Counting complexity and oracles 🧞 in natural computing

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The first and second machine classes

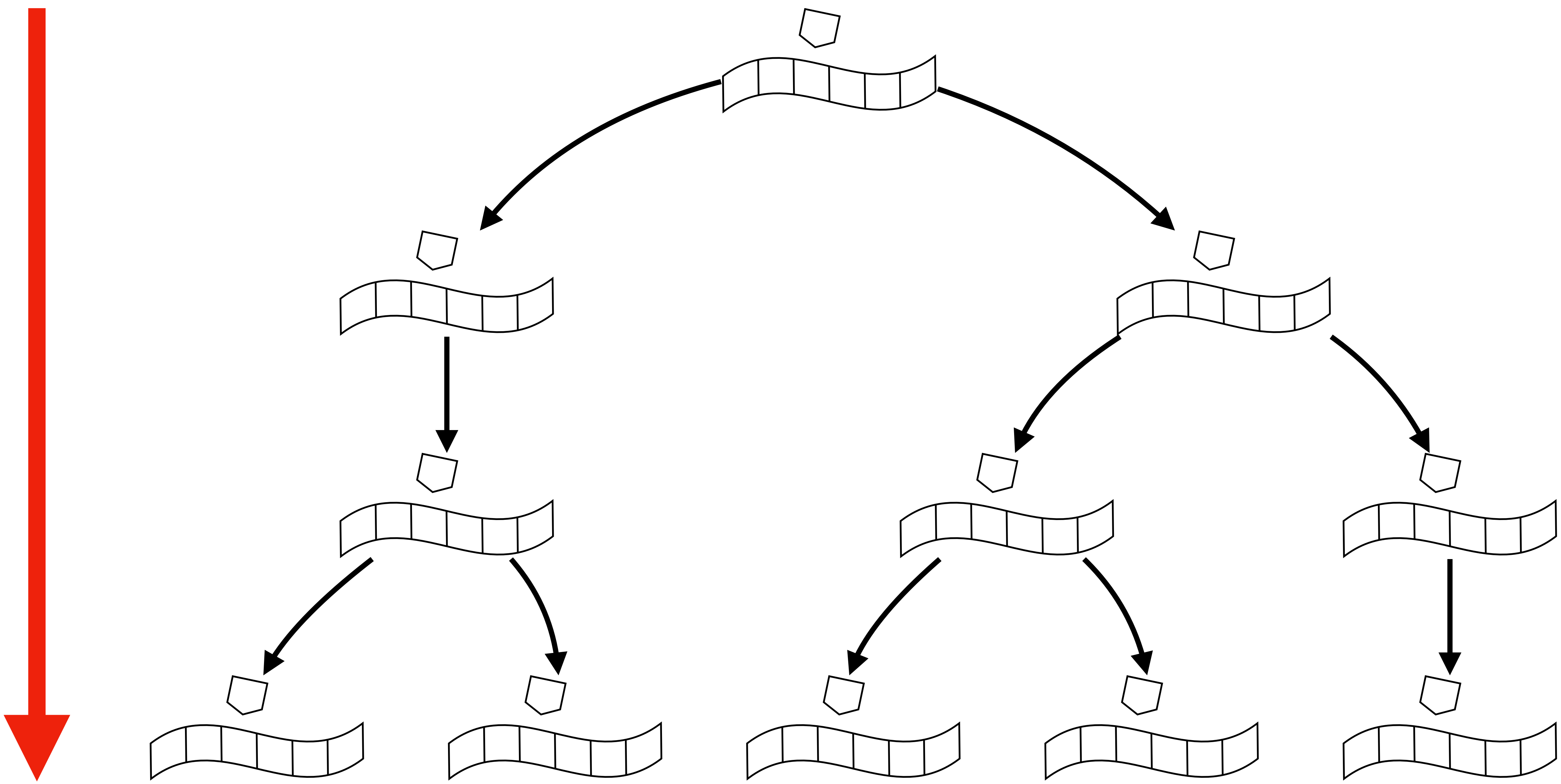
The first machine class and P

- Includes the deterministic Turing machine and all models that simulate and are simulated by it efficiently:
  - Random access machines (RAM) with constant-time addition and subtraction
  - Cellular automata with a finite initial configuration

The second machine class and **PSPACE**

- Includes models of computation that solve in polynomial time what a Turing machine solves in polynomial space:
  - Alternating Turing machines
  - Random access machines including constant time multiplication and division
  - Parallel processes generated by `fork(2)` running on an unbounded number of processors
  - Cellular automata over hyperbolic grids

Nondeterministic Turing machines: **NP**



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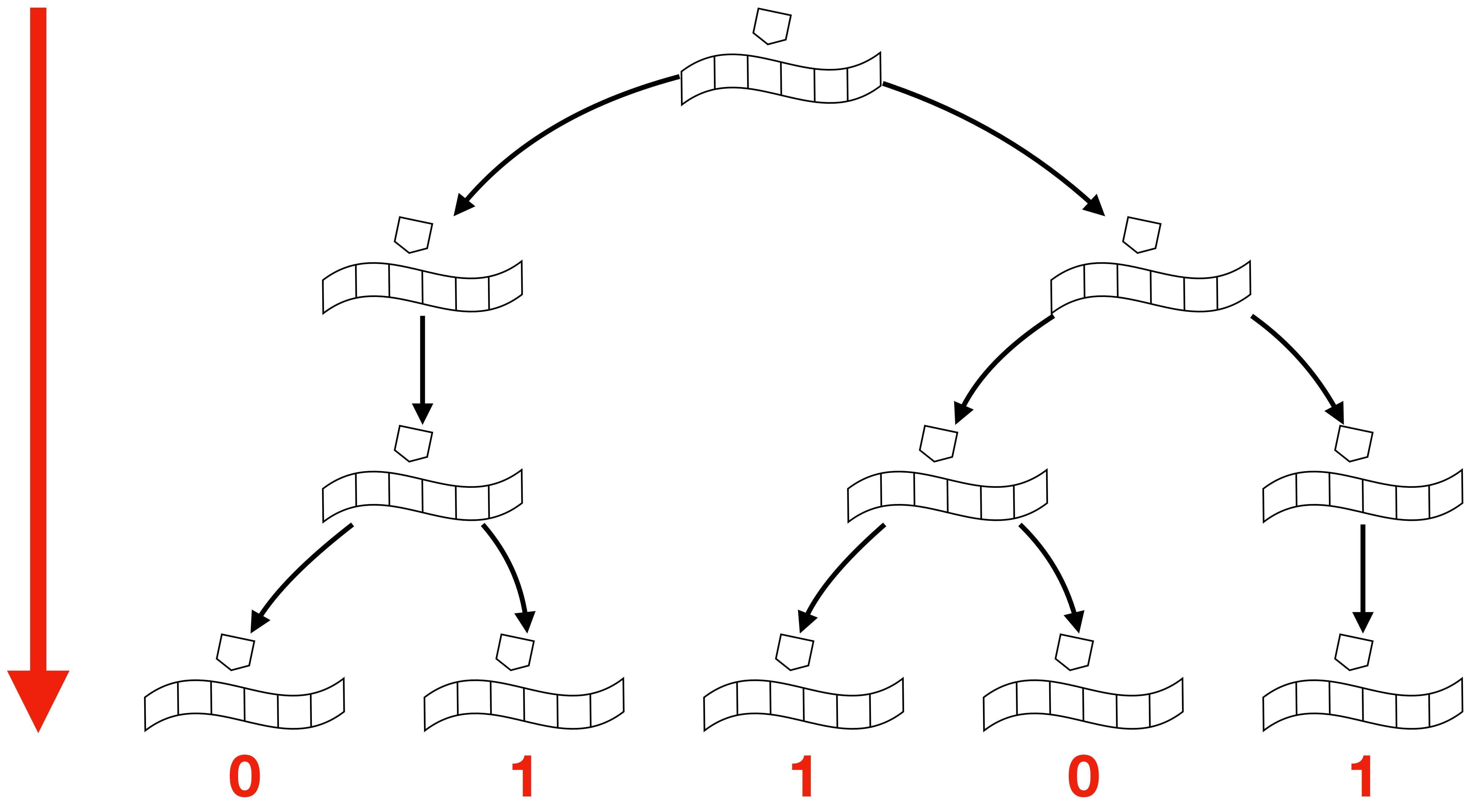

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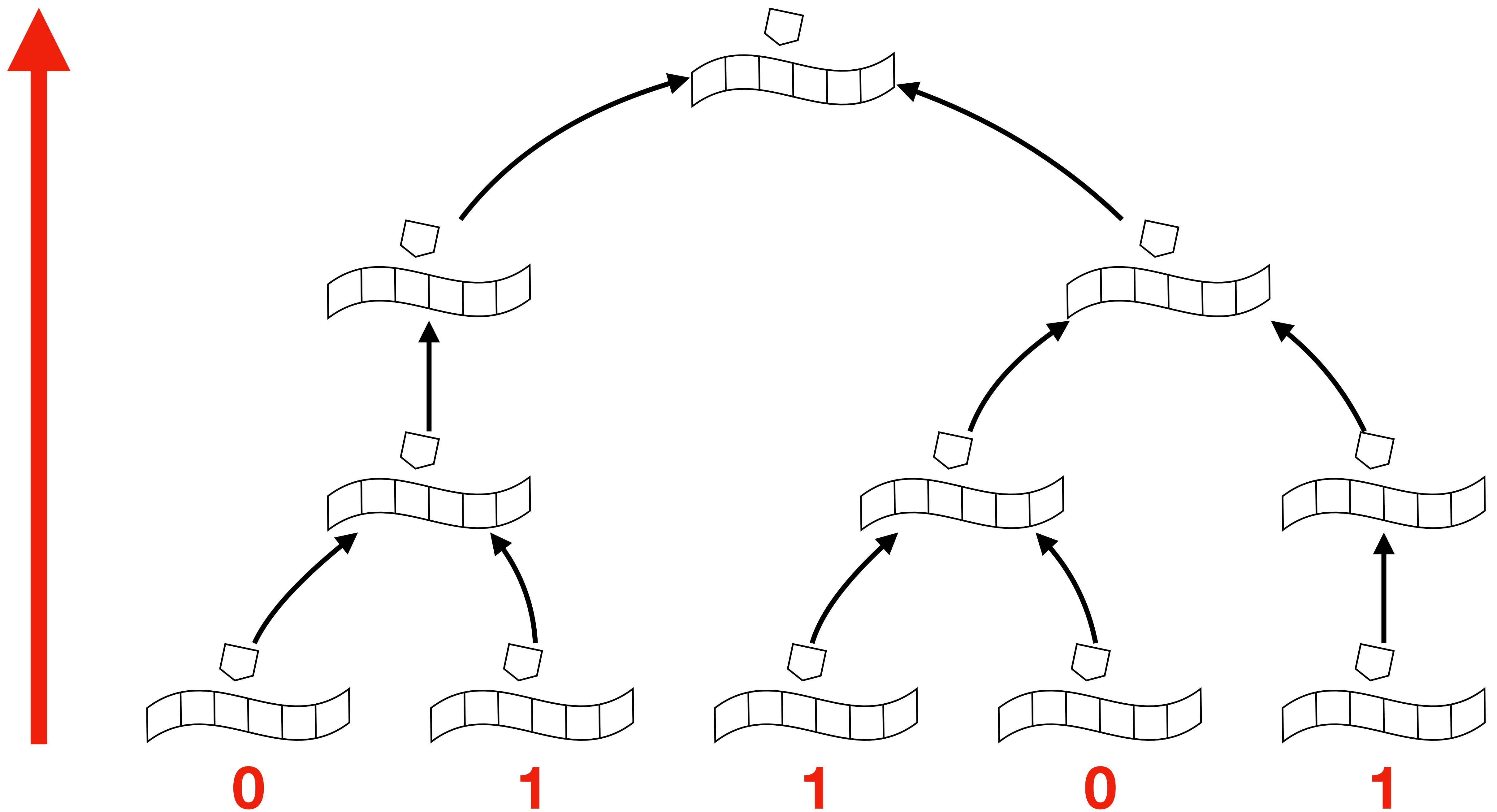

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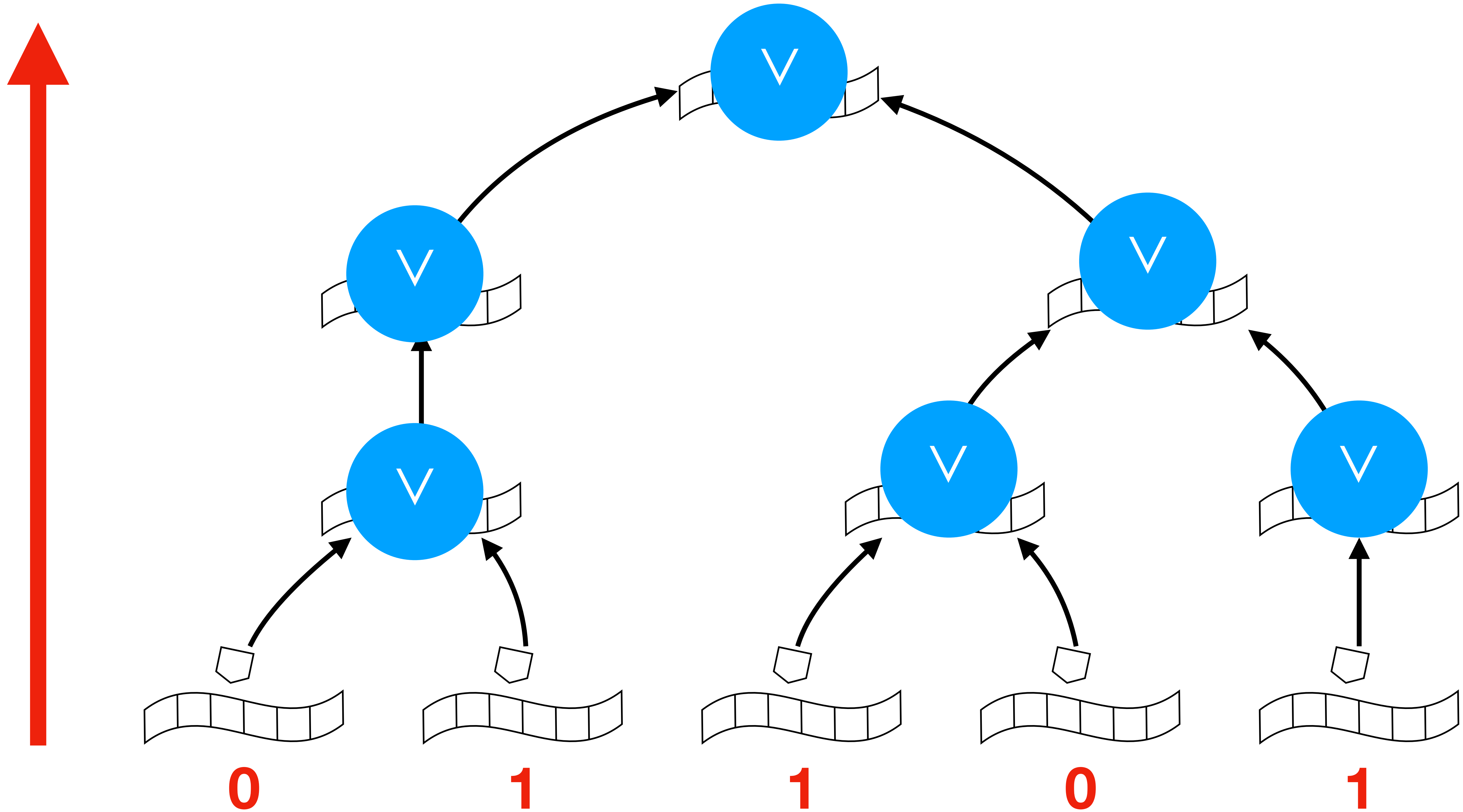

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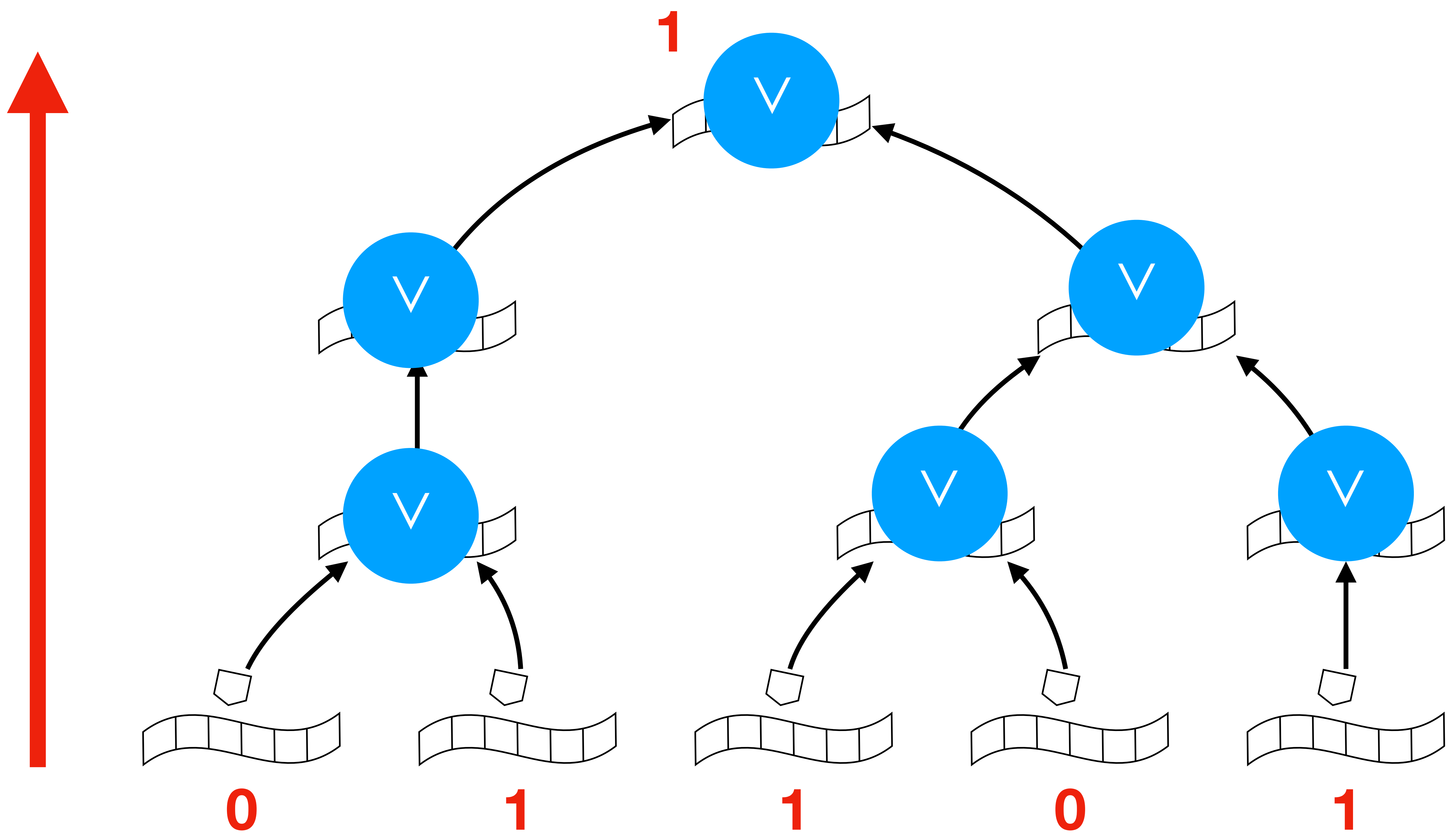
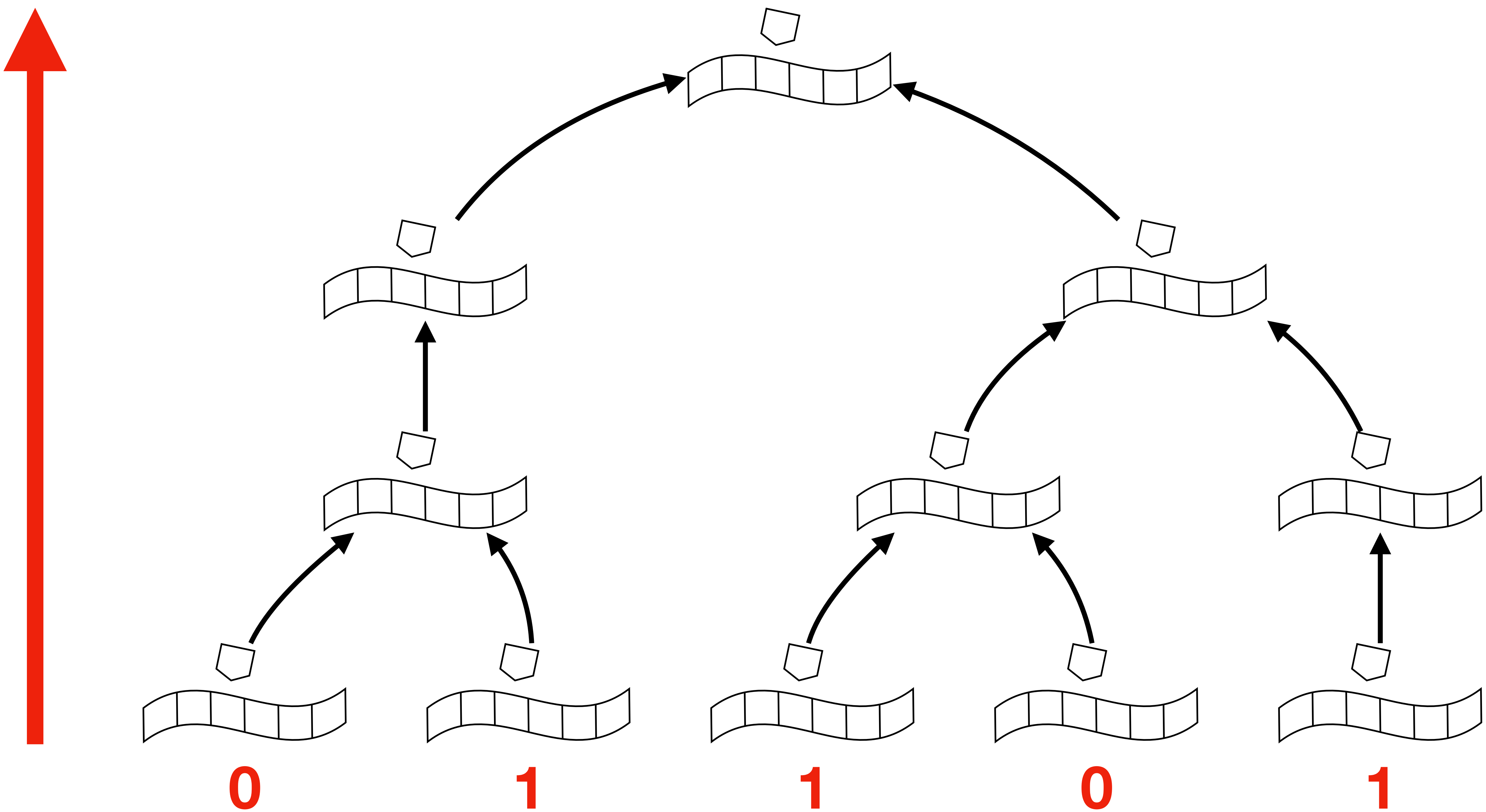


Alternating Turing machines: **PSPACE**



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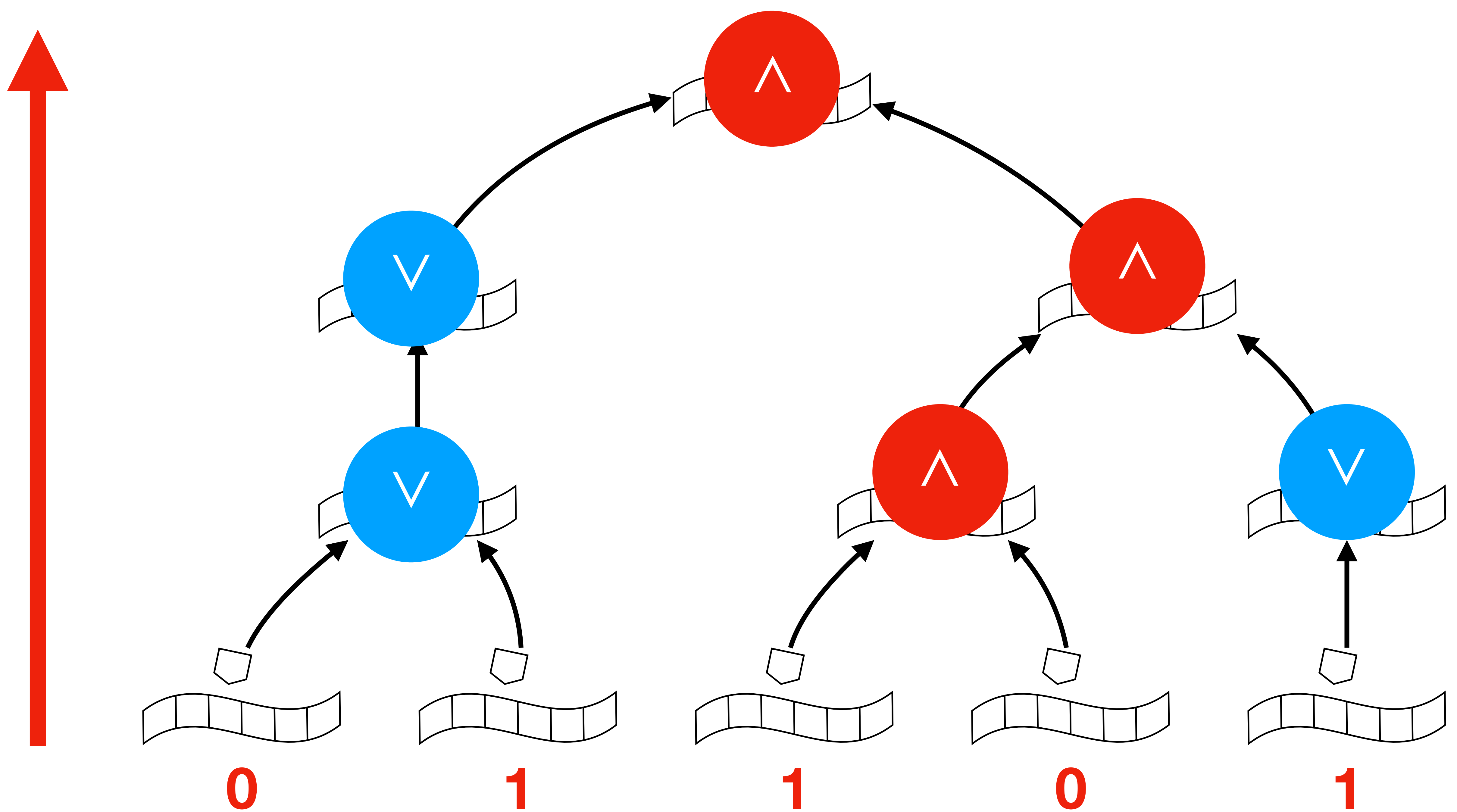

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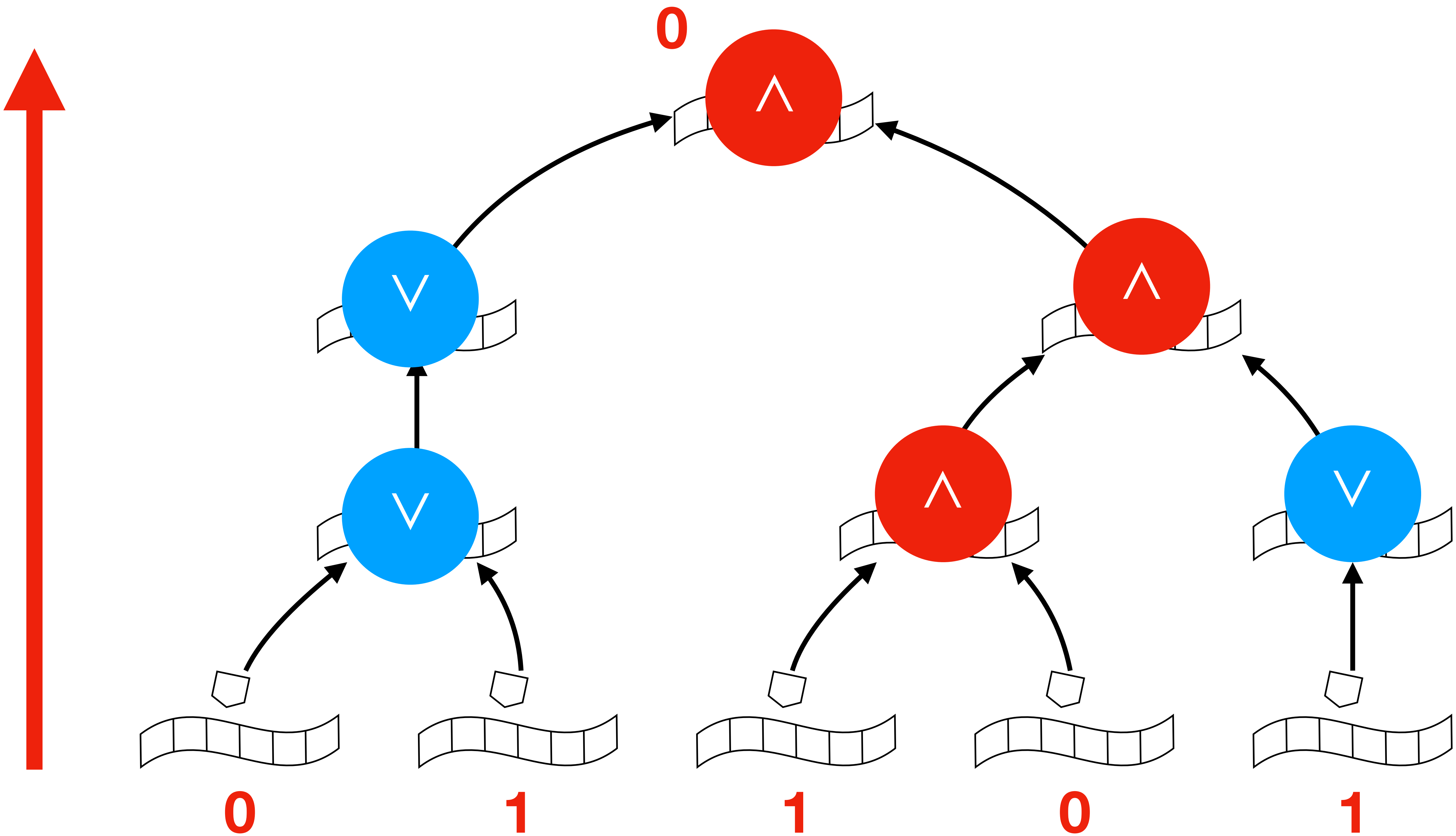
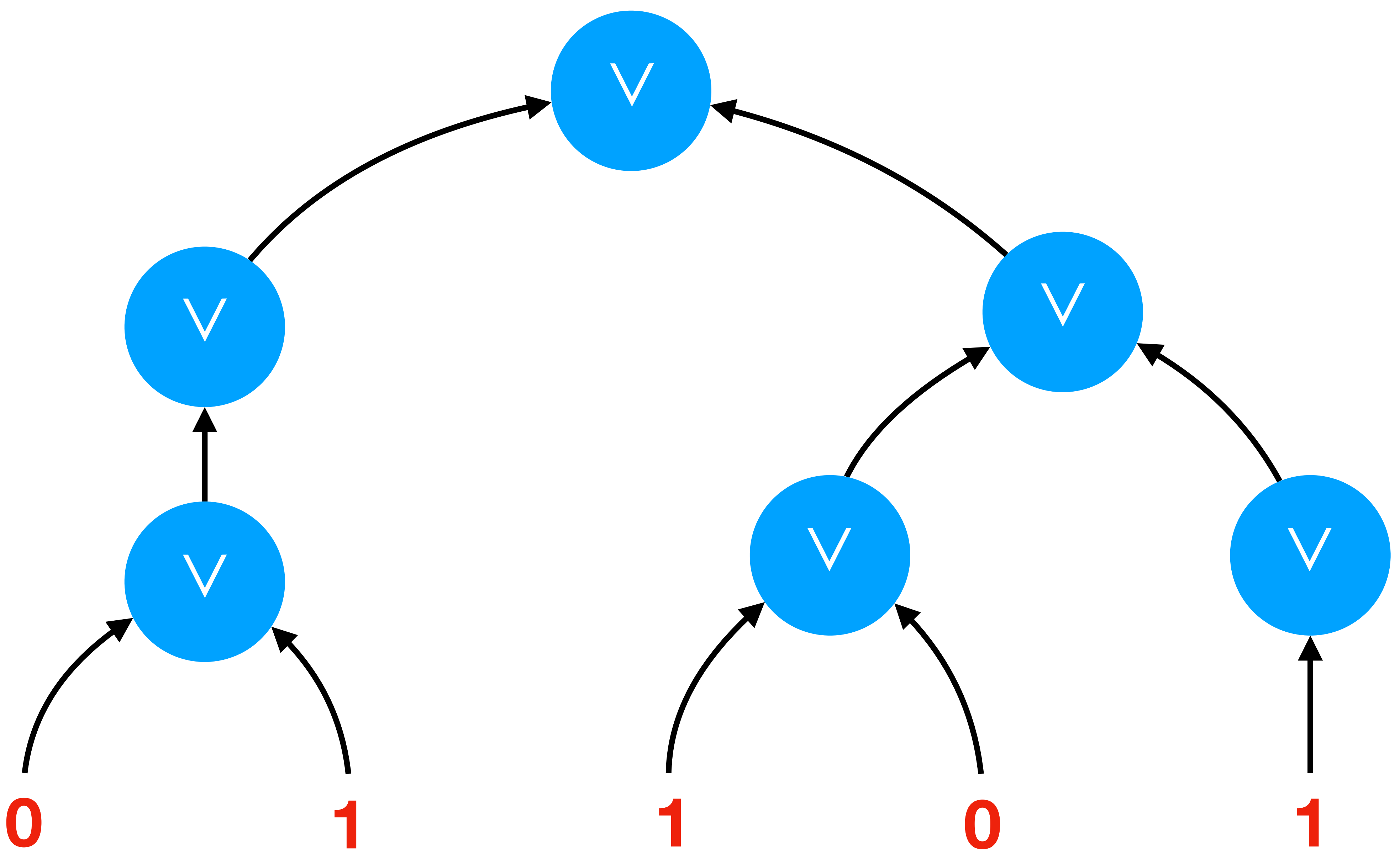


