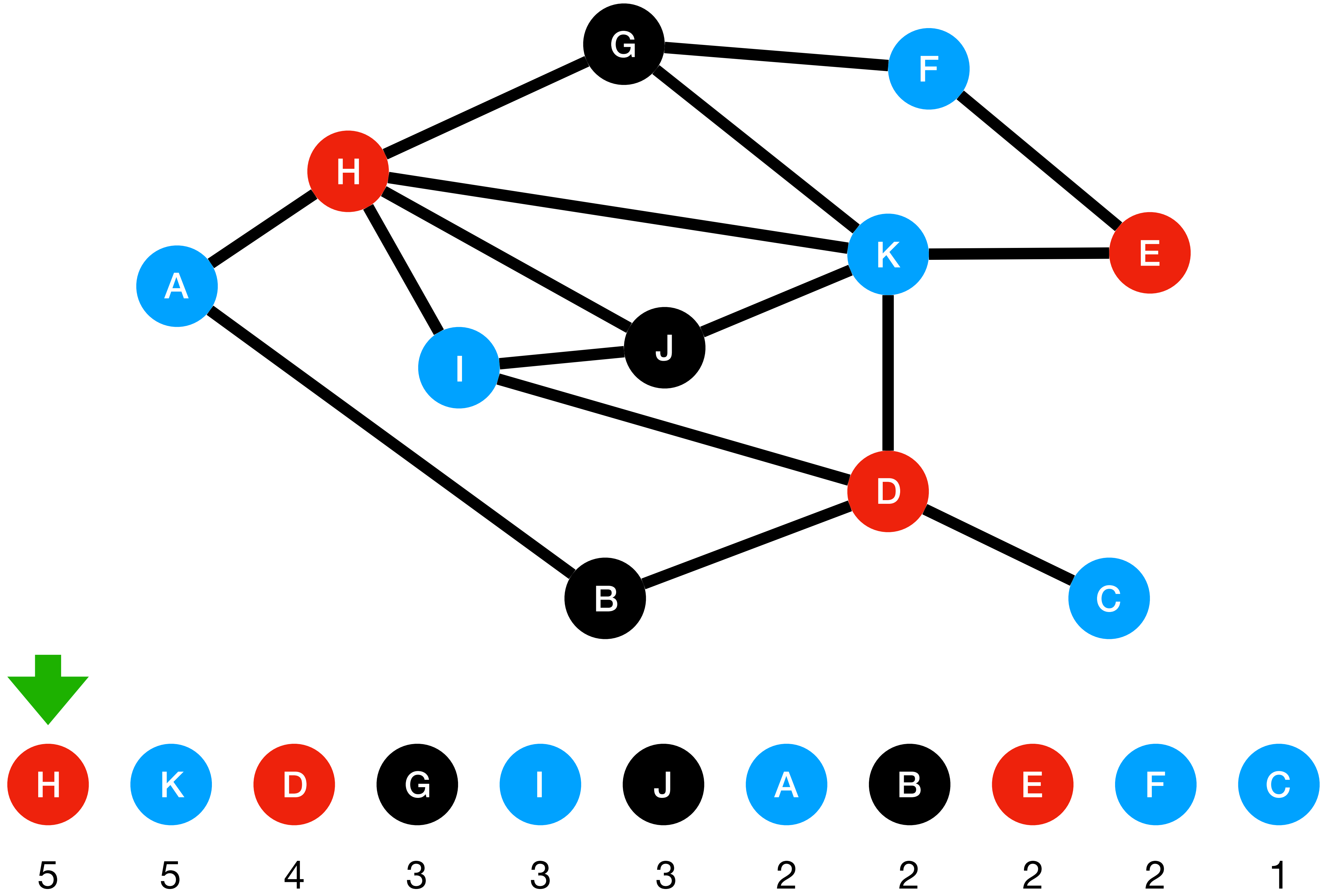

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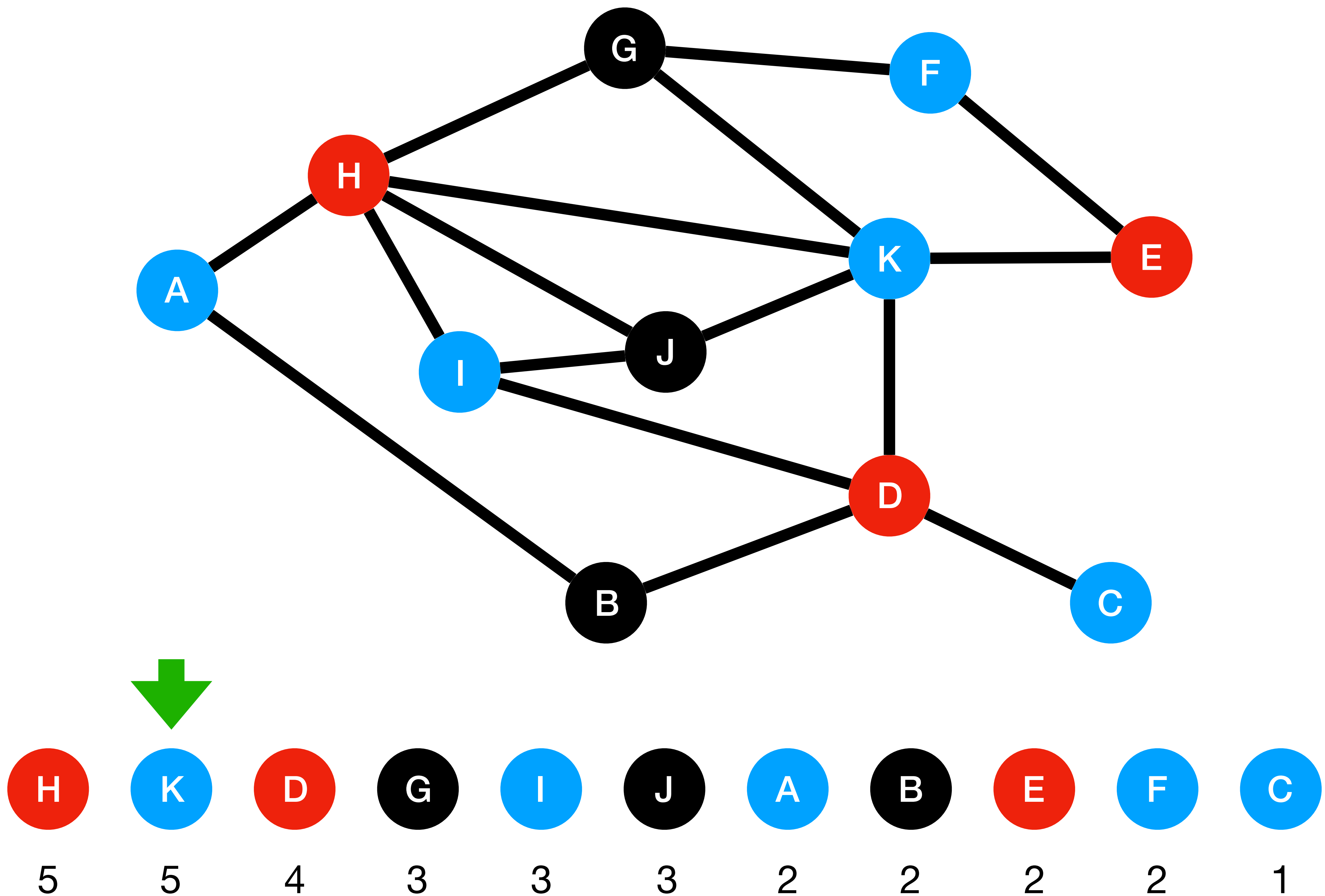

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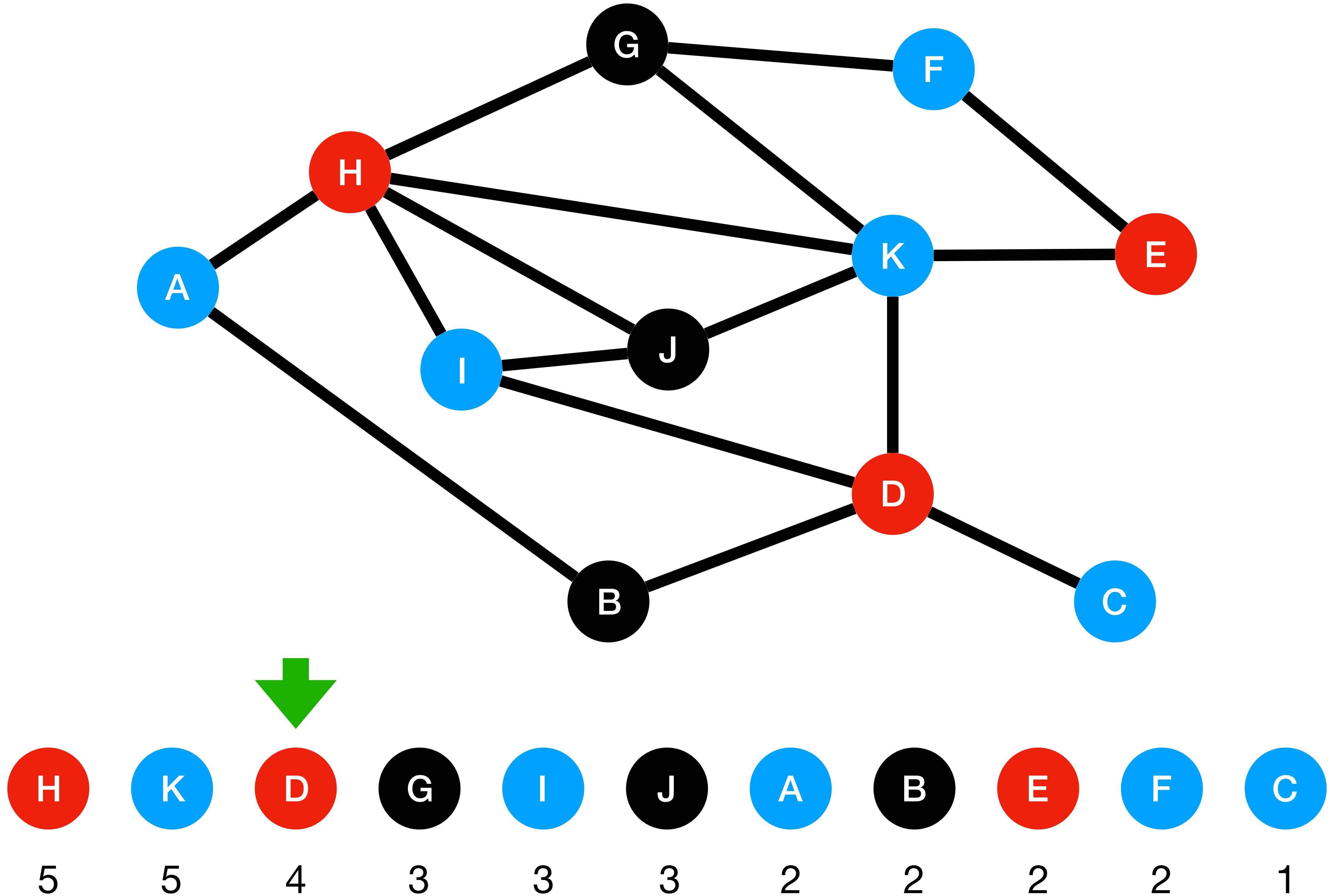

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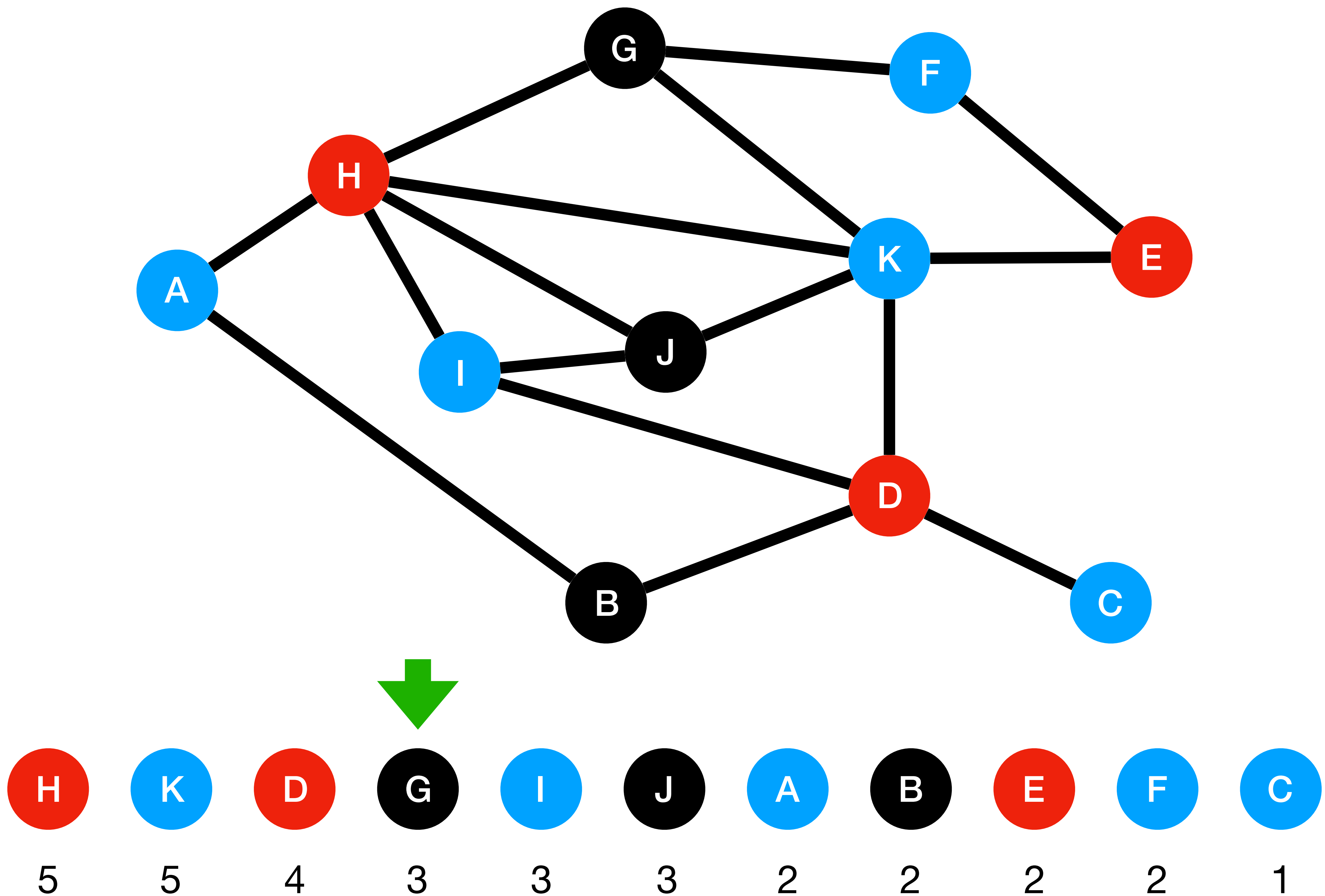

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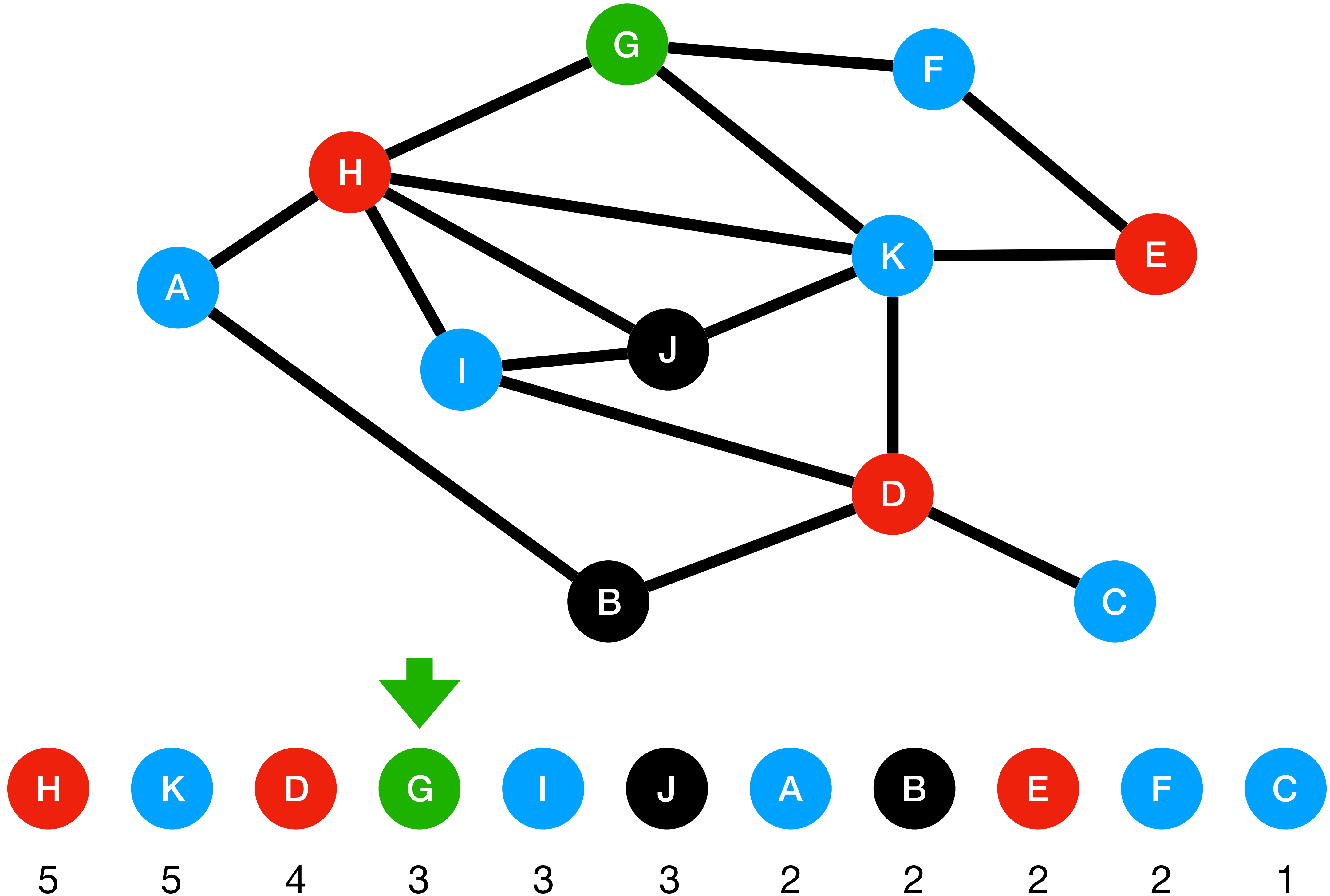

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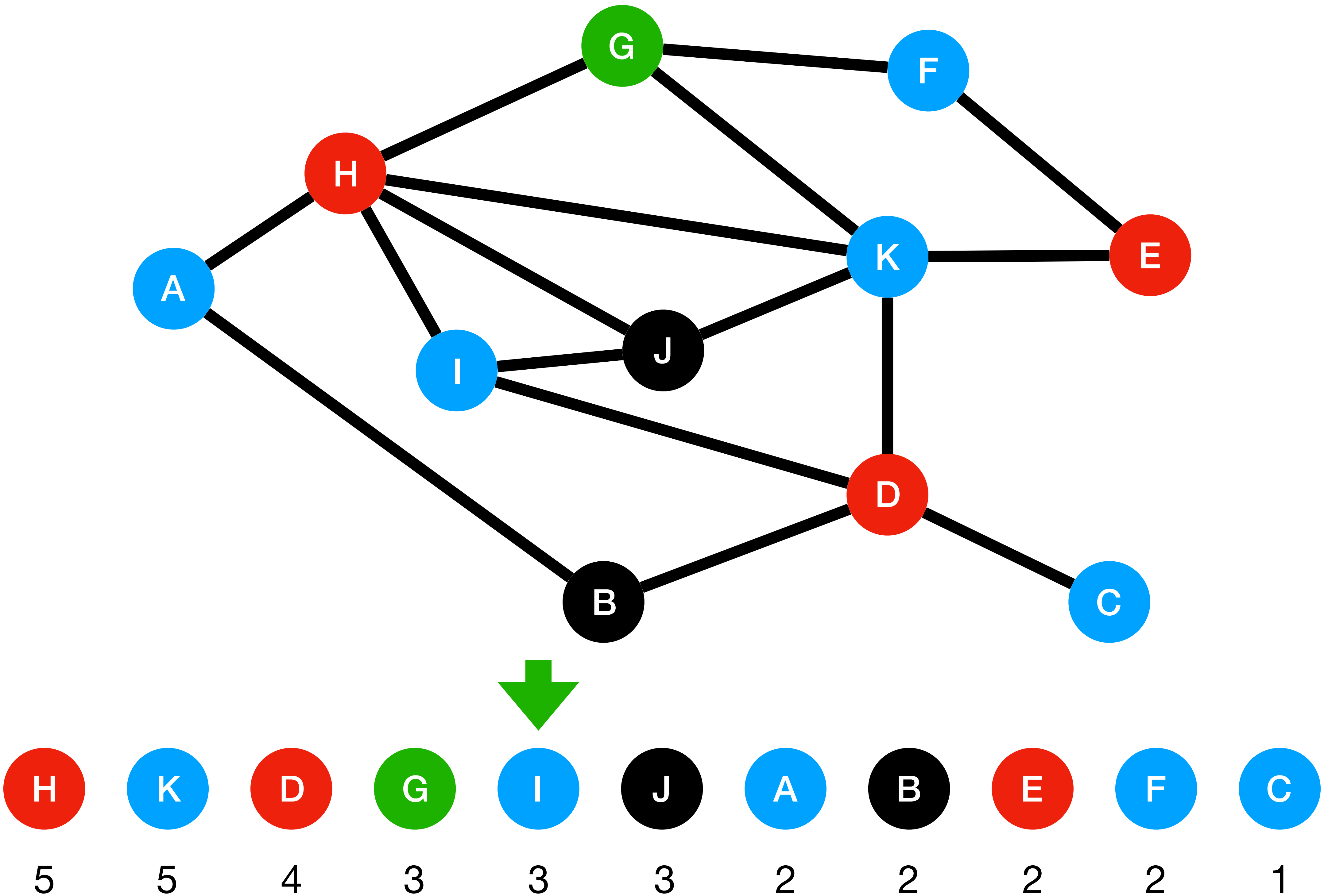