Flattening ∨-circuits



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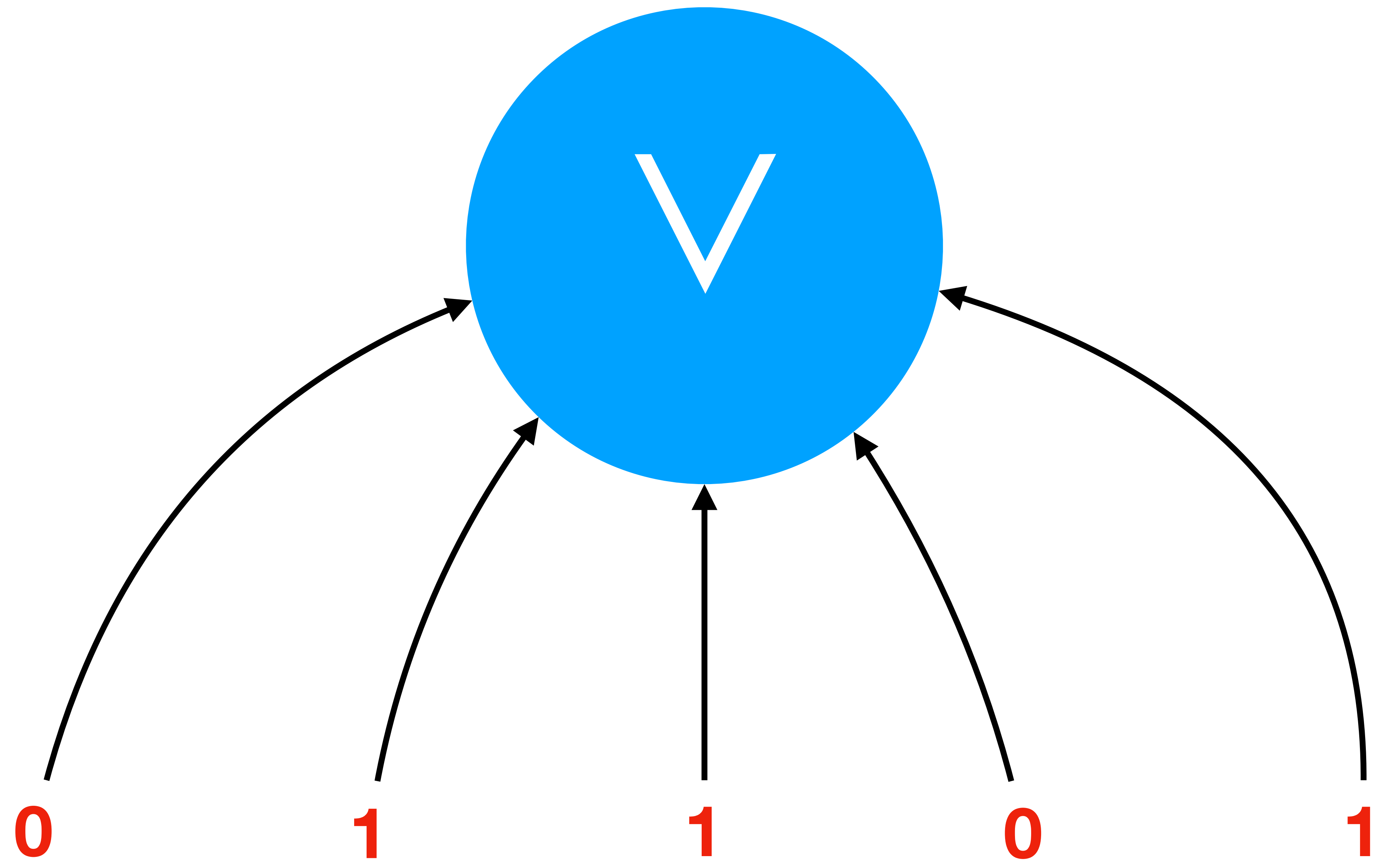
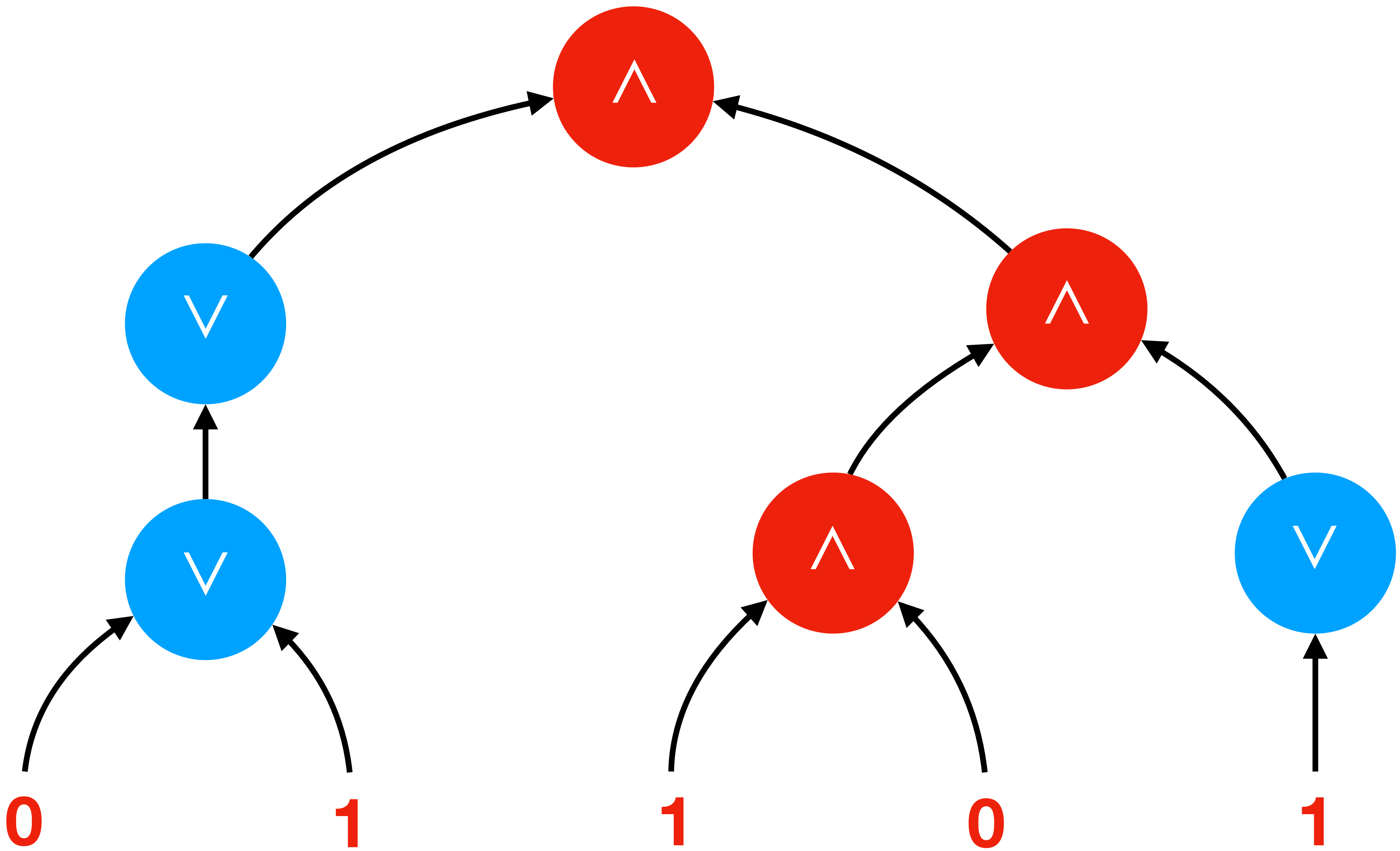
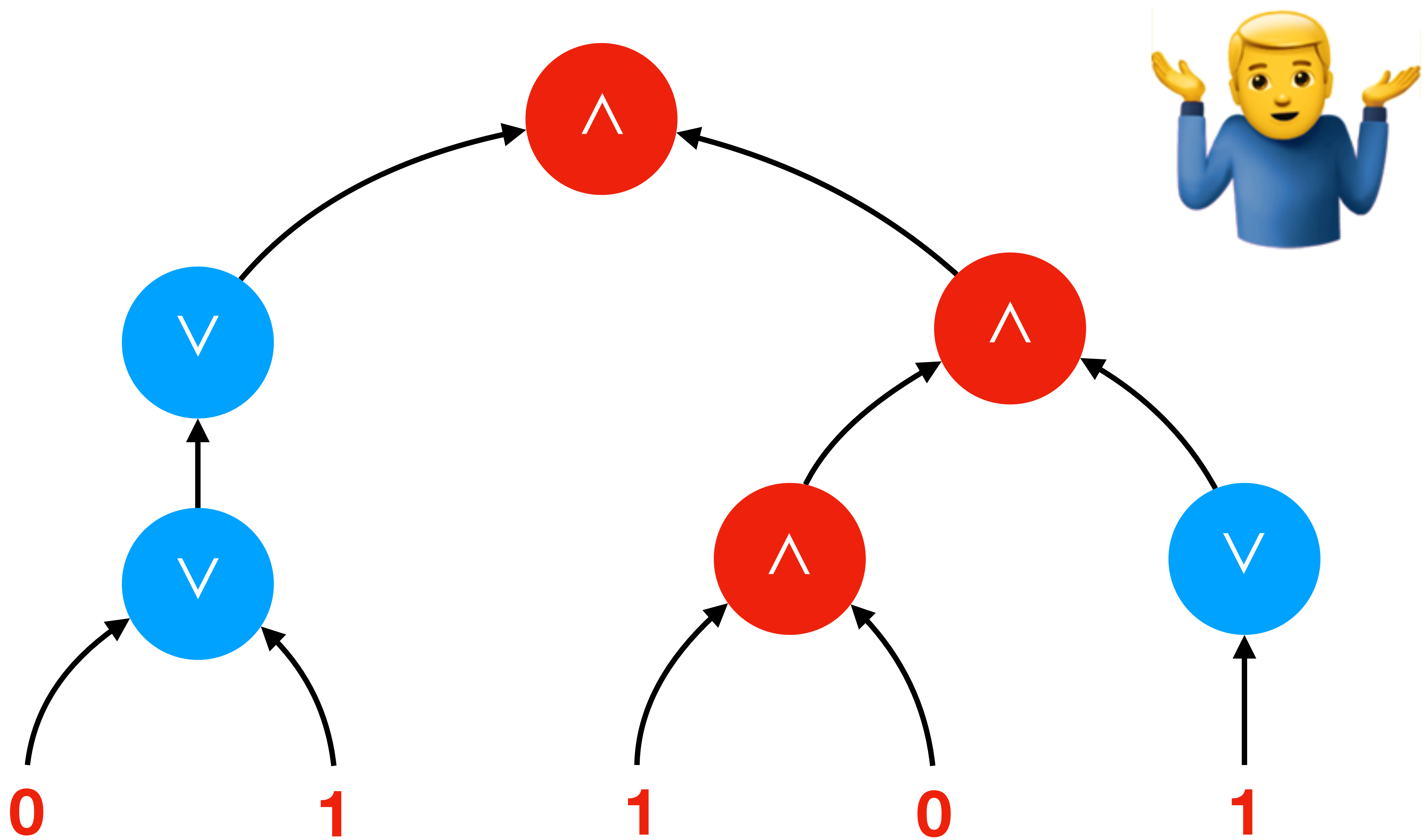


Flattening $\wedge\vee$-circuits?



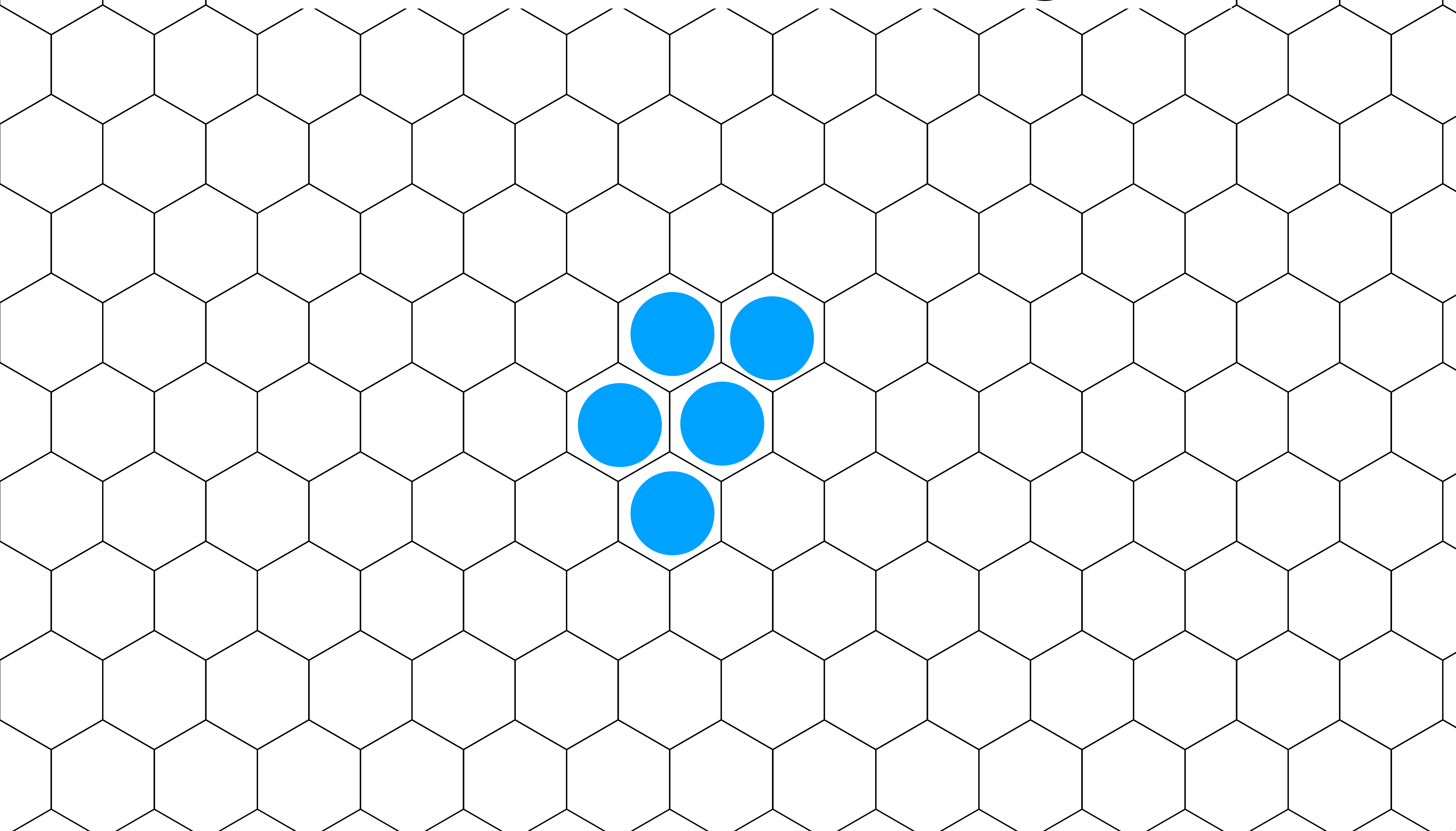
Flattening ∧∨-circuits?



Computation space
vs computation efficiency

Cellular automata over a Euclidean grid

Cellular automata over a Euclidean grid



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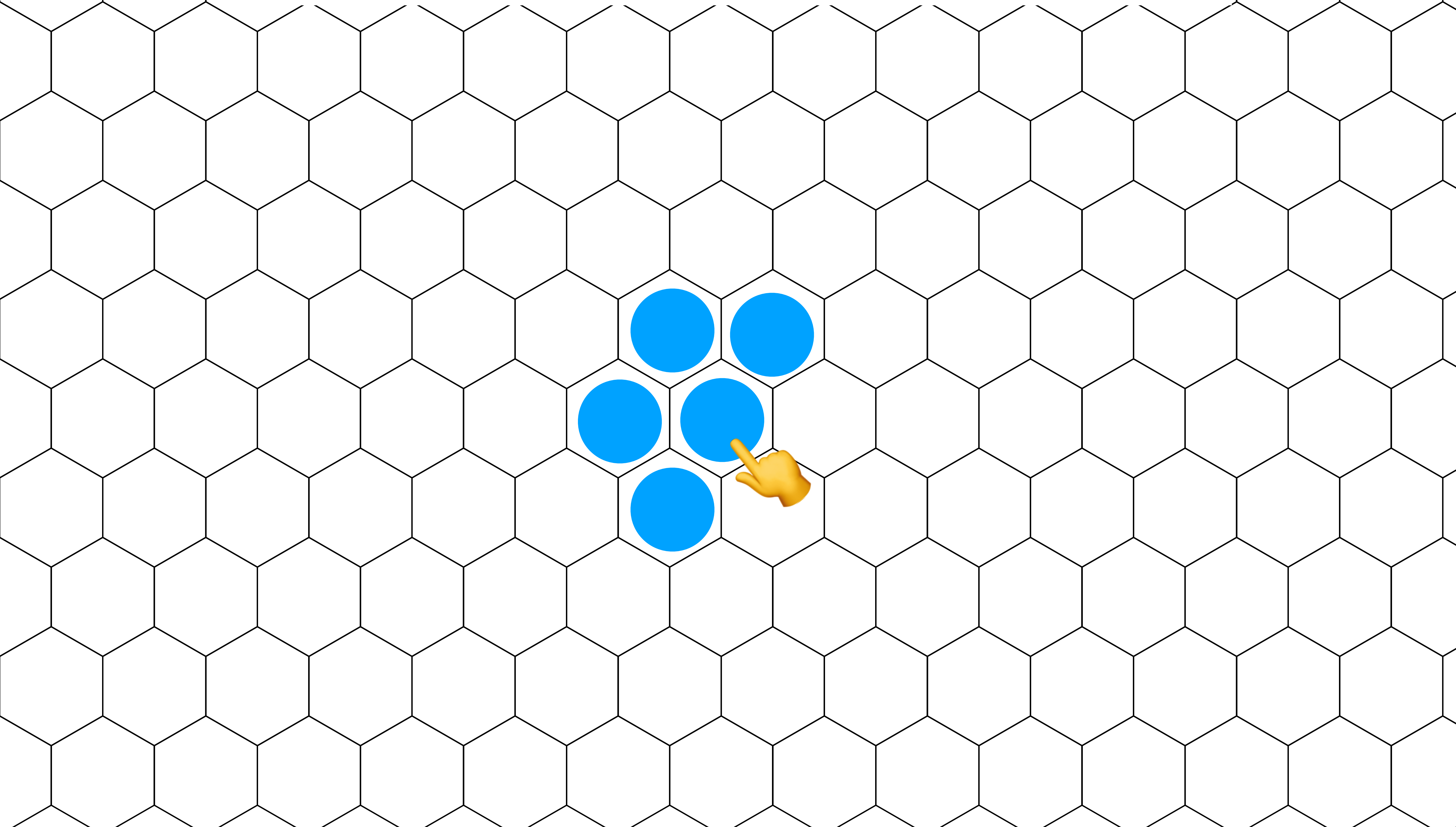

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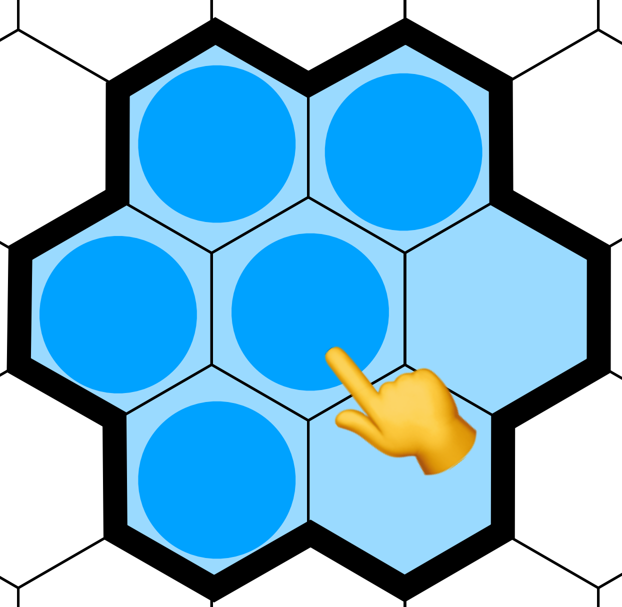

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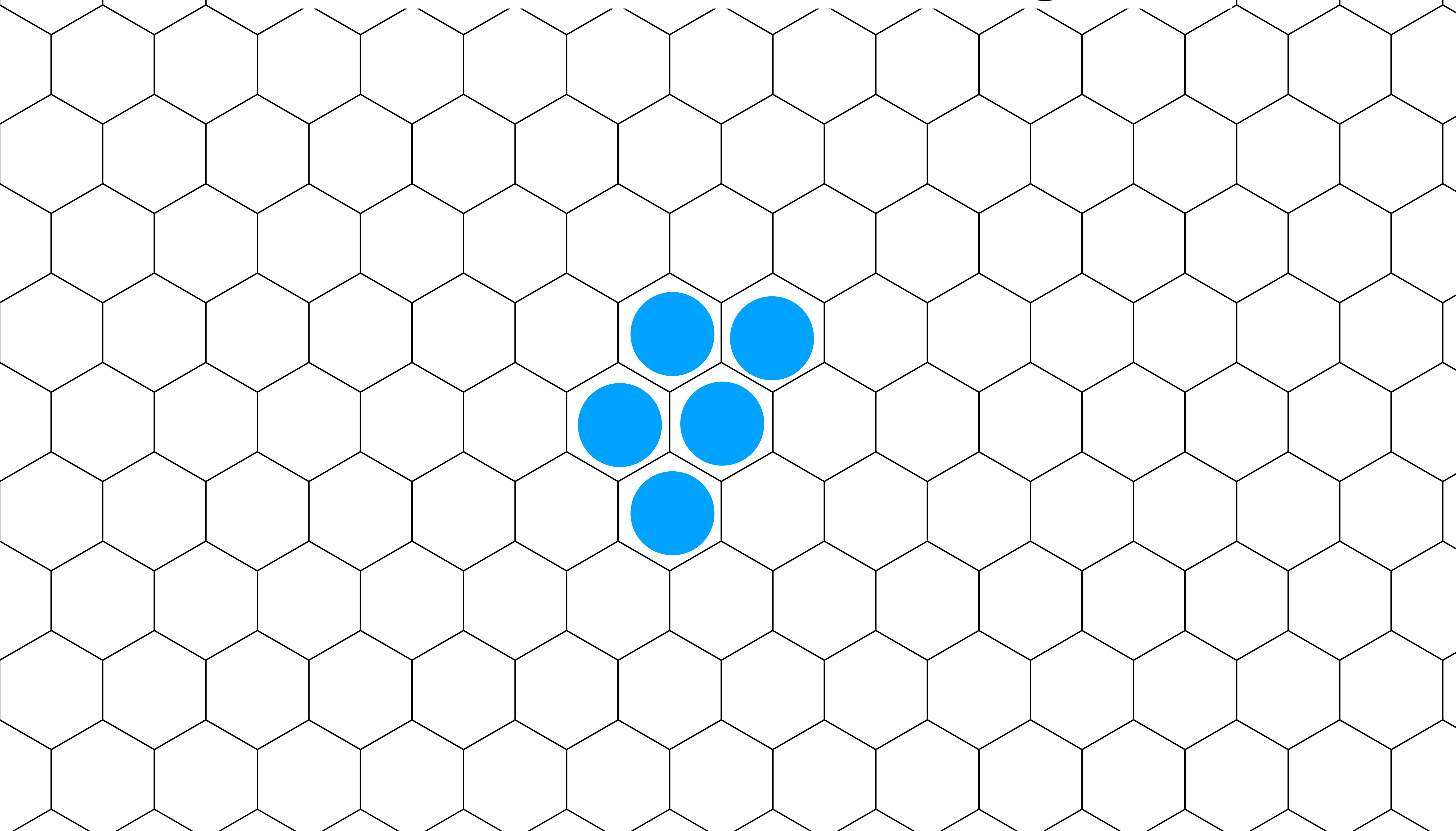

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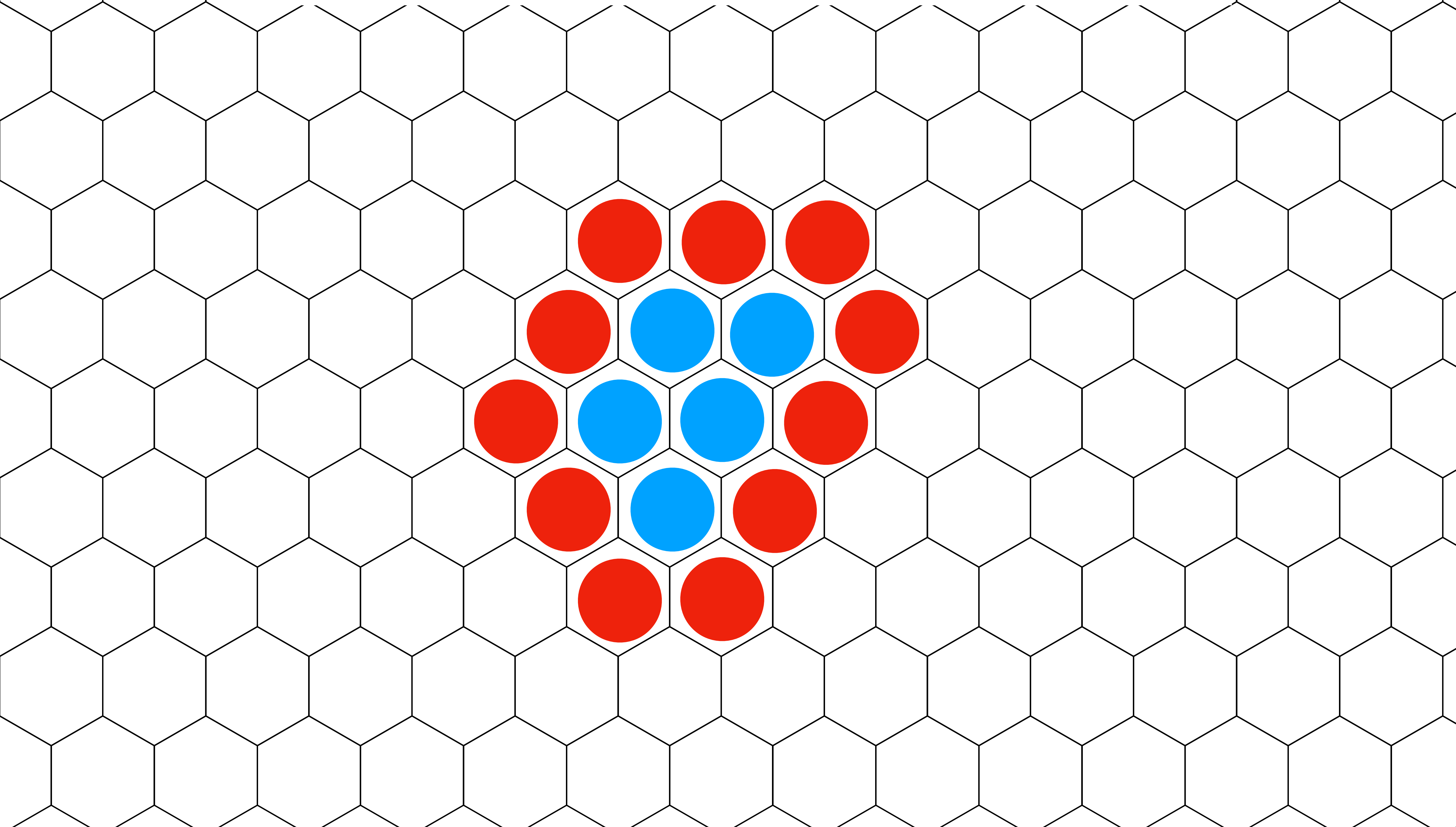

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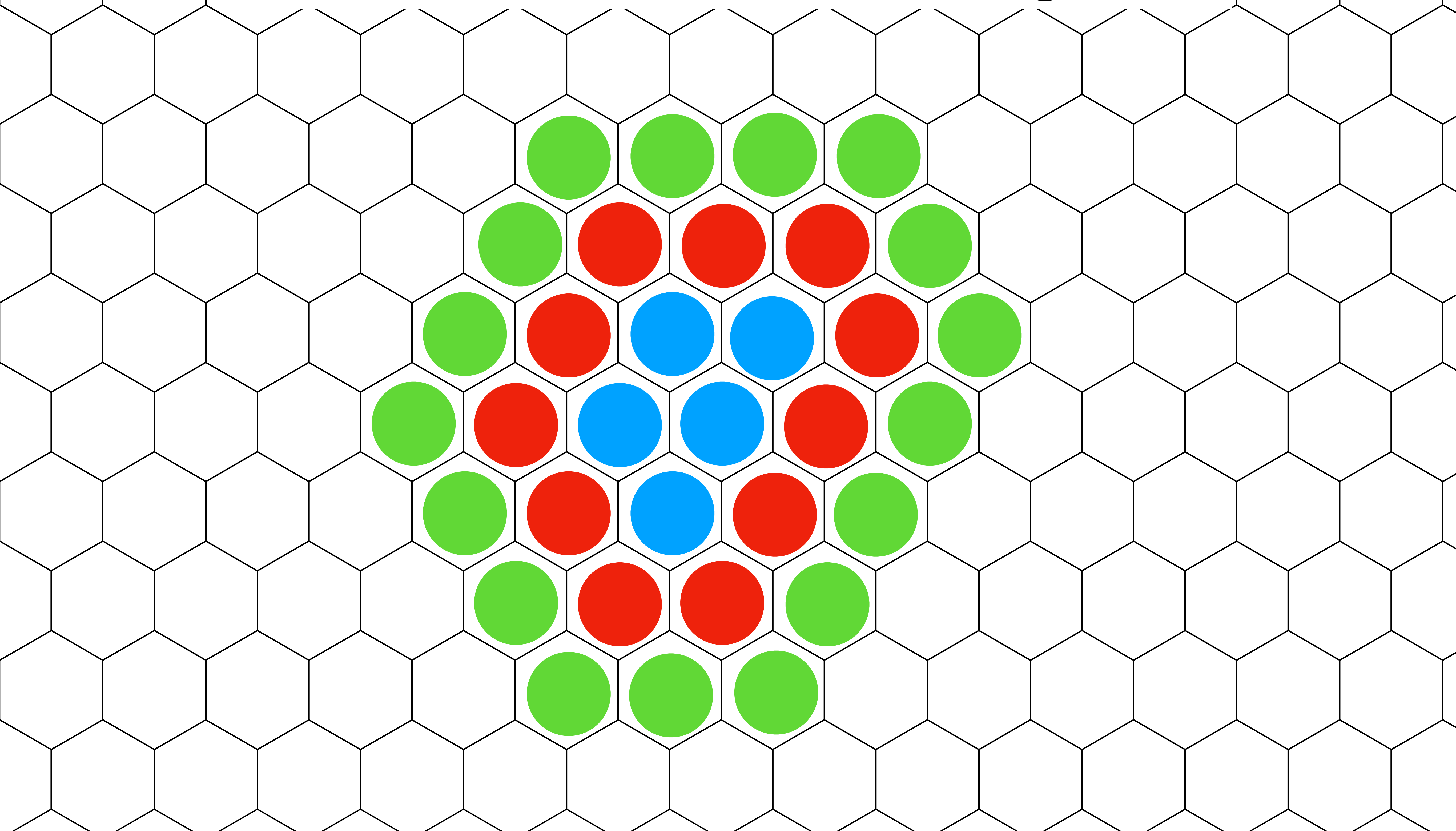