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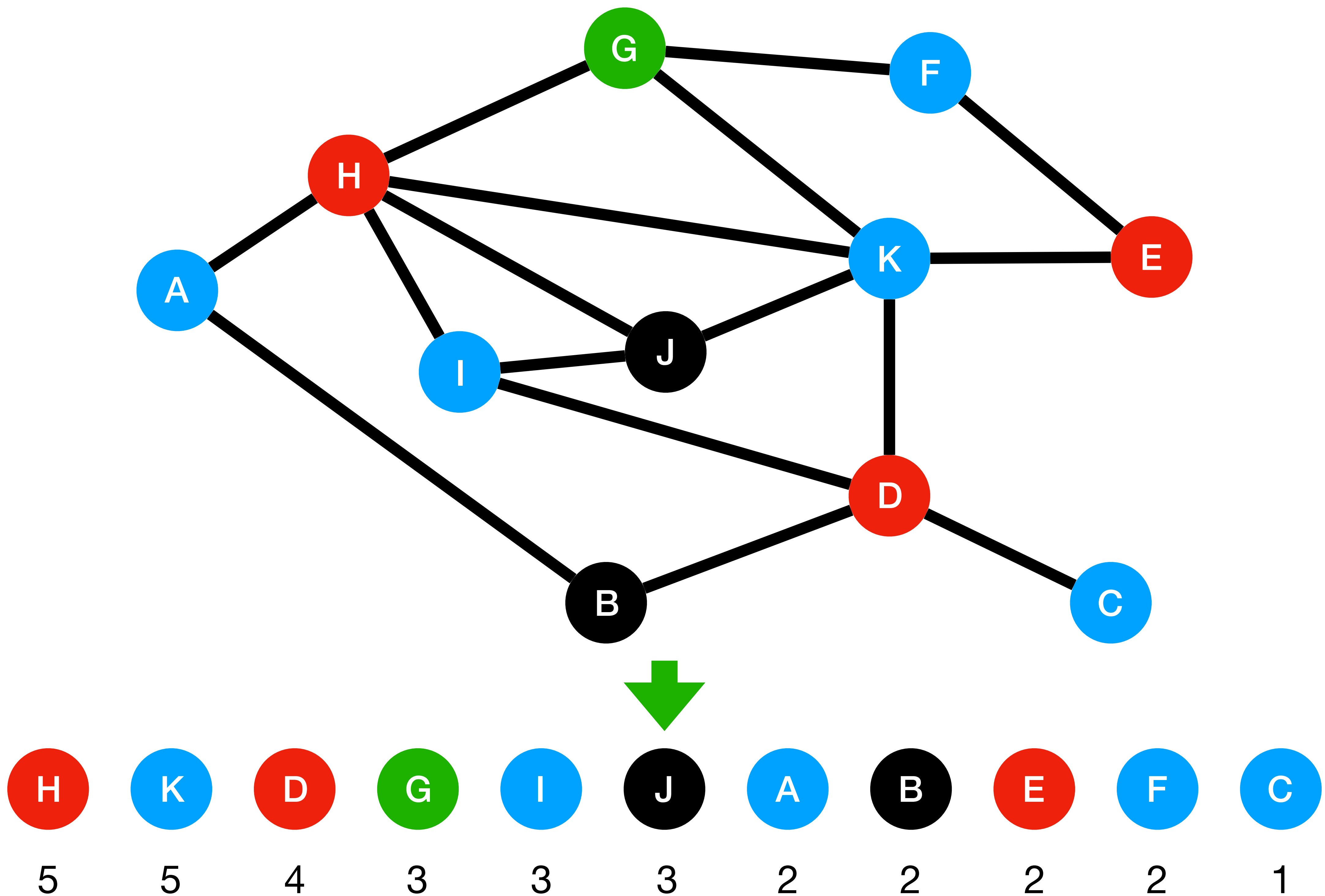

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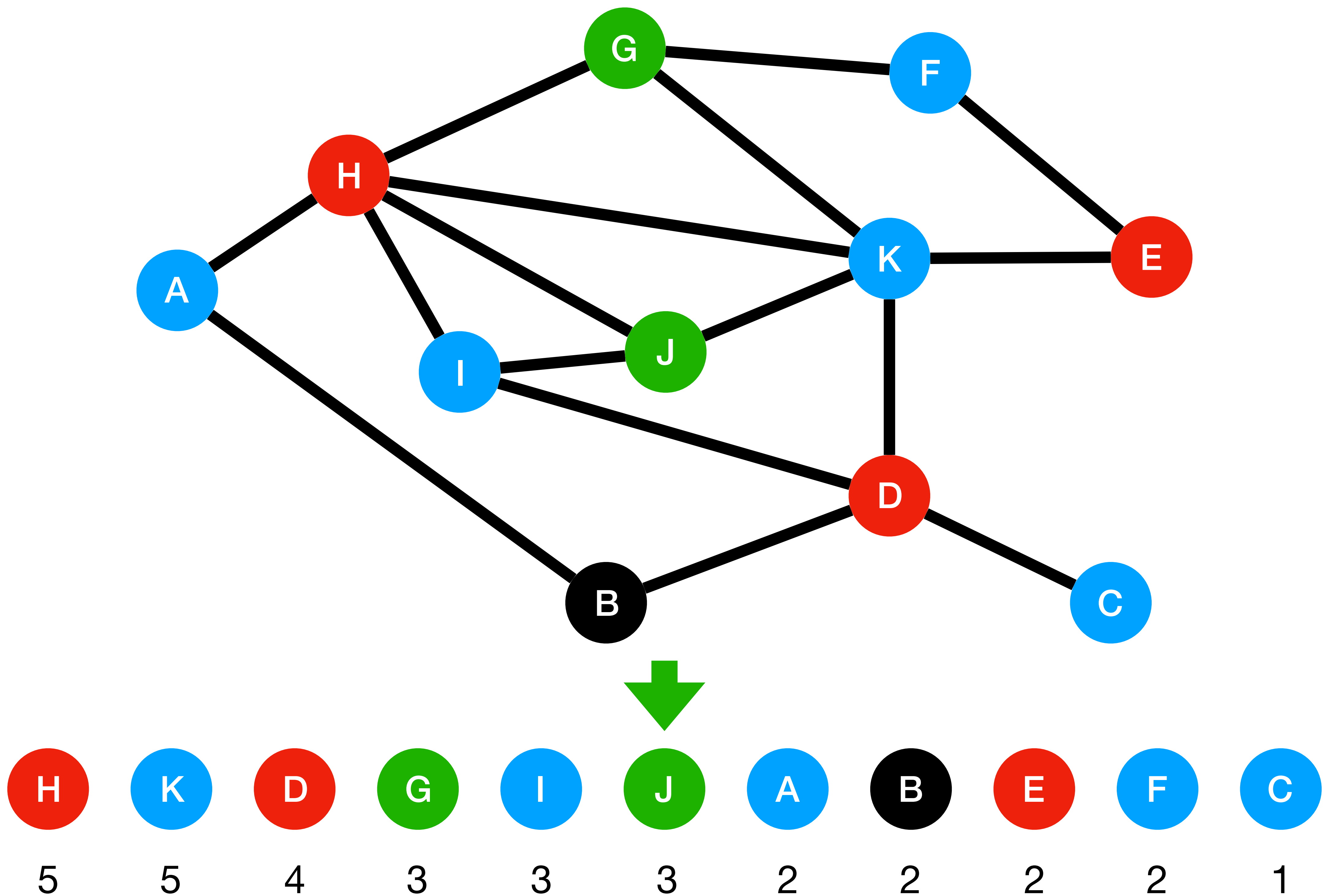

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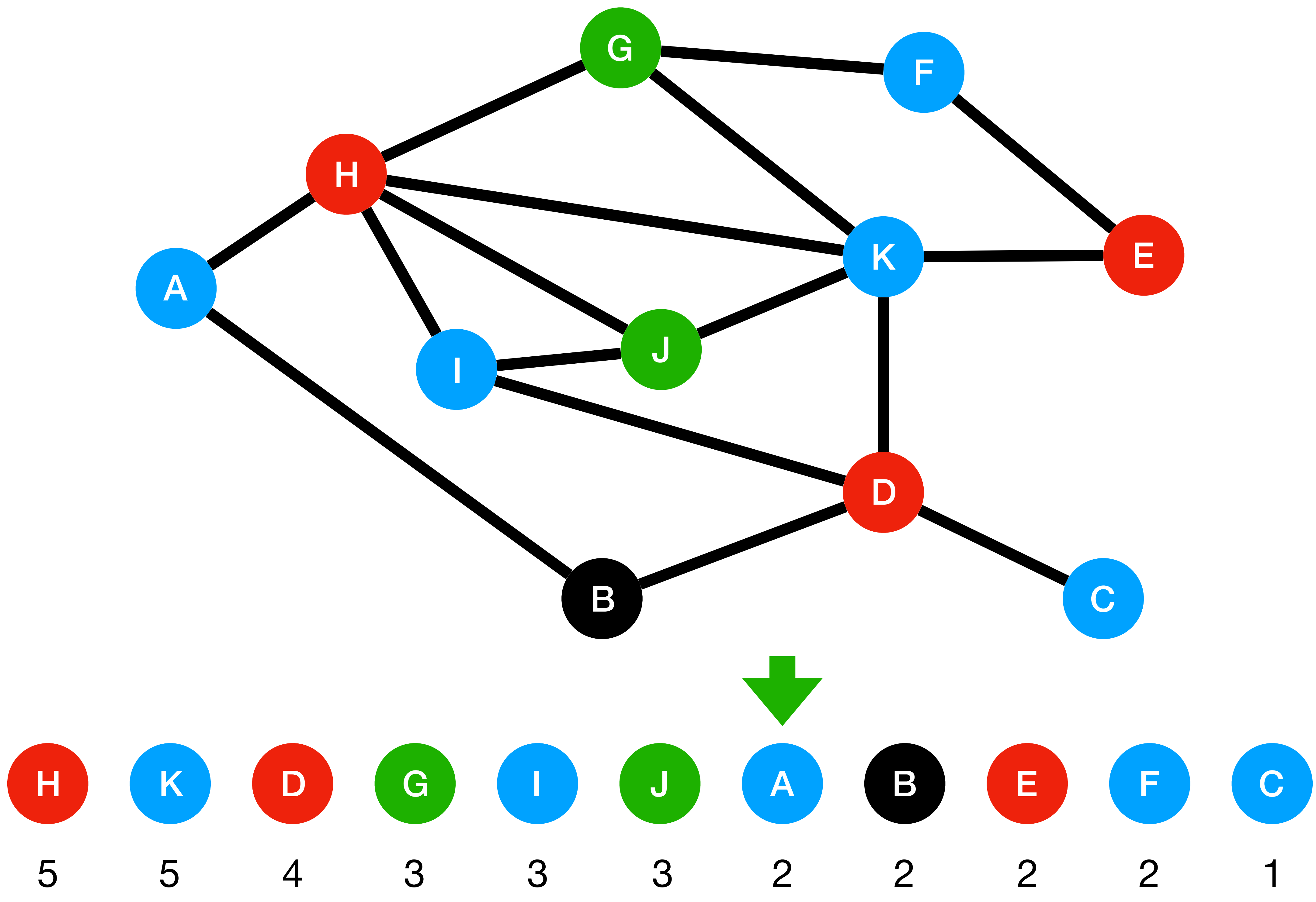

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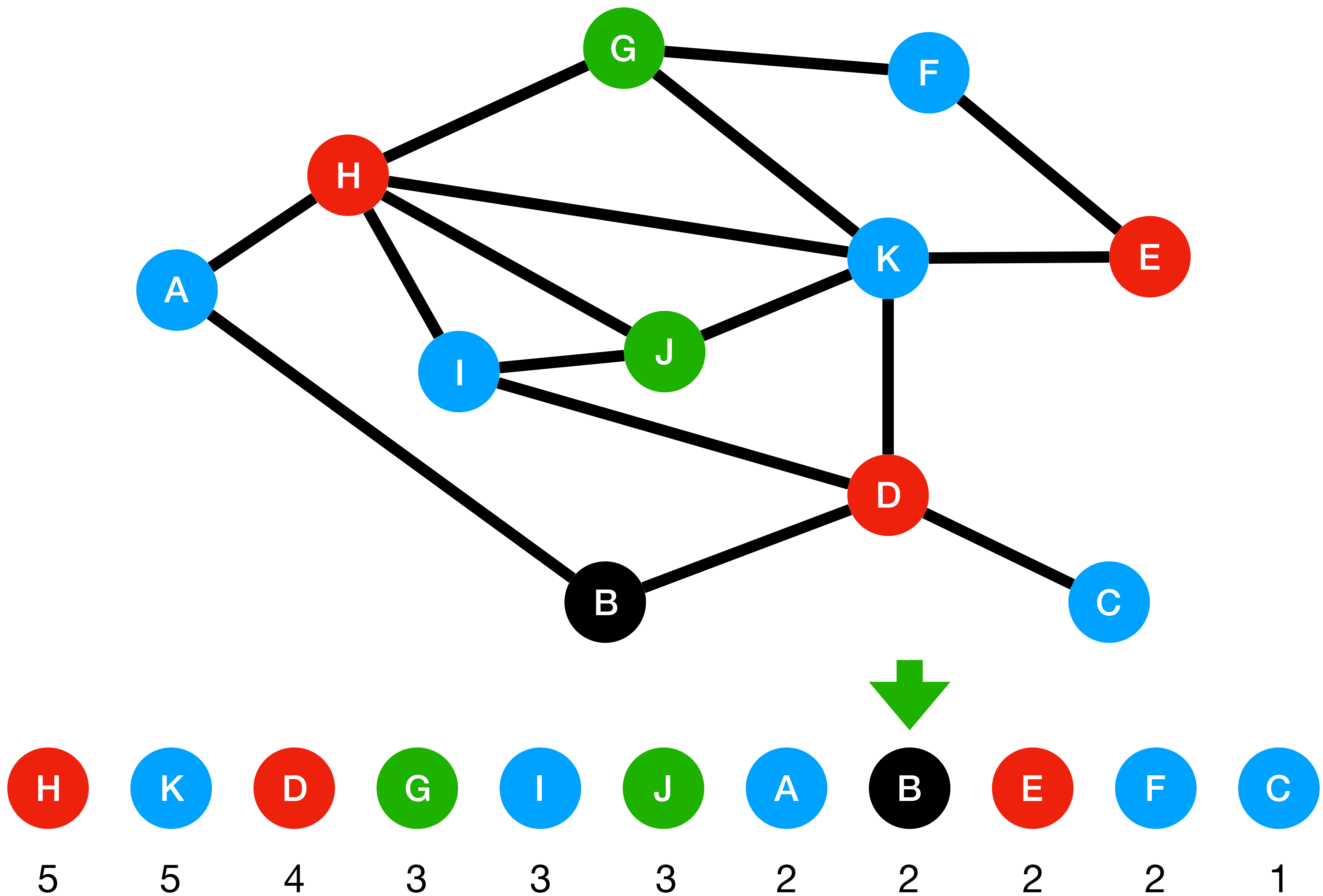

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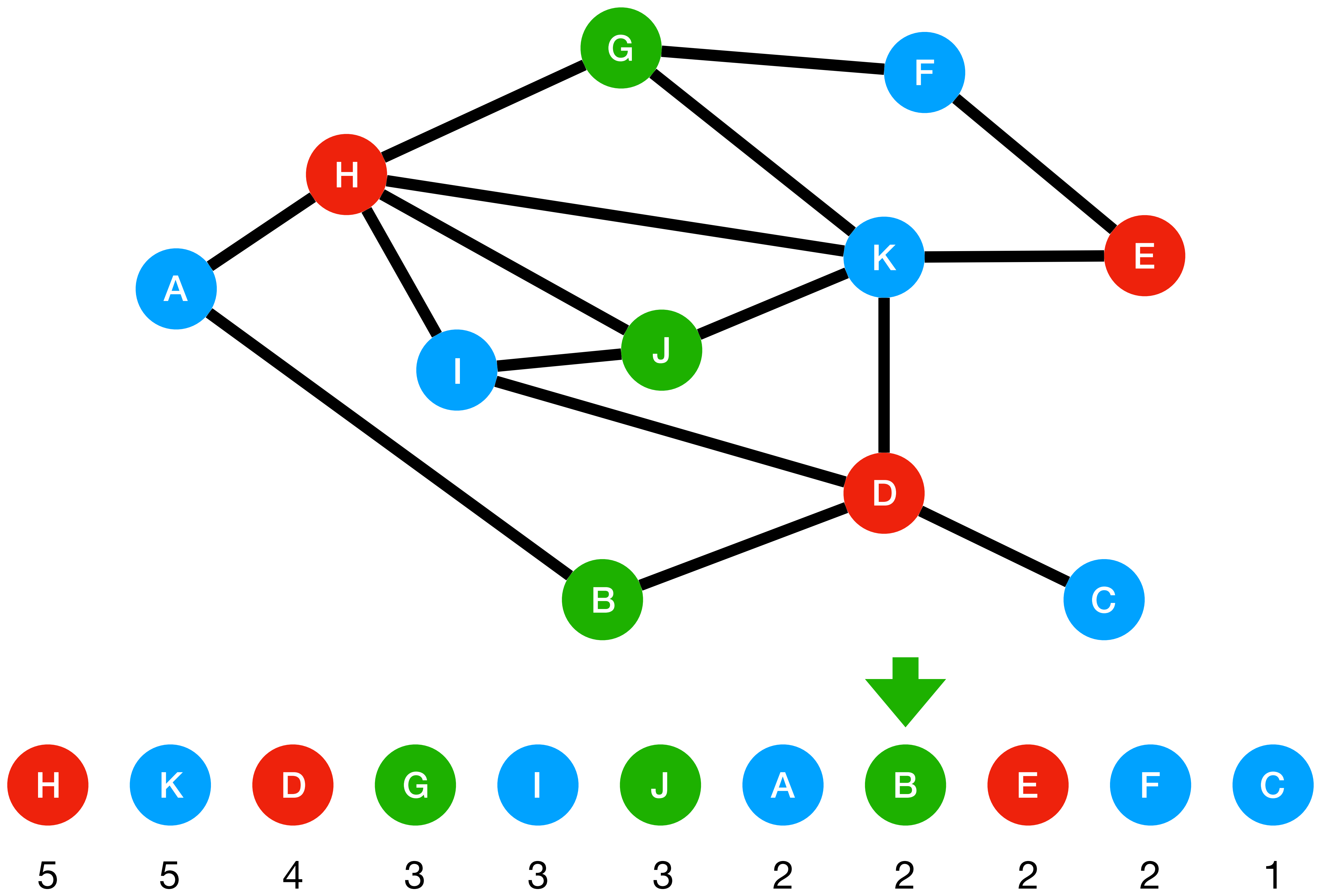

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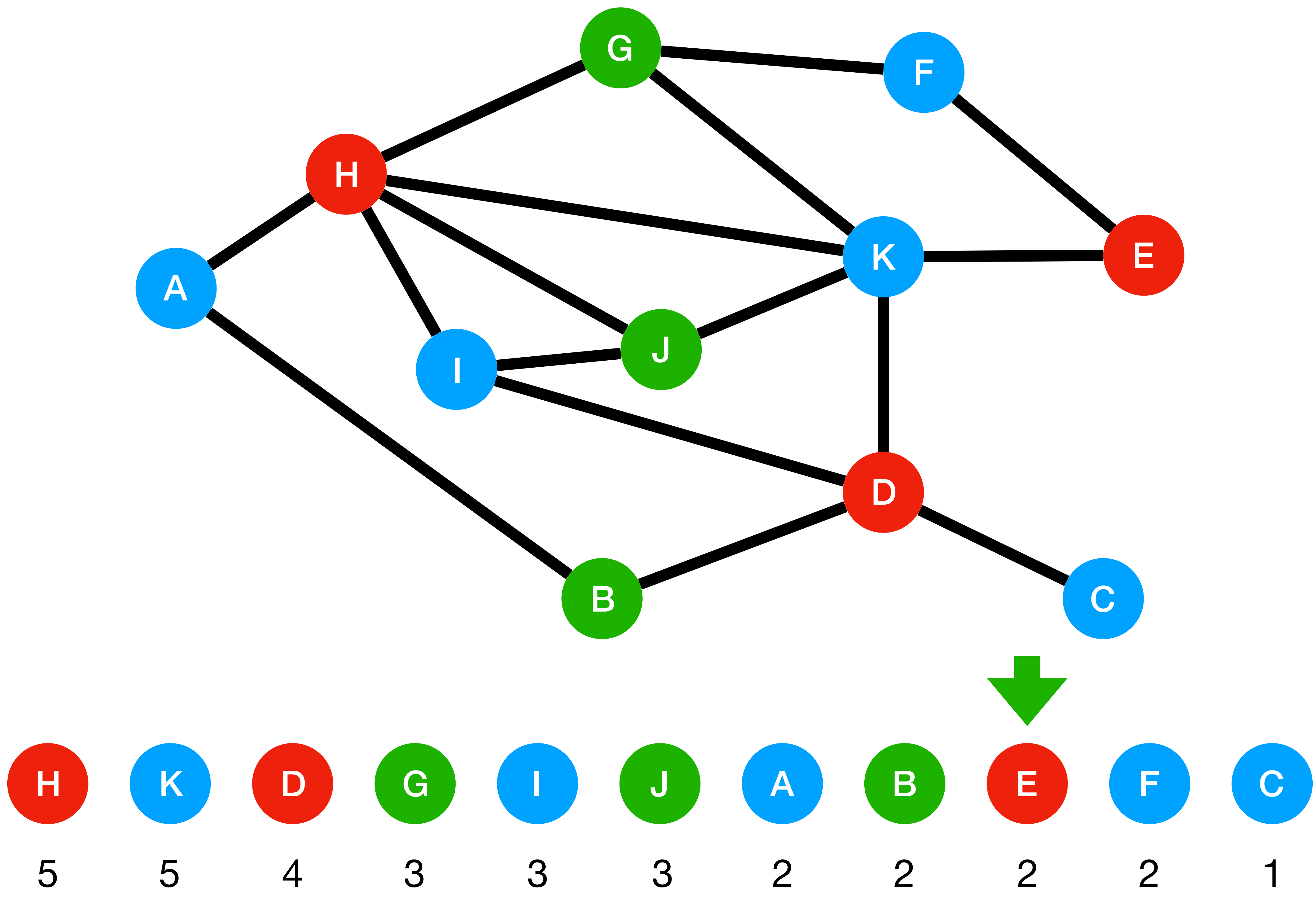

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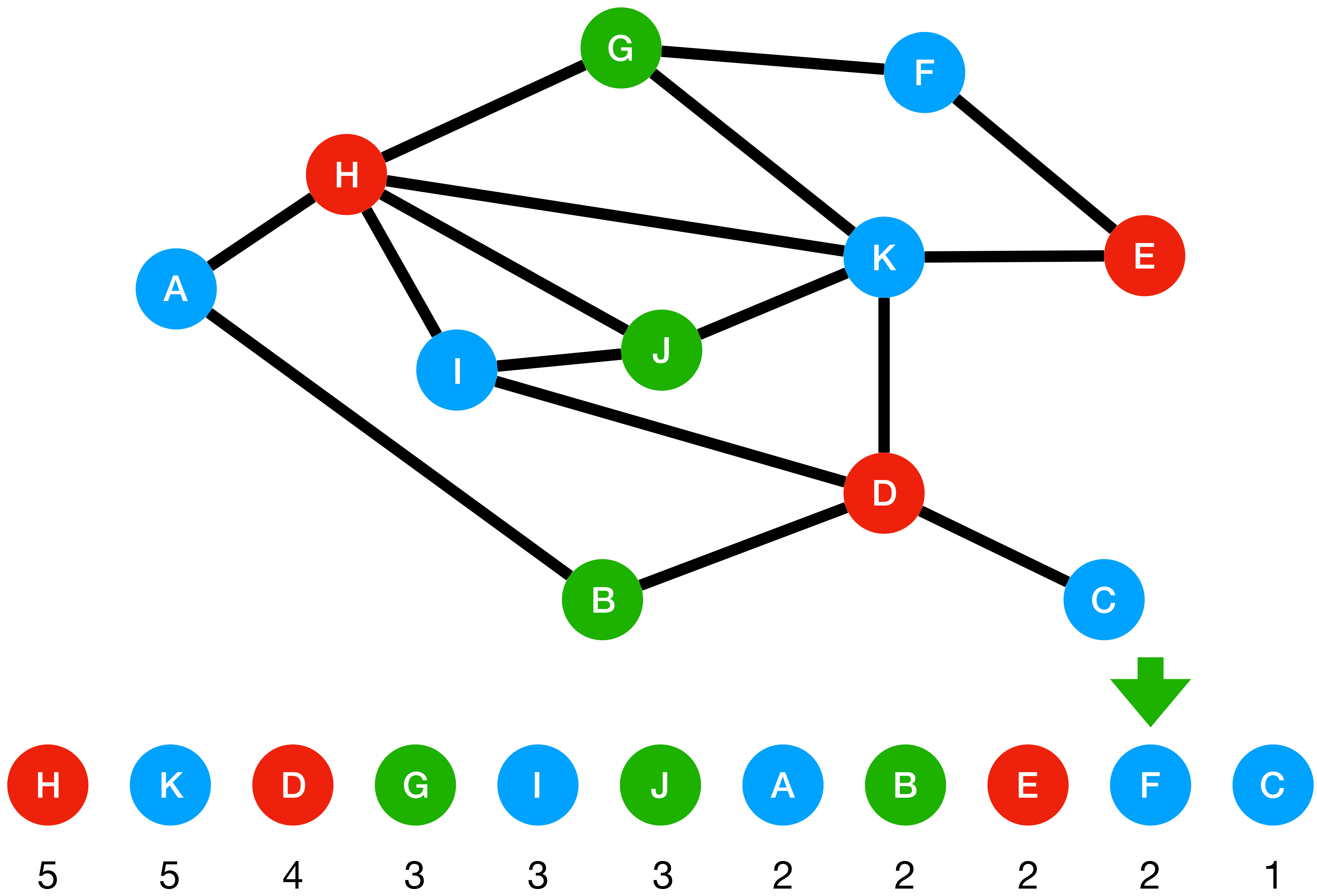