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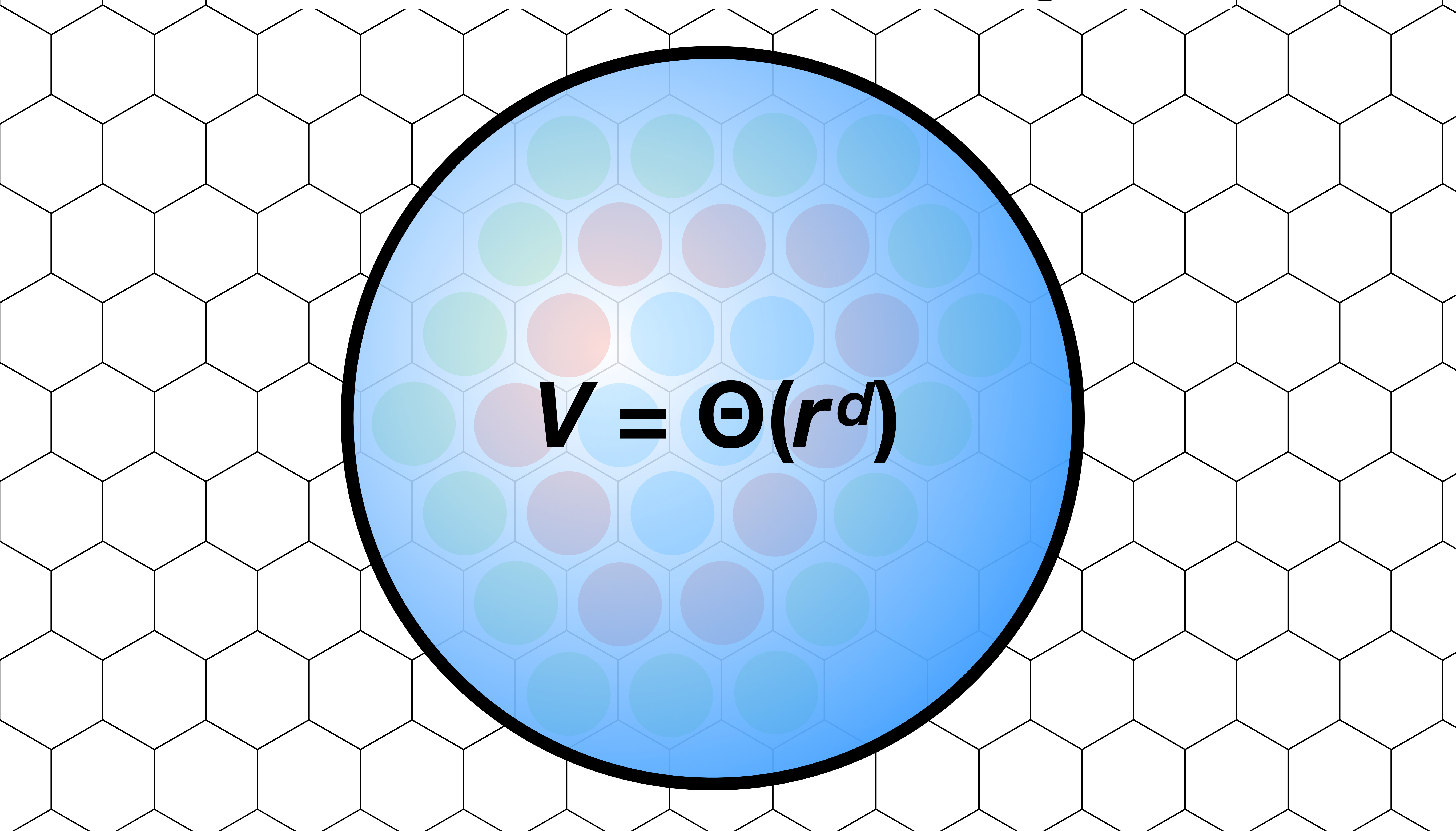
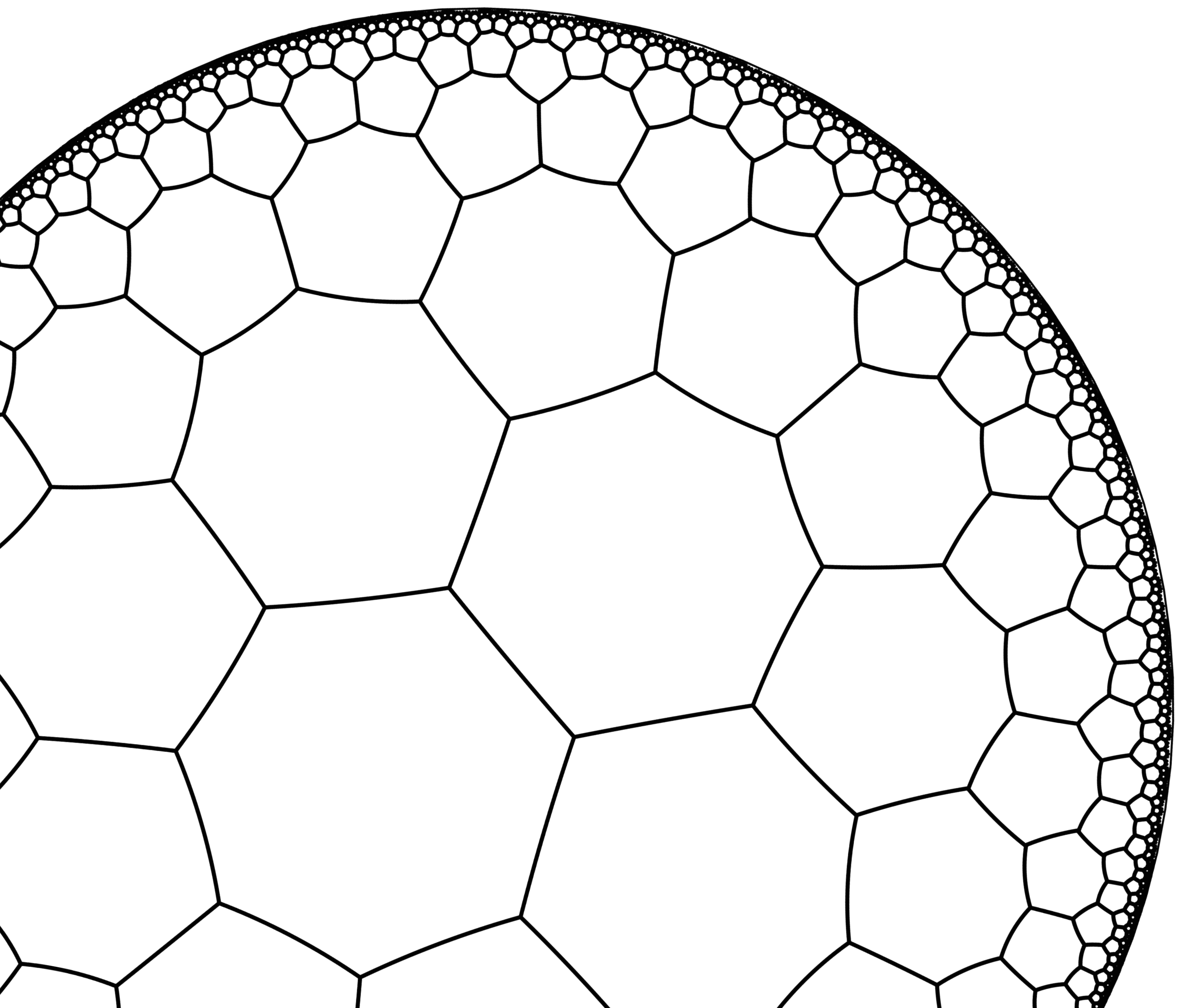
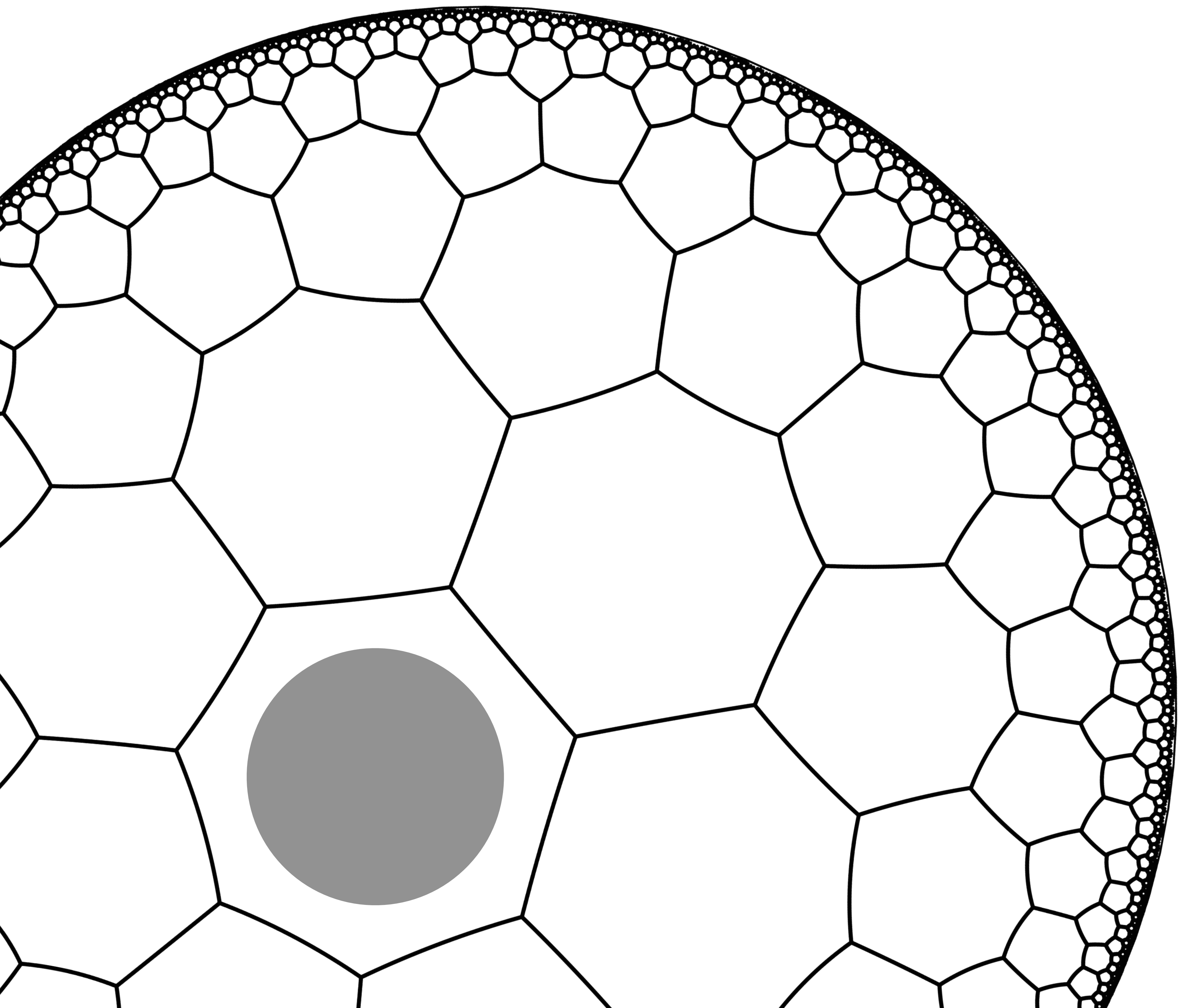


Cellular automata over a hyperbolic grid



Pavage du plan hyperbolique par des heptagones, dans le modèle du disque de Poincaré. By Theon, used under CC BY-SA 3.0 https:/en.wikipedia.org/wiki/File:PavageHypPoincare2.svg

Cellular automata over a hyperbolic grid



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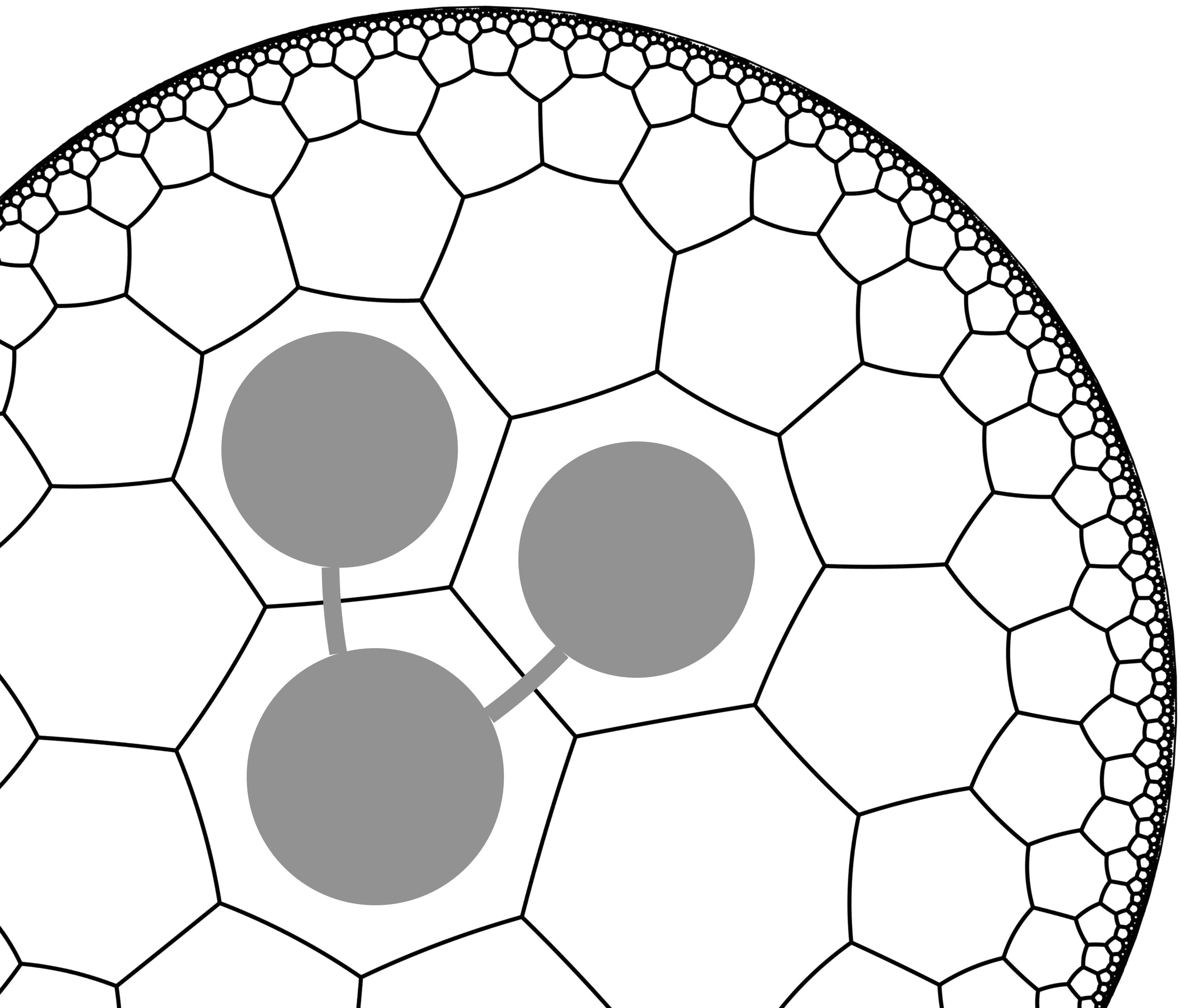

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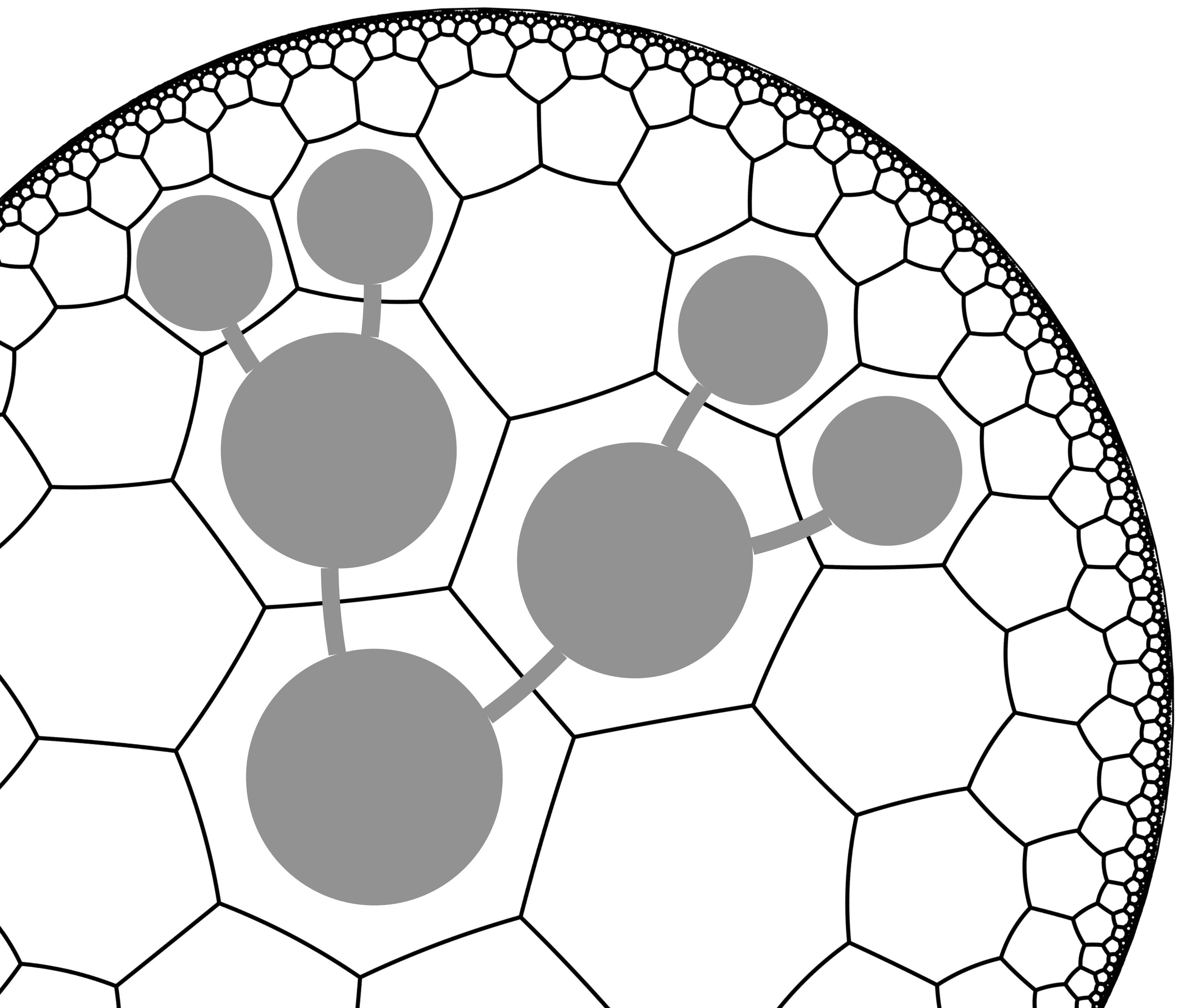

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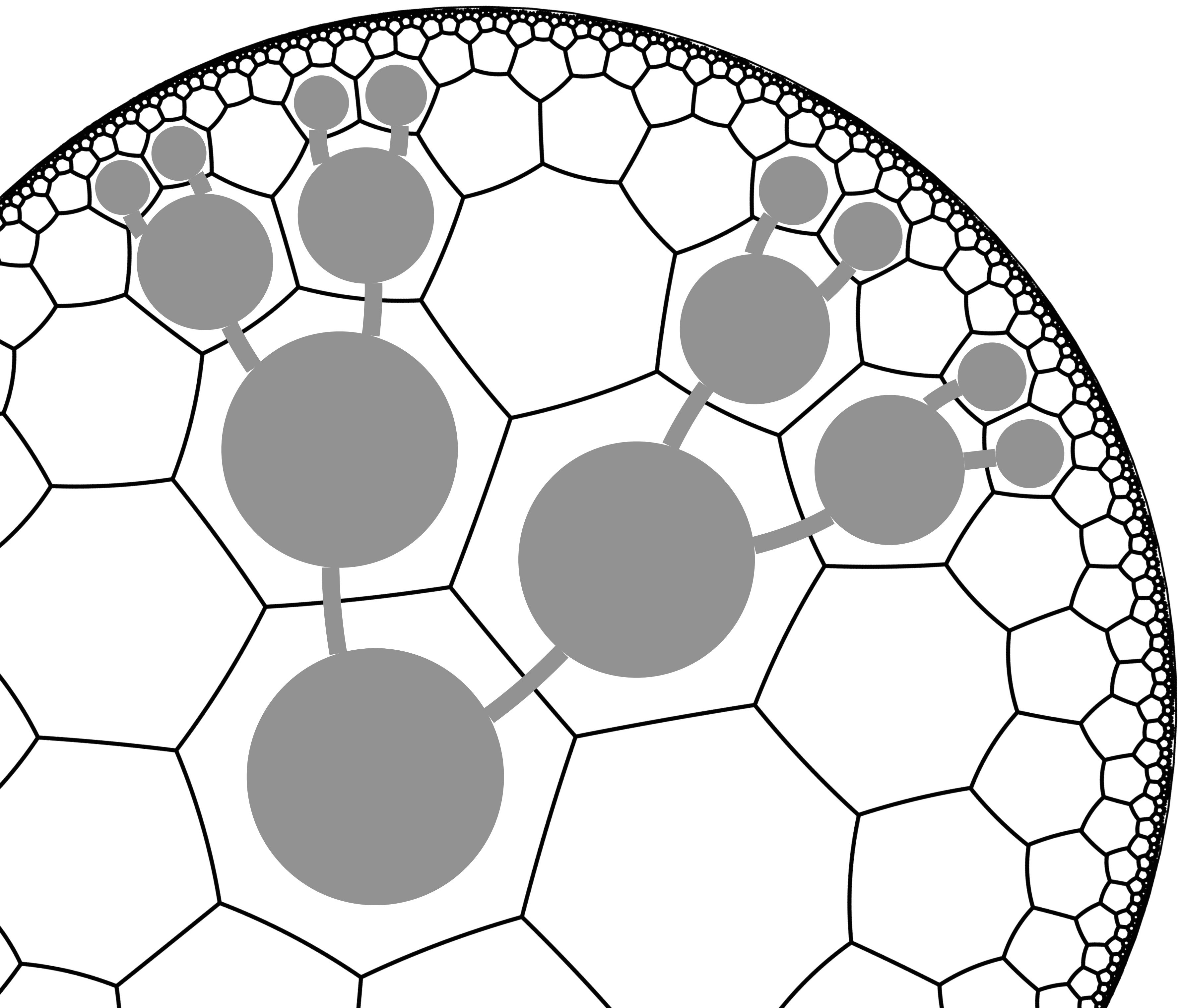

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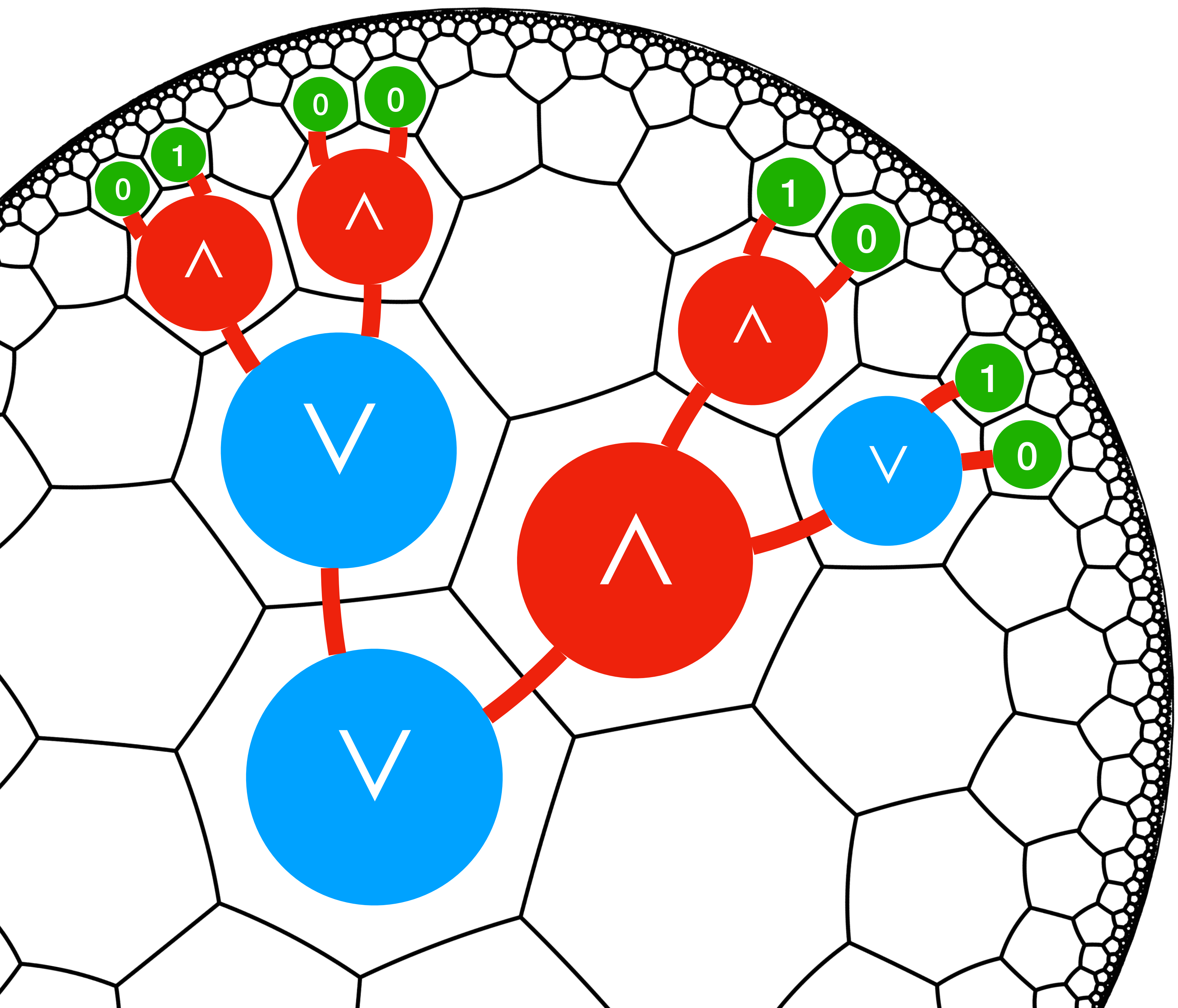

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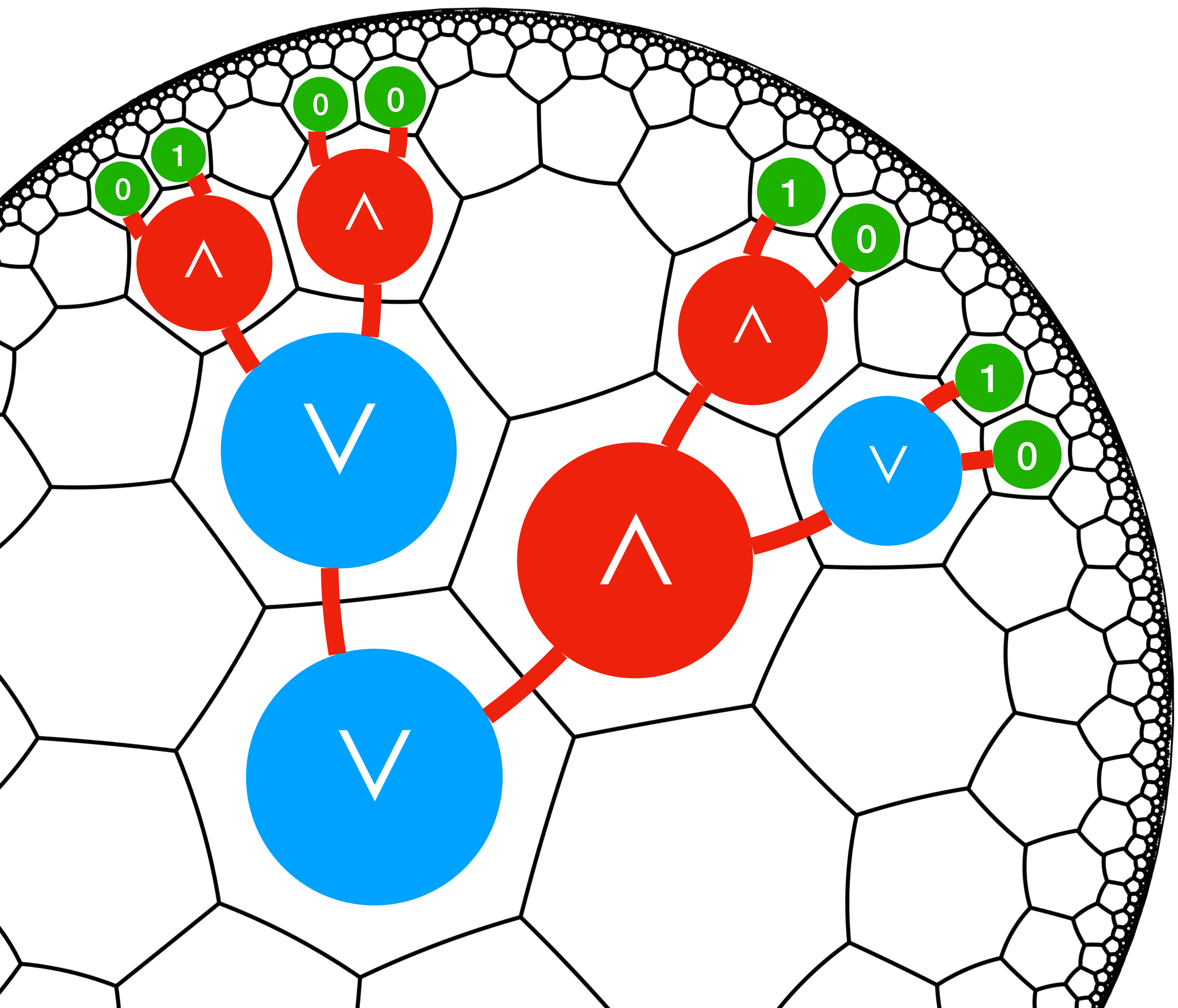

$$V = \Omega(2^r)$$

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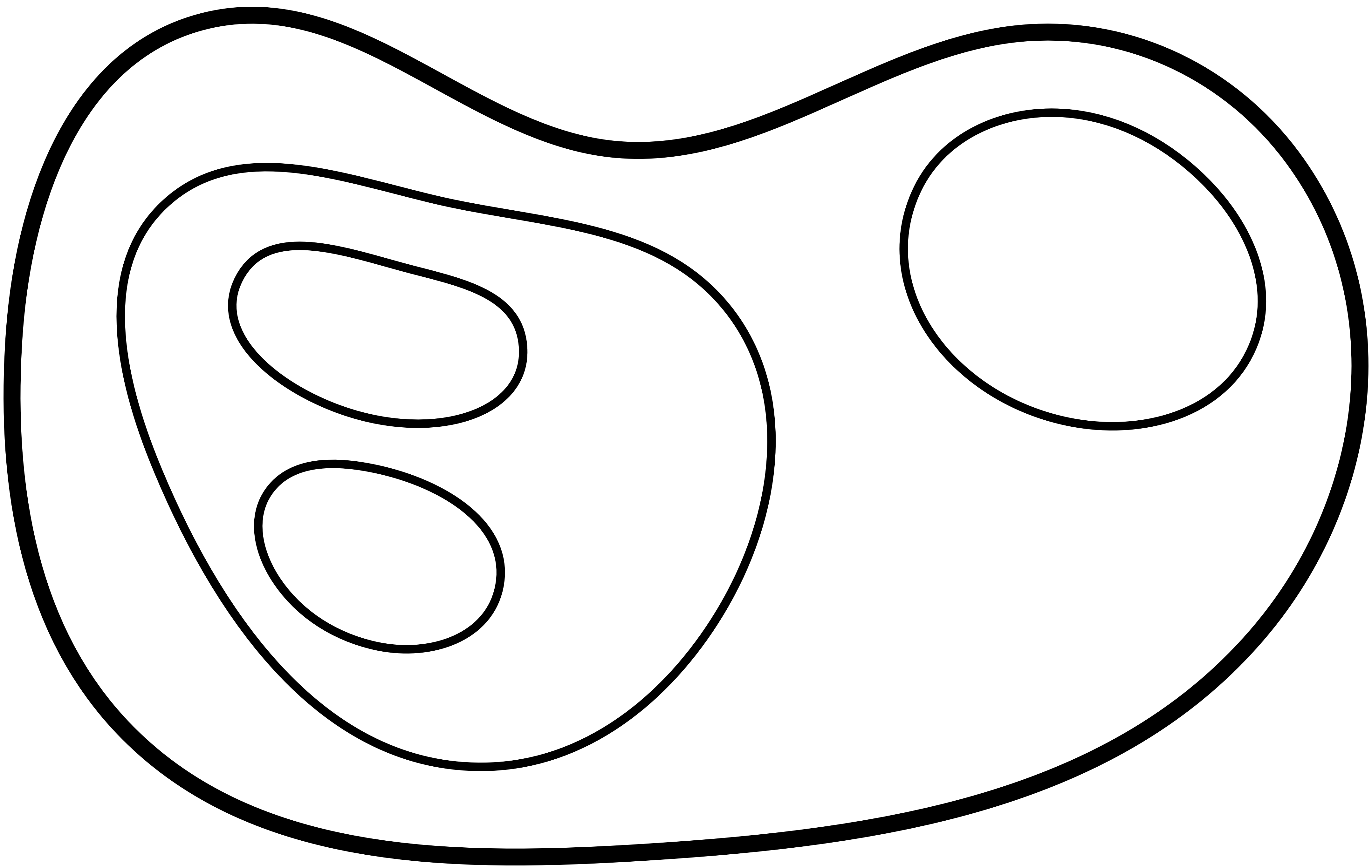
Rule of thumb 👍

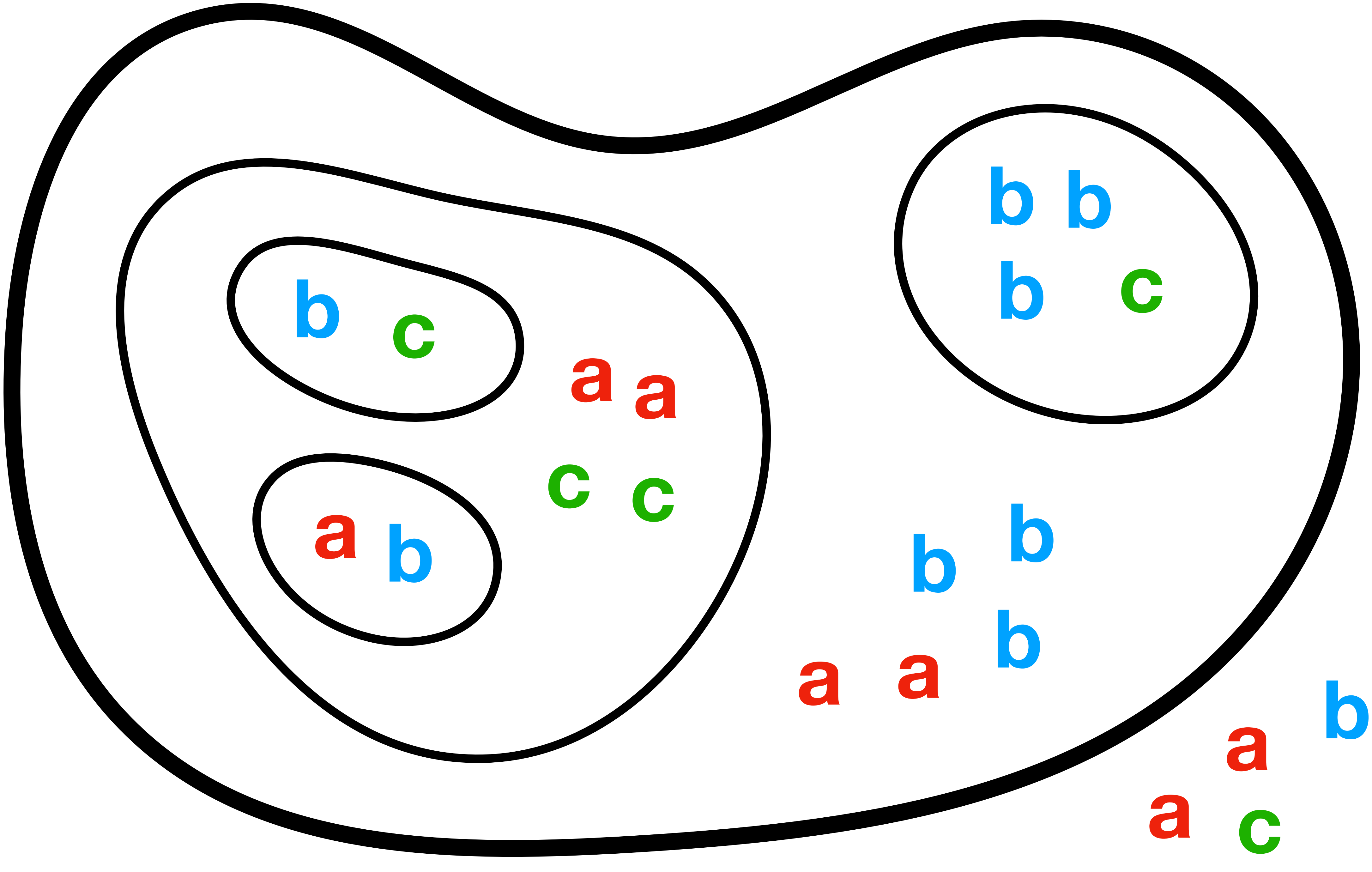
- Sequential machines are first class
- (Constant or polynomial) bounded parallel machines are also first class
- Unbounded (or exponential bounded) parallel machines are second class
- Apparently, this holds even for unconventional computing models ☹️

A “more unconventional” model of computation: membrane systems

Membrane systems



Membrane systems



Evolution rules

$[ab \rightarrow cd]$

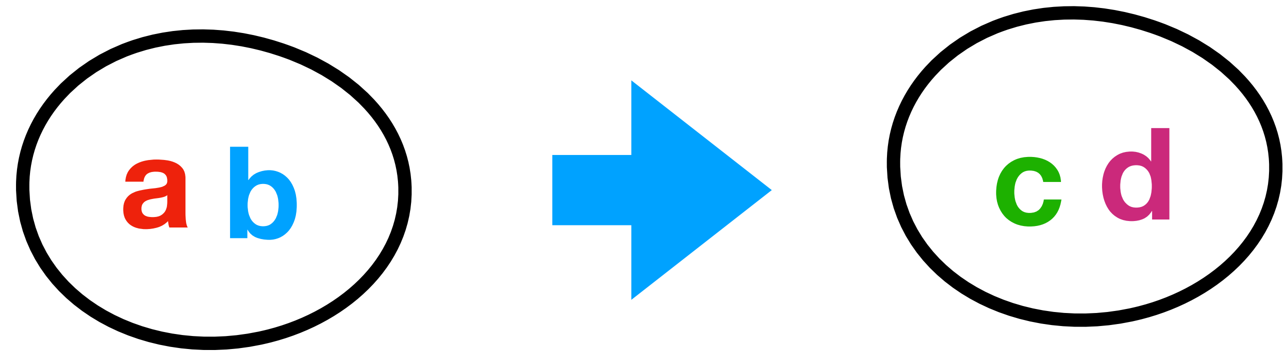
$[a] \rightarrow [\ ]\ b$

$a\ [\ ] \rightarrow [b]$

$[a] \rightarrow [b]\ [c]$

Evolution rules

[ab → cd]