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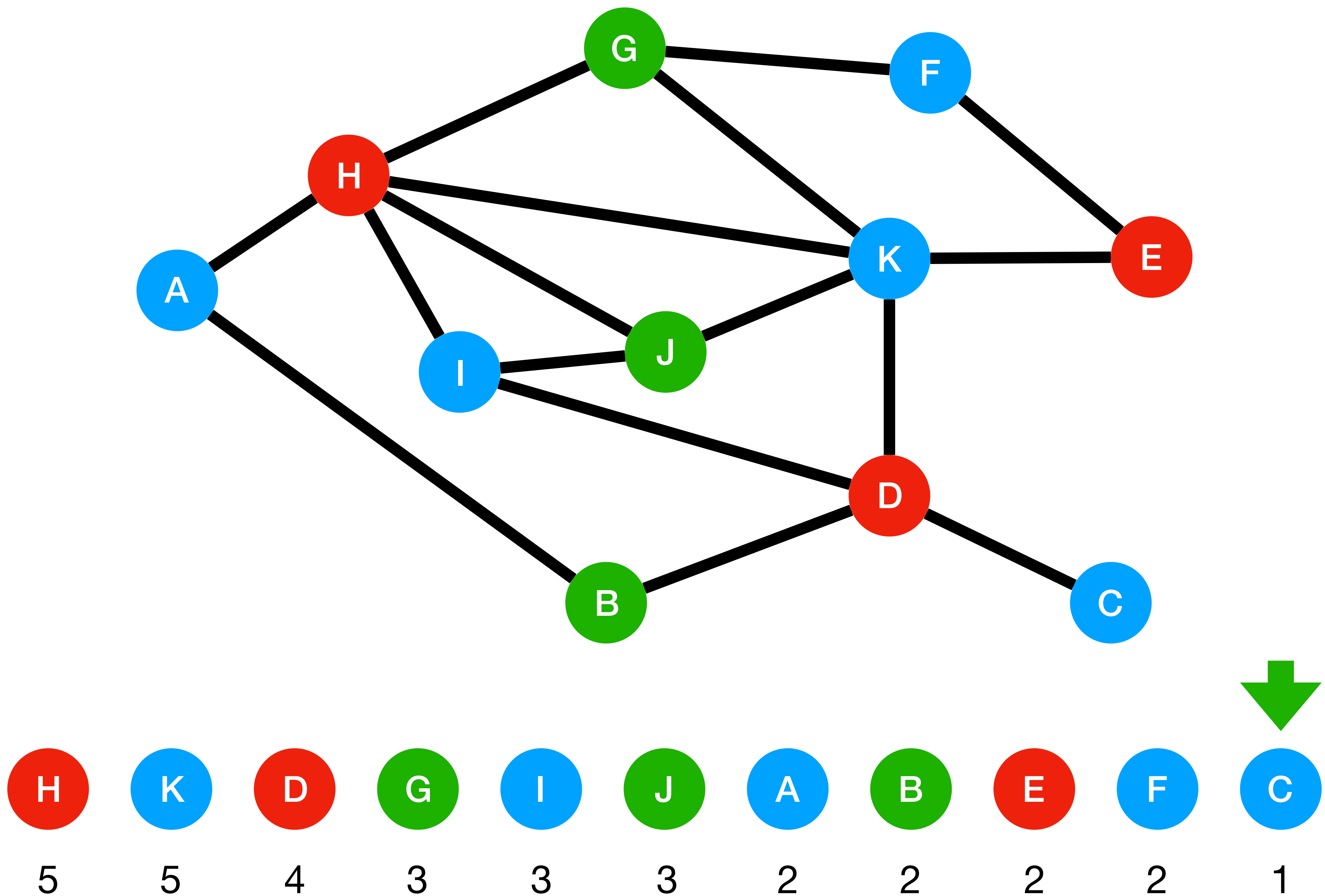

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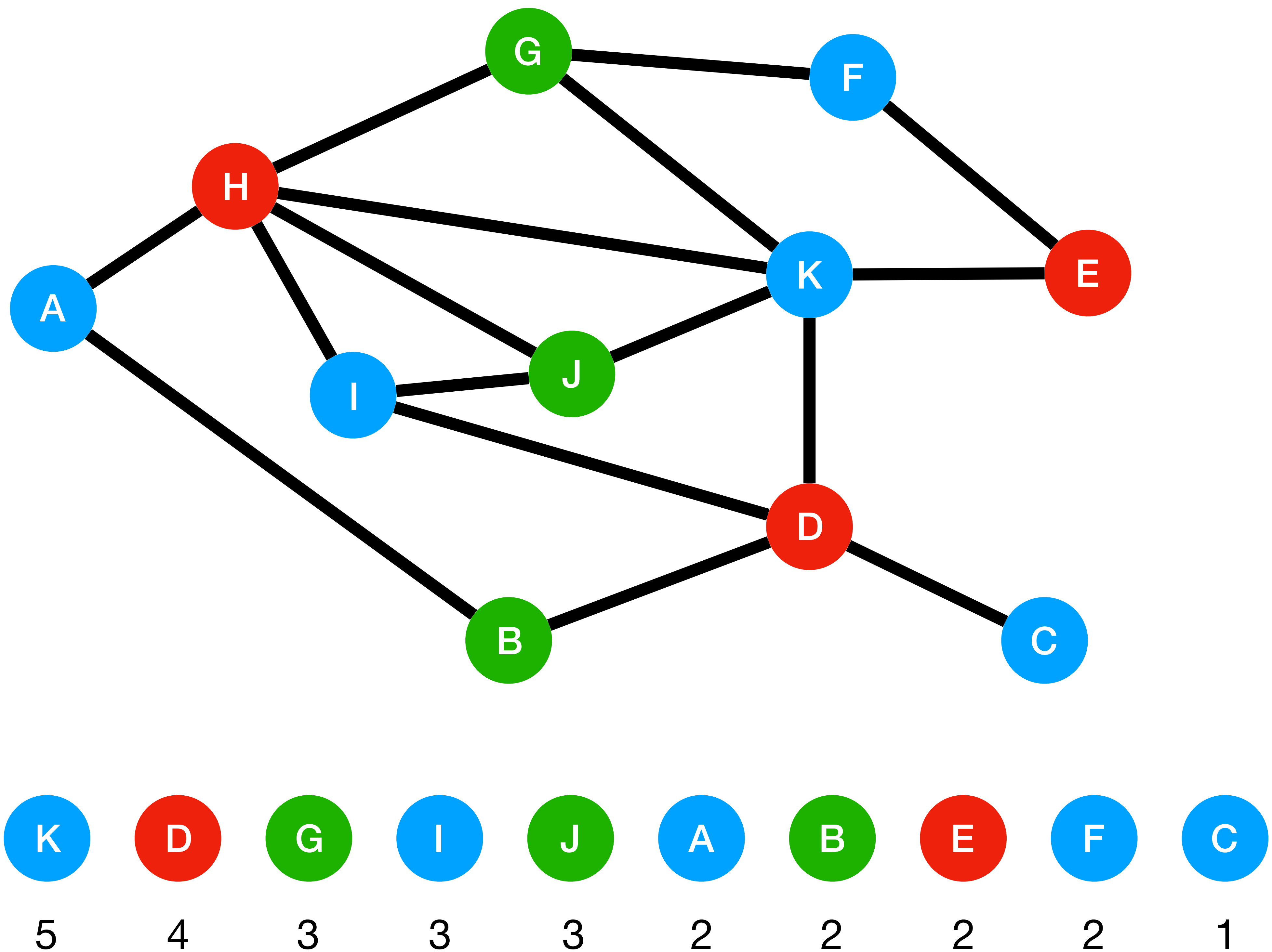

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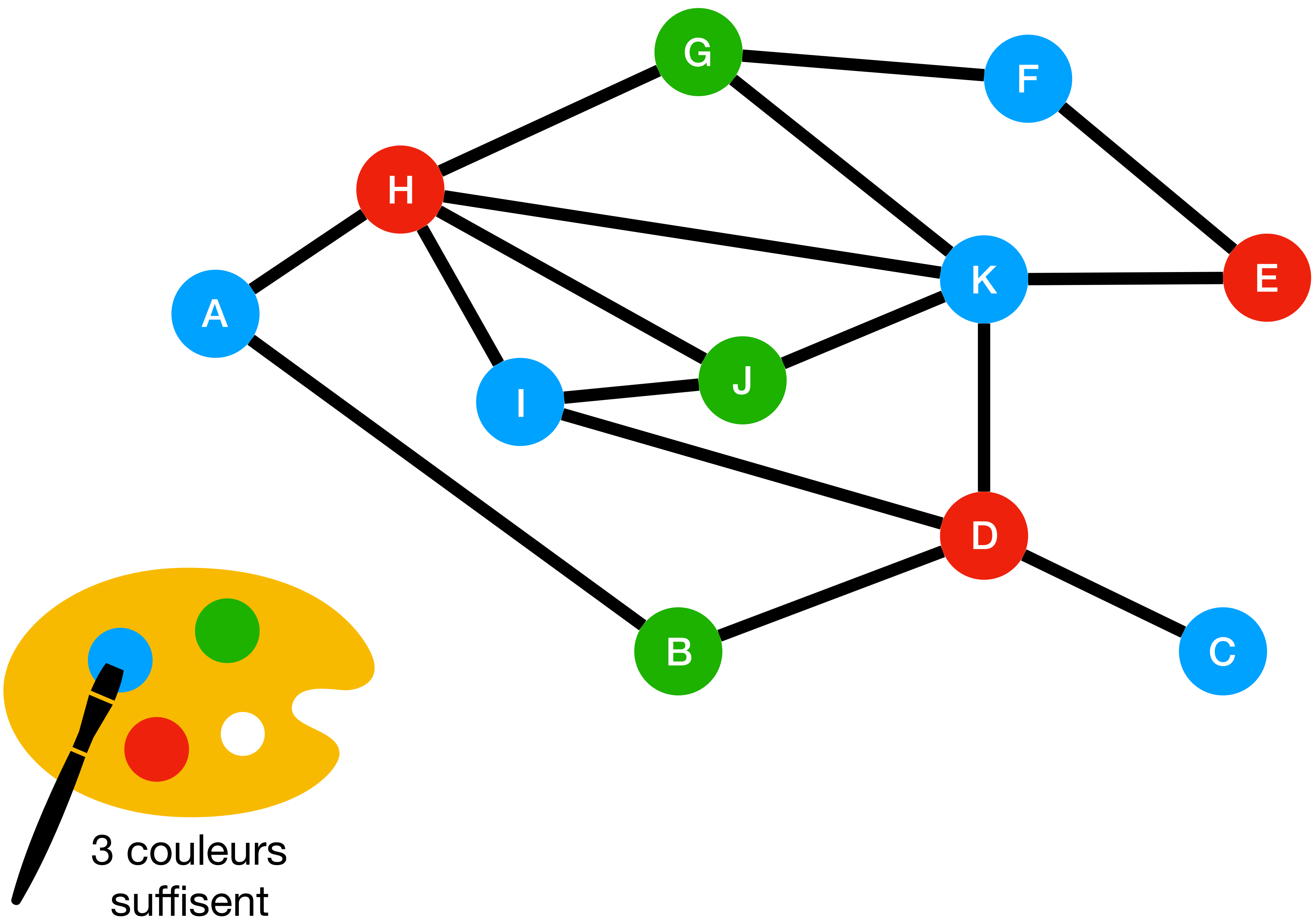

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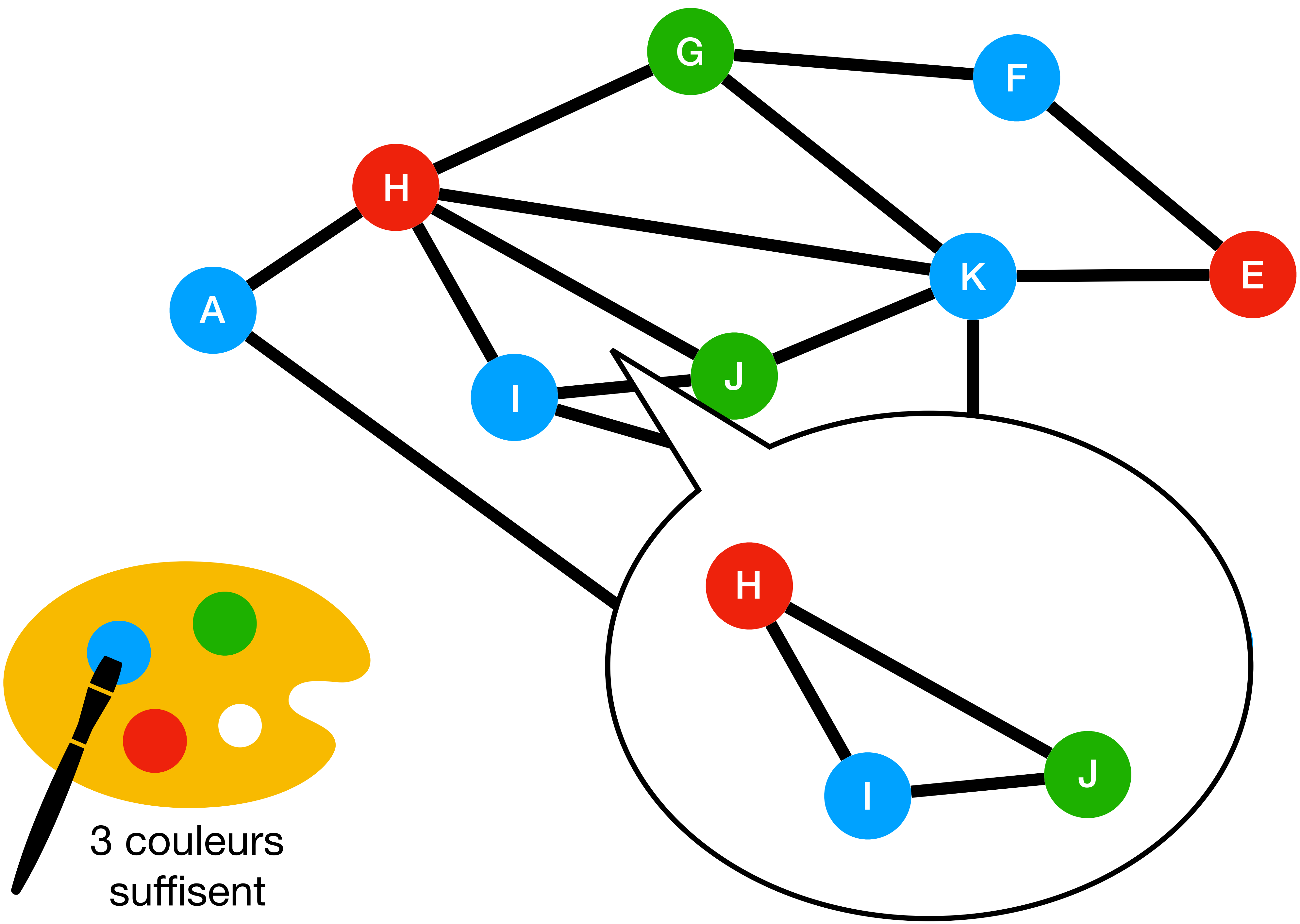

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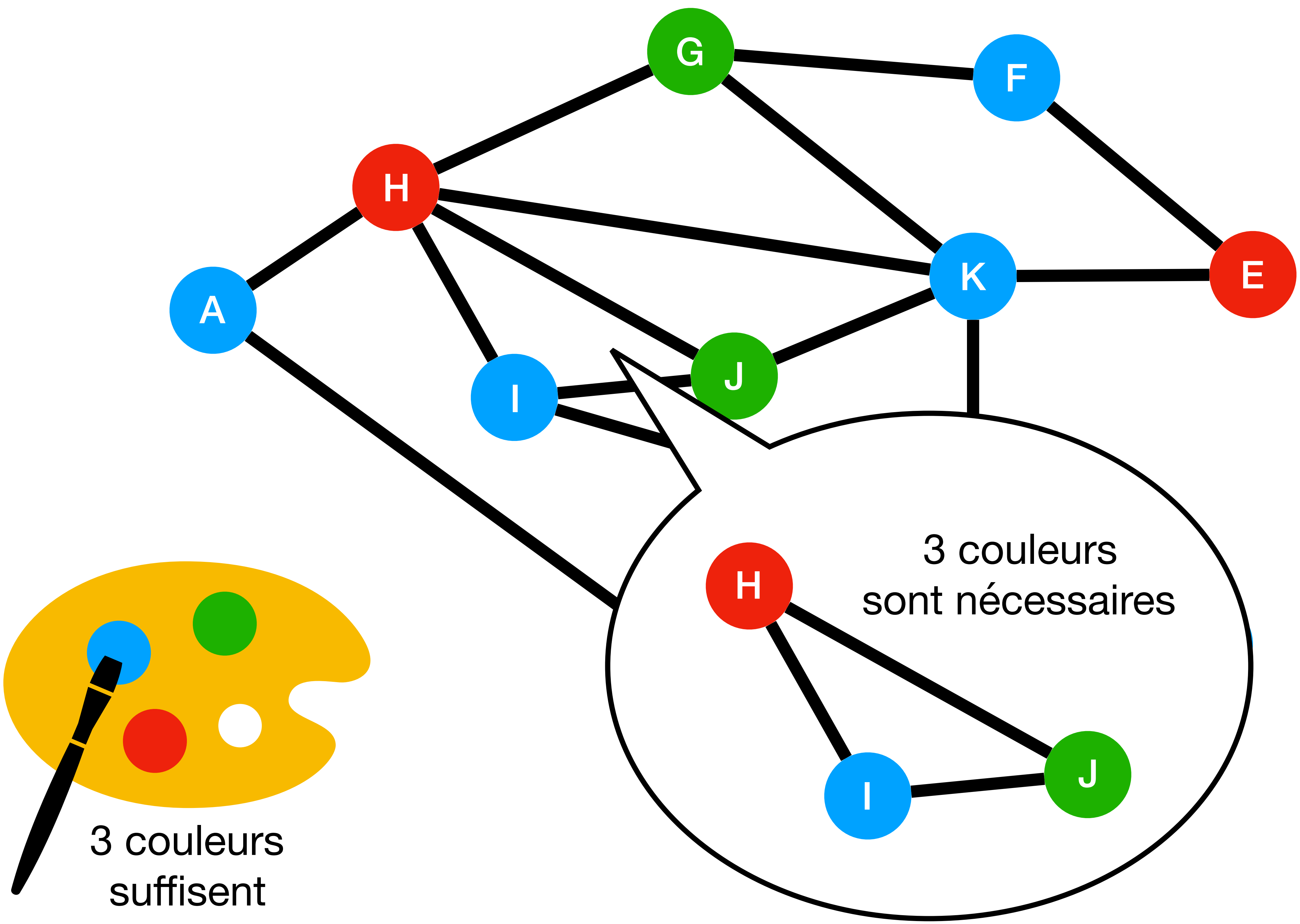
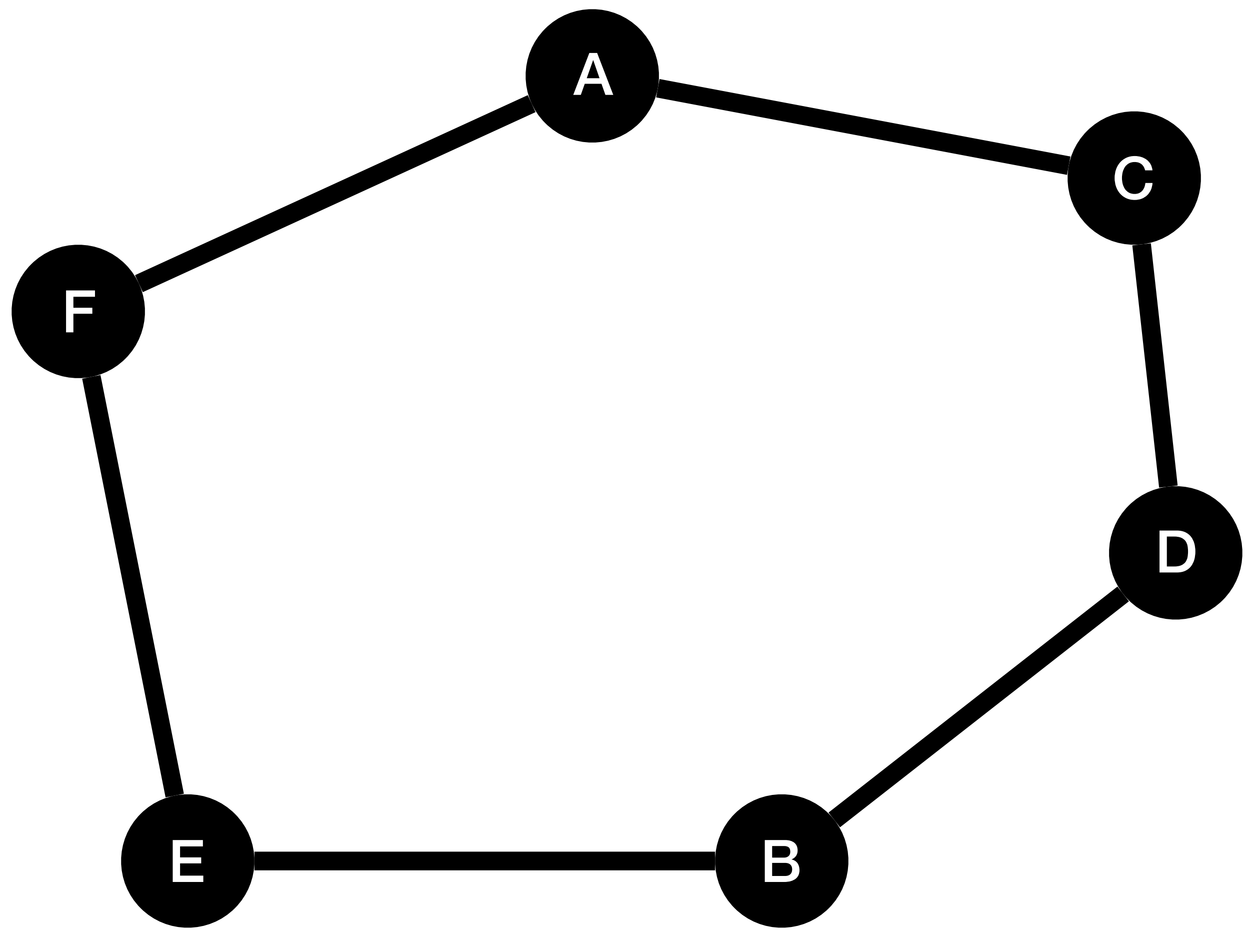