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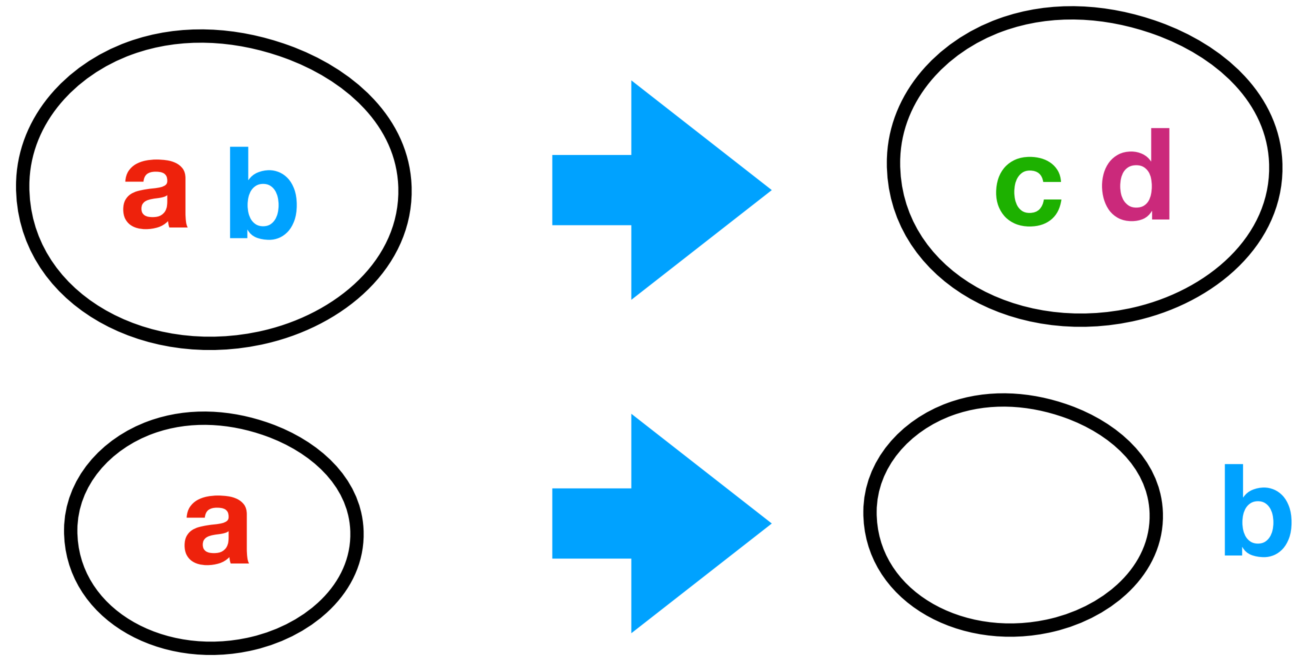

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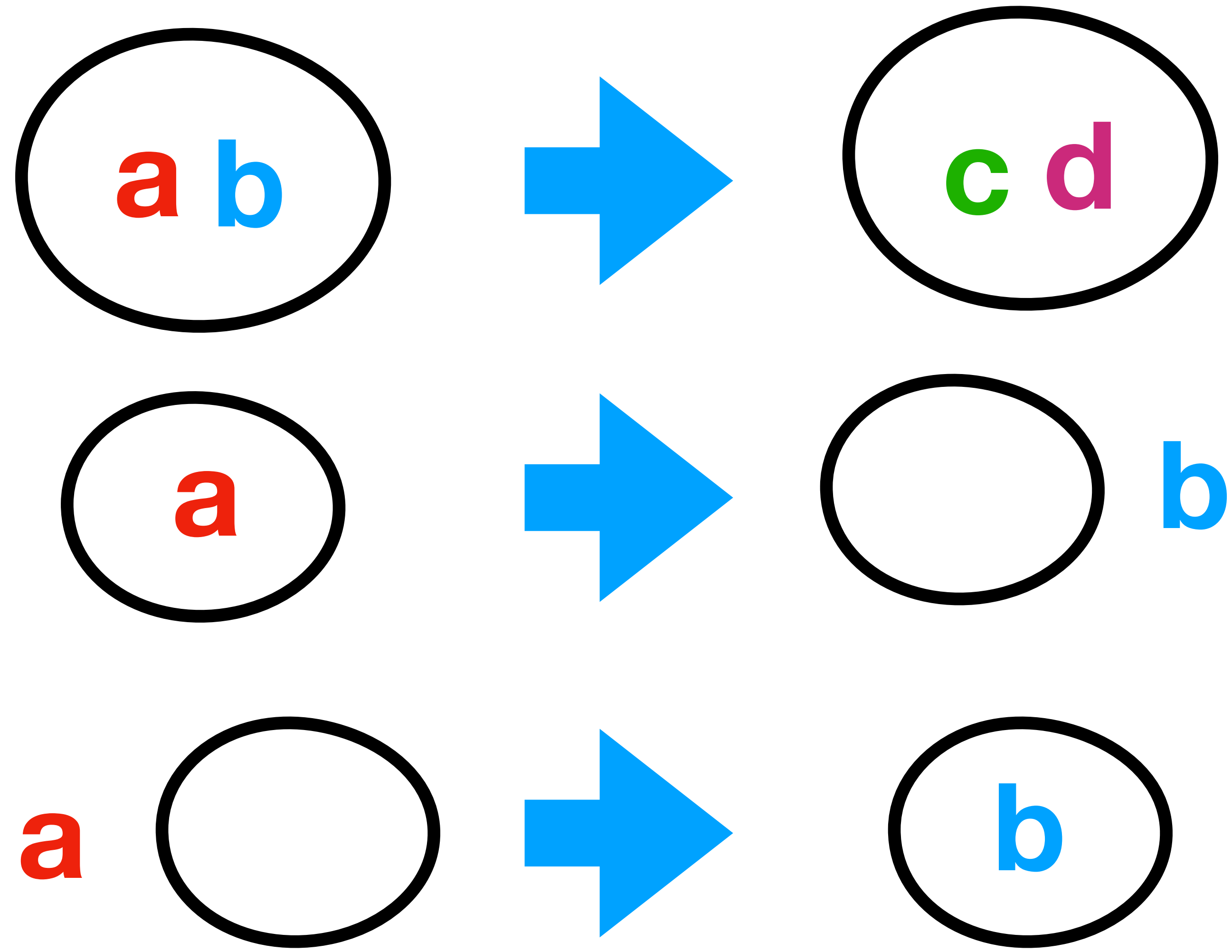

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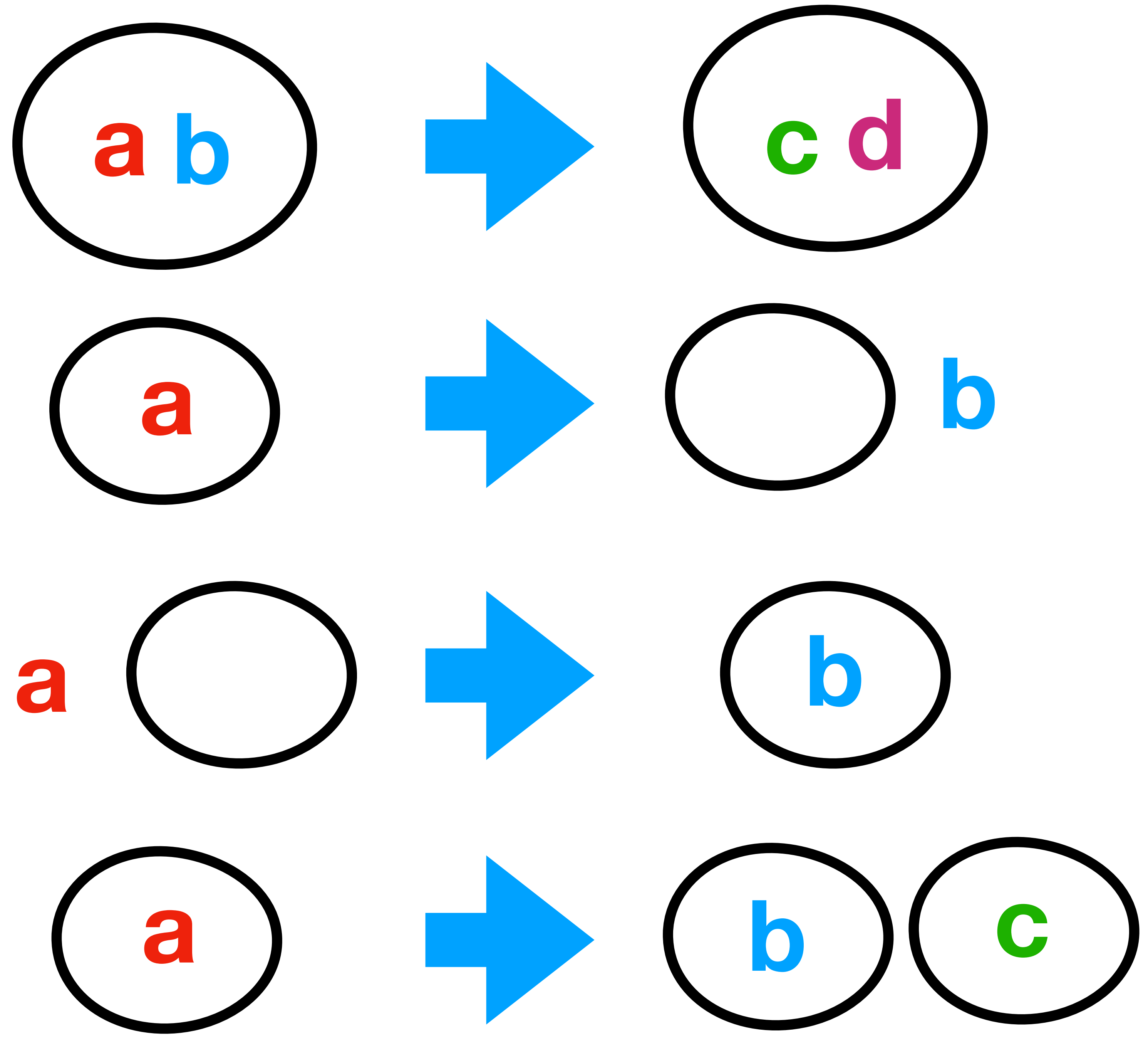


Evolution rules

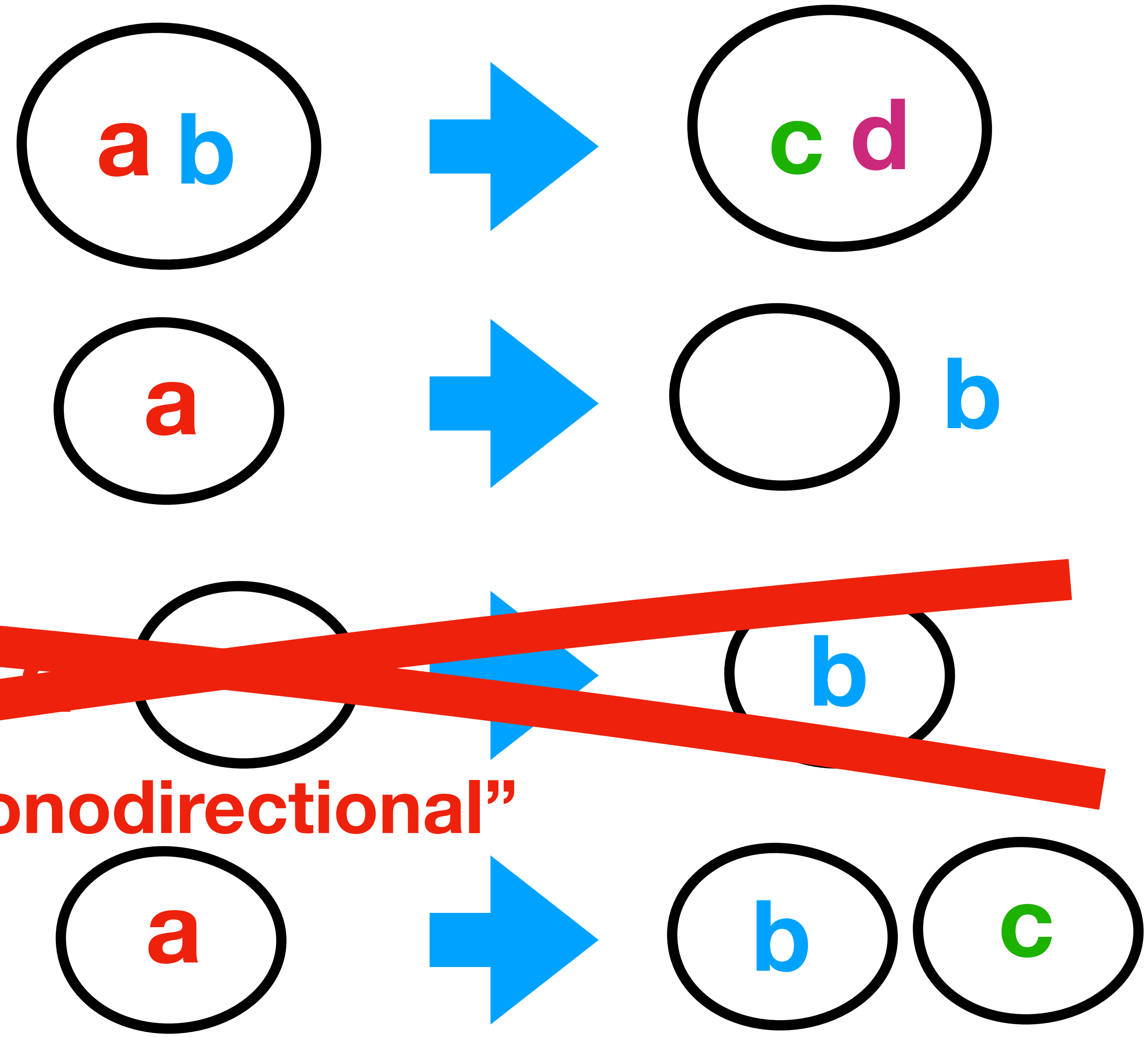
[ab → cd]

[a] → [ ] b

~~a [ ] → [b]~~

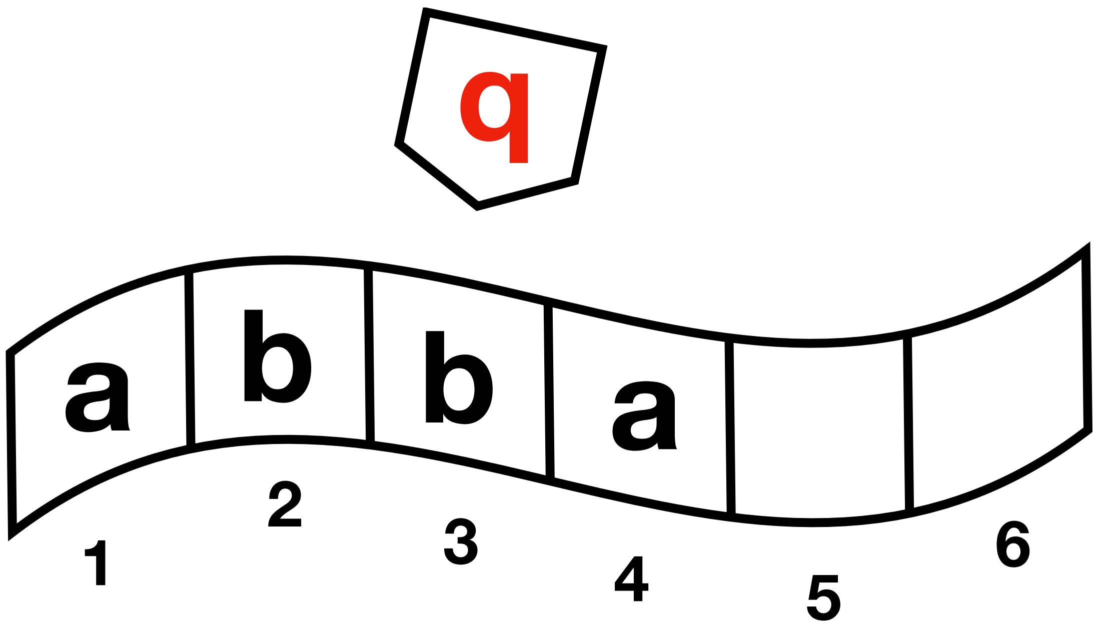
“monodirectional”

[a] → [b] [c]

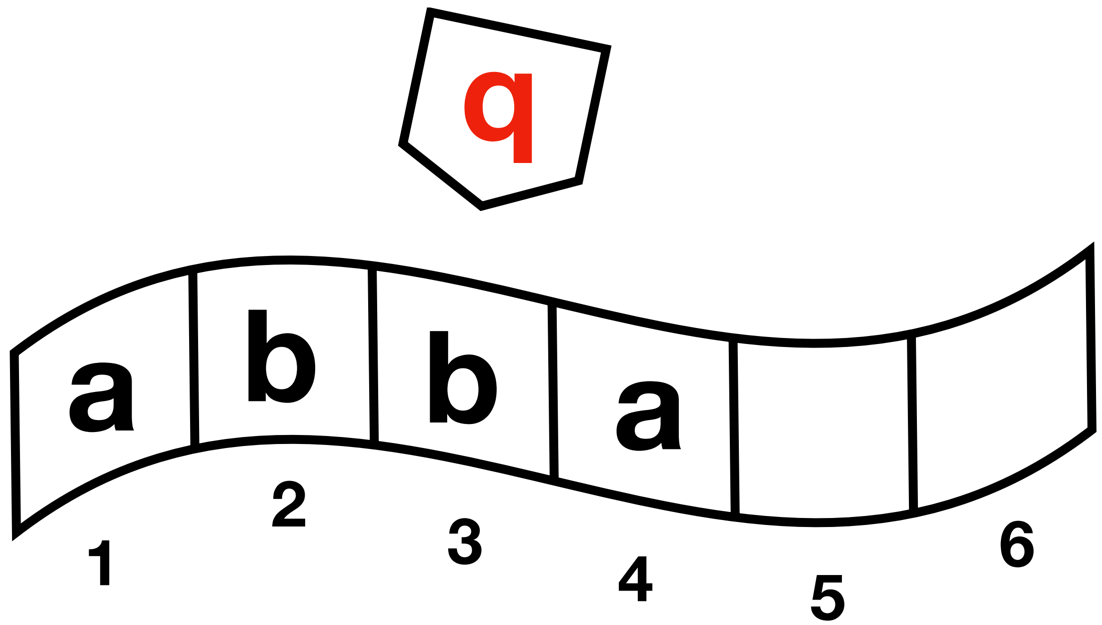


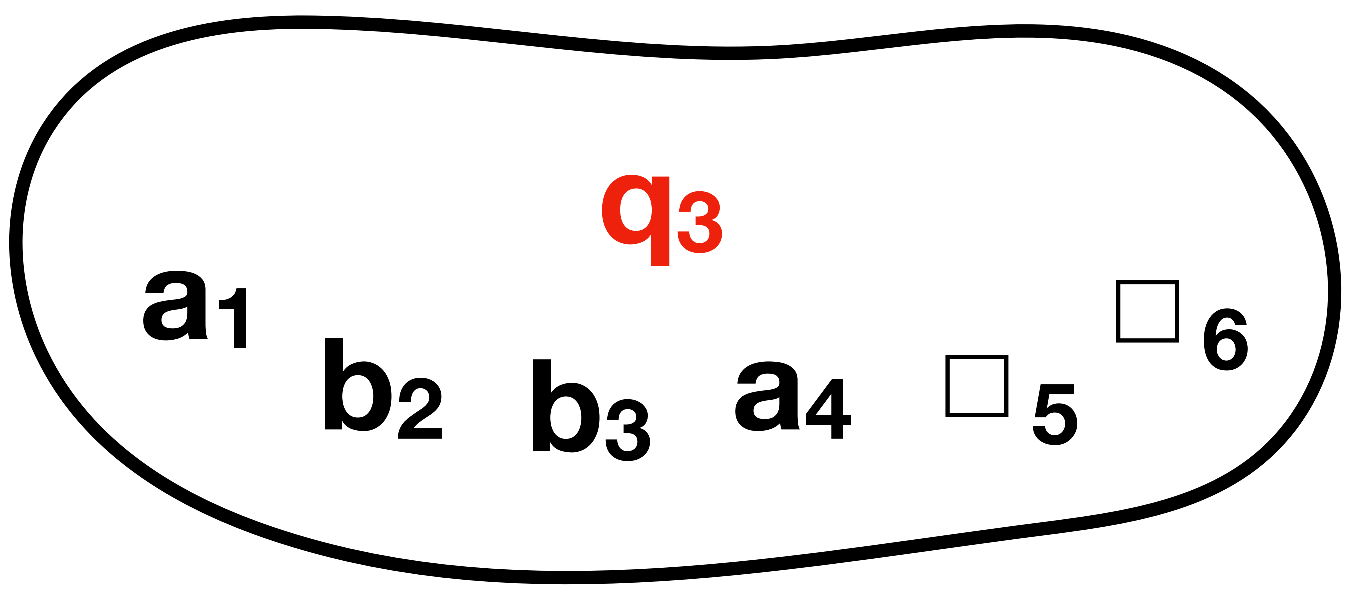
Simulating Turing machines with membrane systems

Encoding the configuration



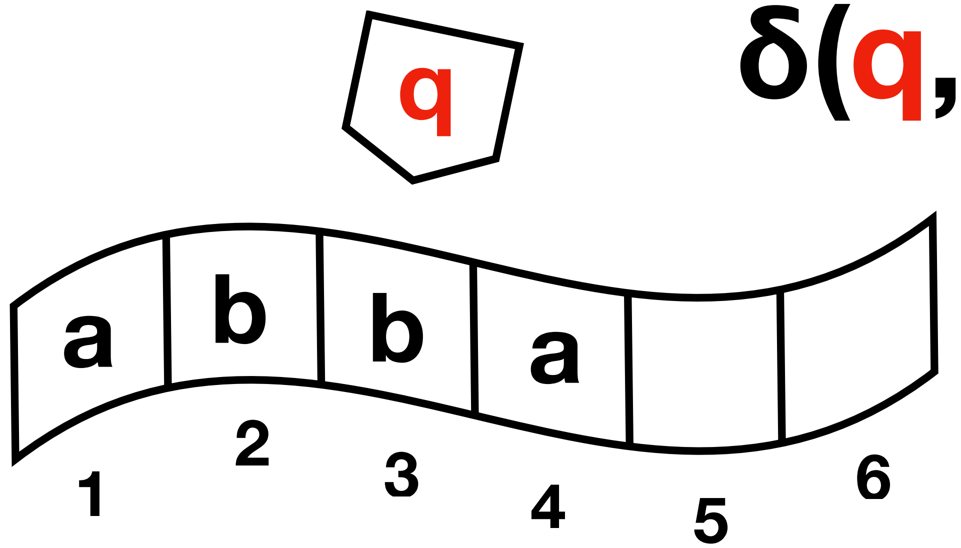
Encoding the configuration

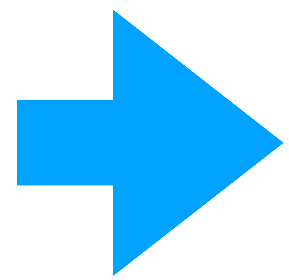


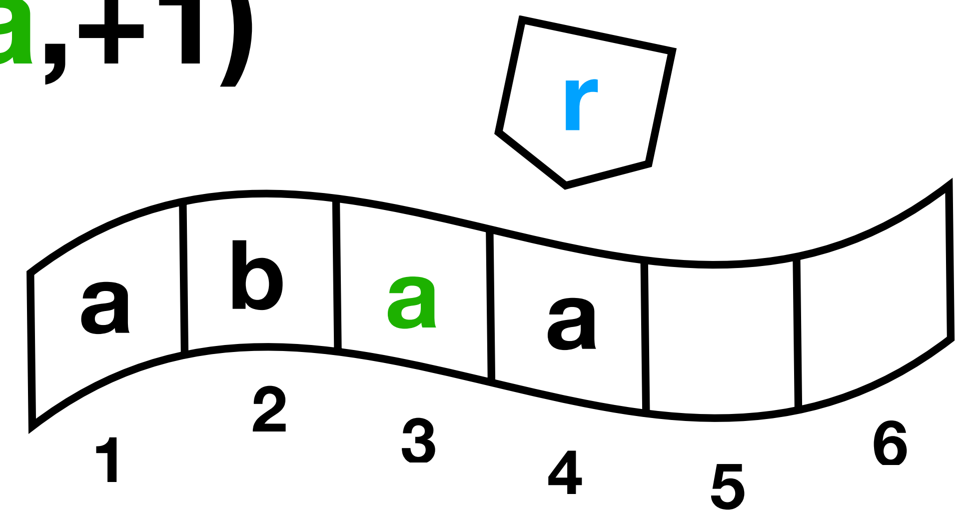


Simulating transitions

$\delta(q,b) = (r,a,+1)$







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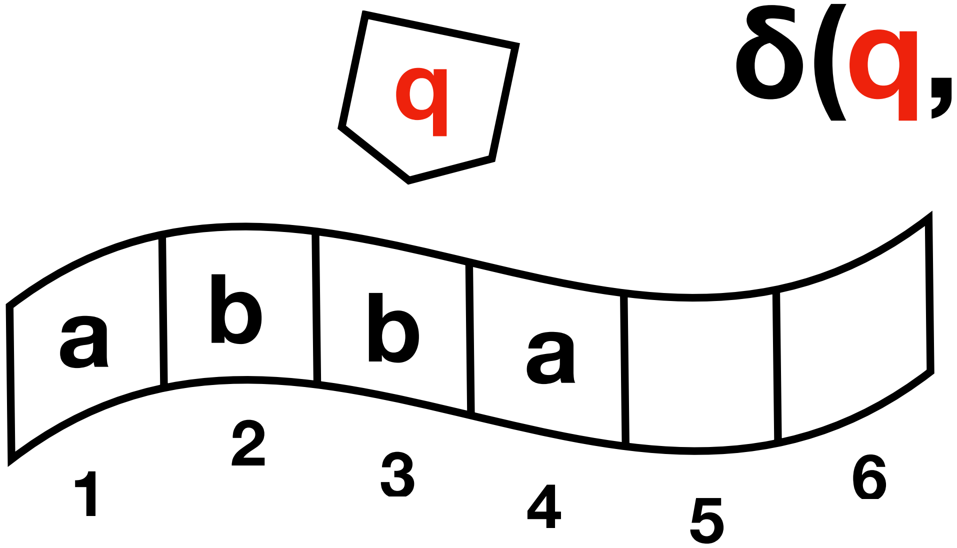

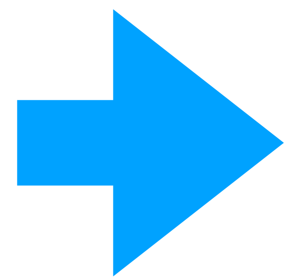

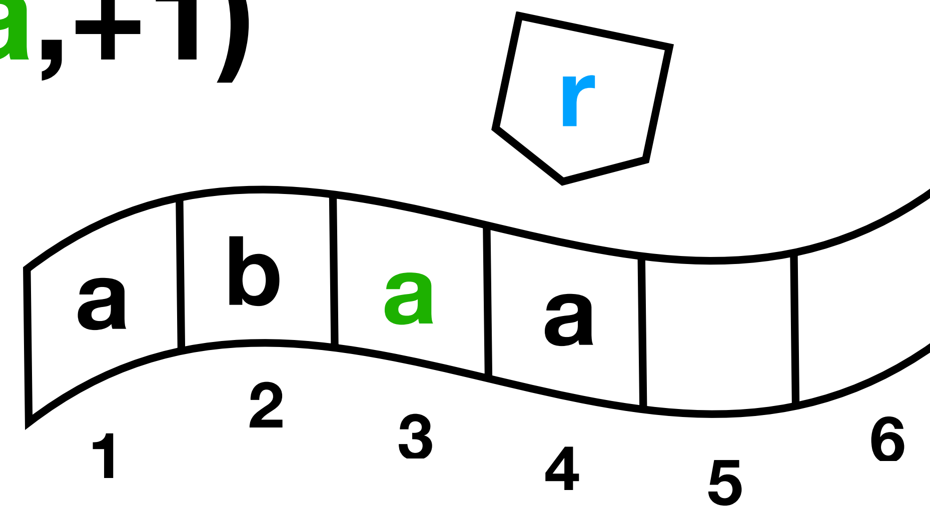

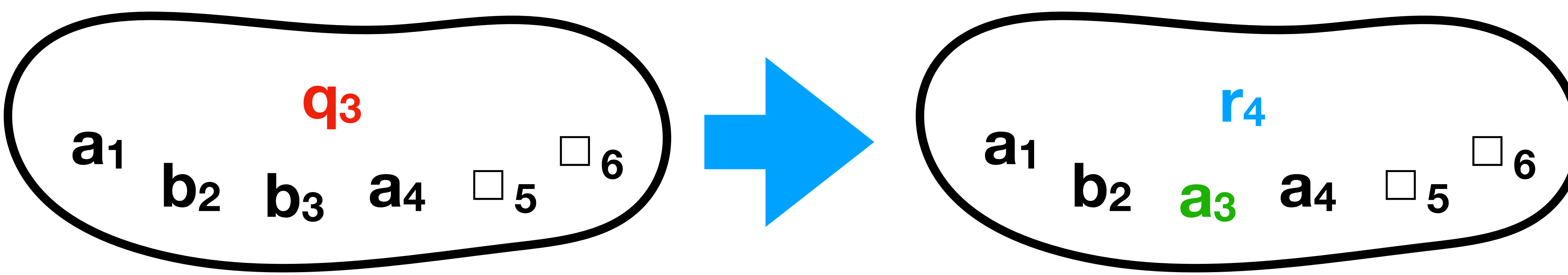

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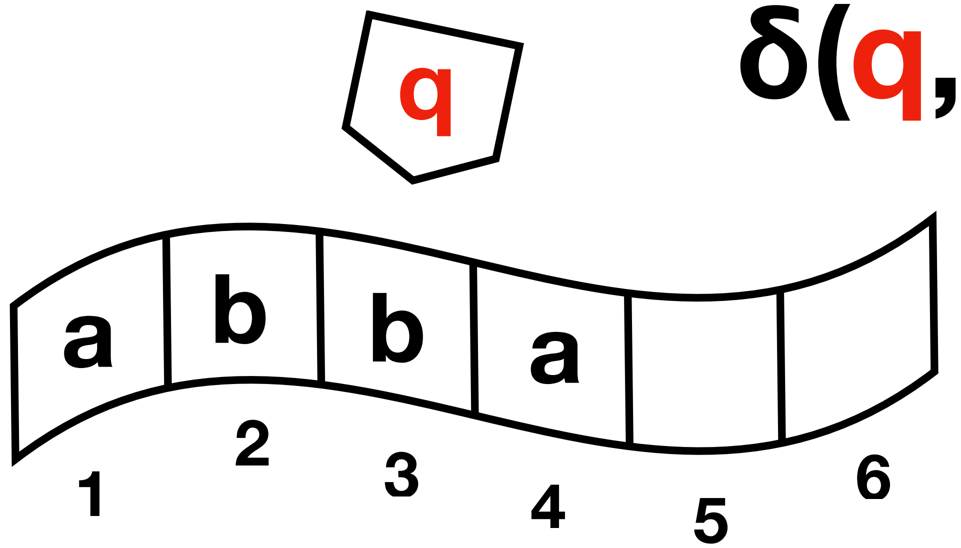
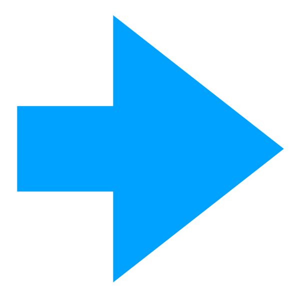
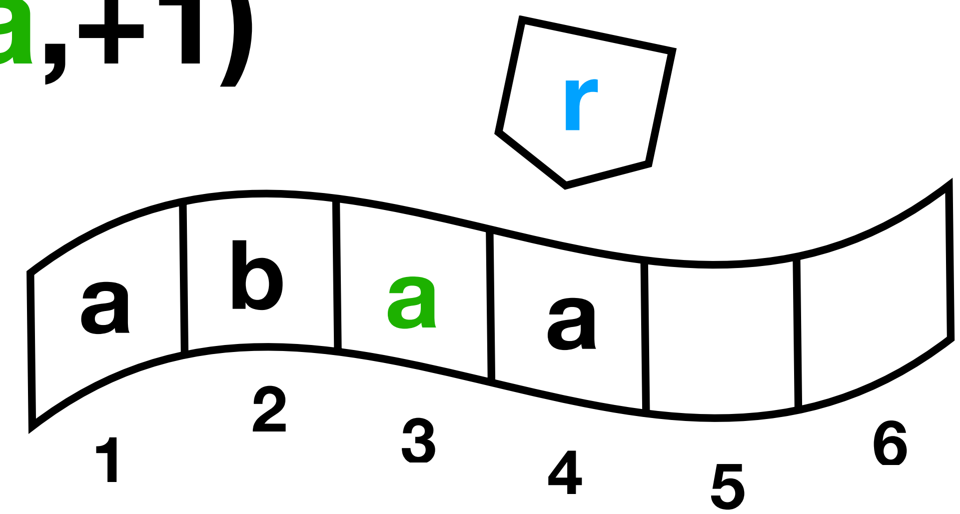
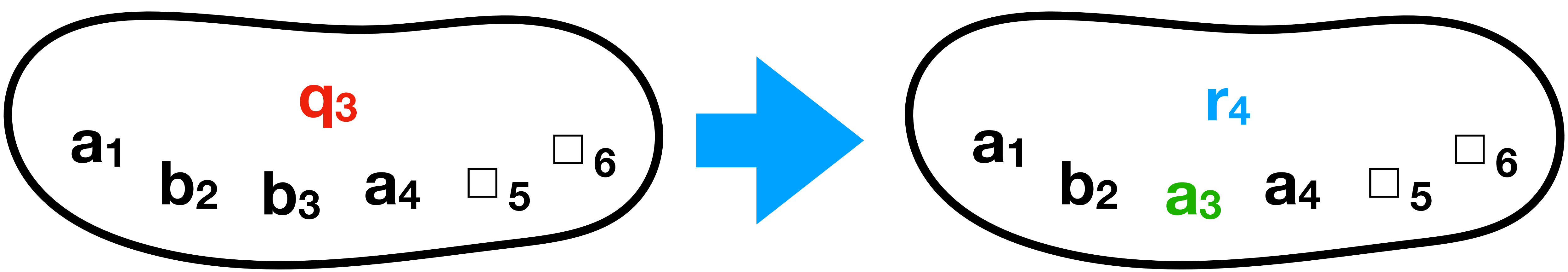