Algorithme de Welsh-Powell

- Trier le sommets du graphe par ordre de degré (nombre de voisins) décroissant
- **couleur** ≔ rouge
- **Tant qu**’il y a encore des sommets en noir **faire**
    - Parcourir la liste triée des sommets et colorer en **couleur** les sommets en noir qui ne sont pas connectés à d’autres sommets de la même couleur
    - Choisir une nouvelle **couleur**

Colorations non optimales



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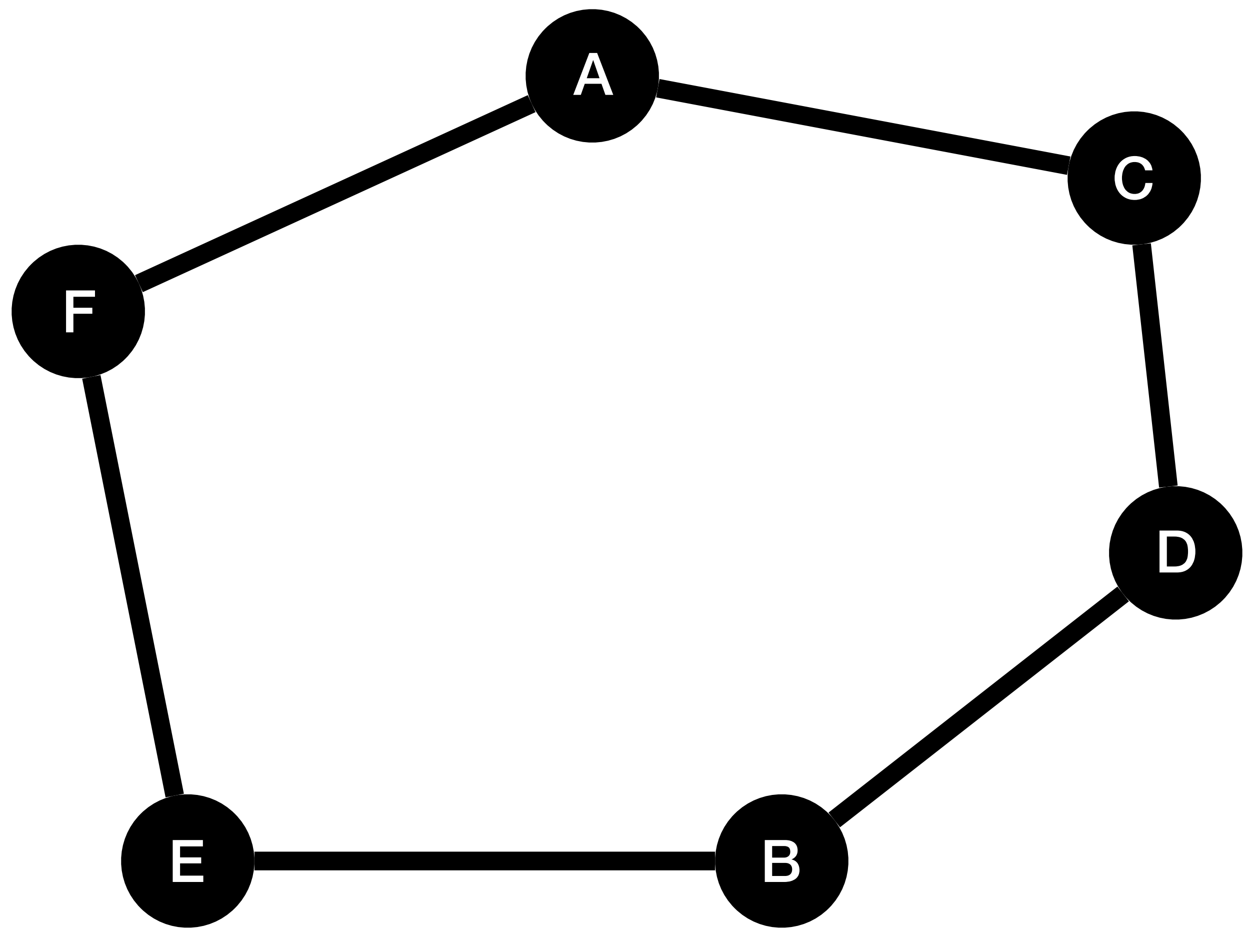

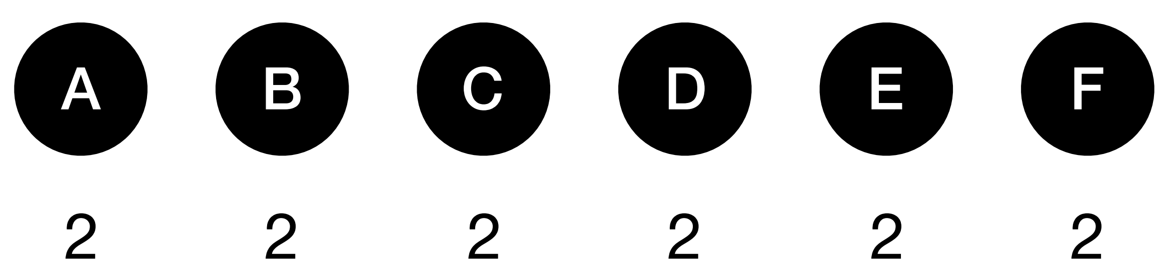

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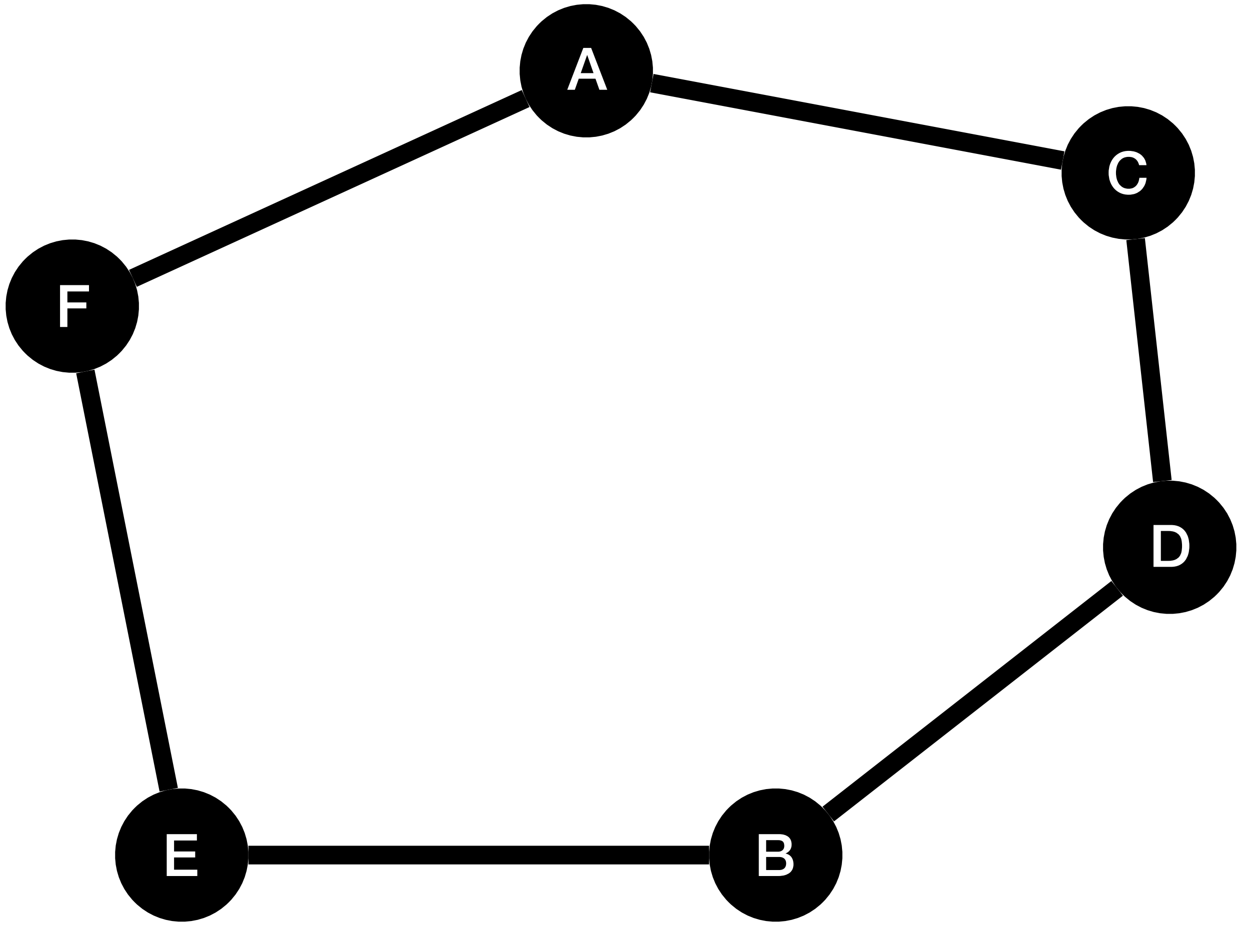

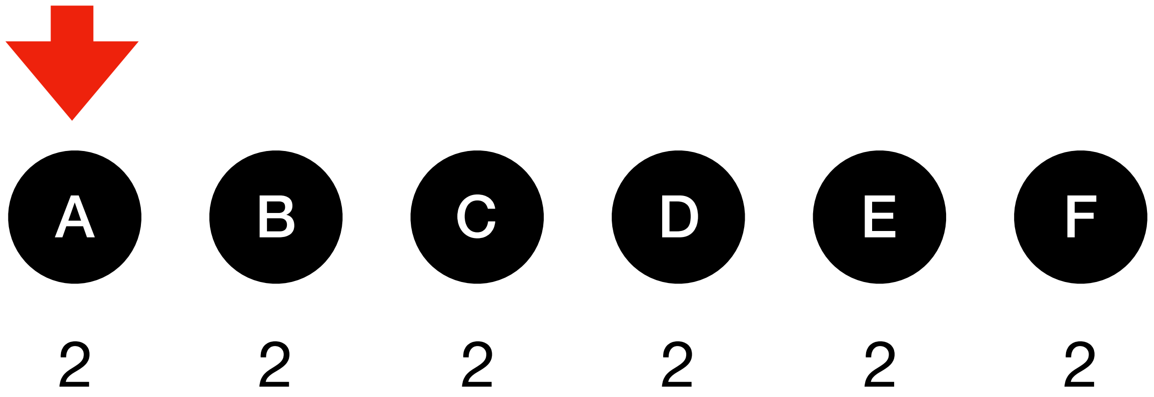

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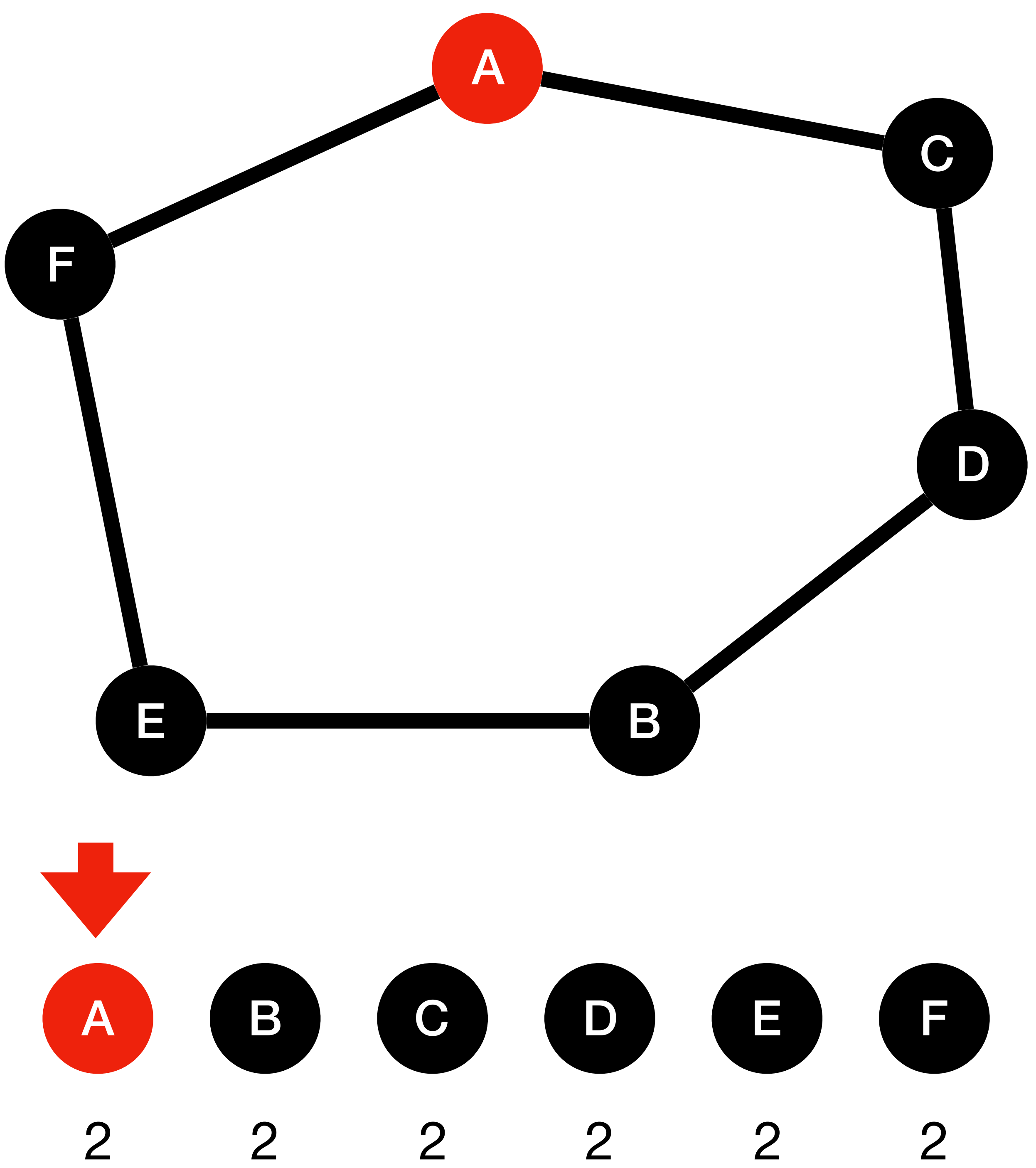

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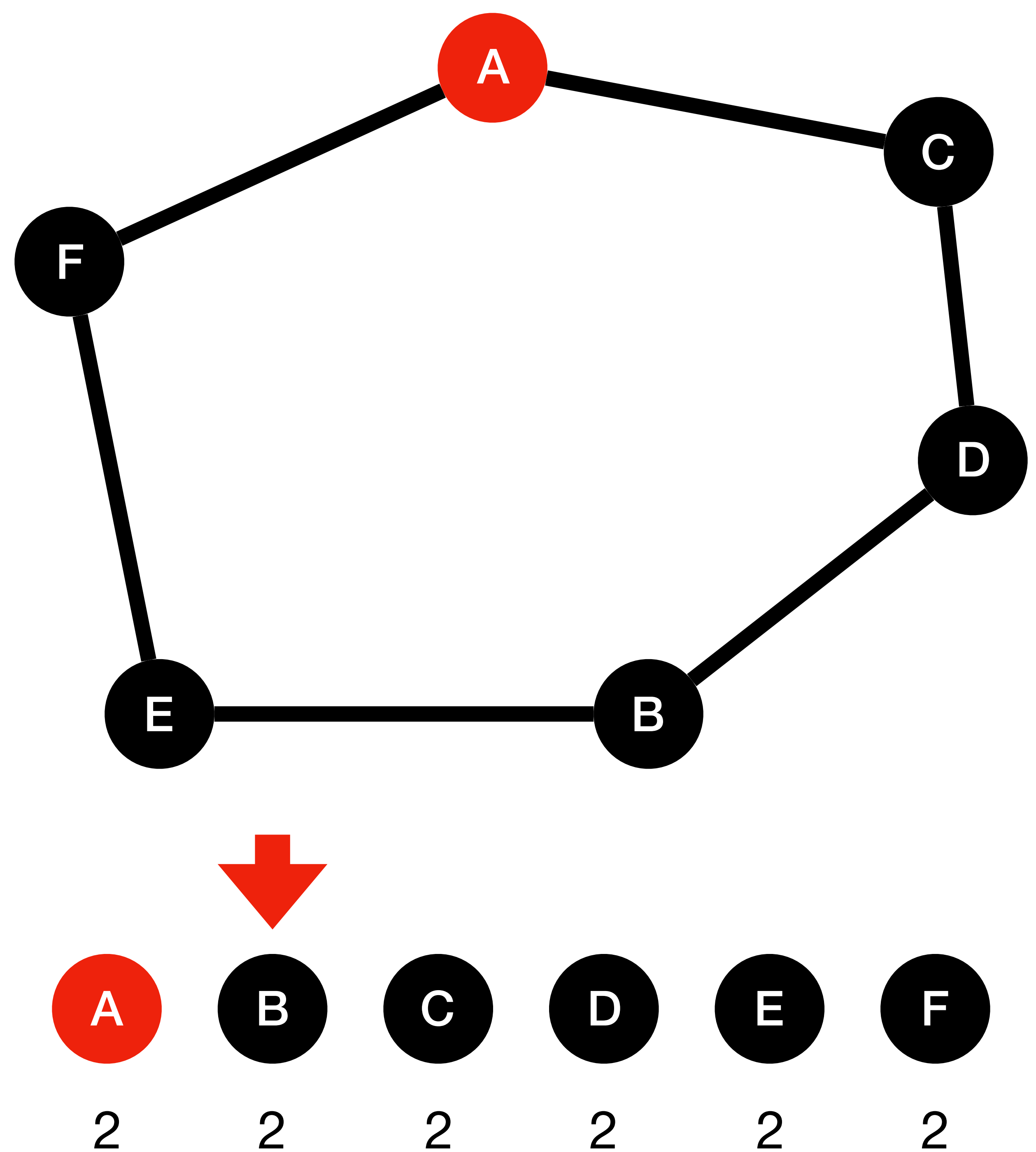

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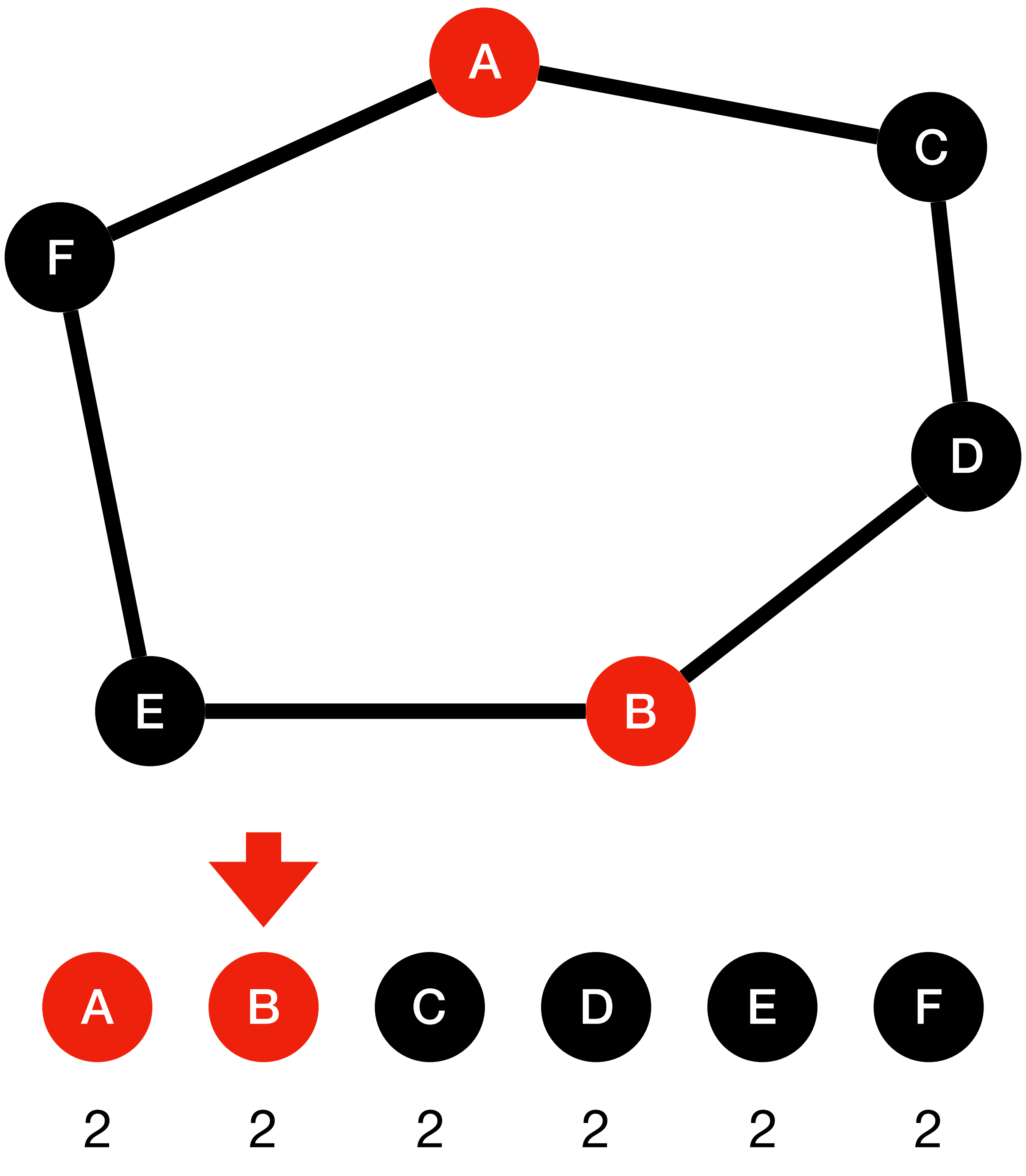

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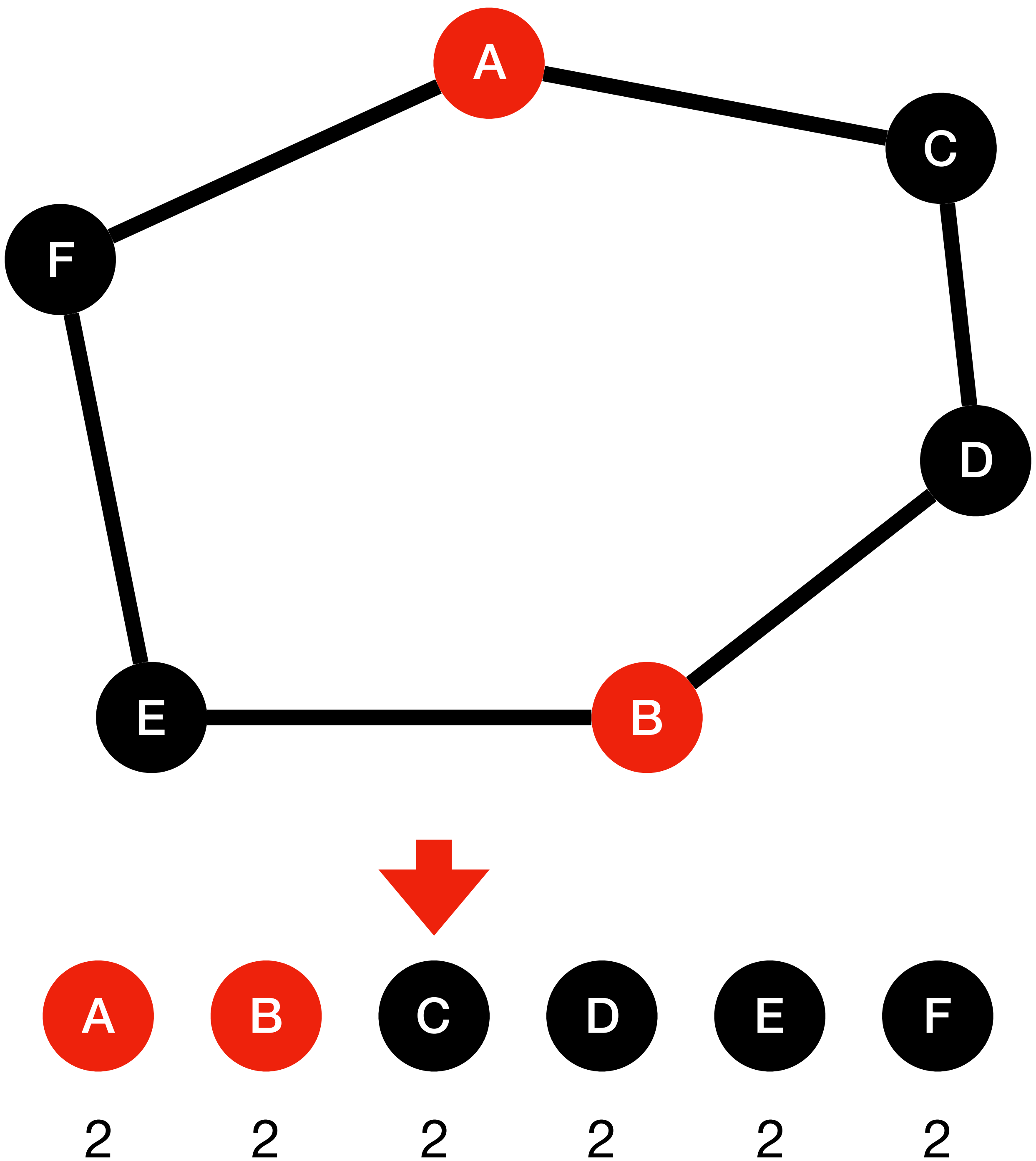

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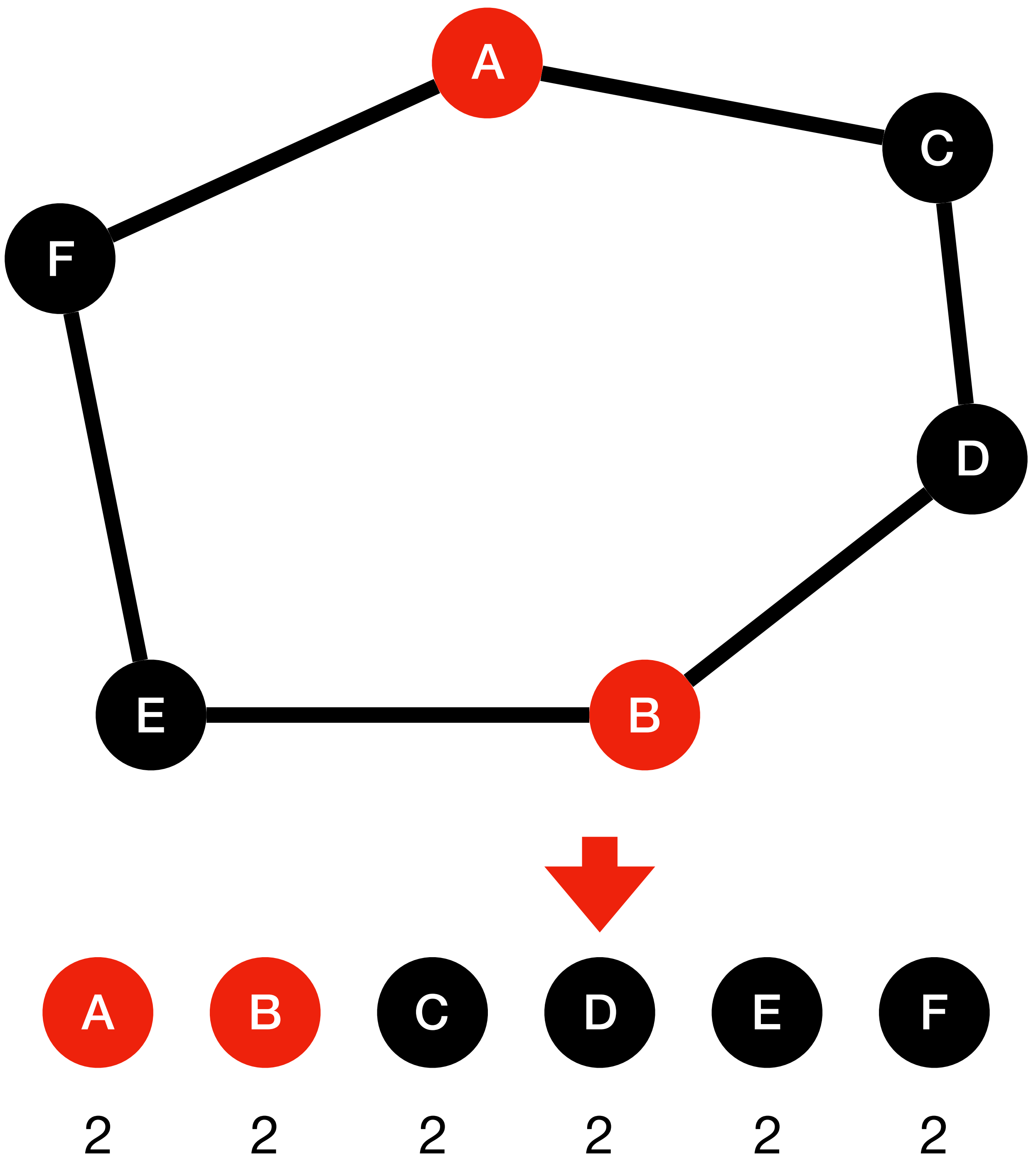

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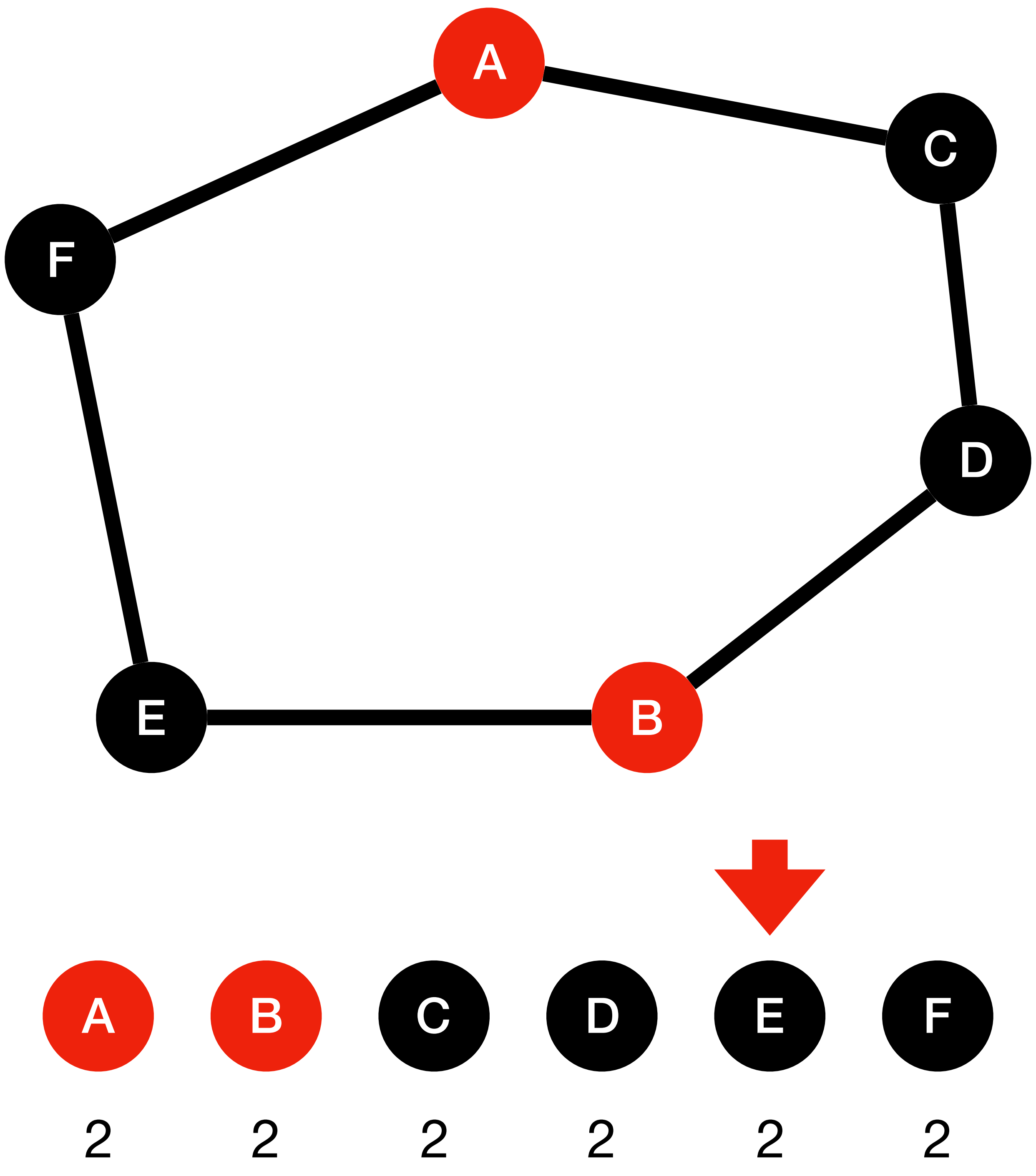

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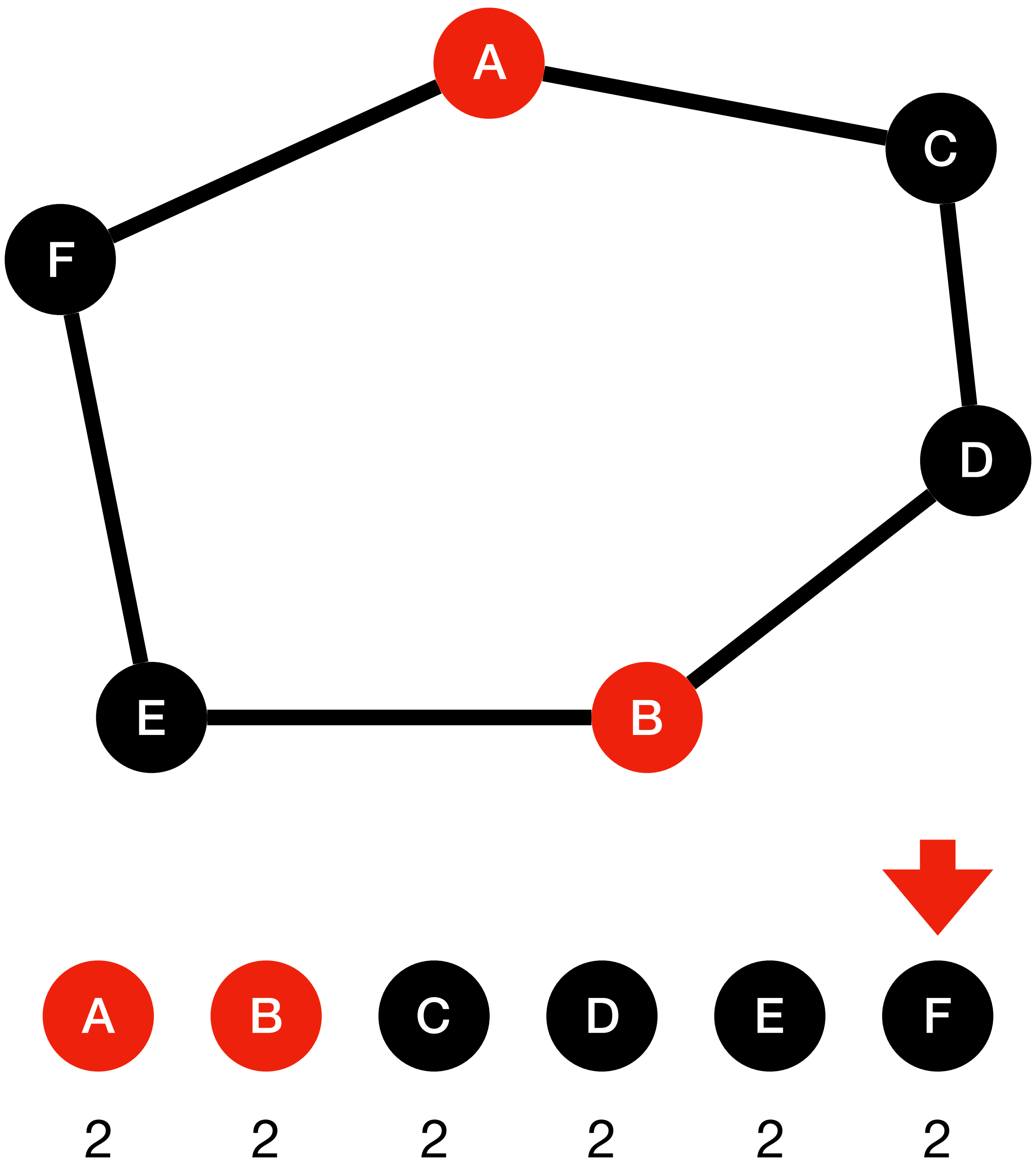

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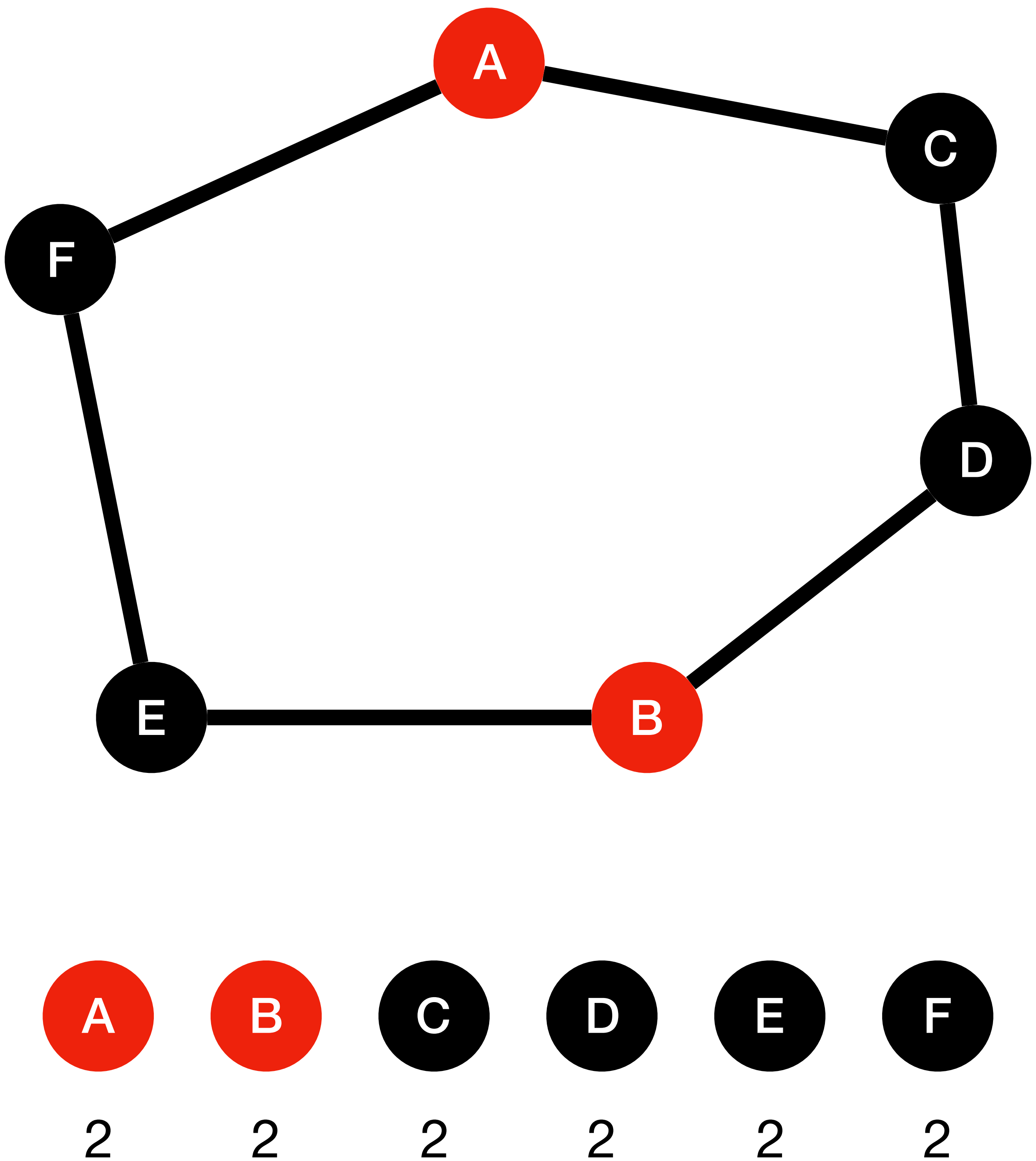

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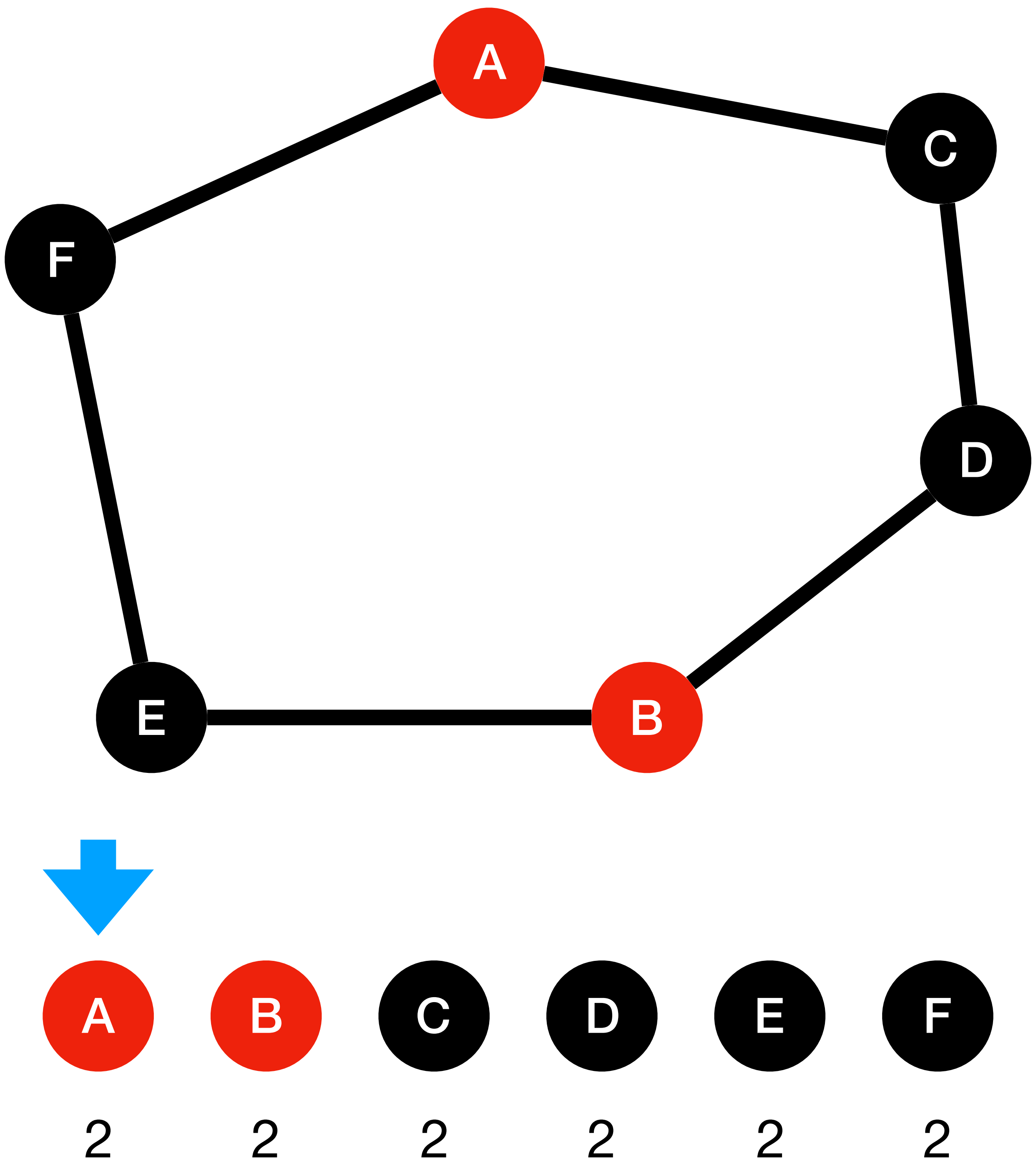

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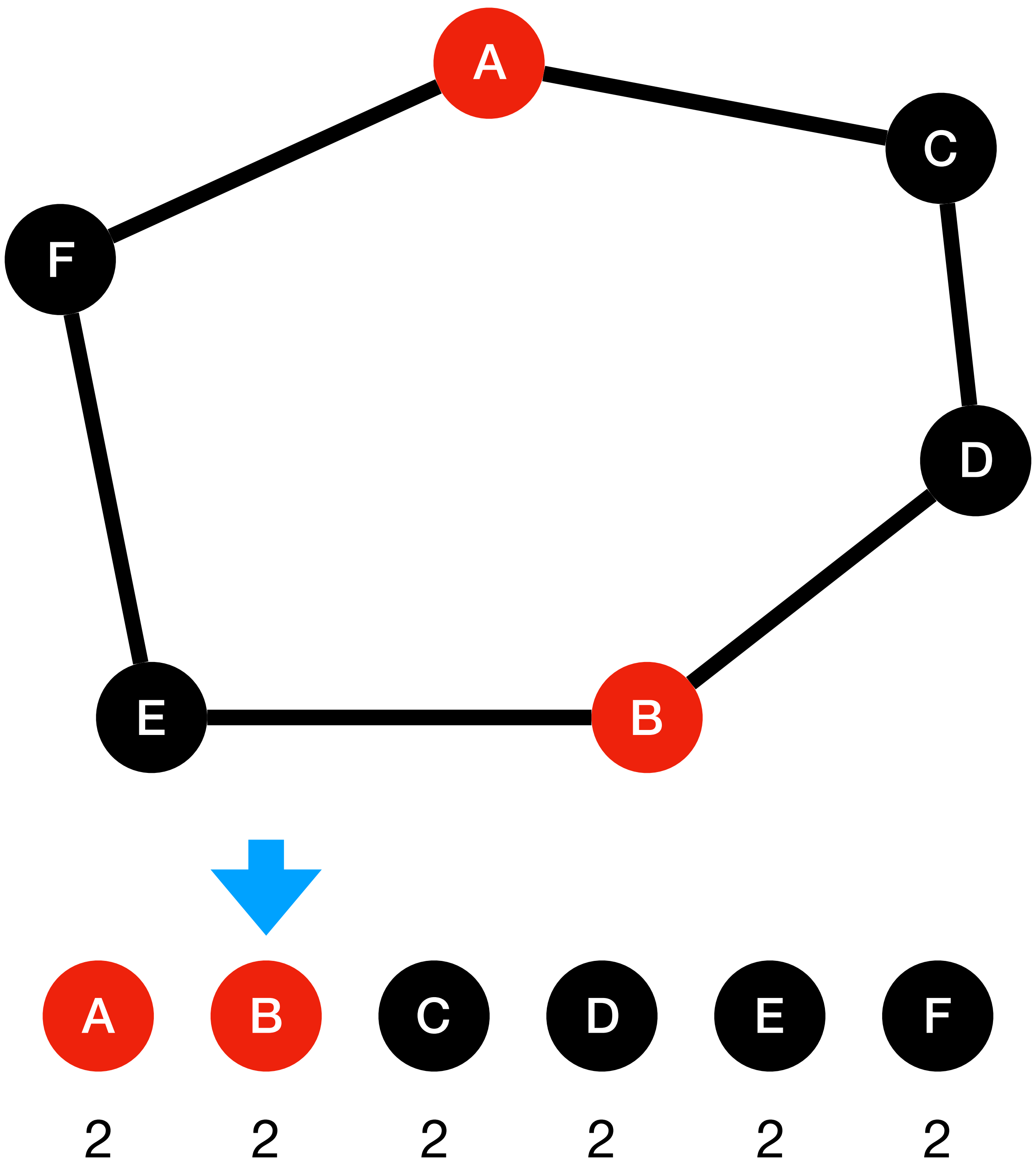