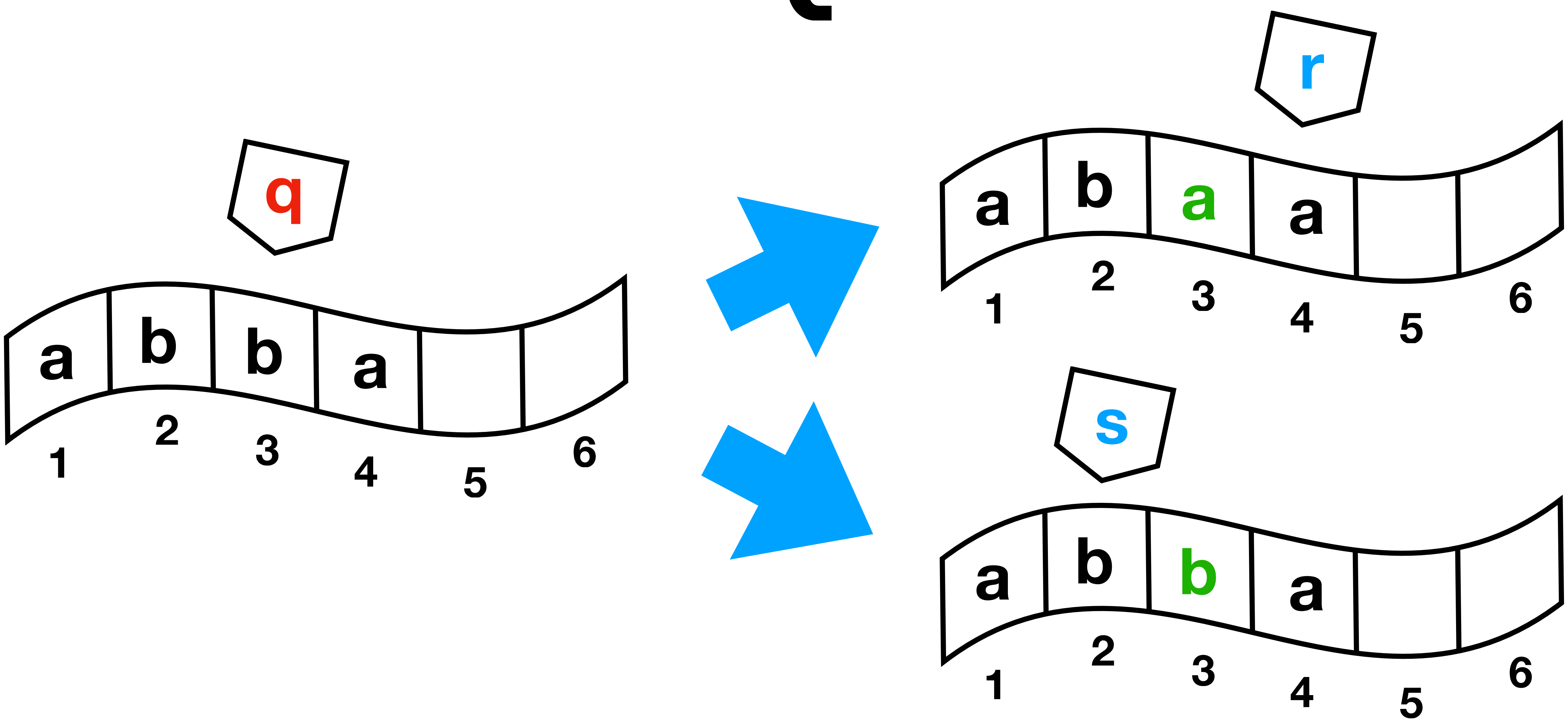


$[q_3\ b_3 \rightarrow r_4\ a_3]$



Simulating nondeterminism

$$\delta(q,b) = \begin{cases} (r,a,+1) \\ (s,b,-1) \end{cases}$$



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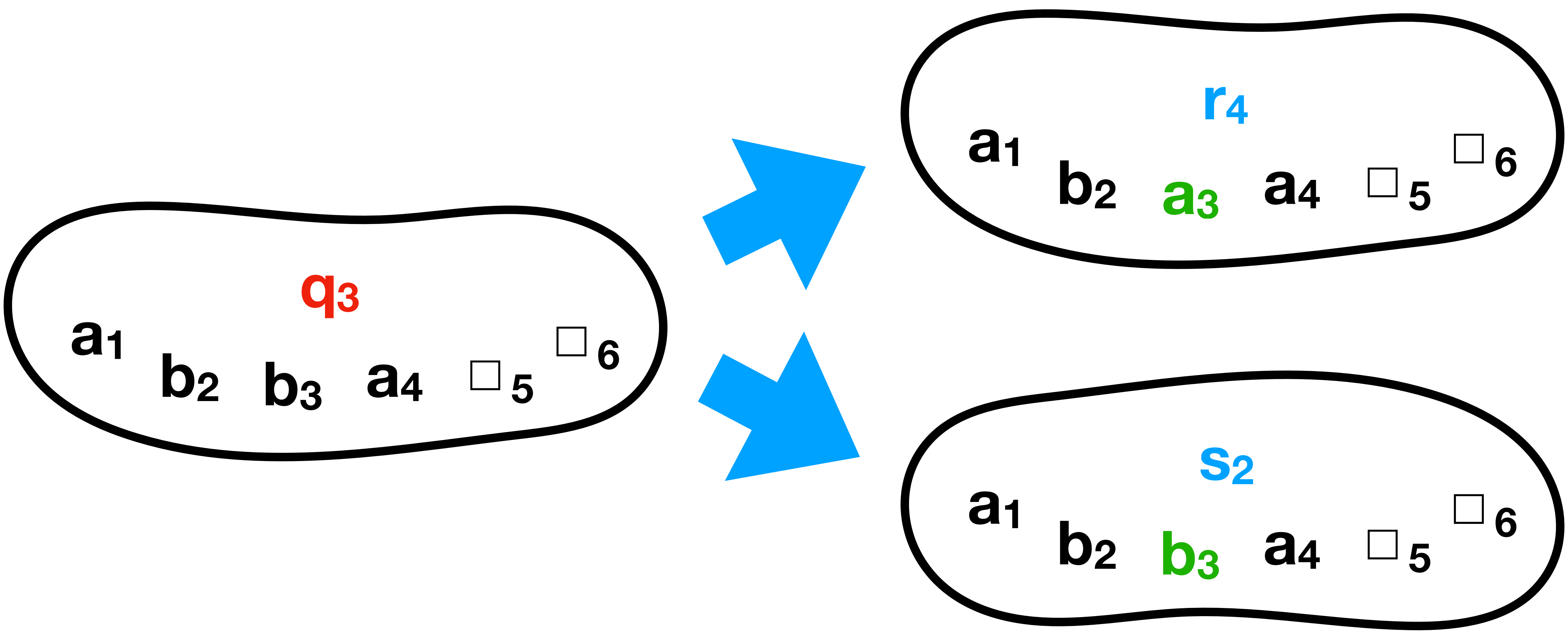

Simulating nondeterminism

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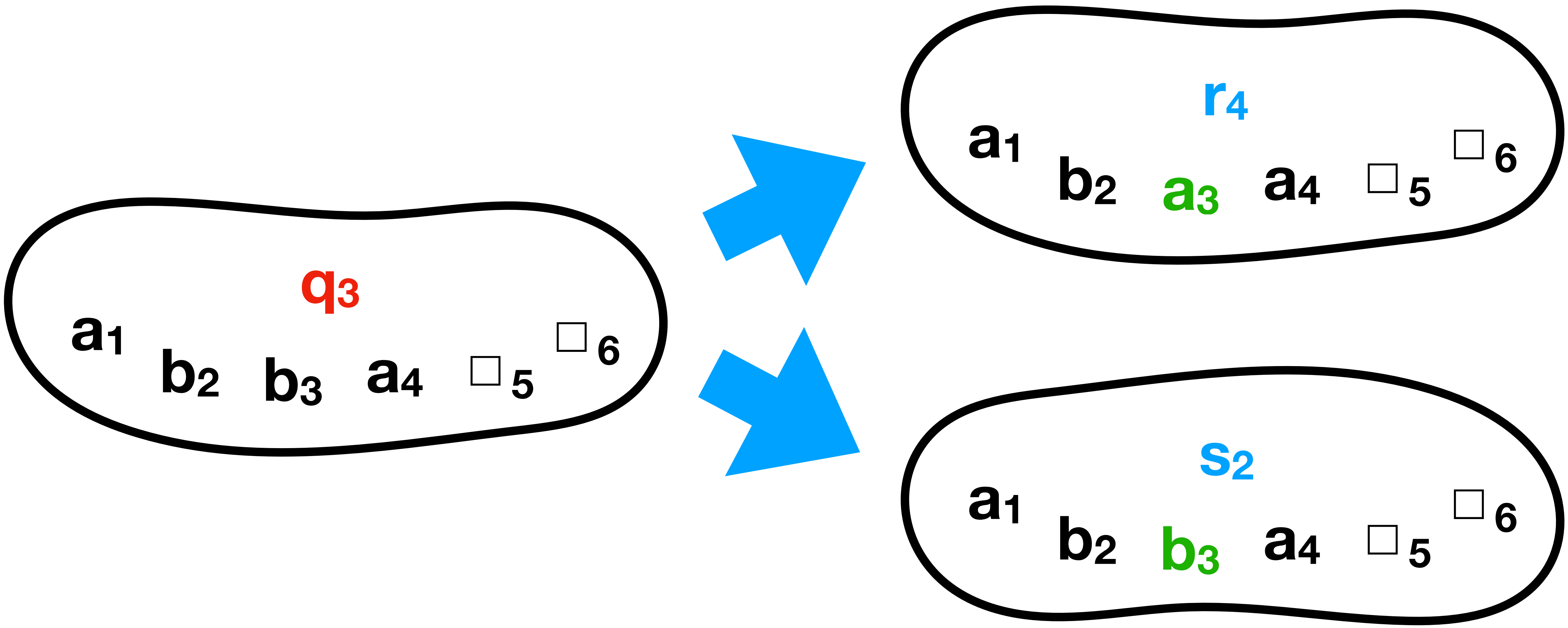
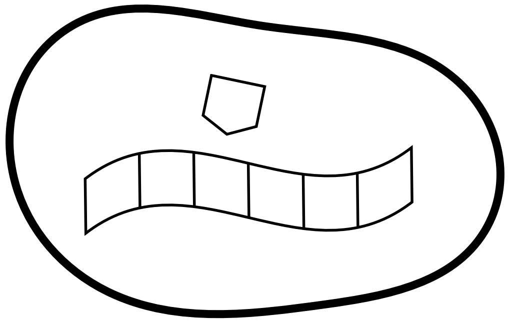
$[q_3 b_3] \rightarrow [r_4 a_3]\ [s_2 b_3]$



Simulating NDTM ⇒ **NP**



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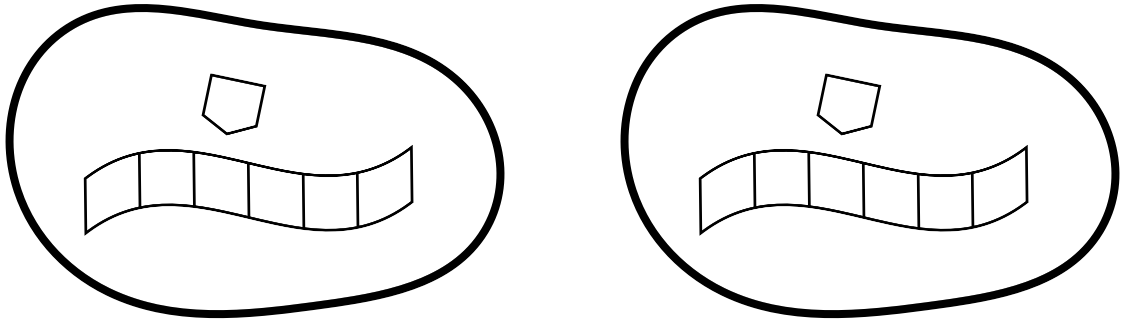

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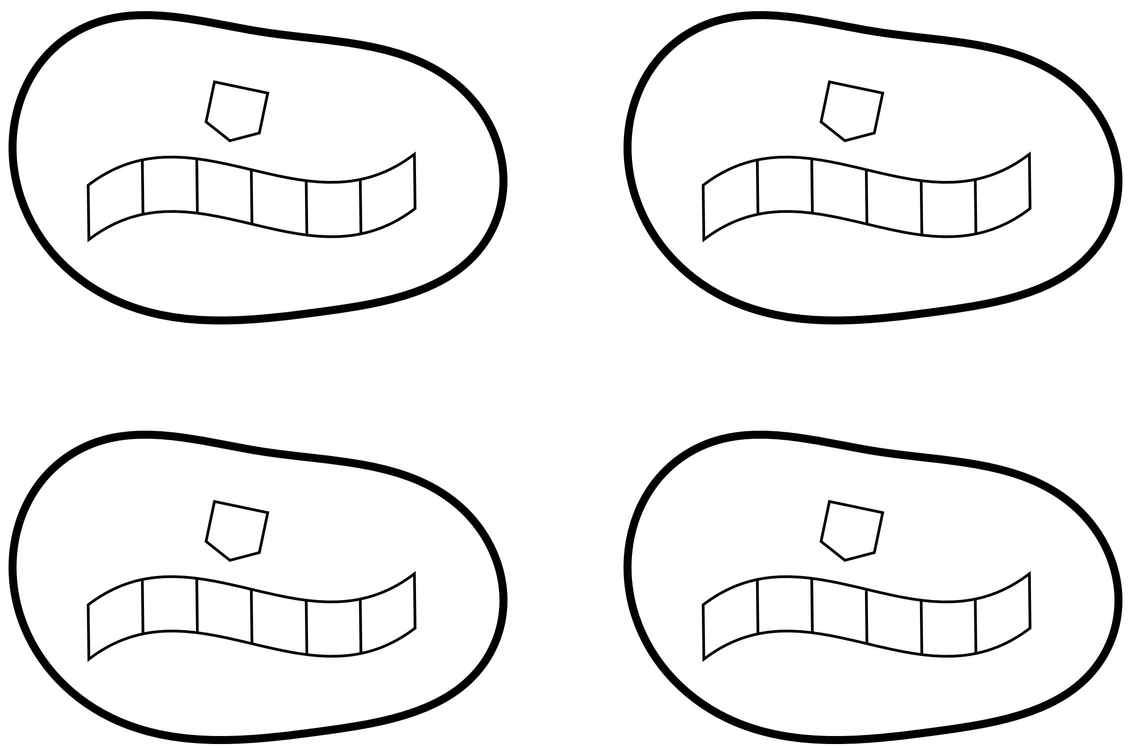

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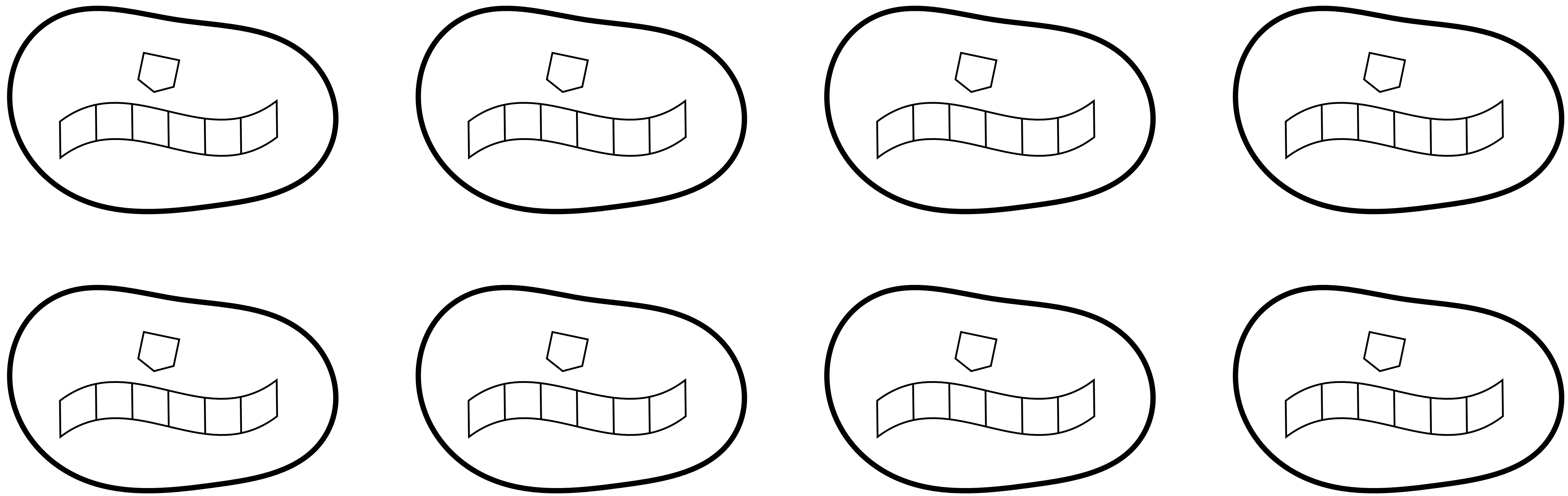

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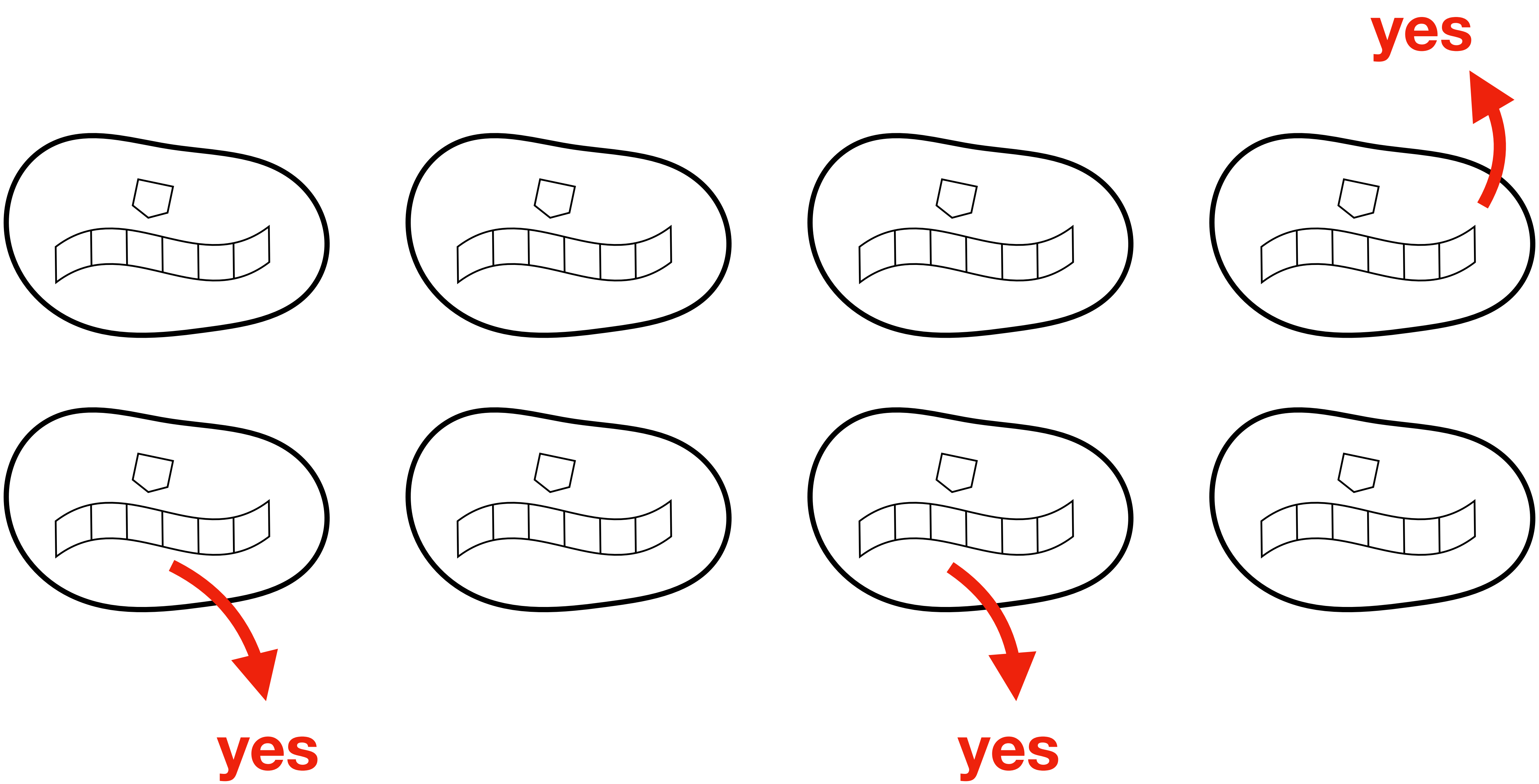

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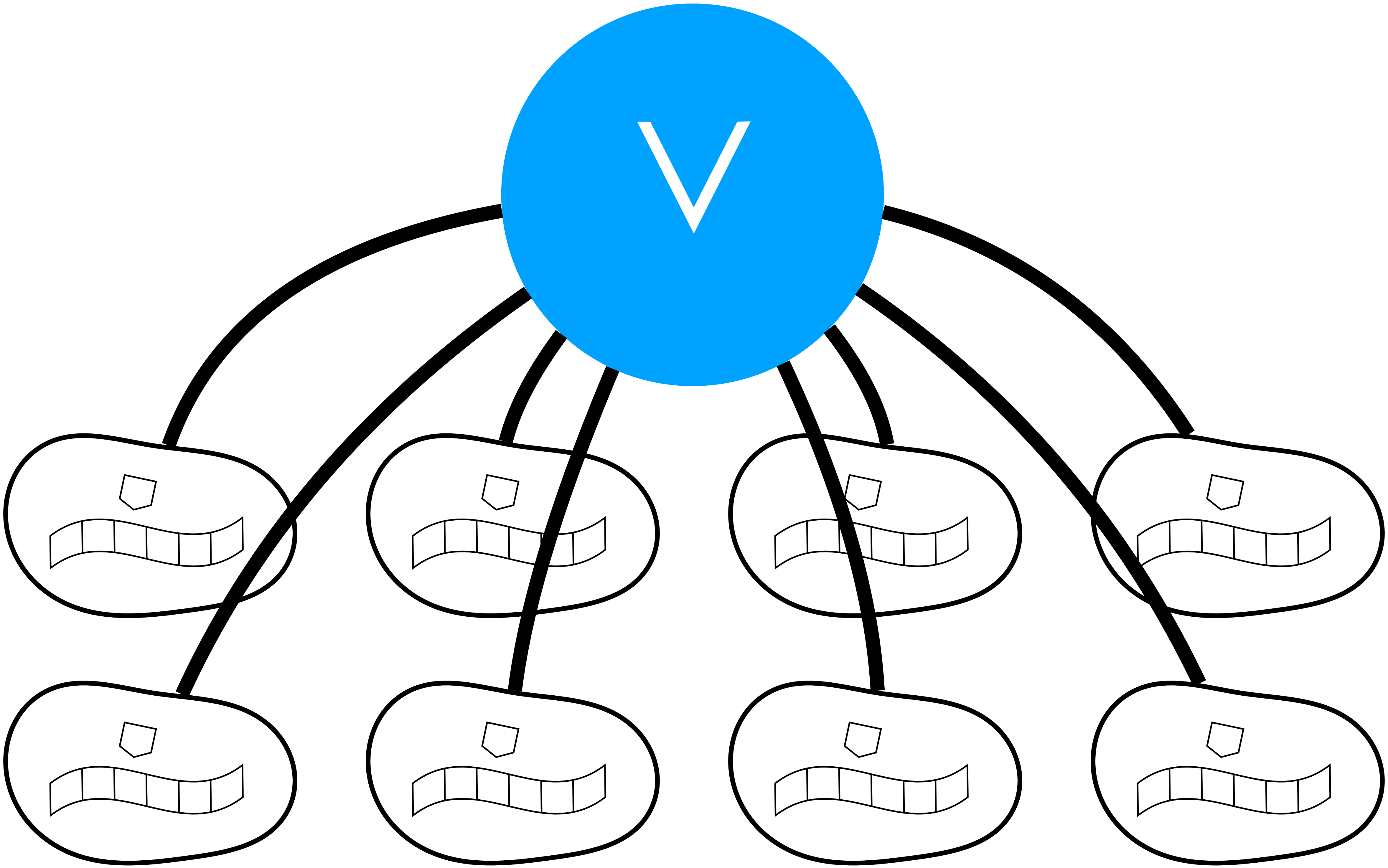
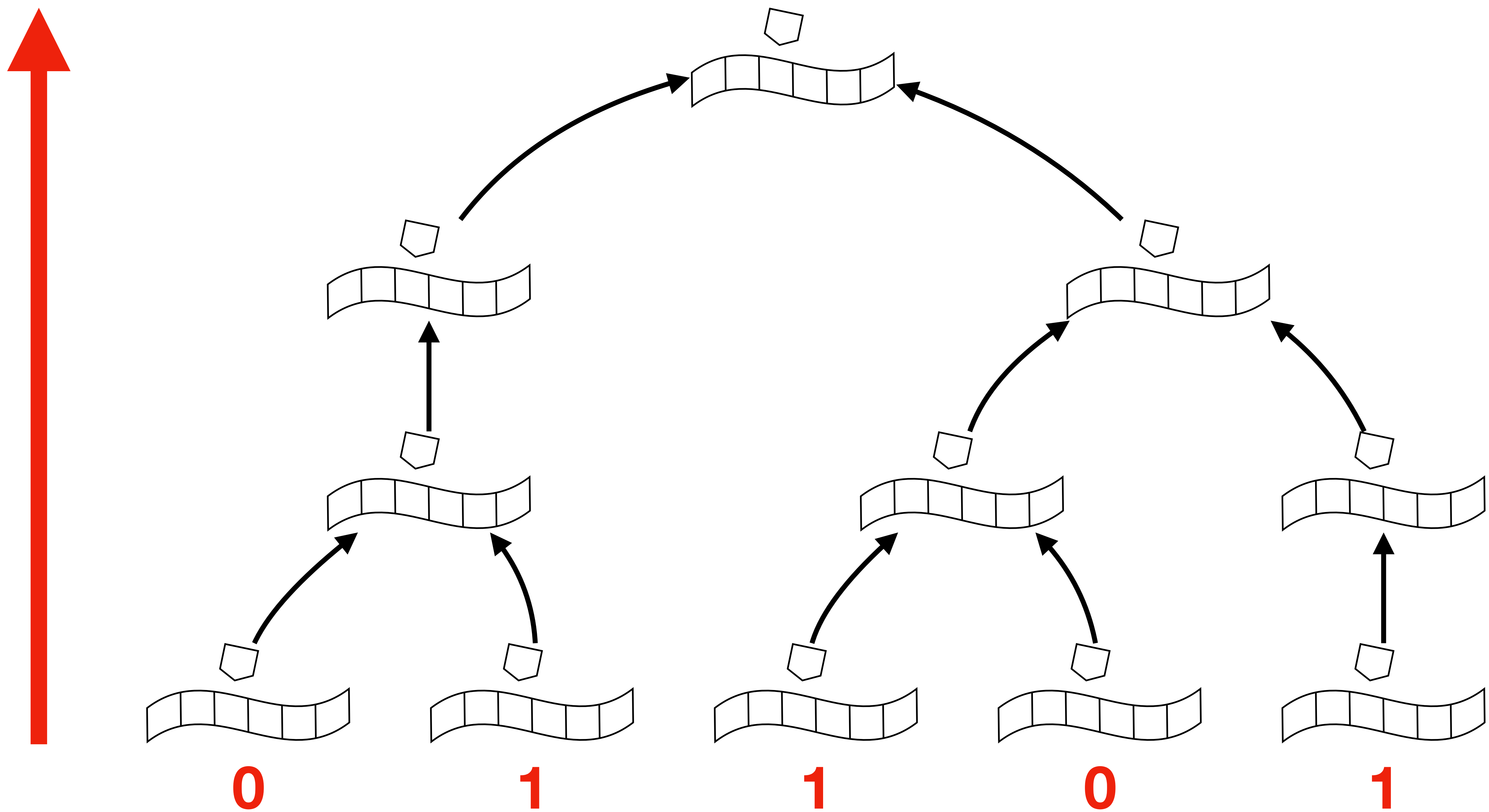


Counting complexity

Counting
Turing machines: #P



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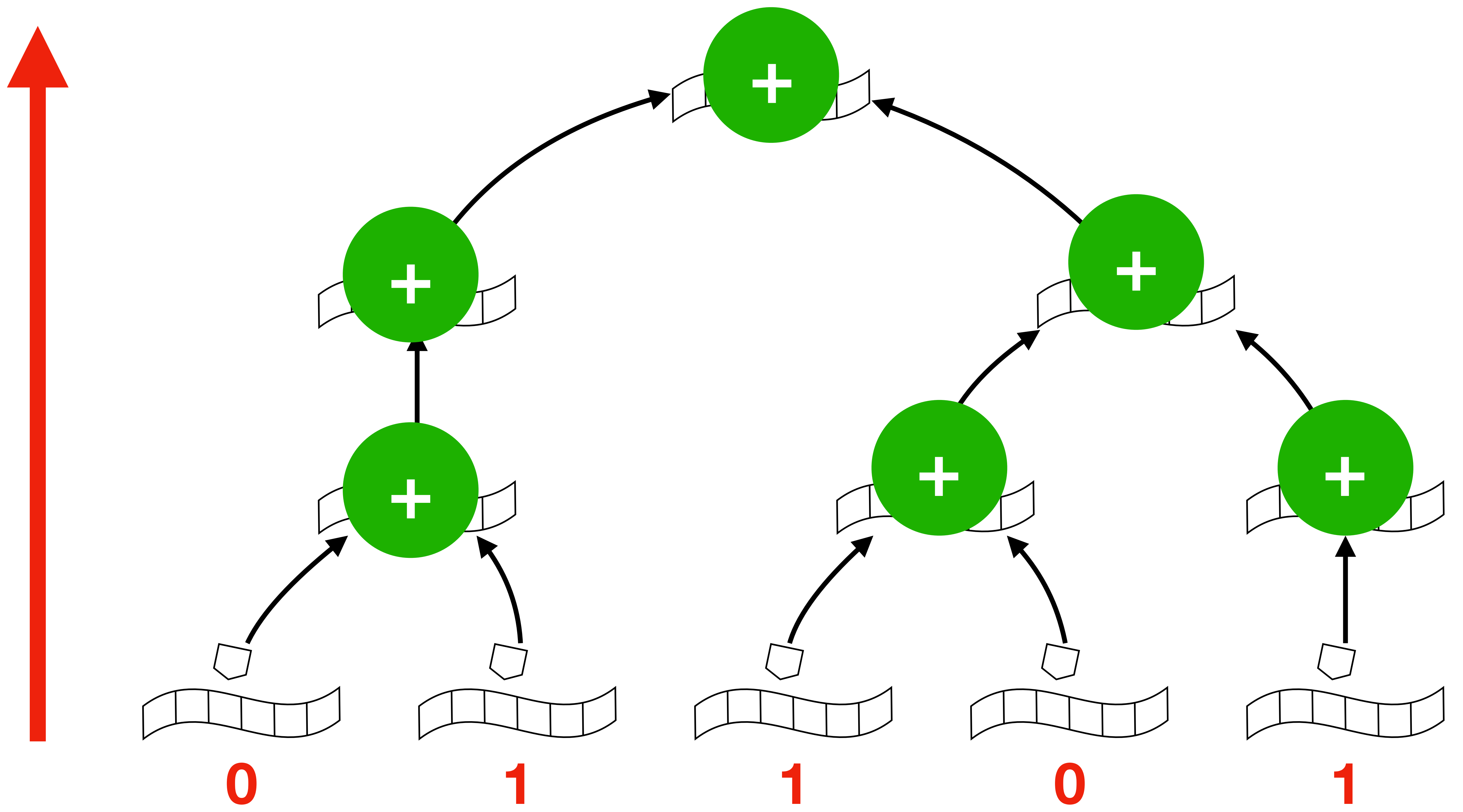

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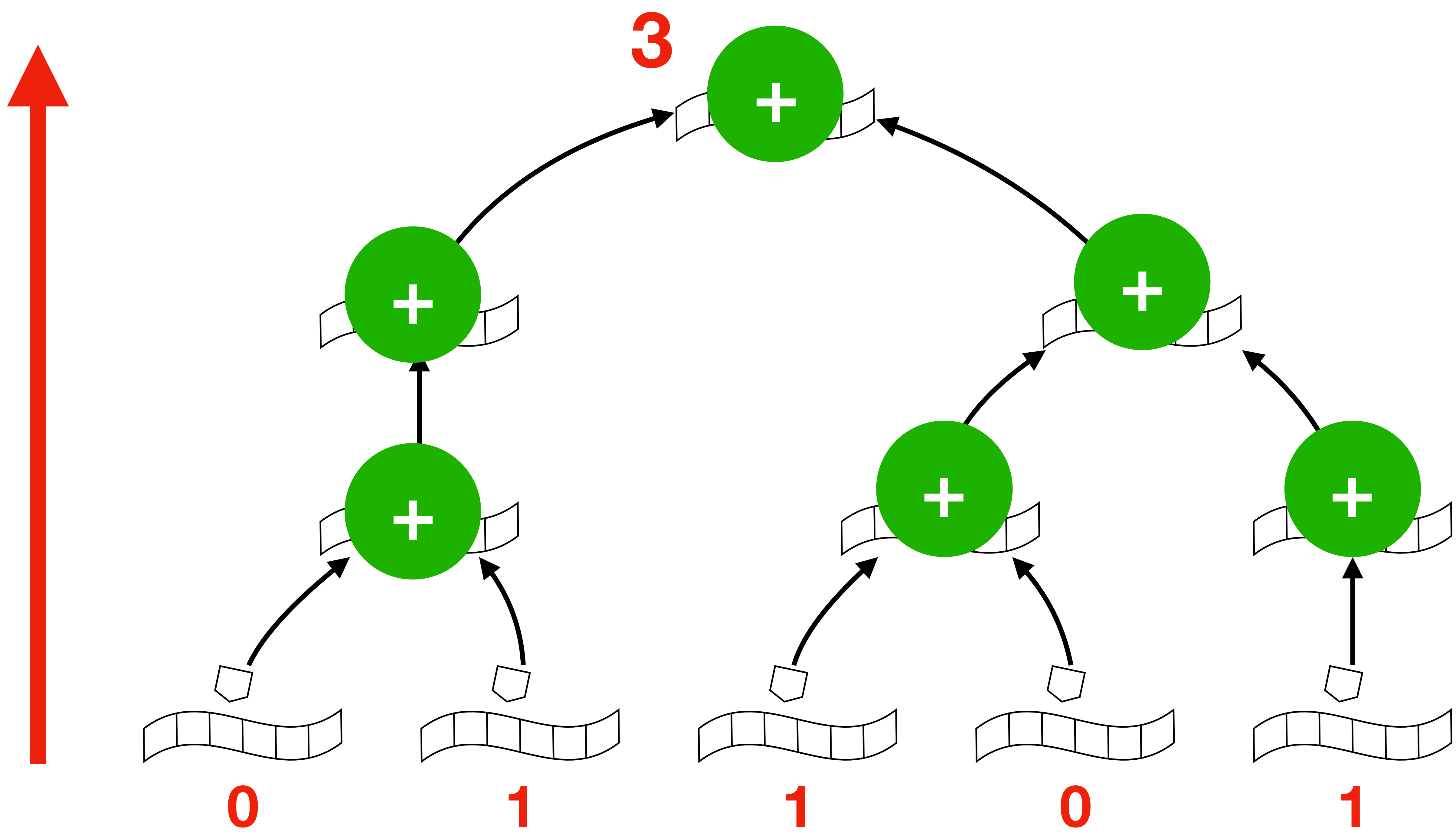
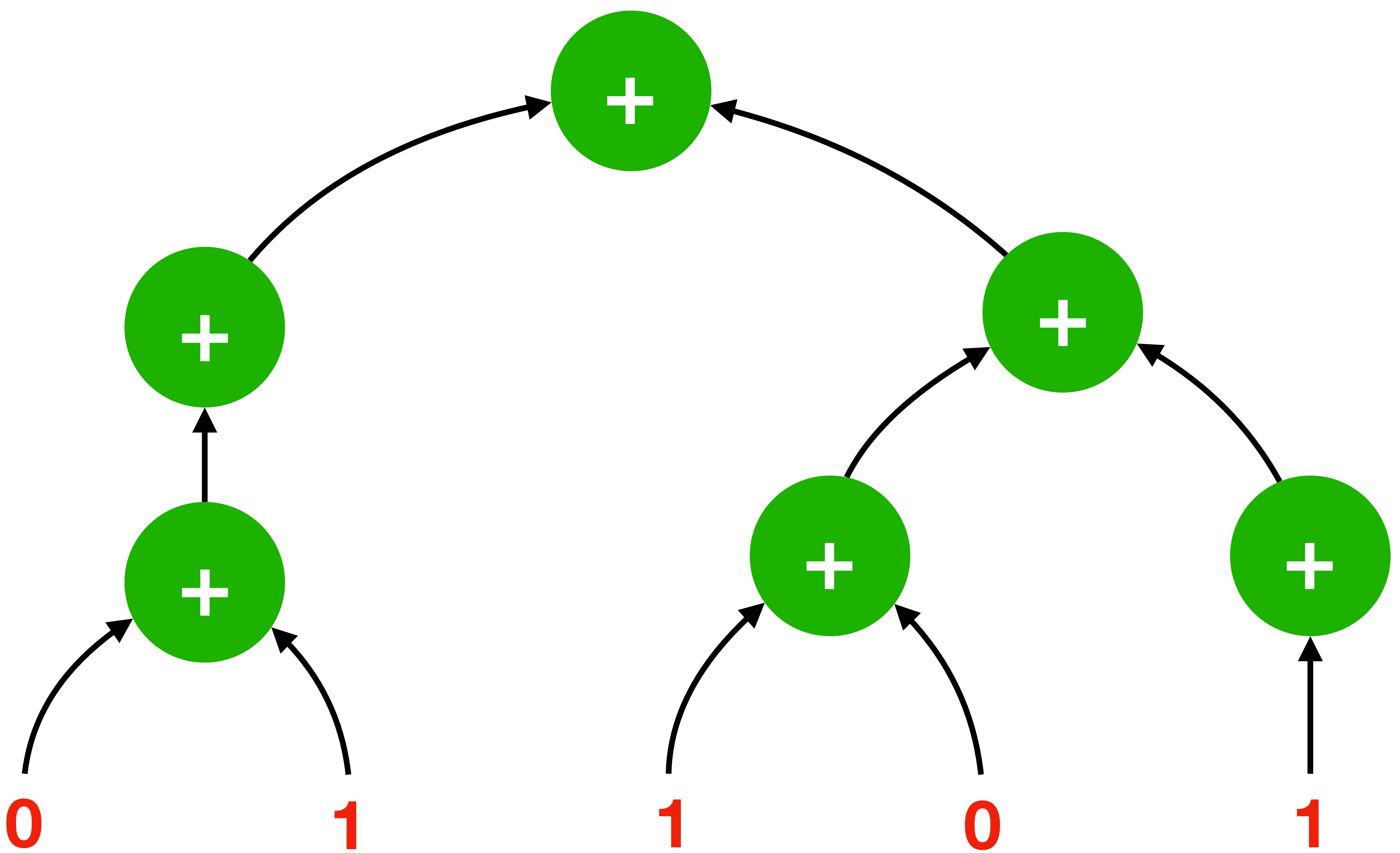


Flattening ⊕-circuits



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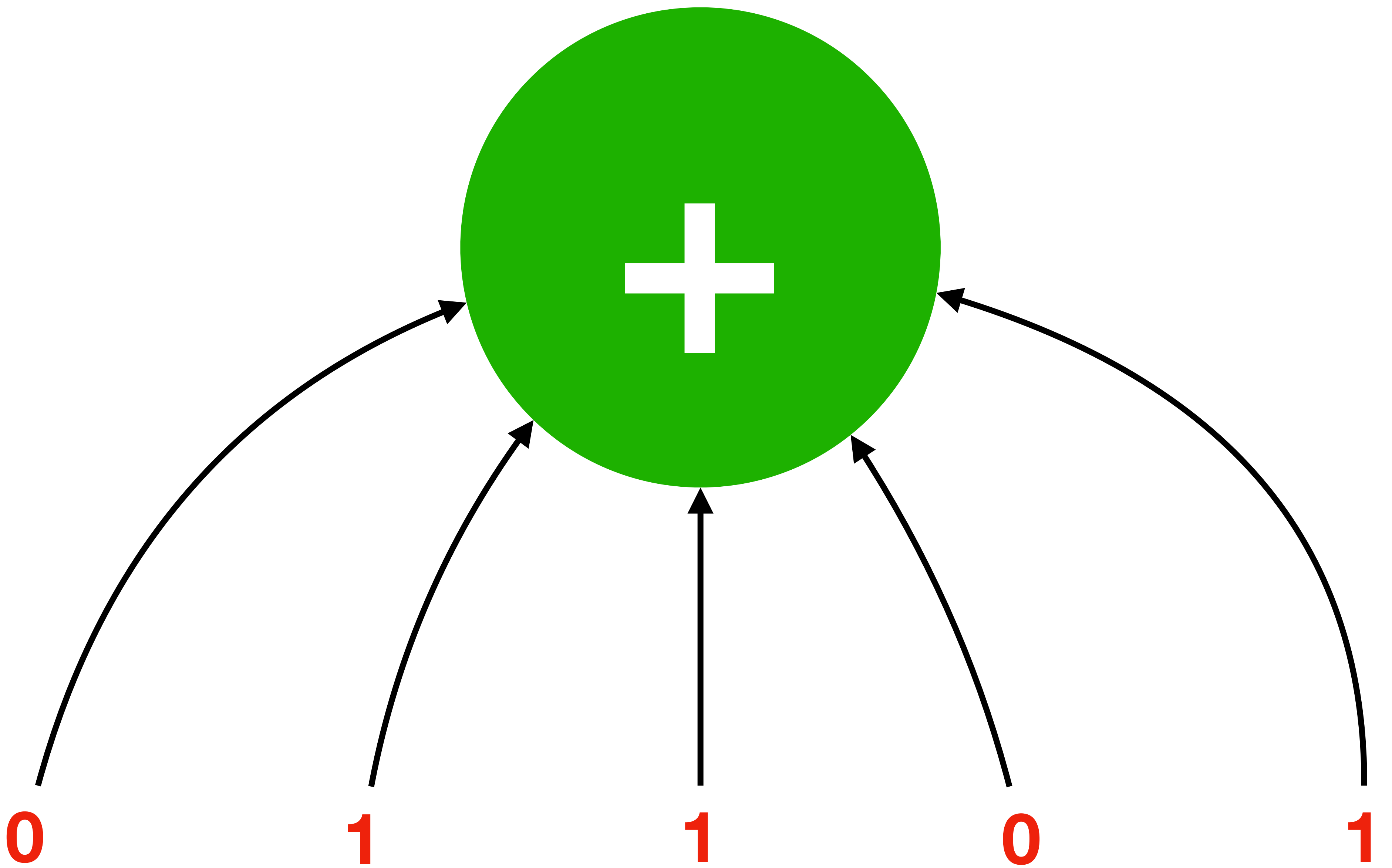
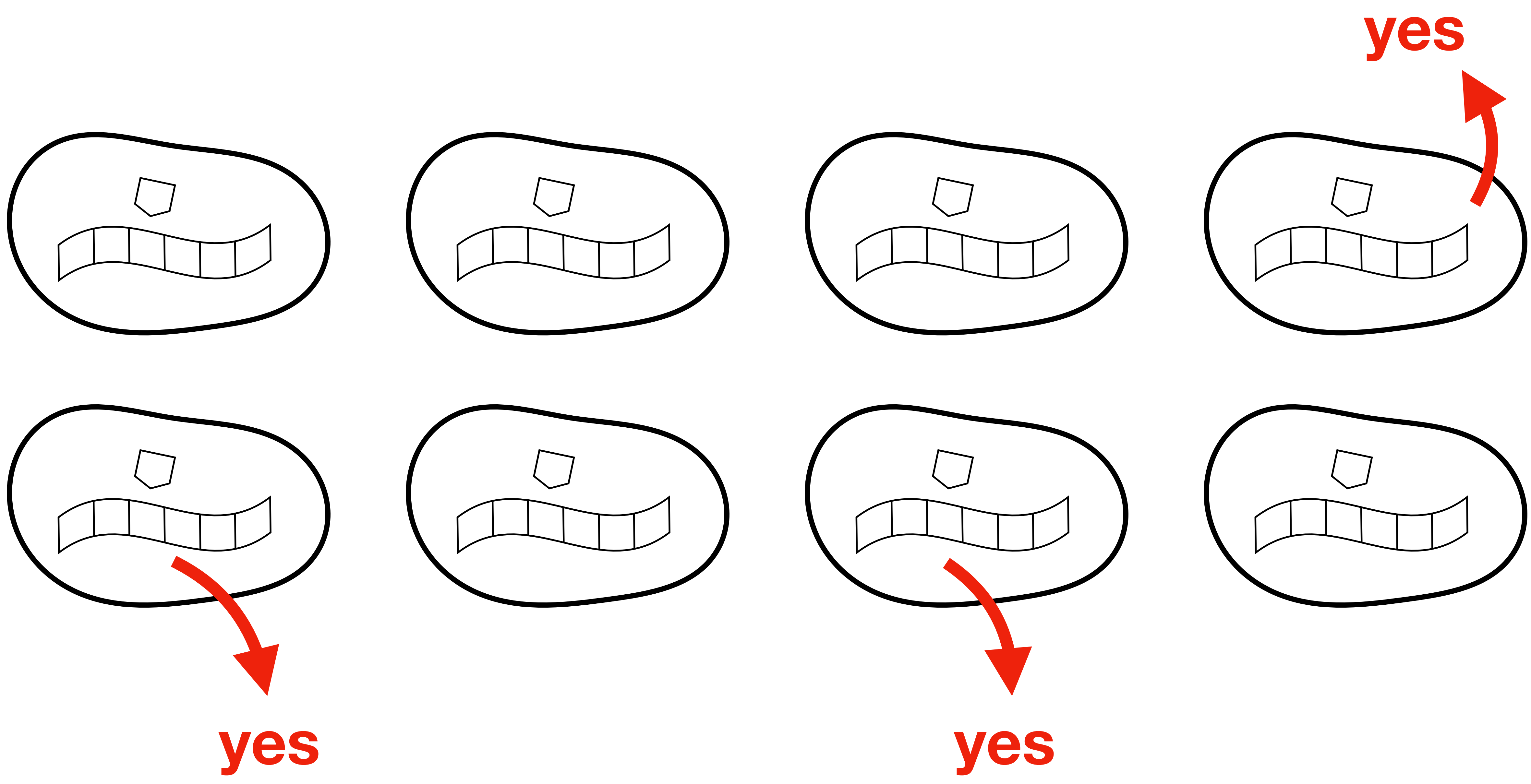


Counting with membrane systems

Simulating CTM ⇒ #P



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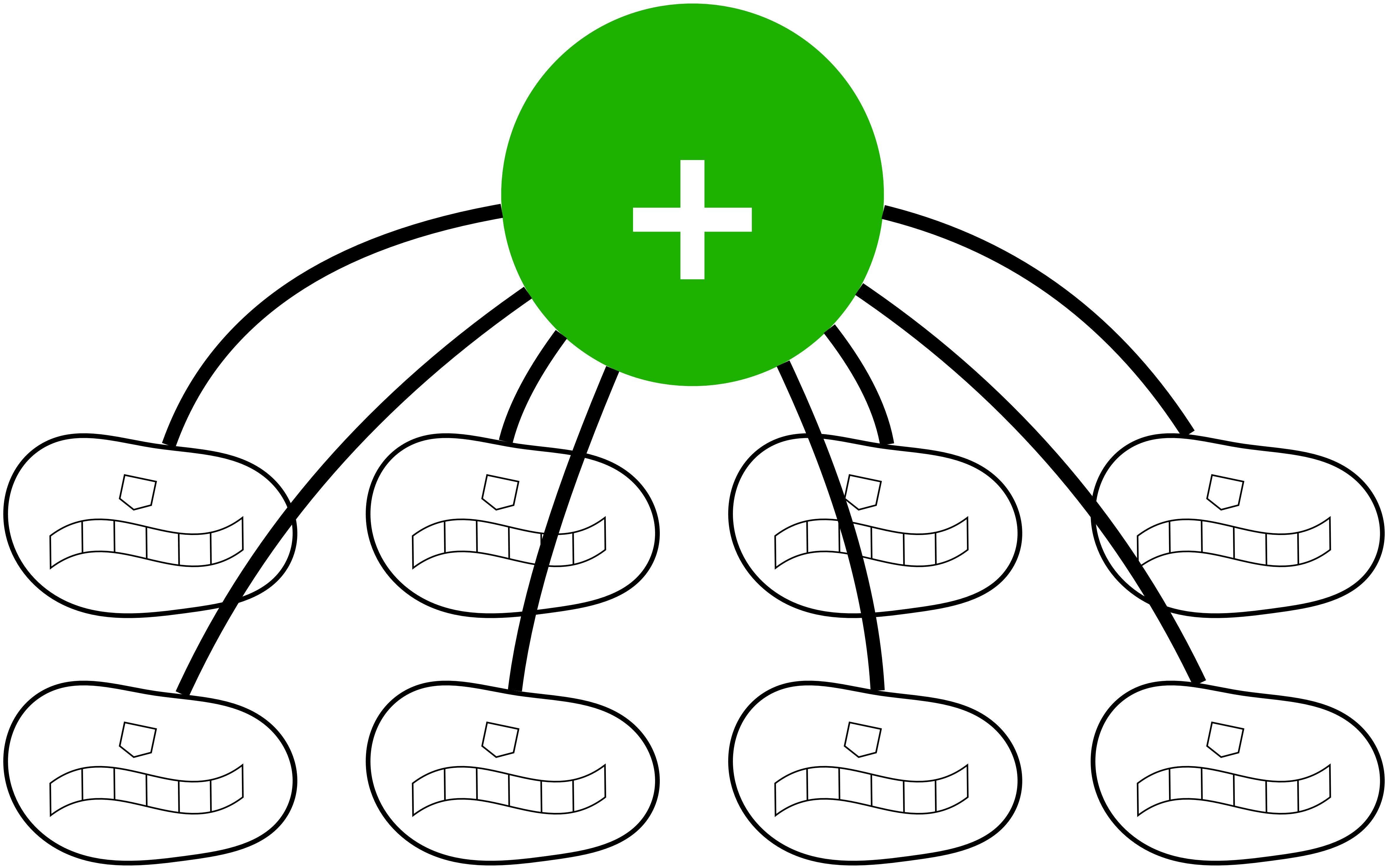
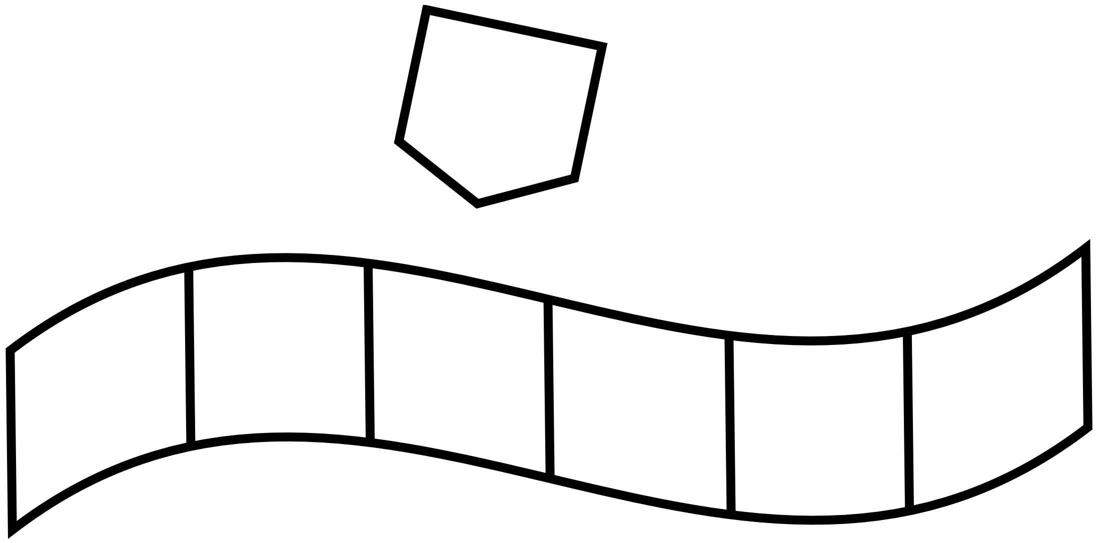
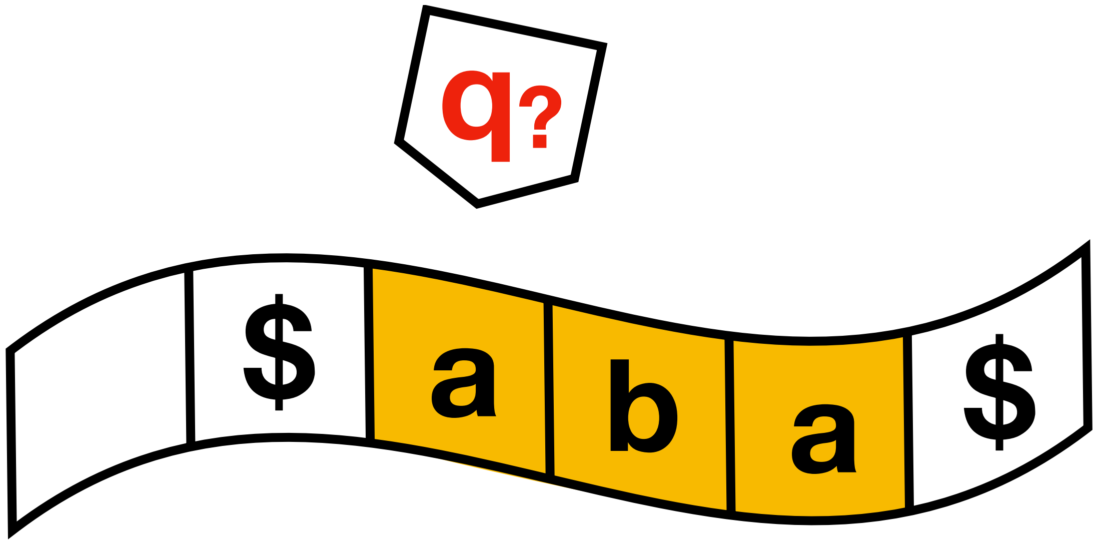


Oracles 🧞‍♀️

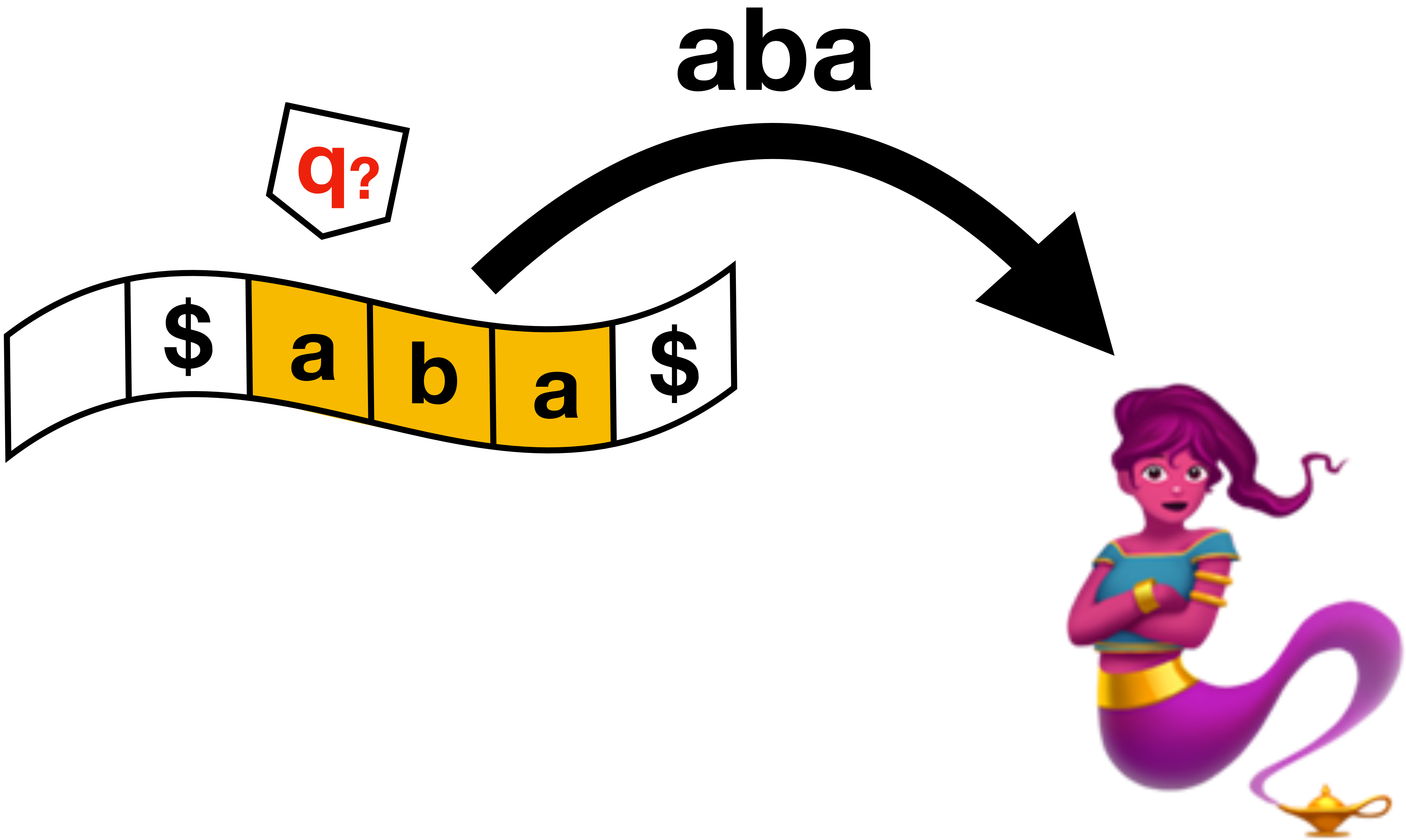
TM with oracle



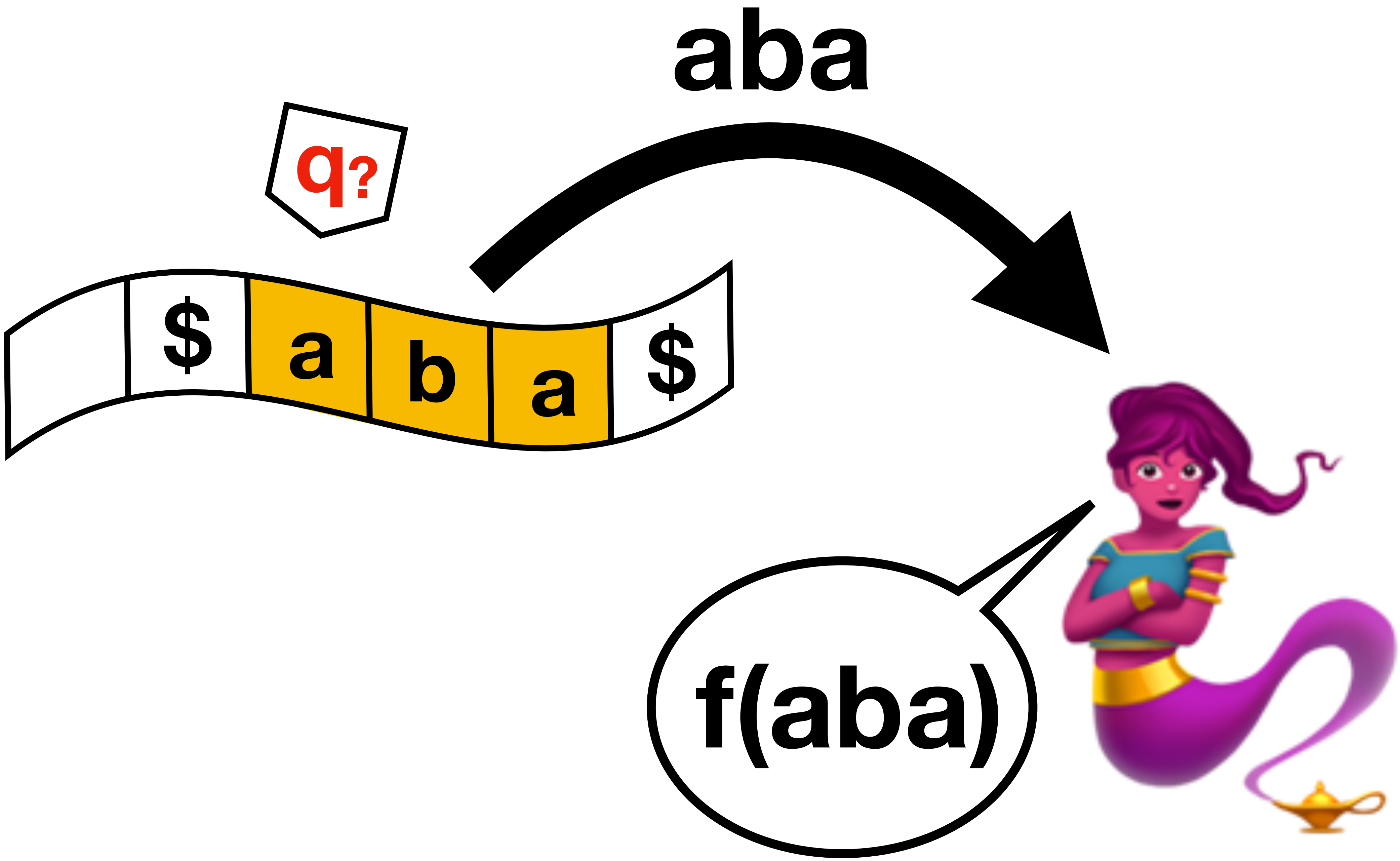
TM with oracle



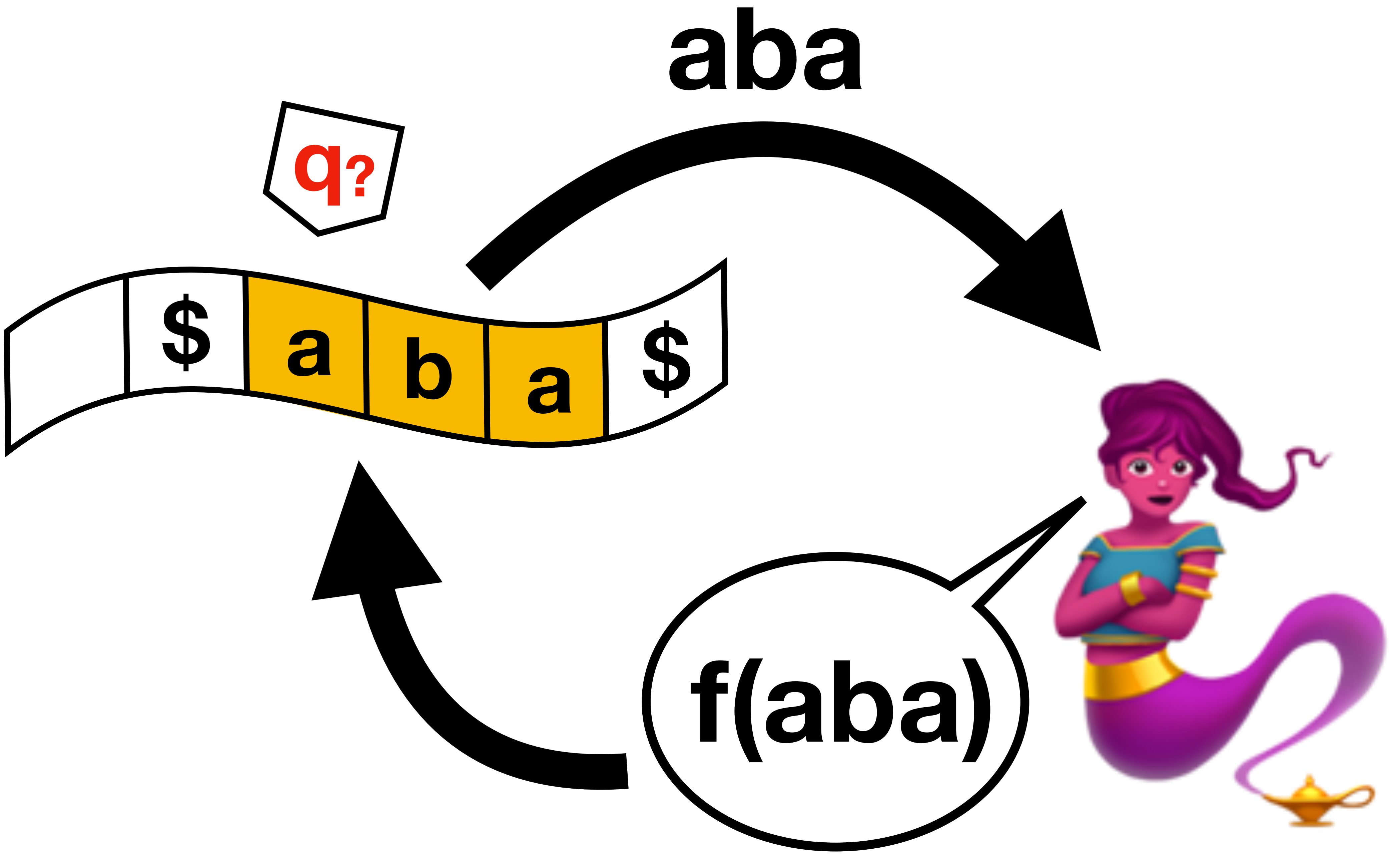
TM with oracle



TM with oracle

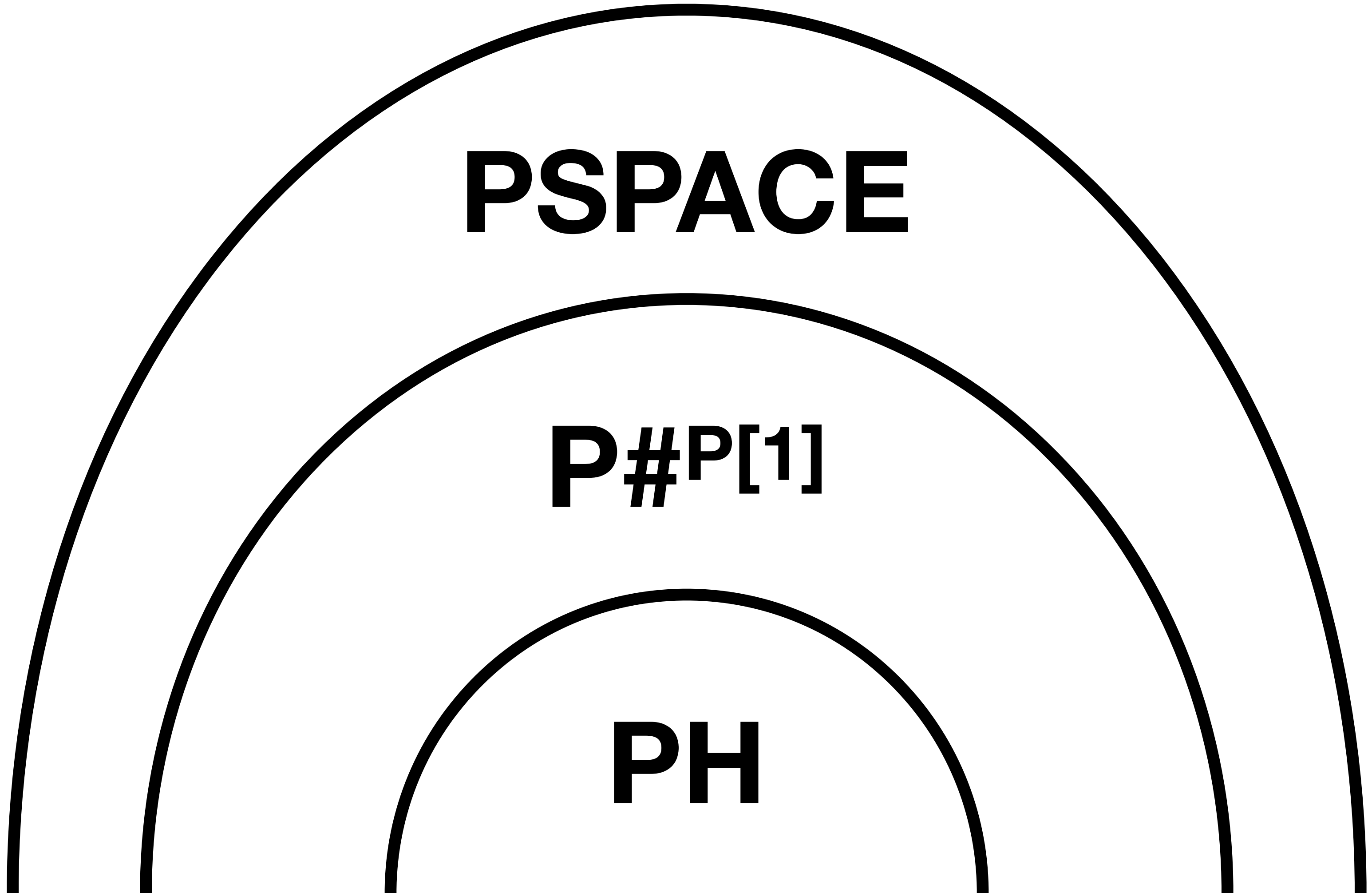


TM with oracle



The complexity class $\mathbf{P}^{\#P[1]}$

The class of problems solved by polynomial-time Turing machines with a single query to an oracle for a **#P**-complete problem