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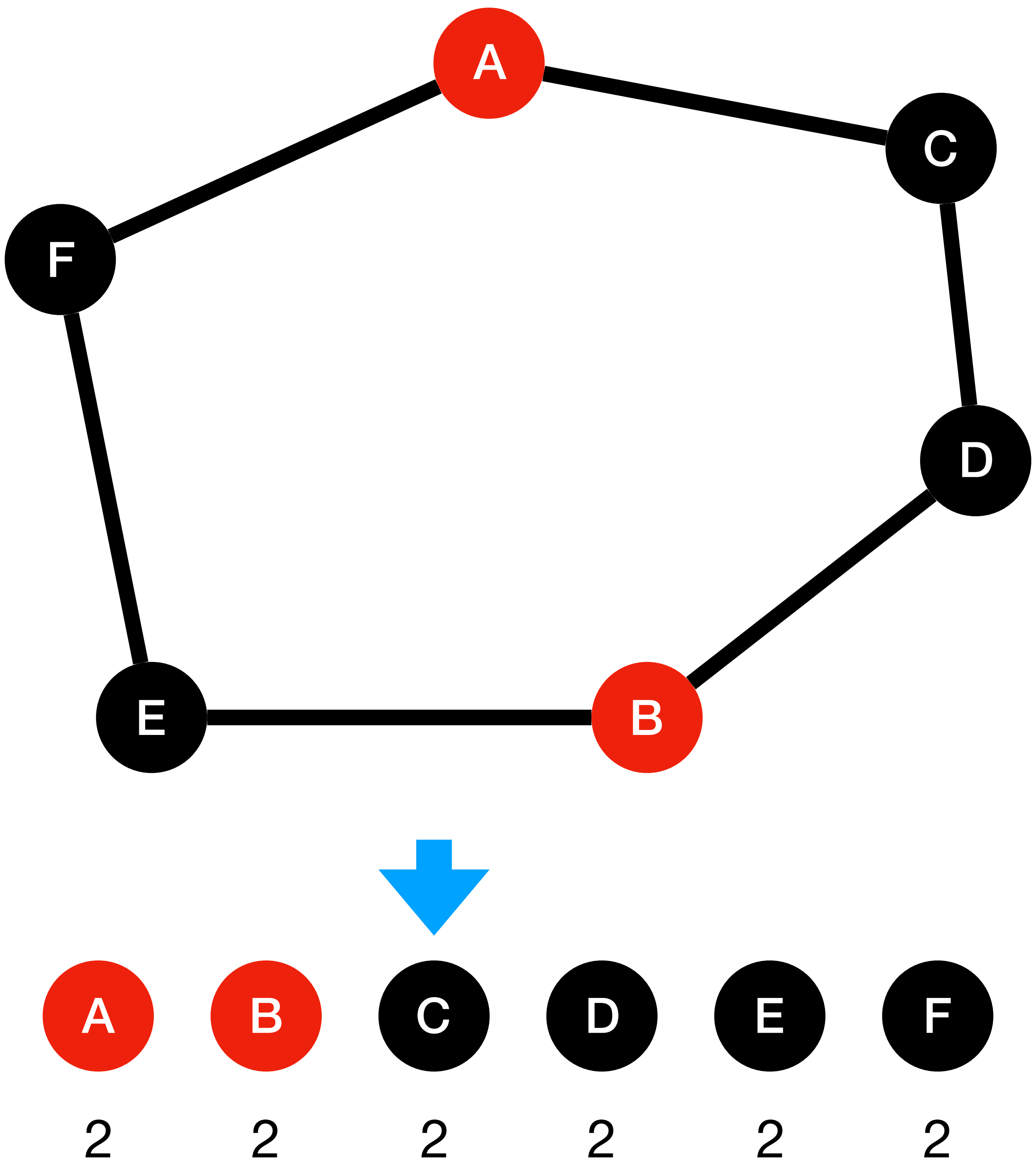

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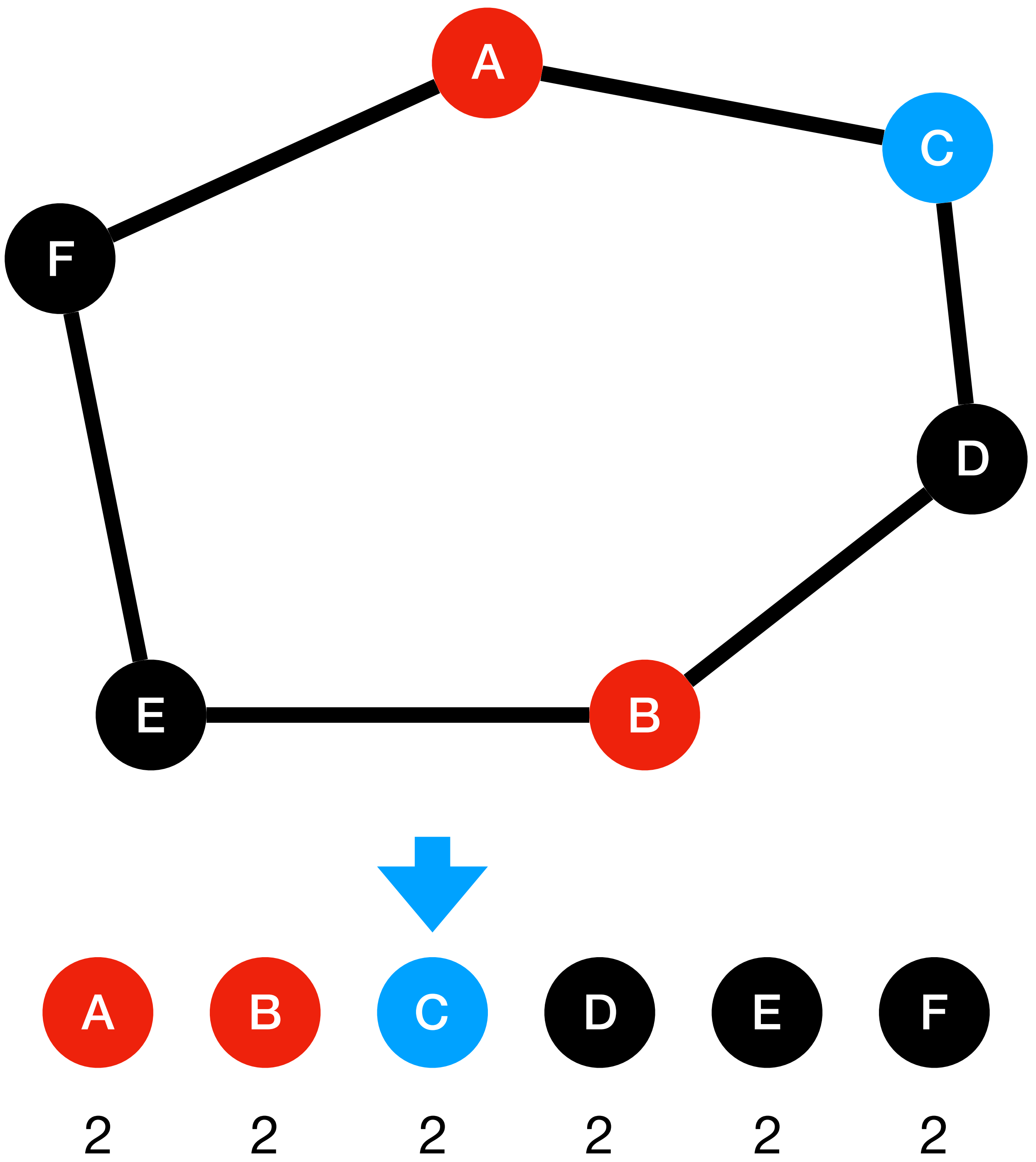

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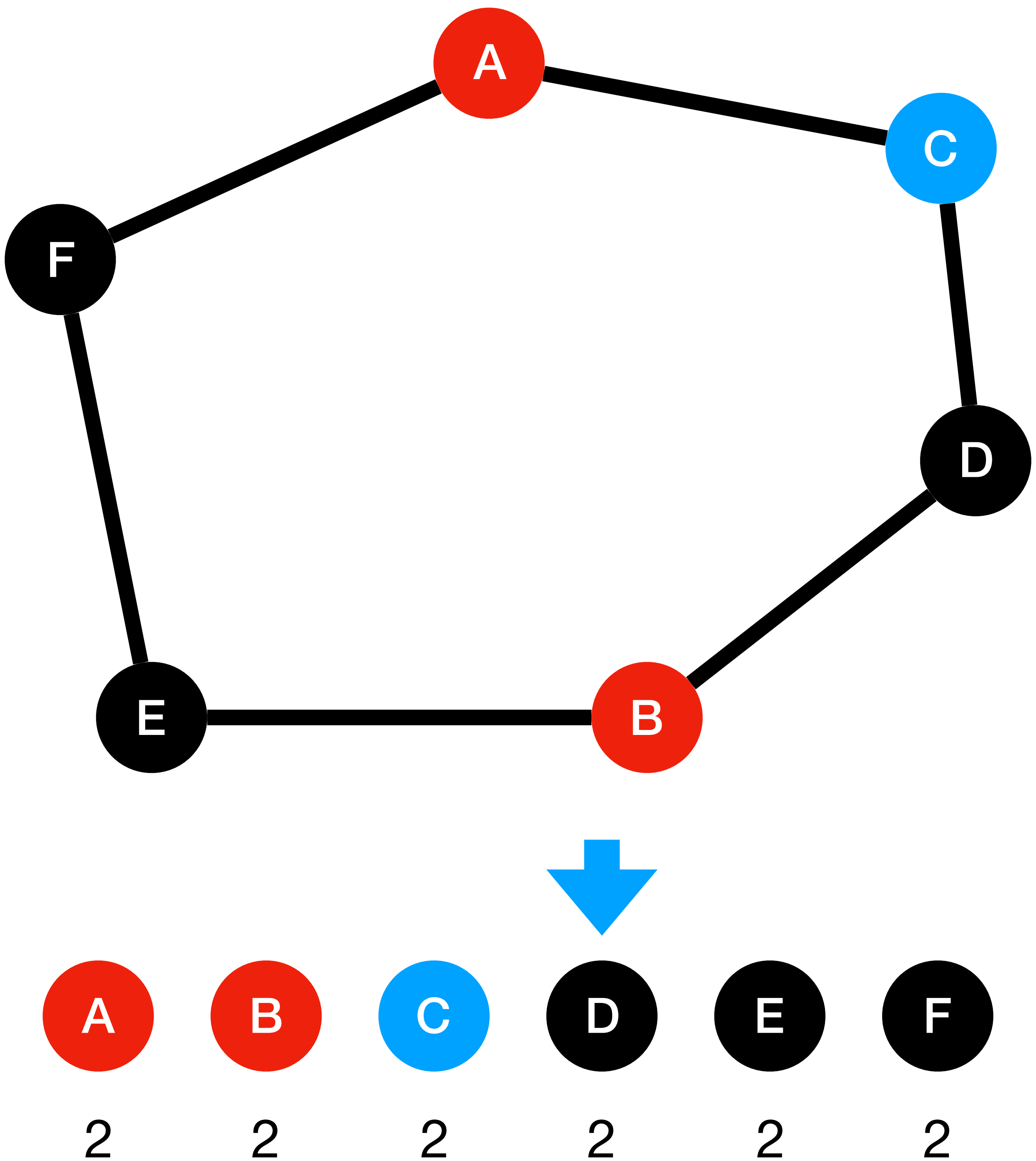

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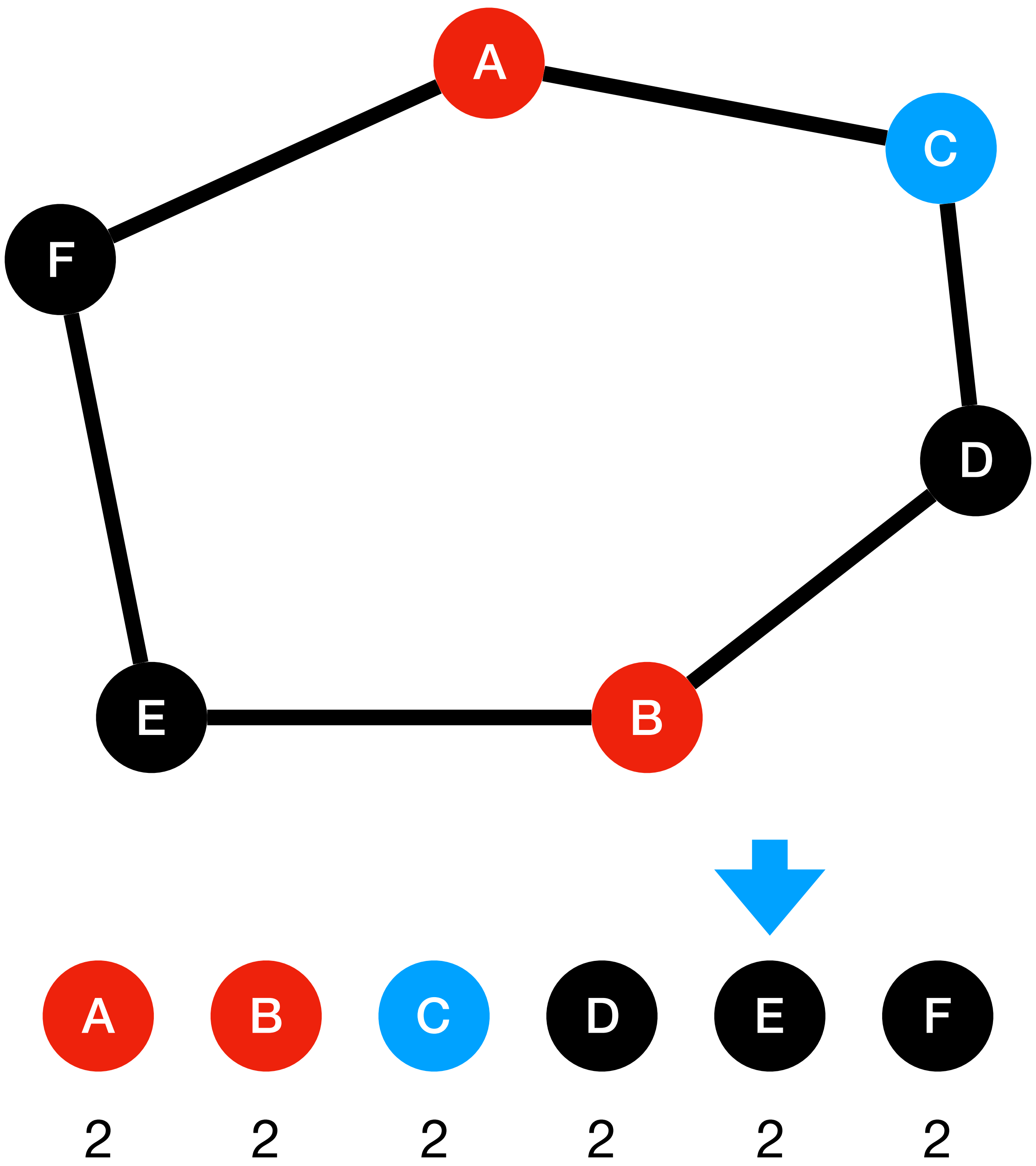

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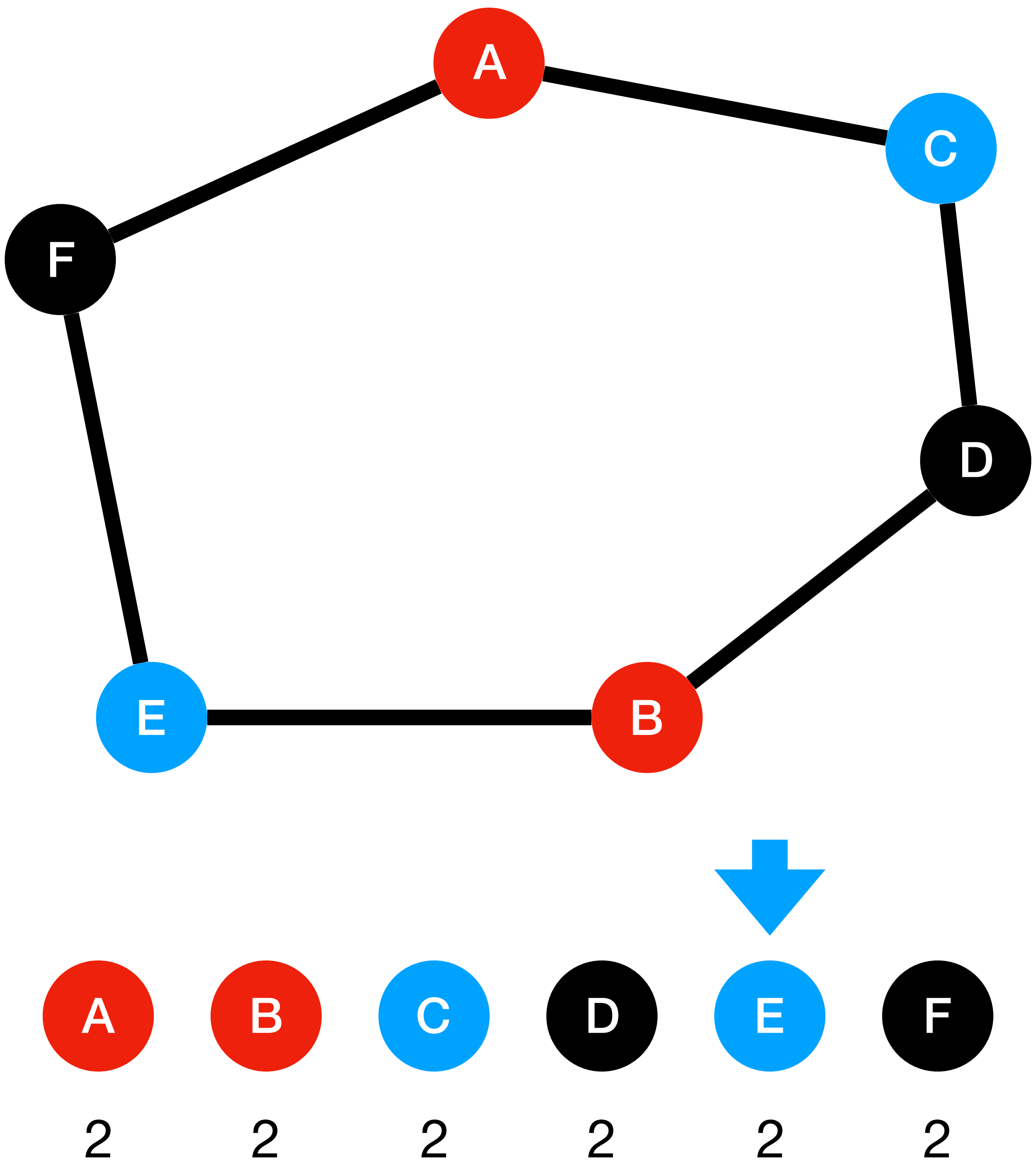

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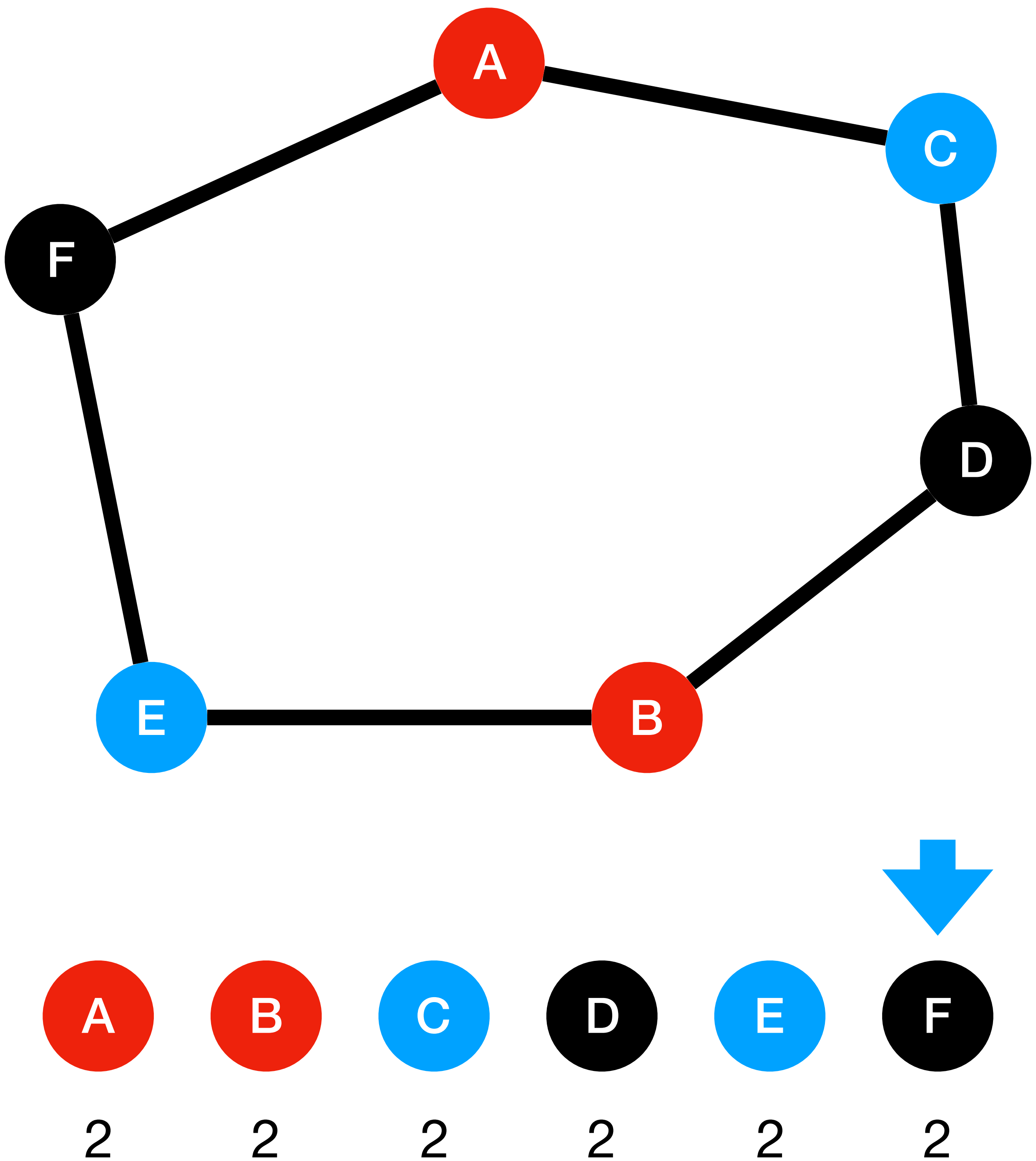