The complexity class $\mathbf{P}^{\#\mathbf{P}[1]}$

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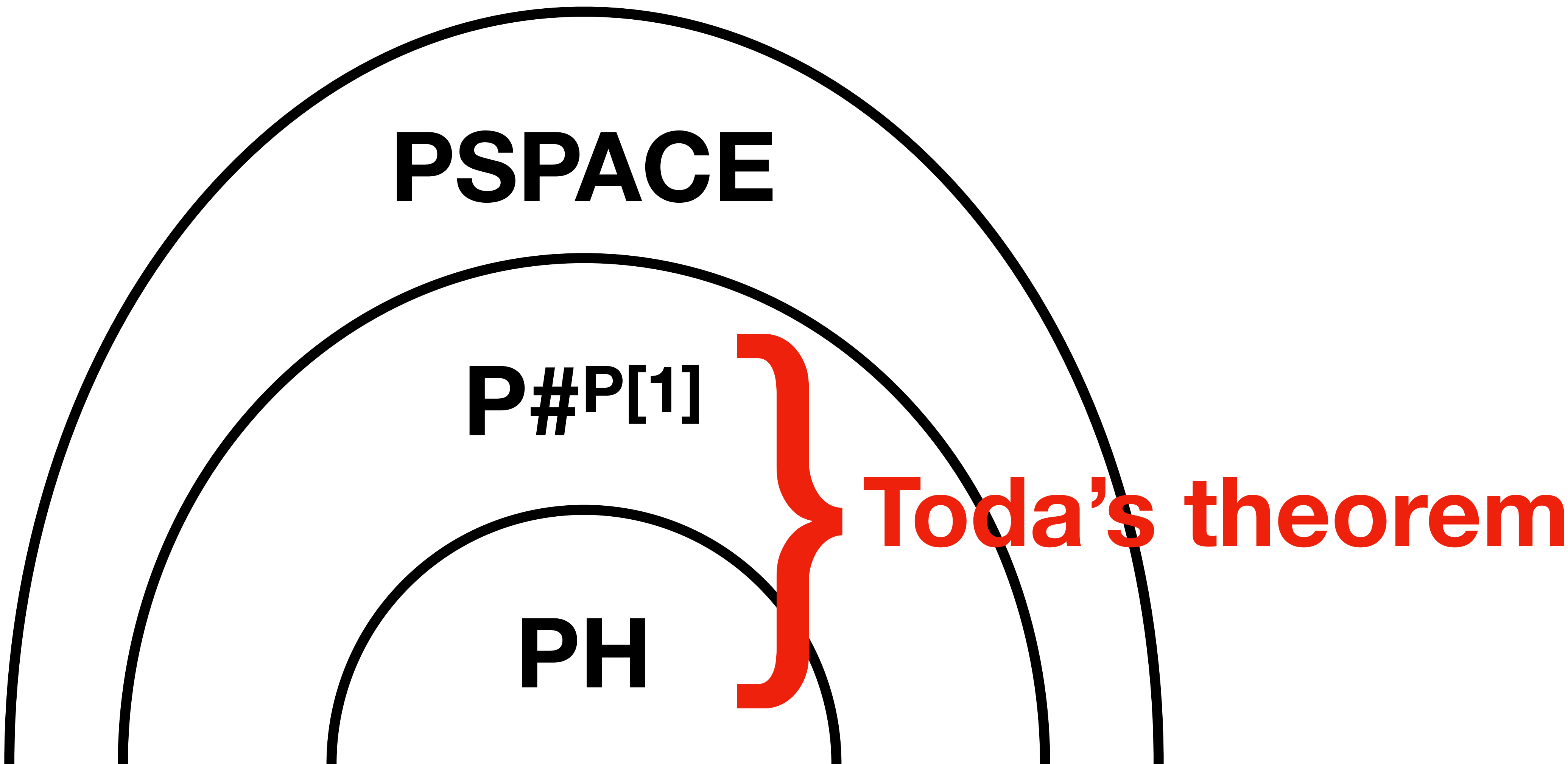
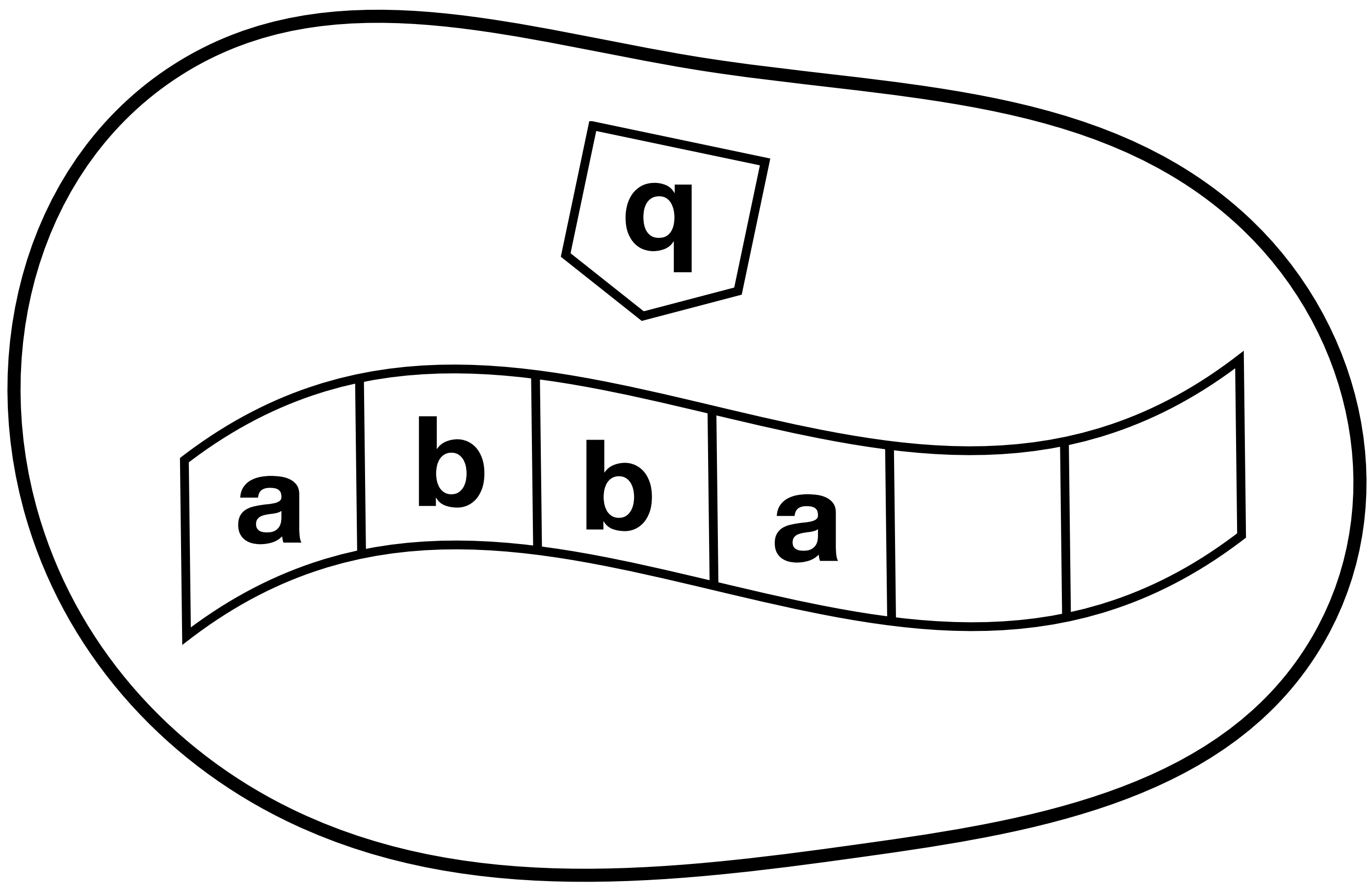
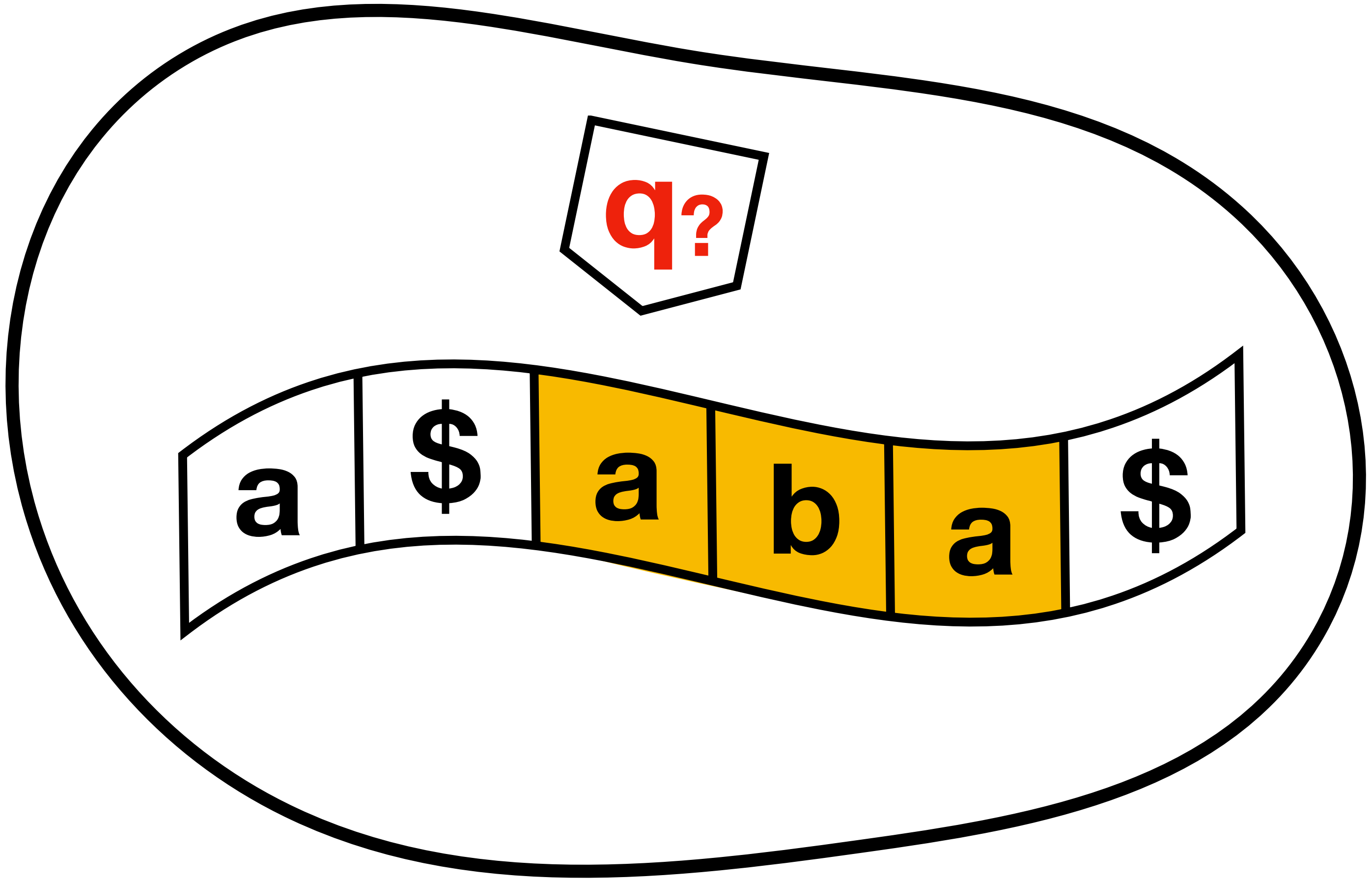


Solving $\mathbf{P}^{\#\mathrm{P}[1]}$ with (monodirectional, shallow) membrane systems

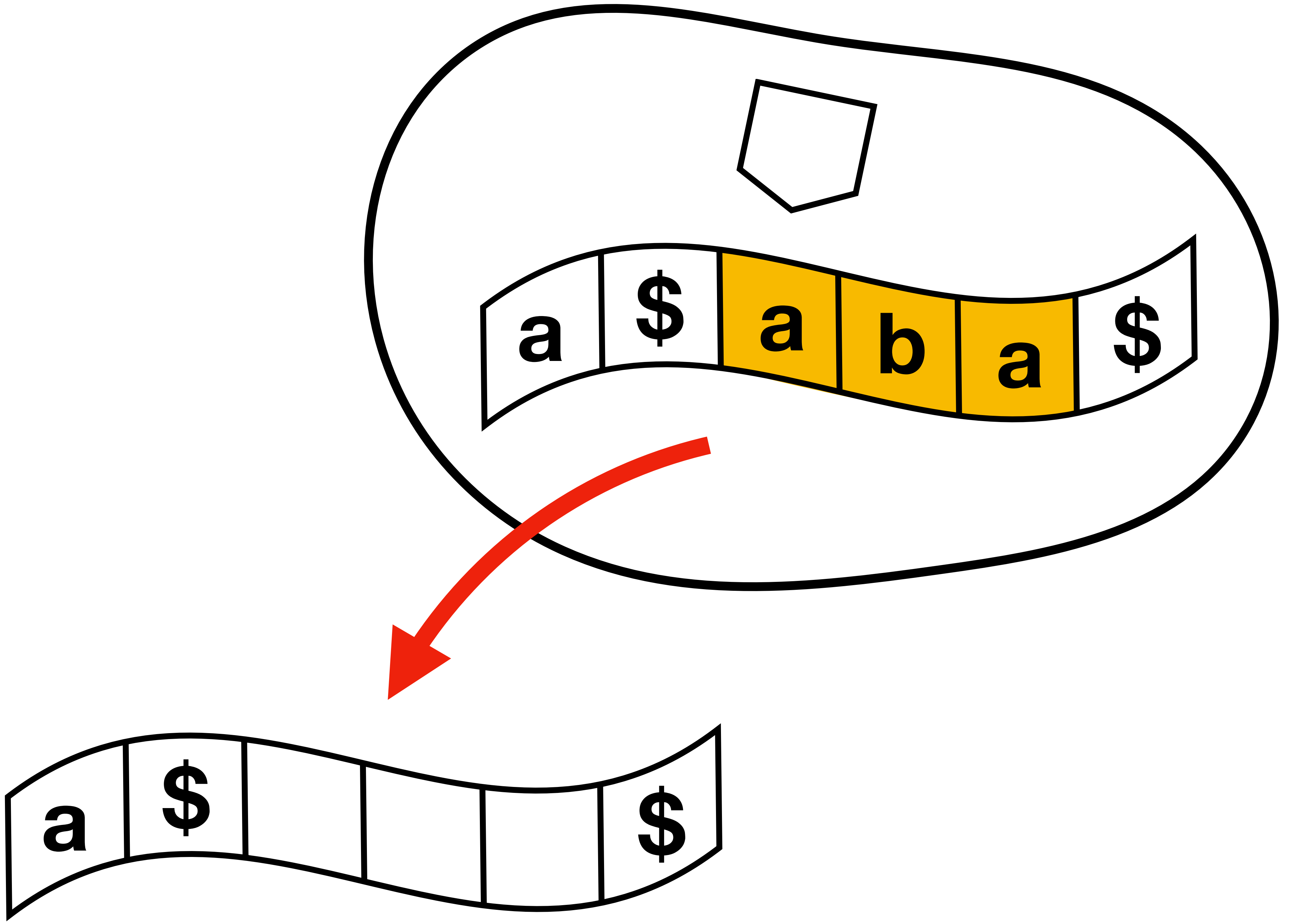
Pre-query computation



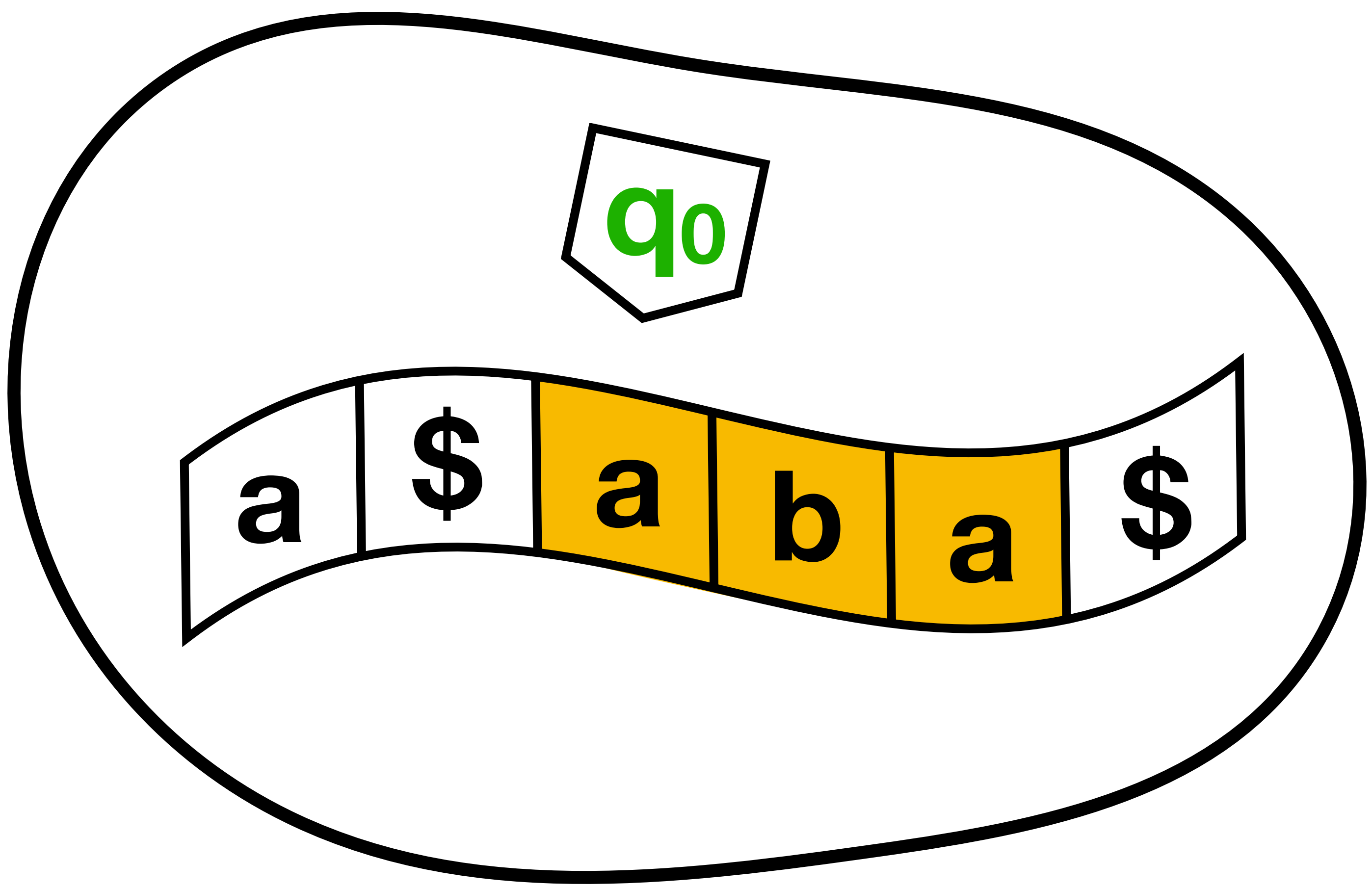
Entering the query state



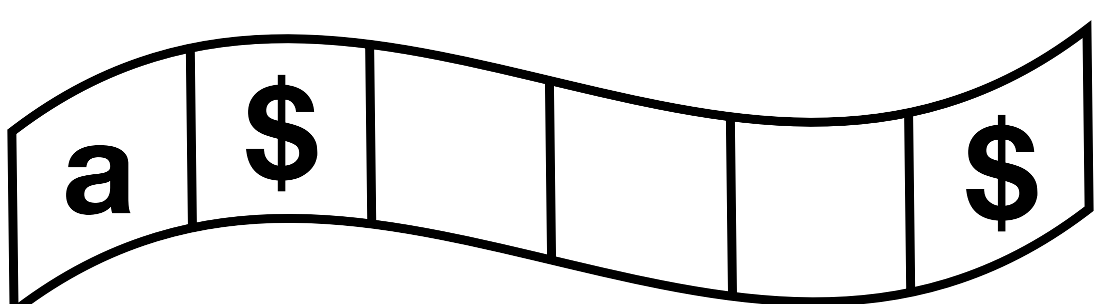
Entering the query state



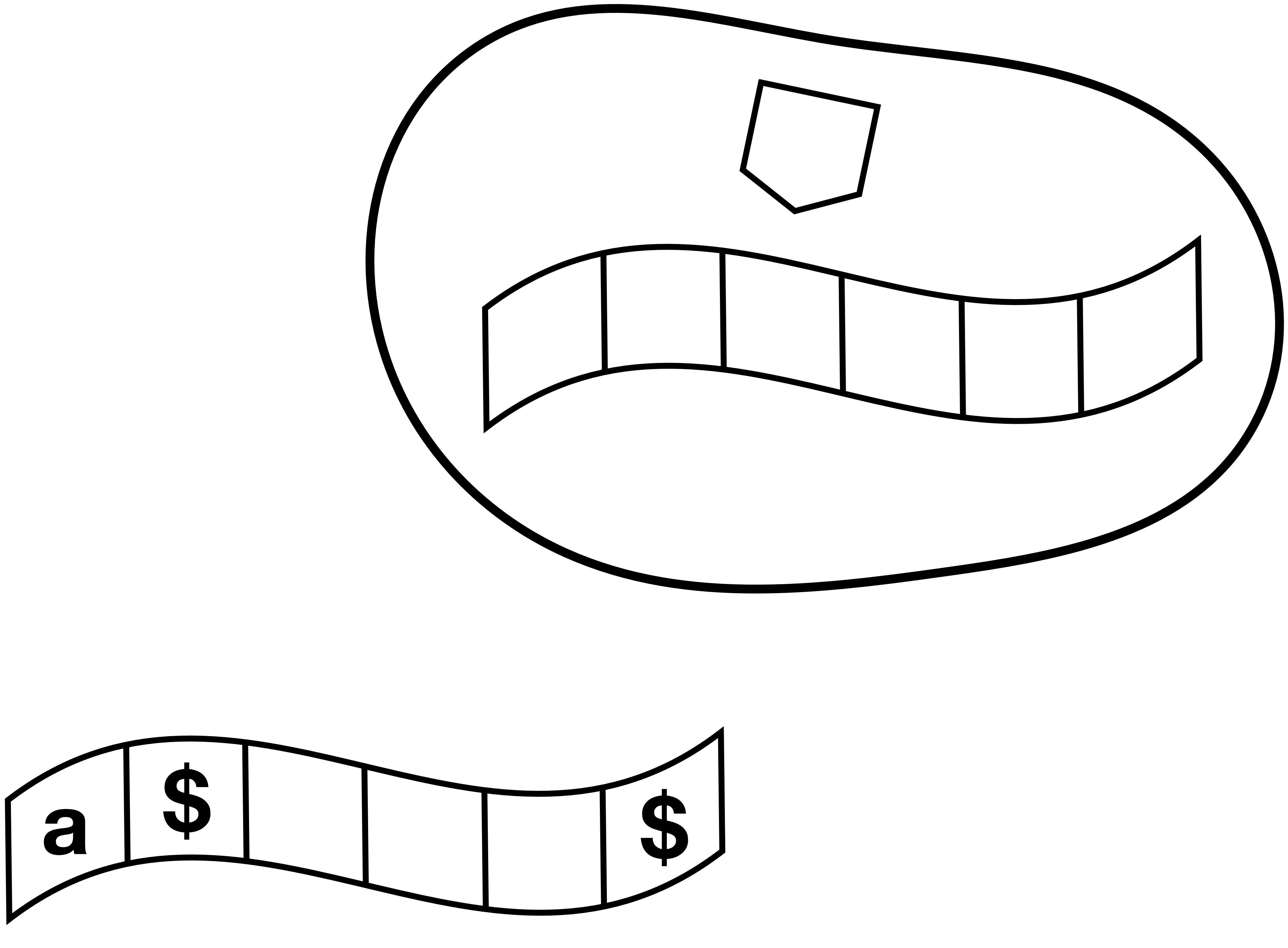
Entering the query state



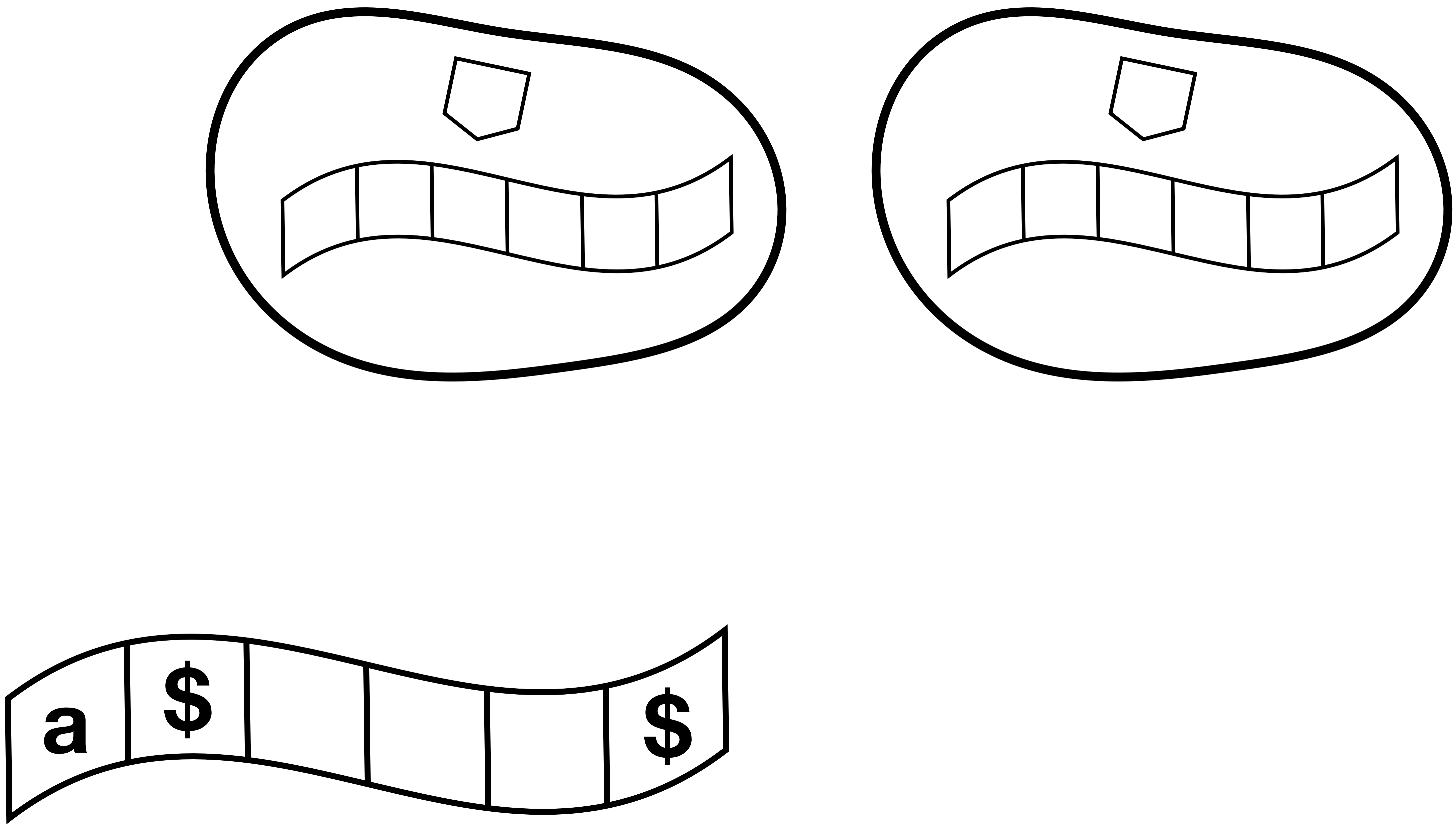
Initial state of a TM solving the #P-complete oracle problem