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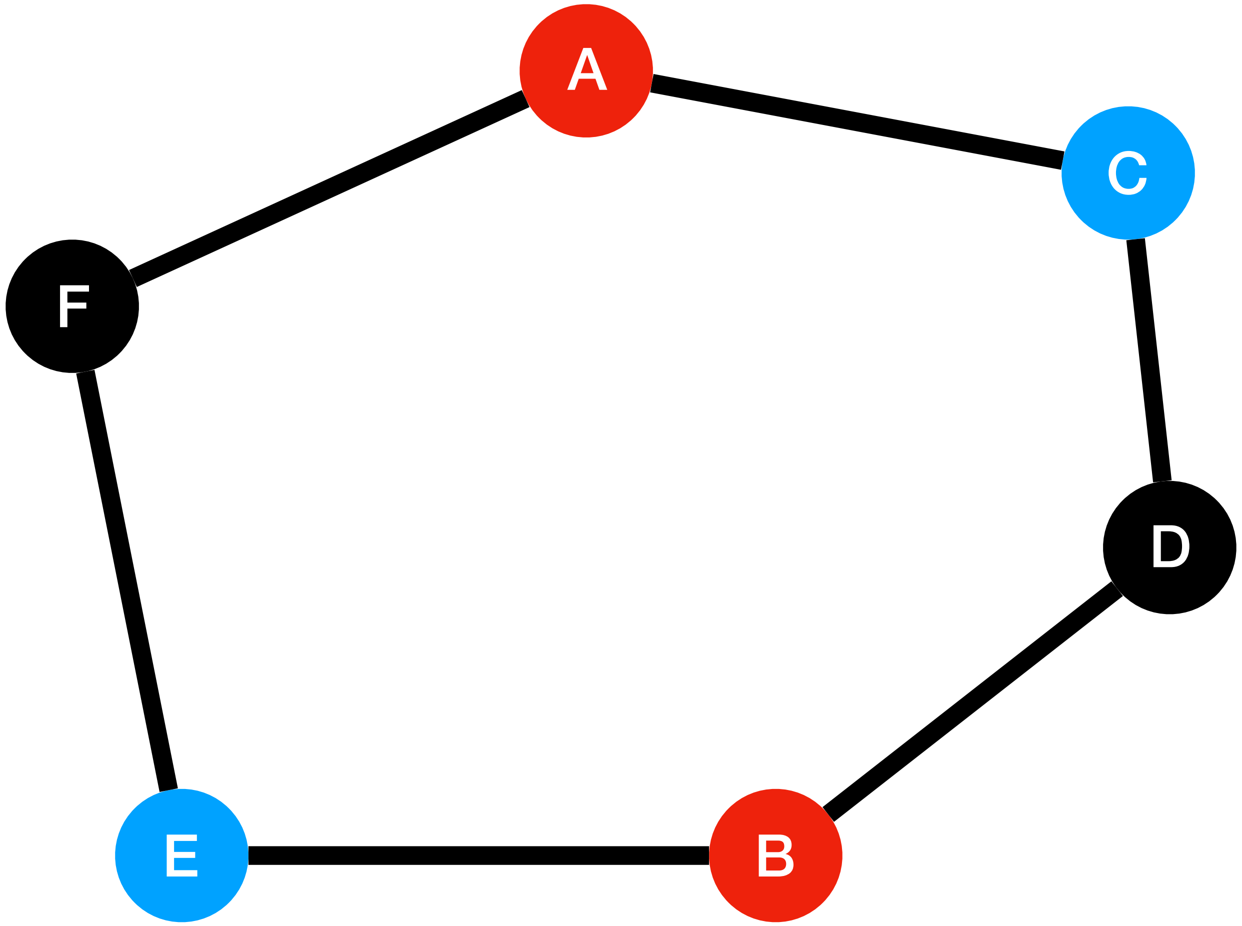

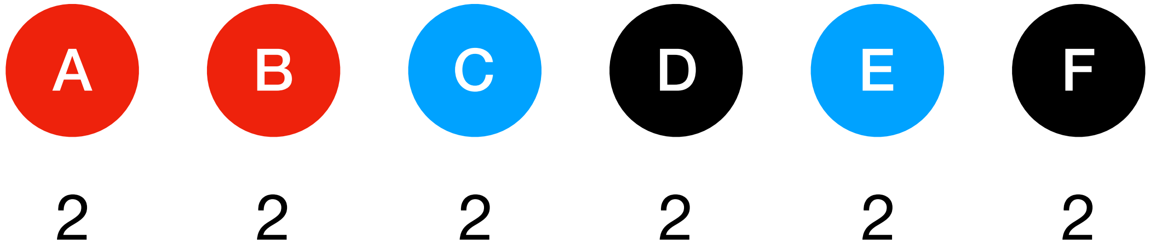

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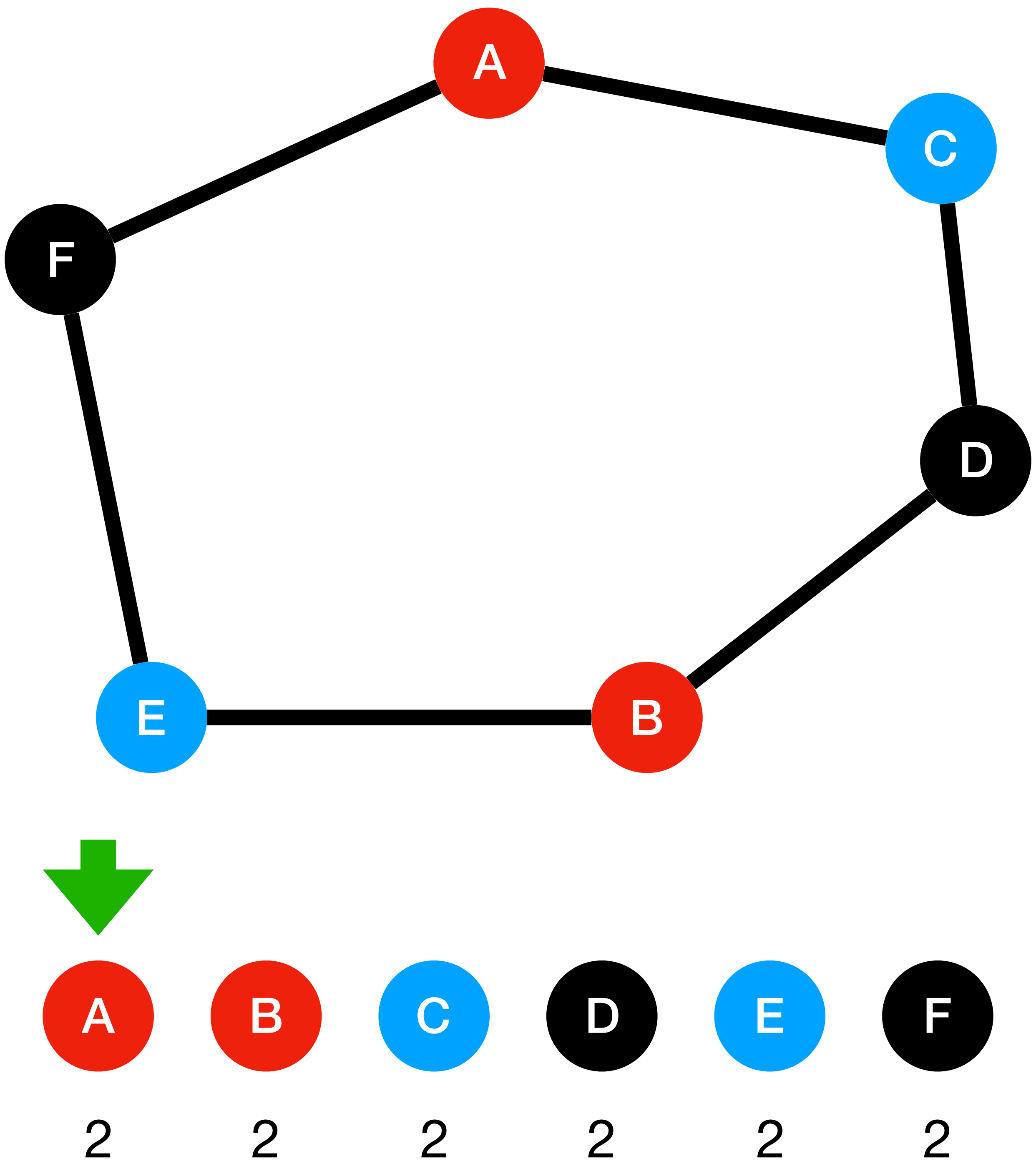

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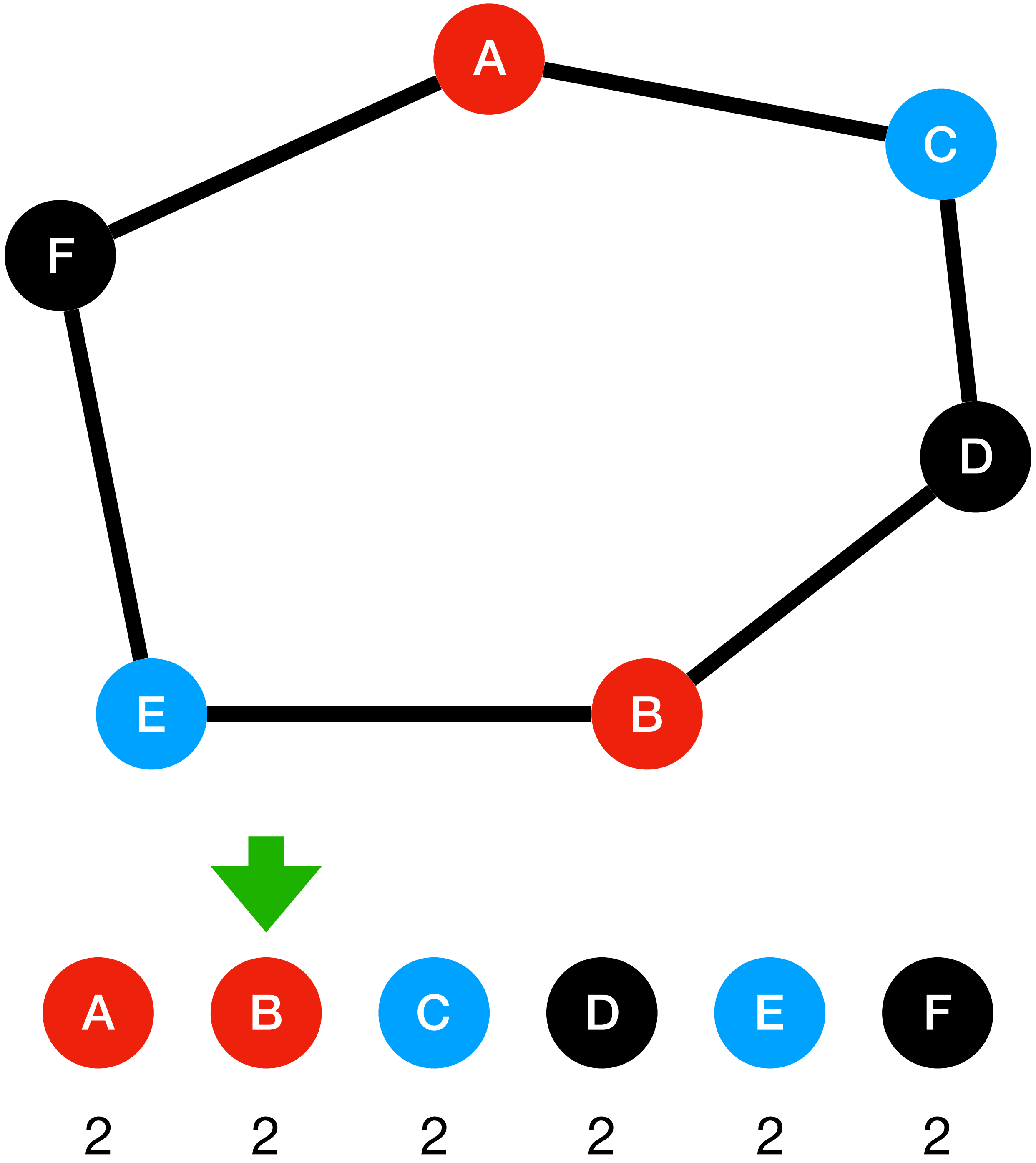

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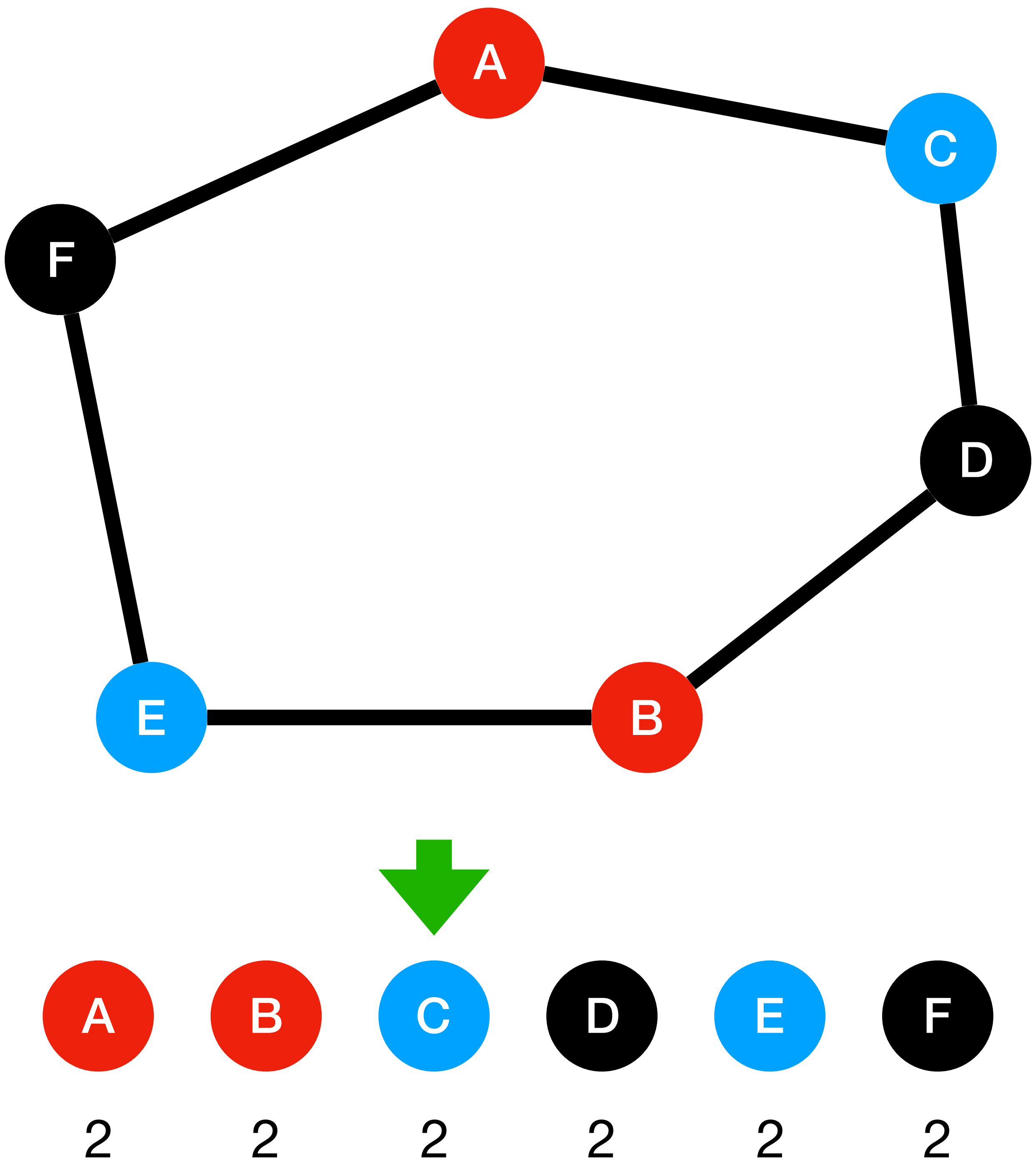

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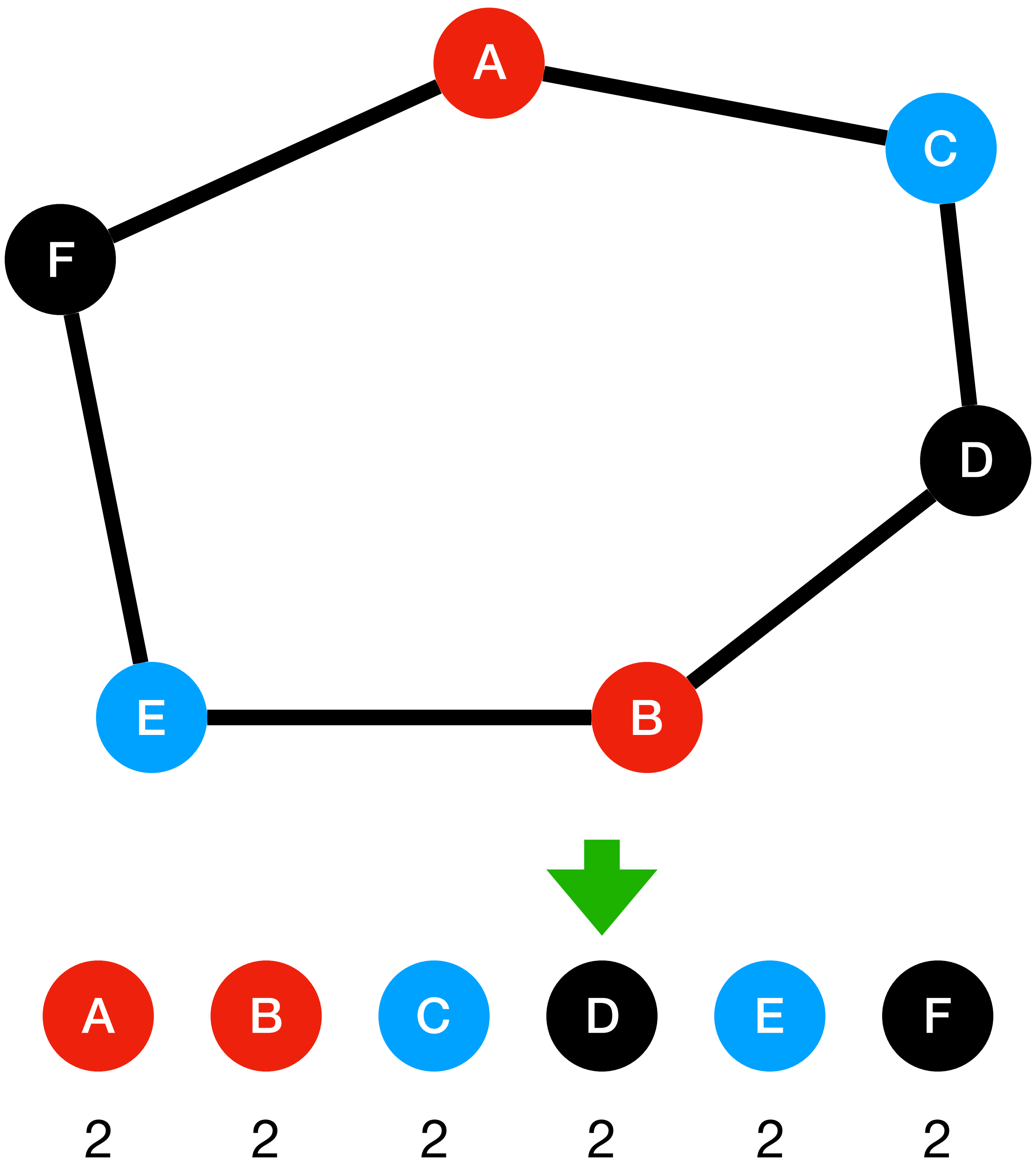

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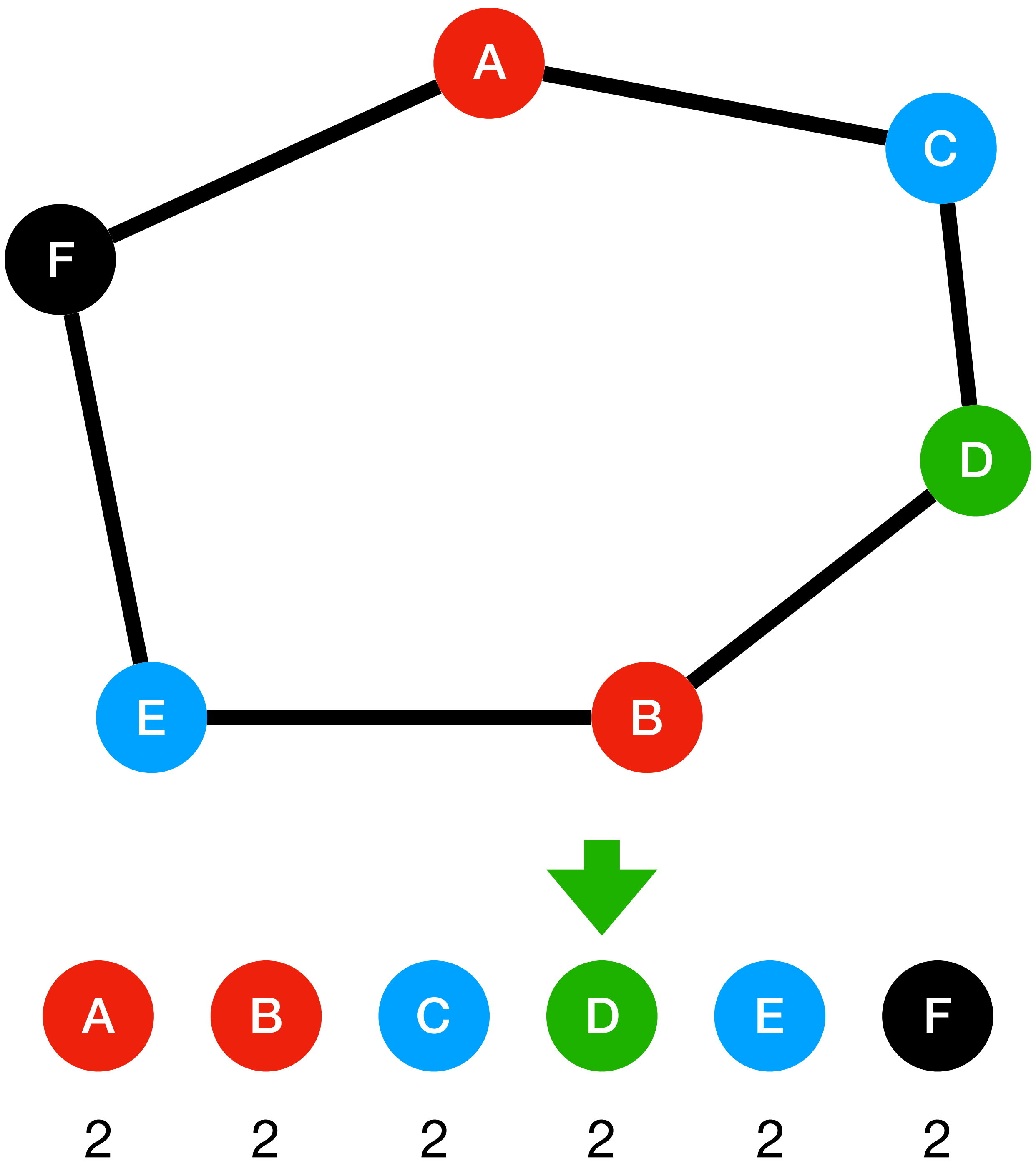

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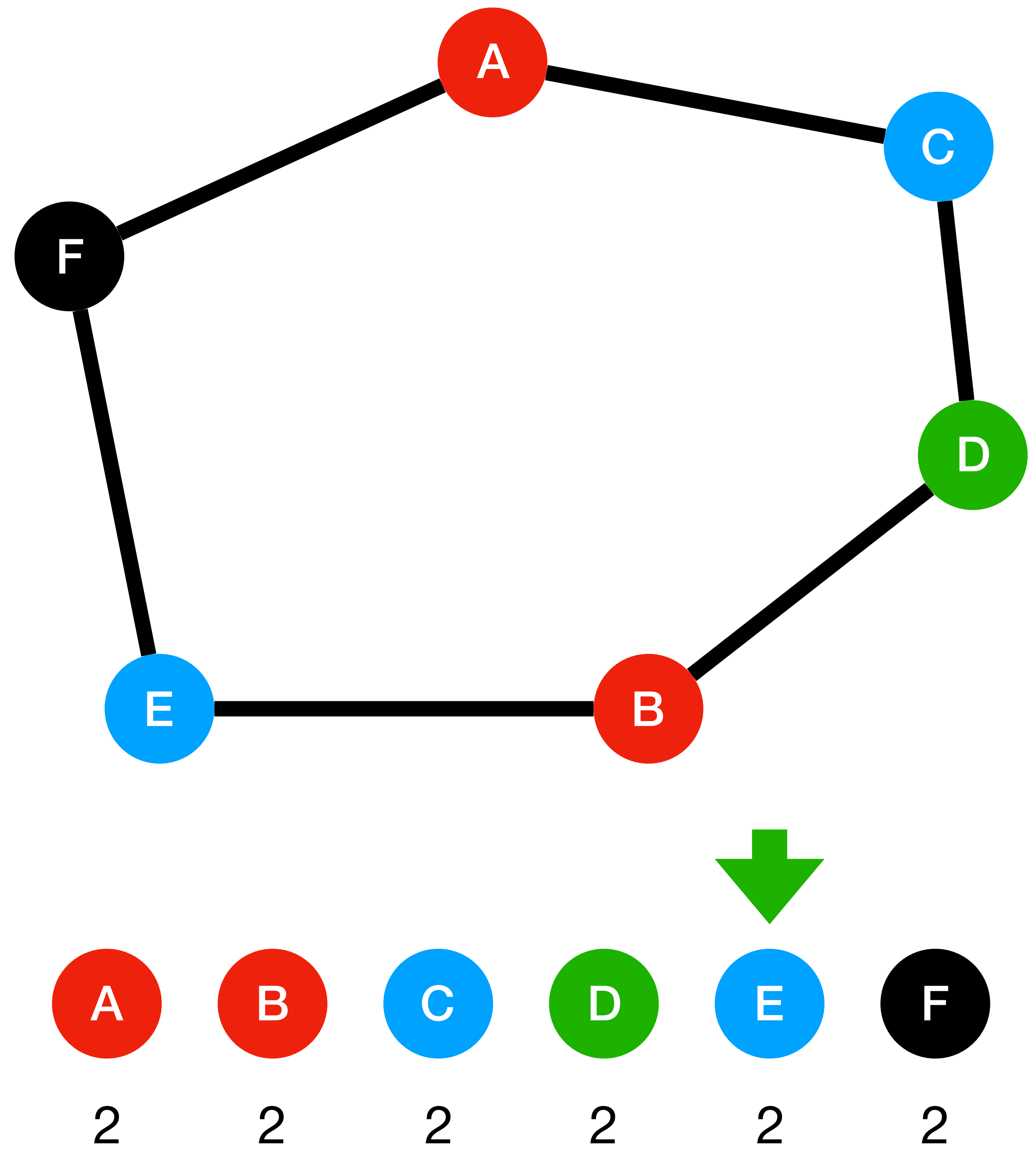

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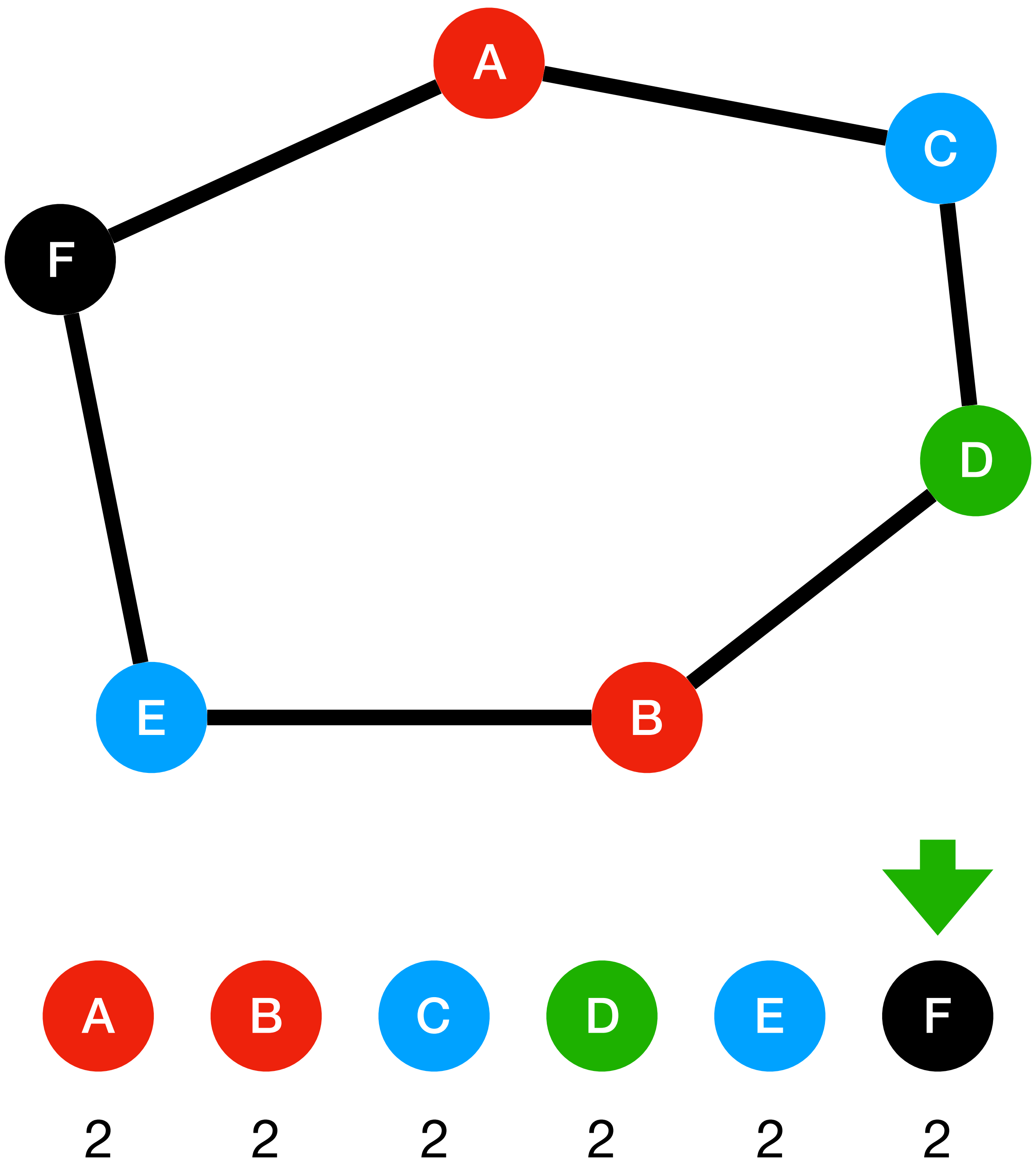

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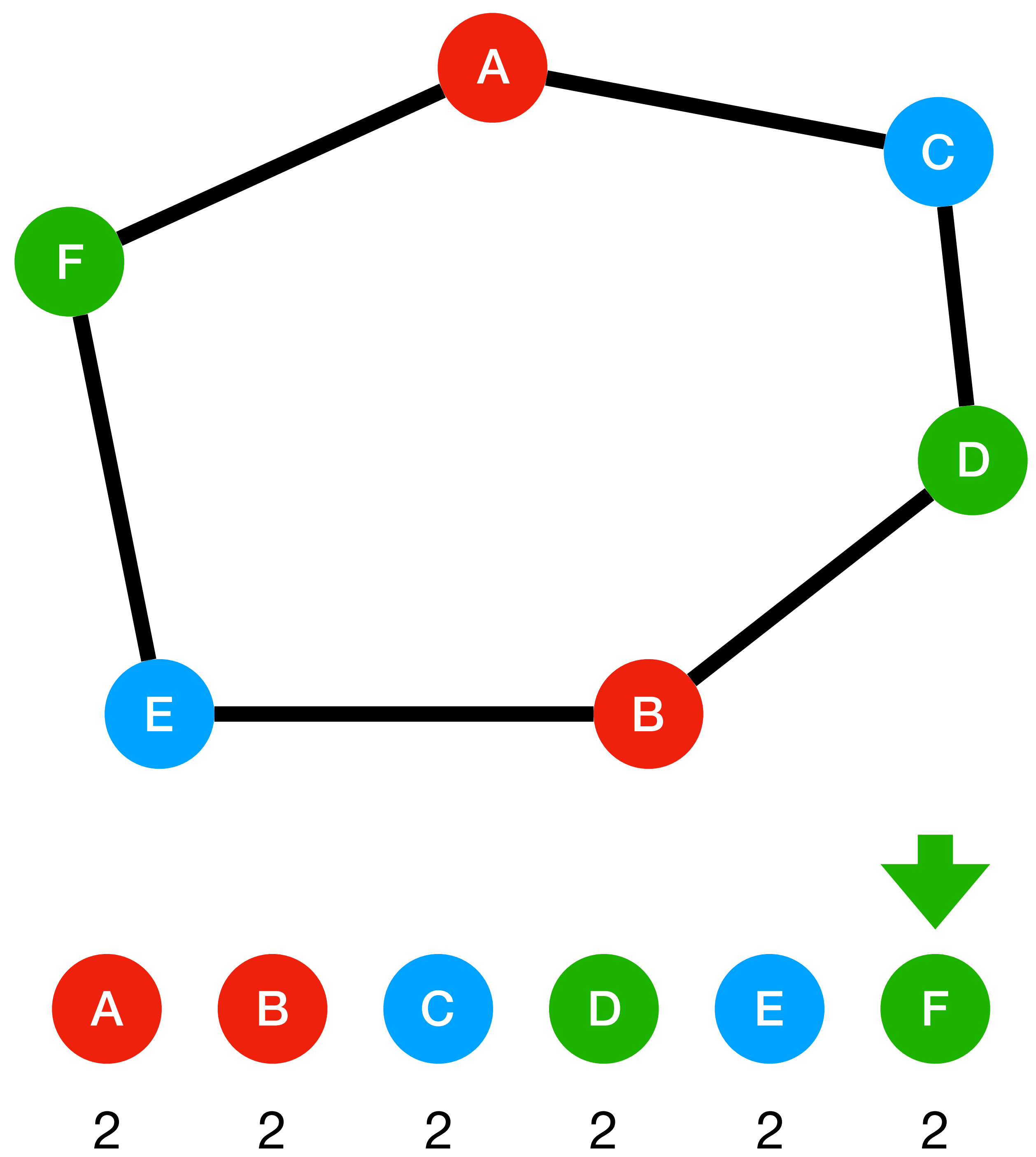

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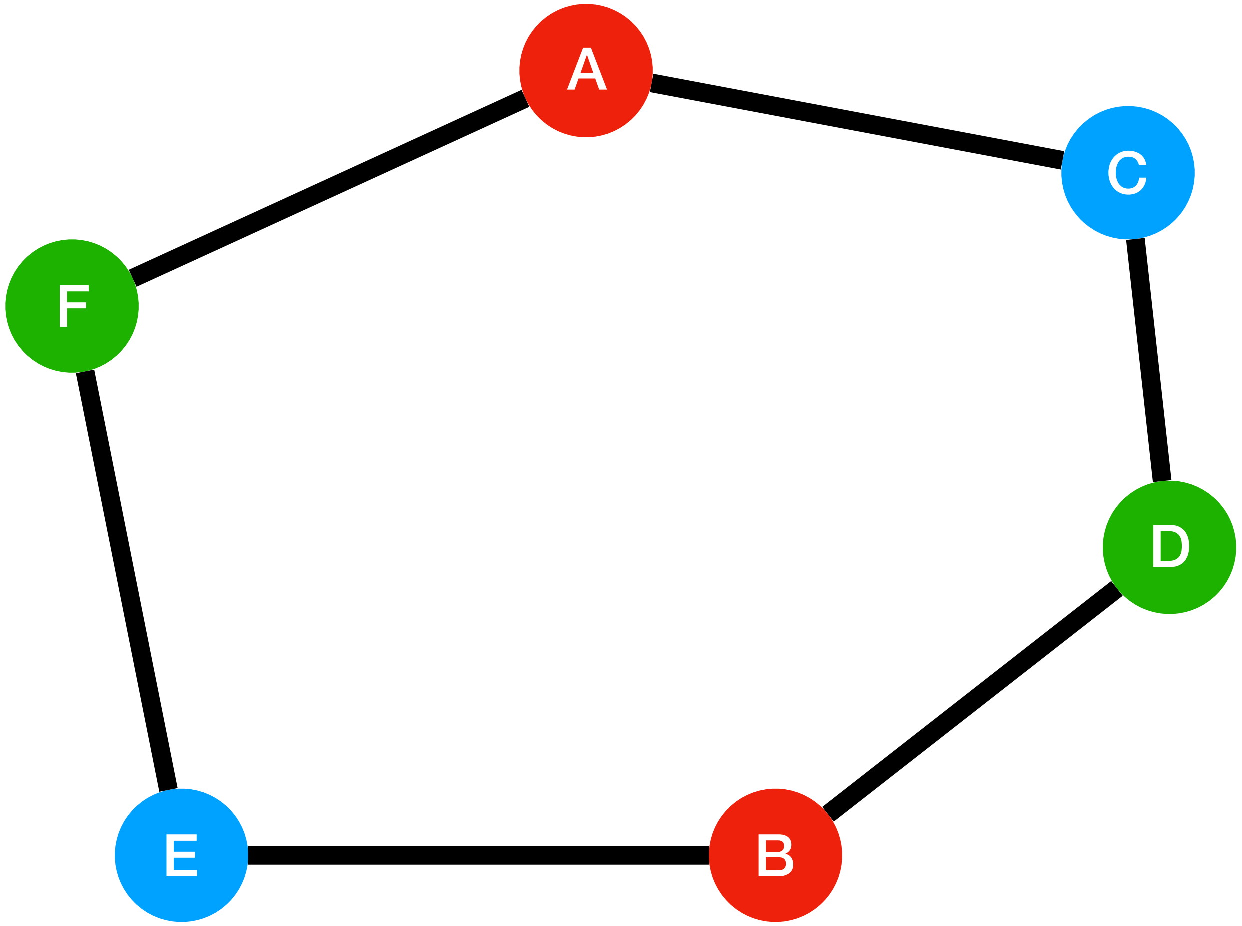

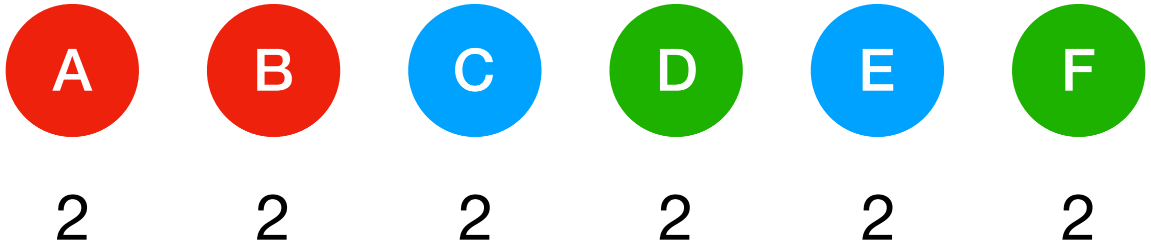

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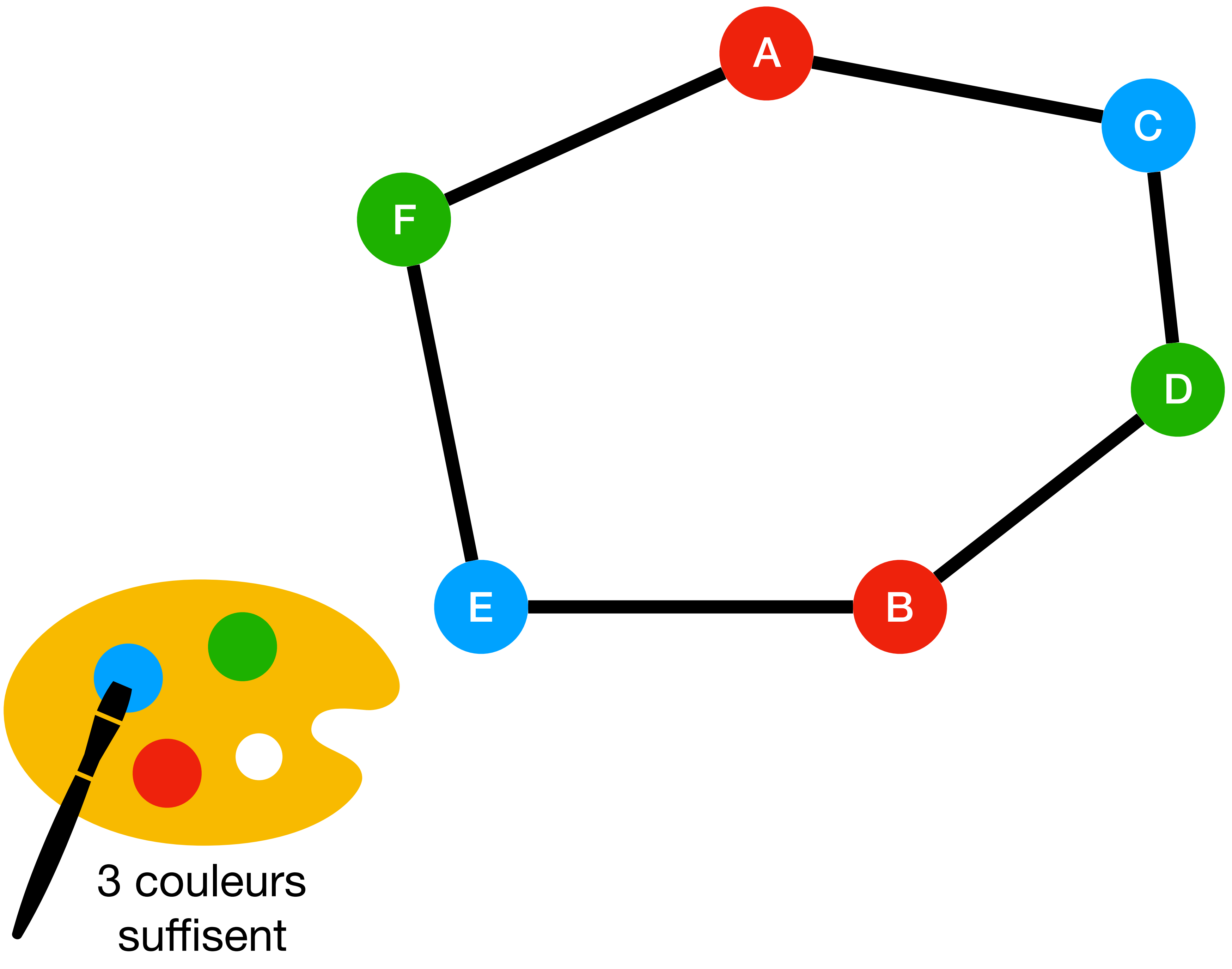
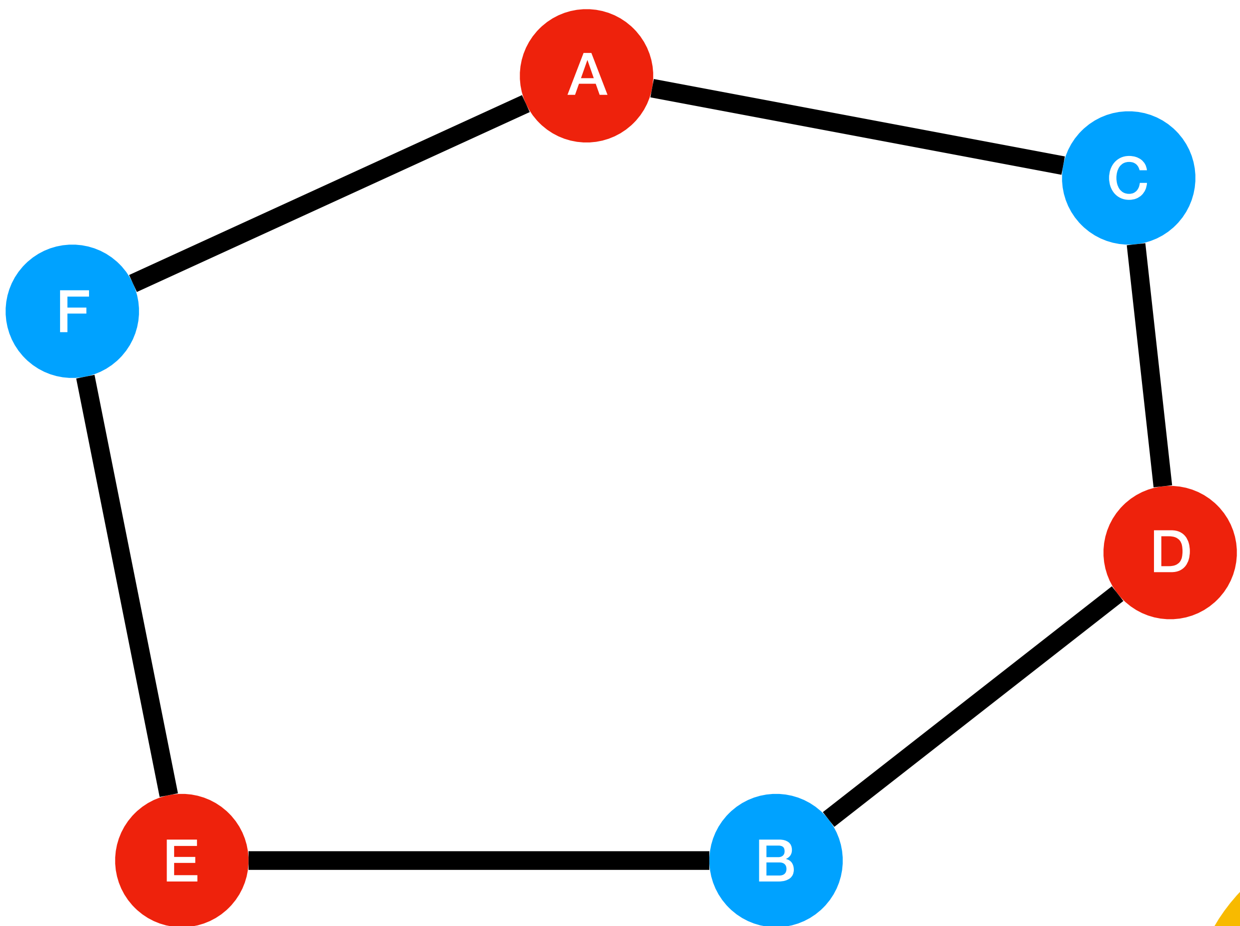
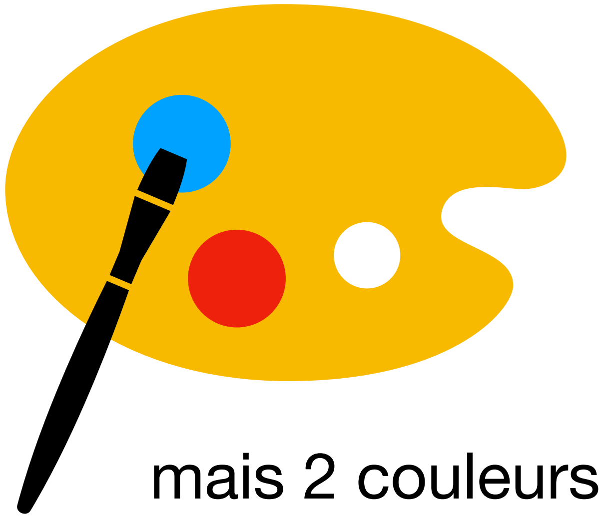


Colorations non optimales





mais 2 couleurs
suffisent aussi !