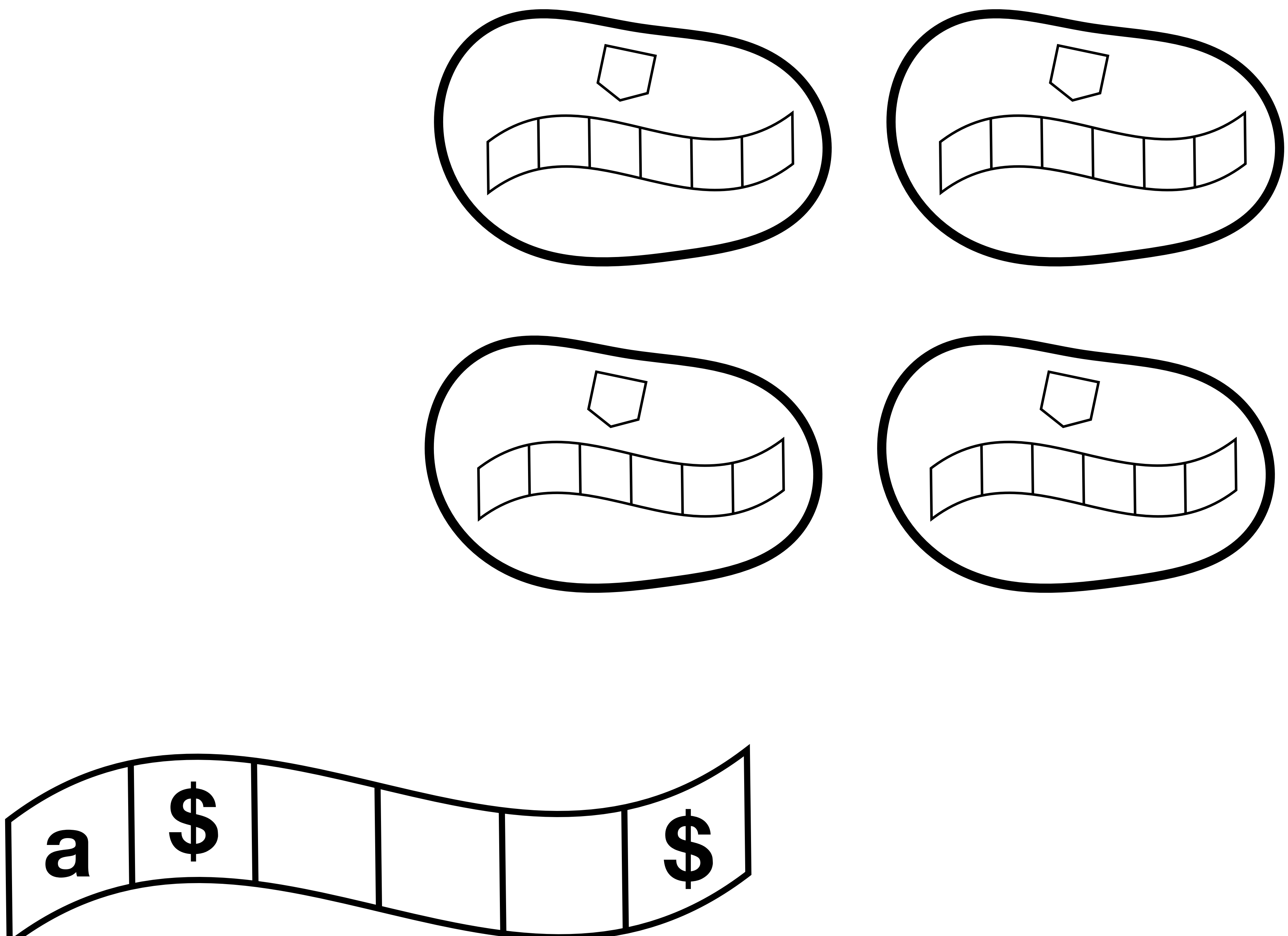
Simulating the auxiliary TM



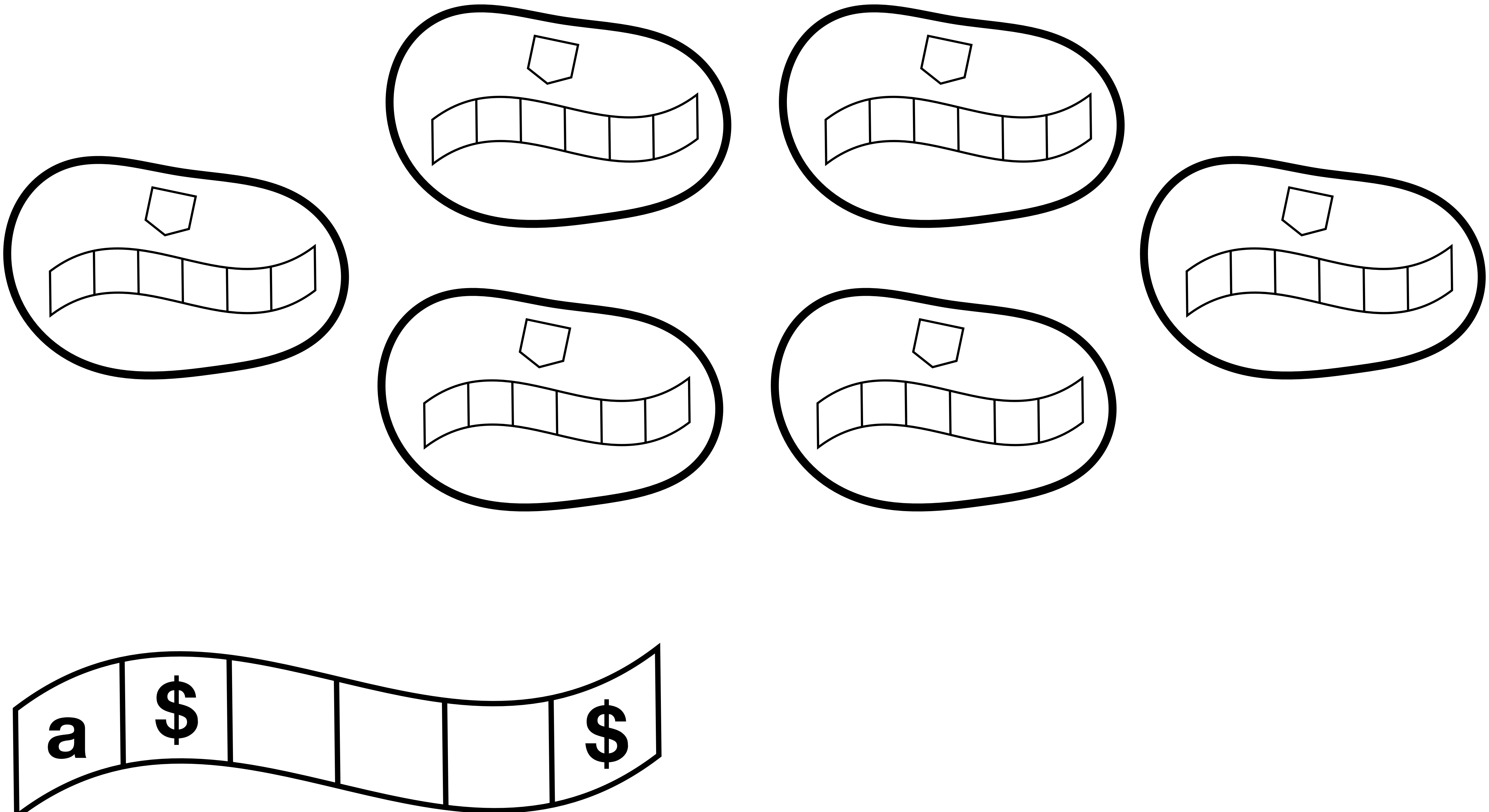
Simulating the auxiliary TM



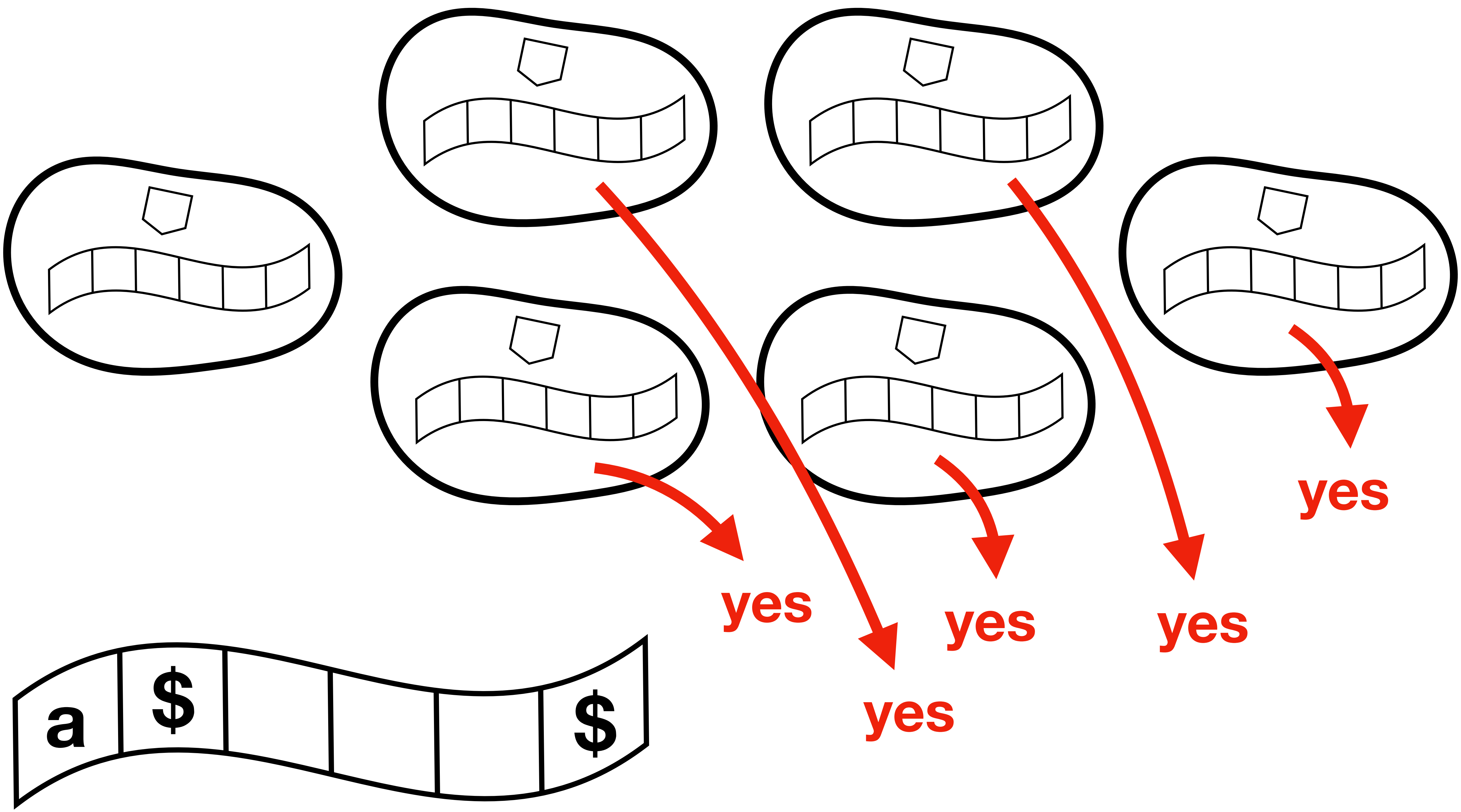
Simulating the auxiliary TM



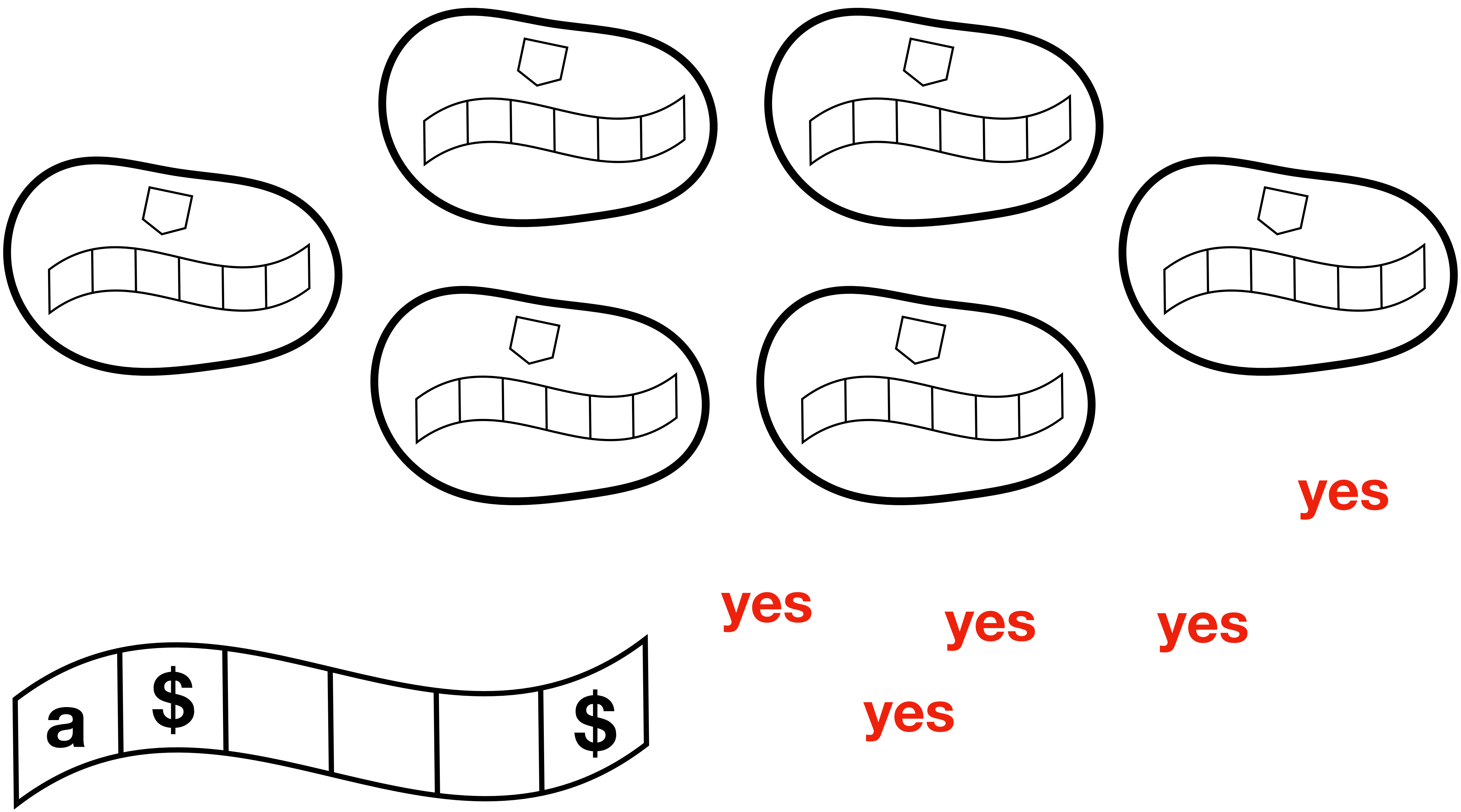
Simulating the auxiliary TM



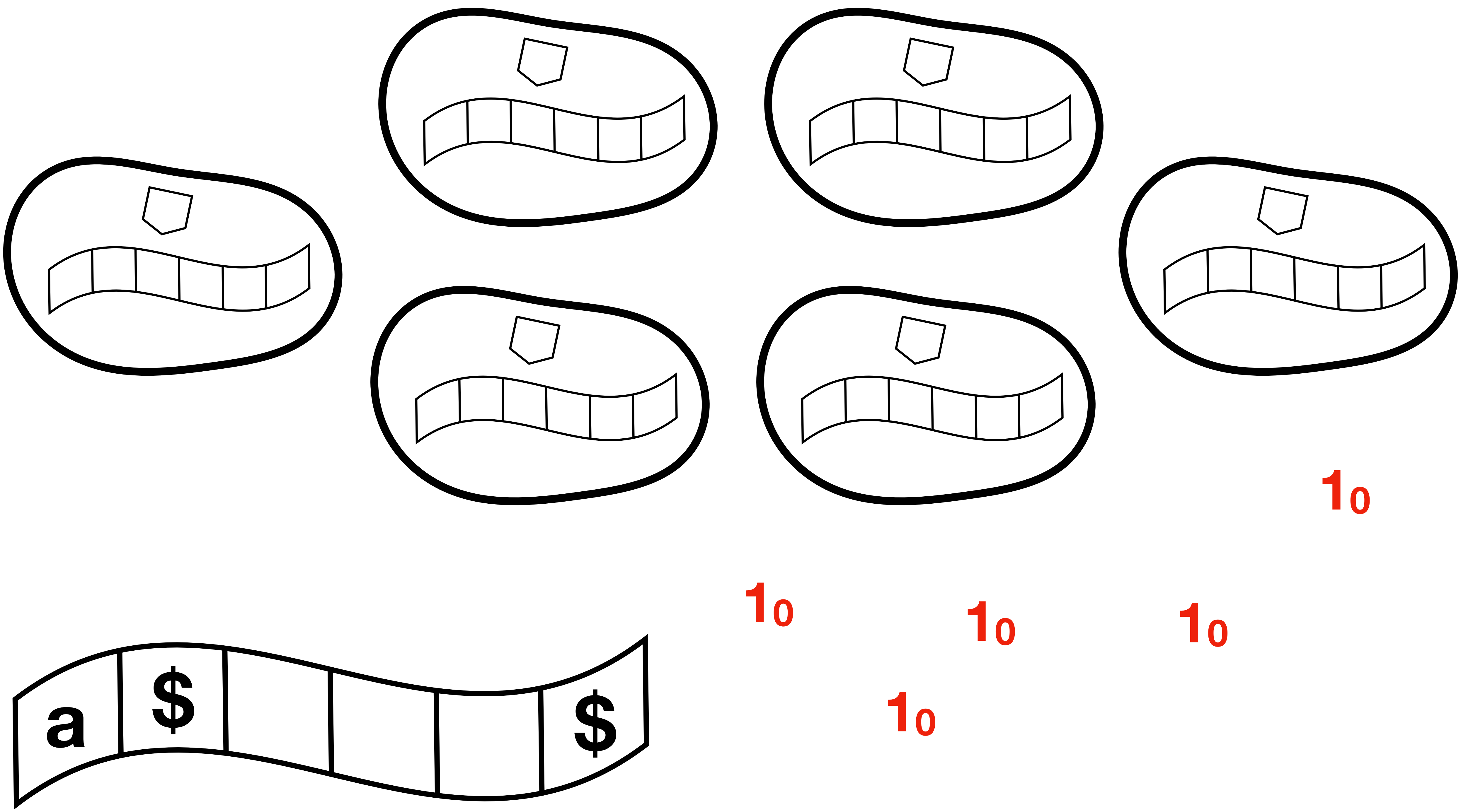
Collecting the output



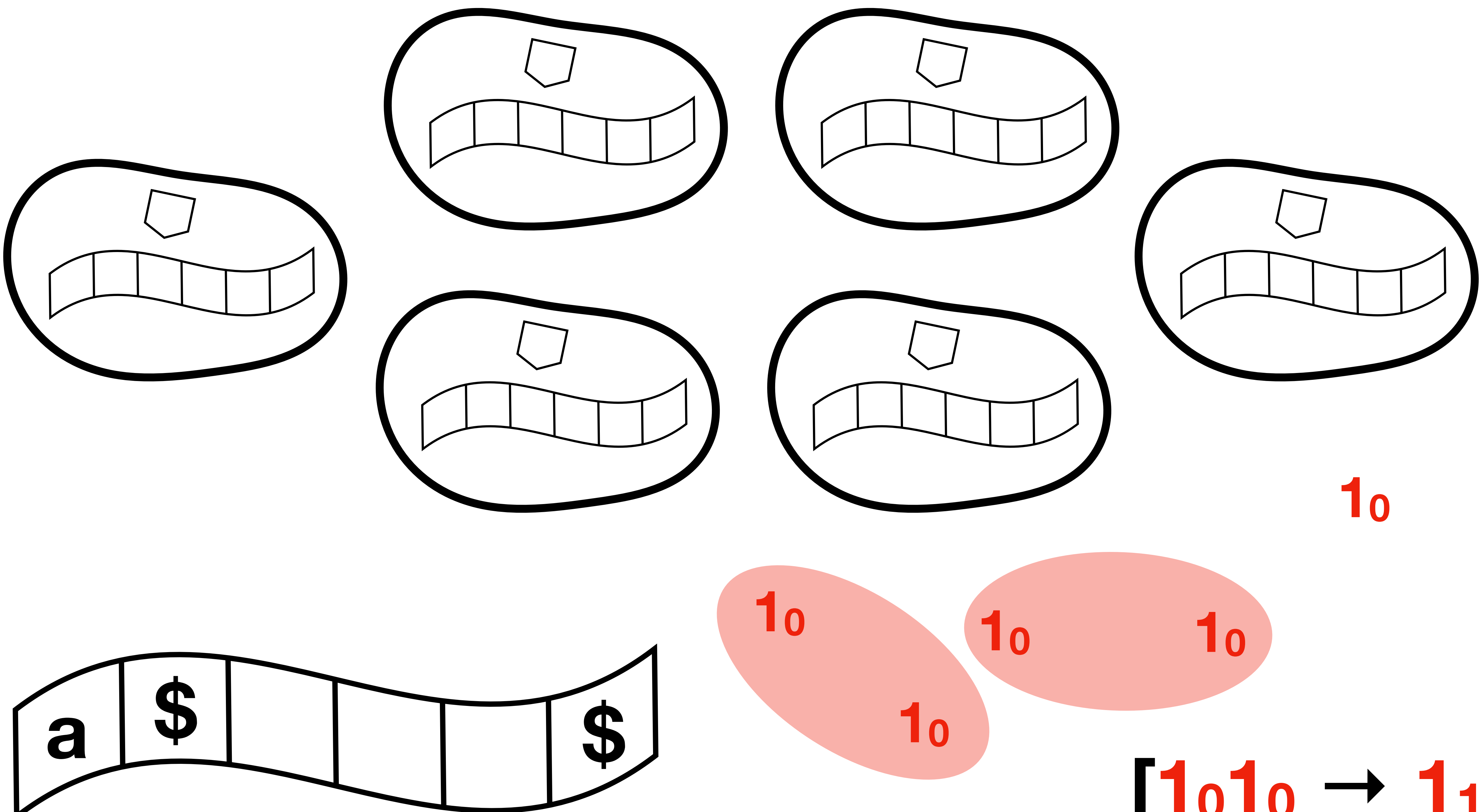
Converting unary to binary



Converting unary to binary

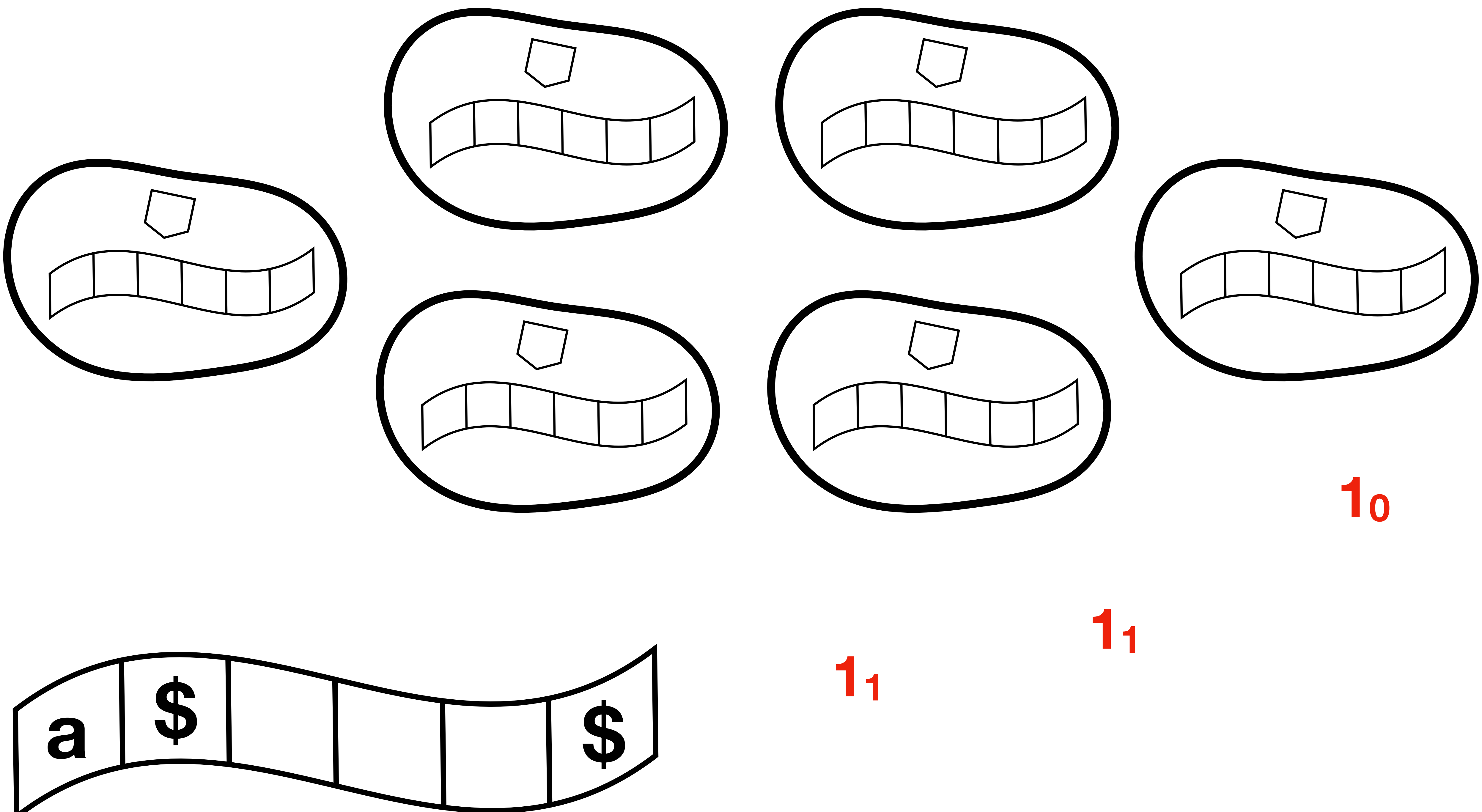


Converting unary to binary

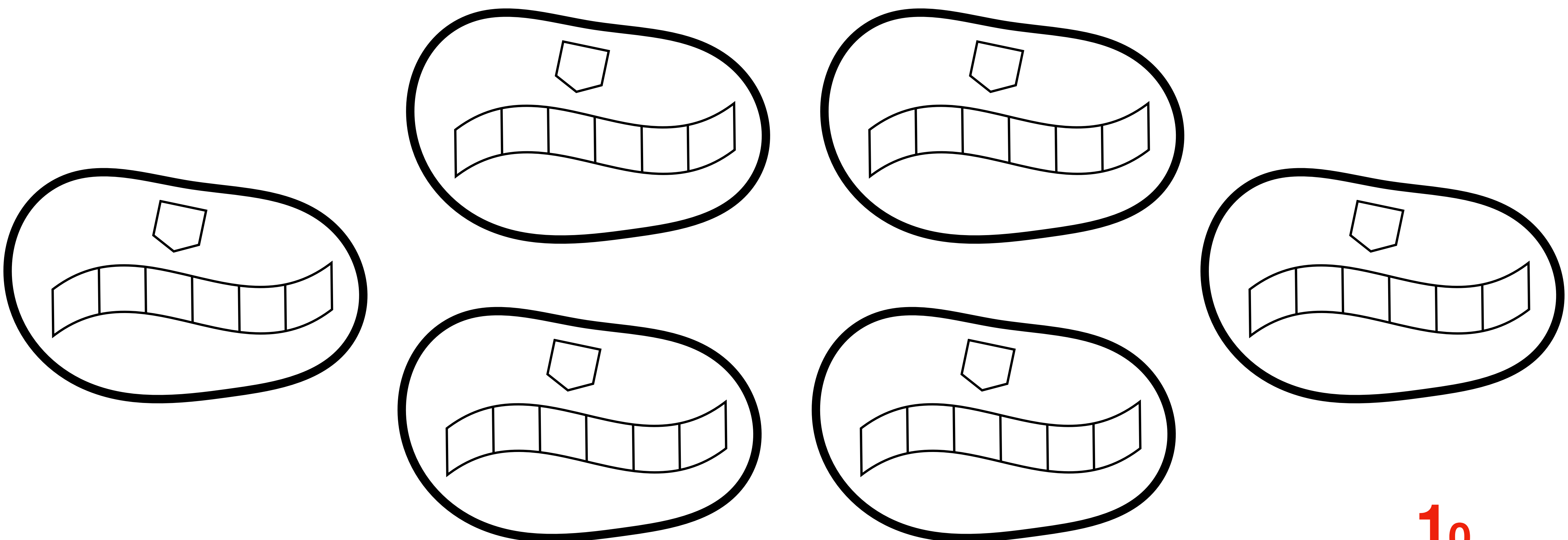


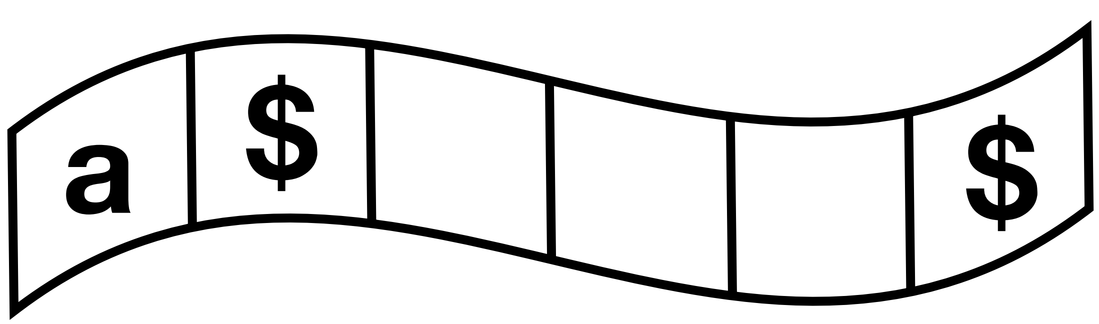
$[1_0 1_0 \rightarrow 1_1]$

Converting unary to binary



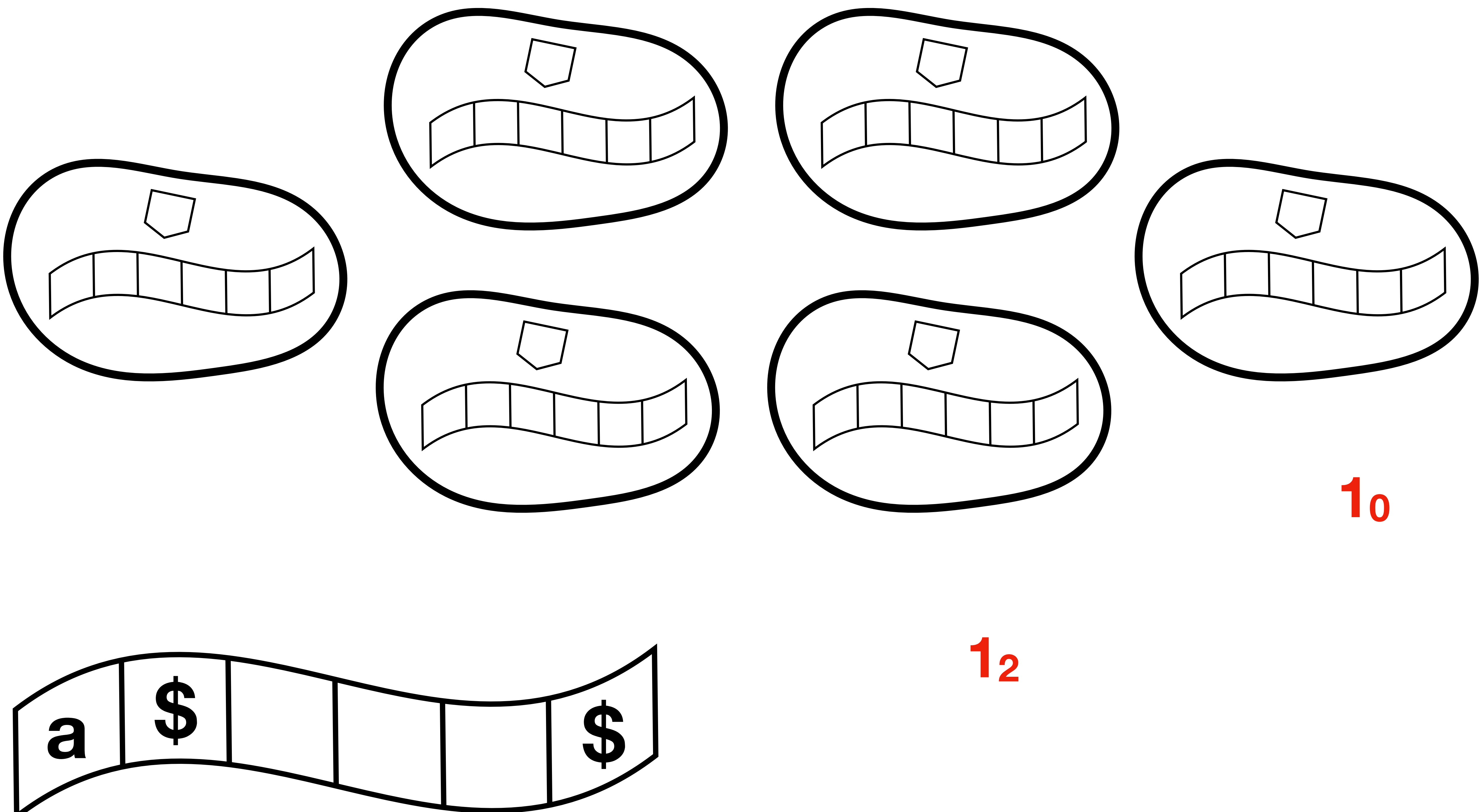
Converting unary to binary



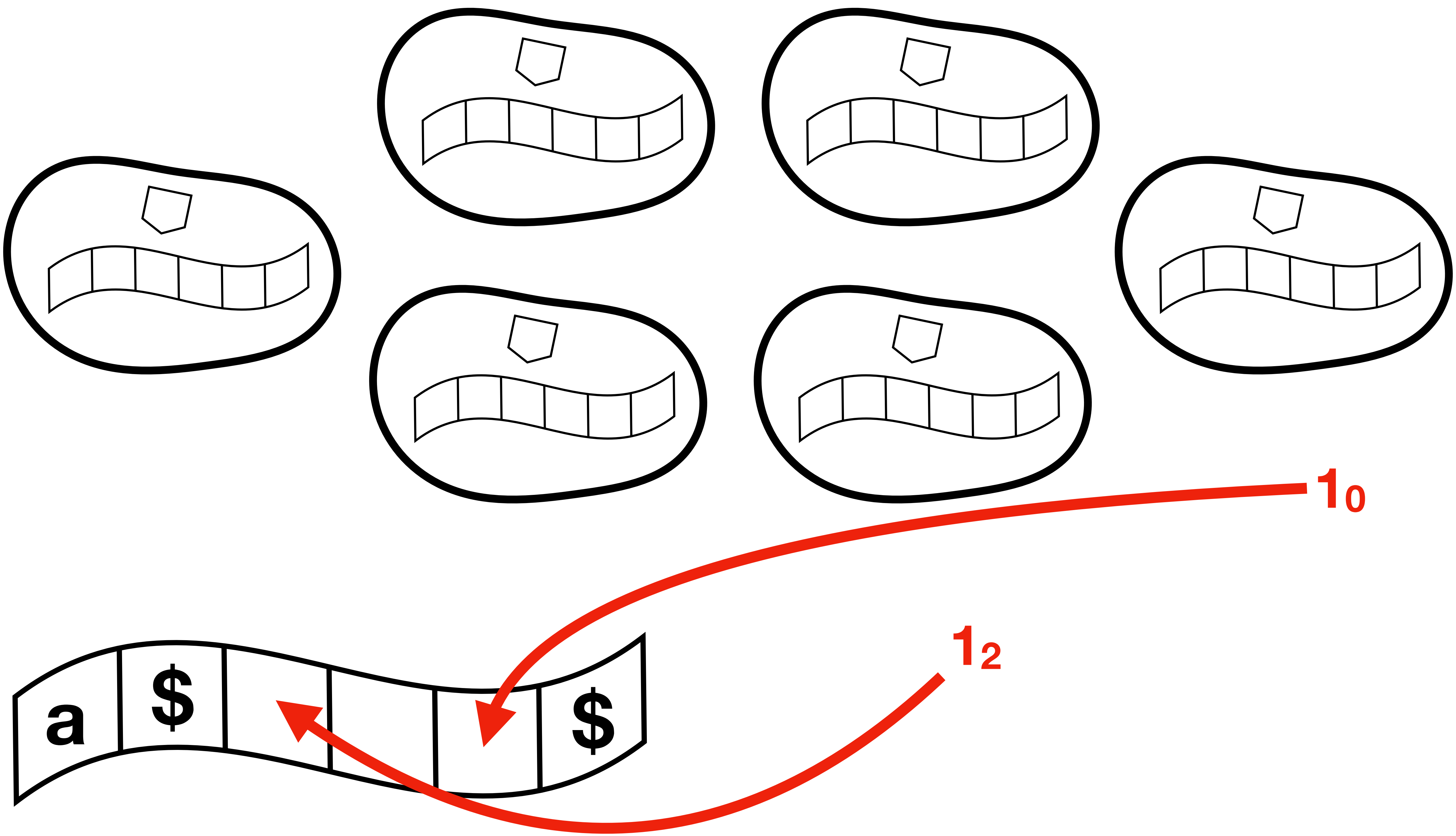


$[1_1 1_1 \rightarrow 1_2]$

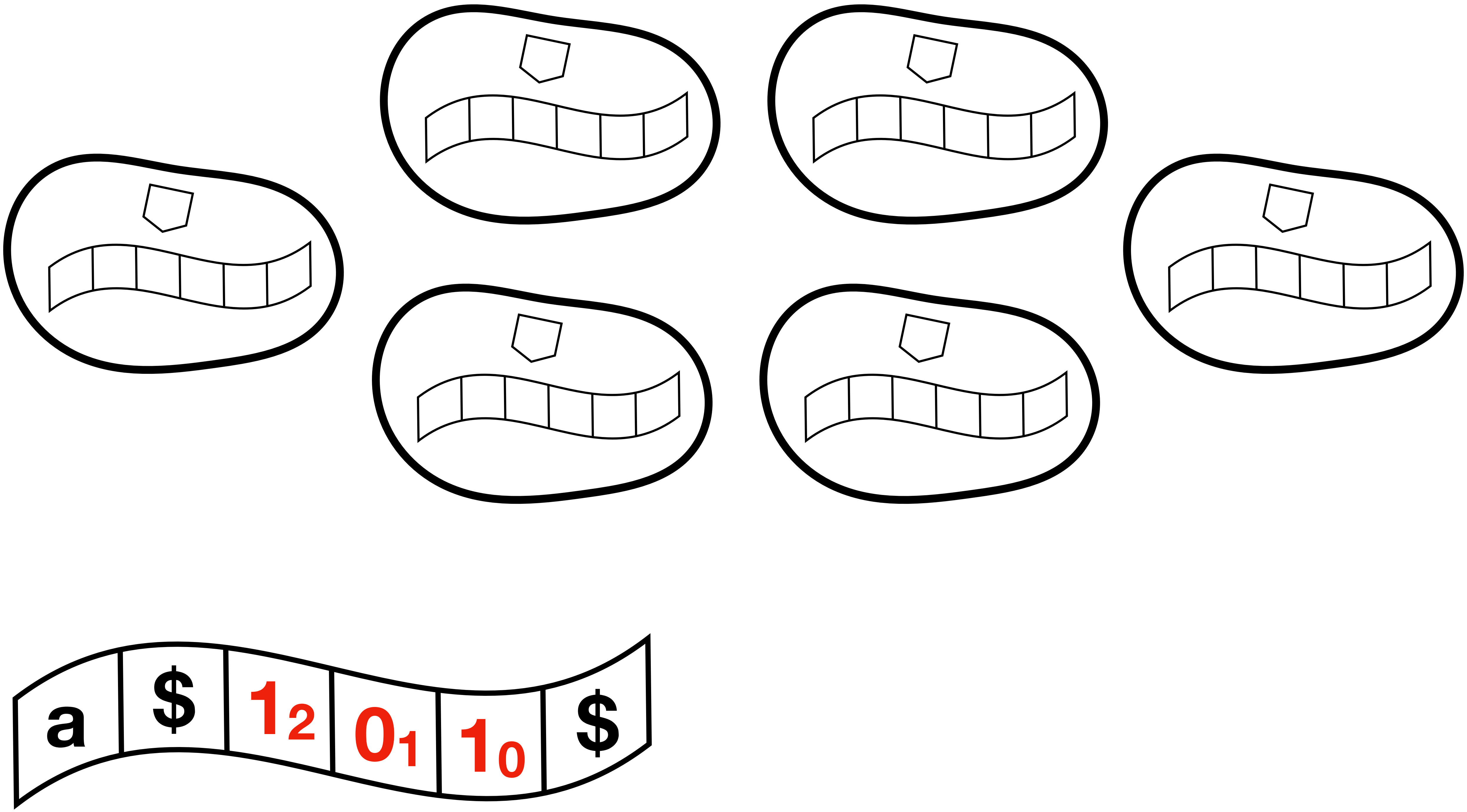
Converting unary to binary



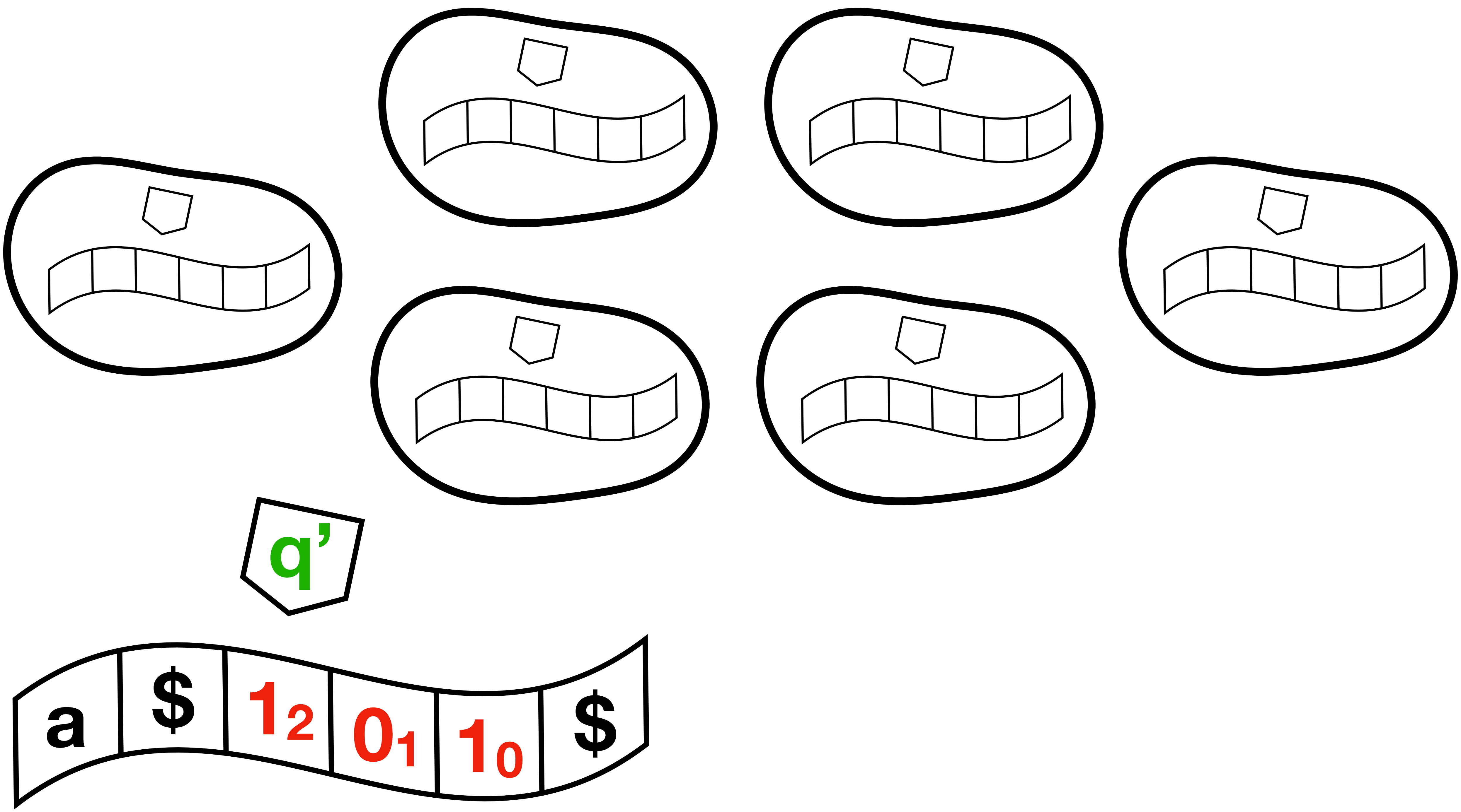
Answer on the tape



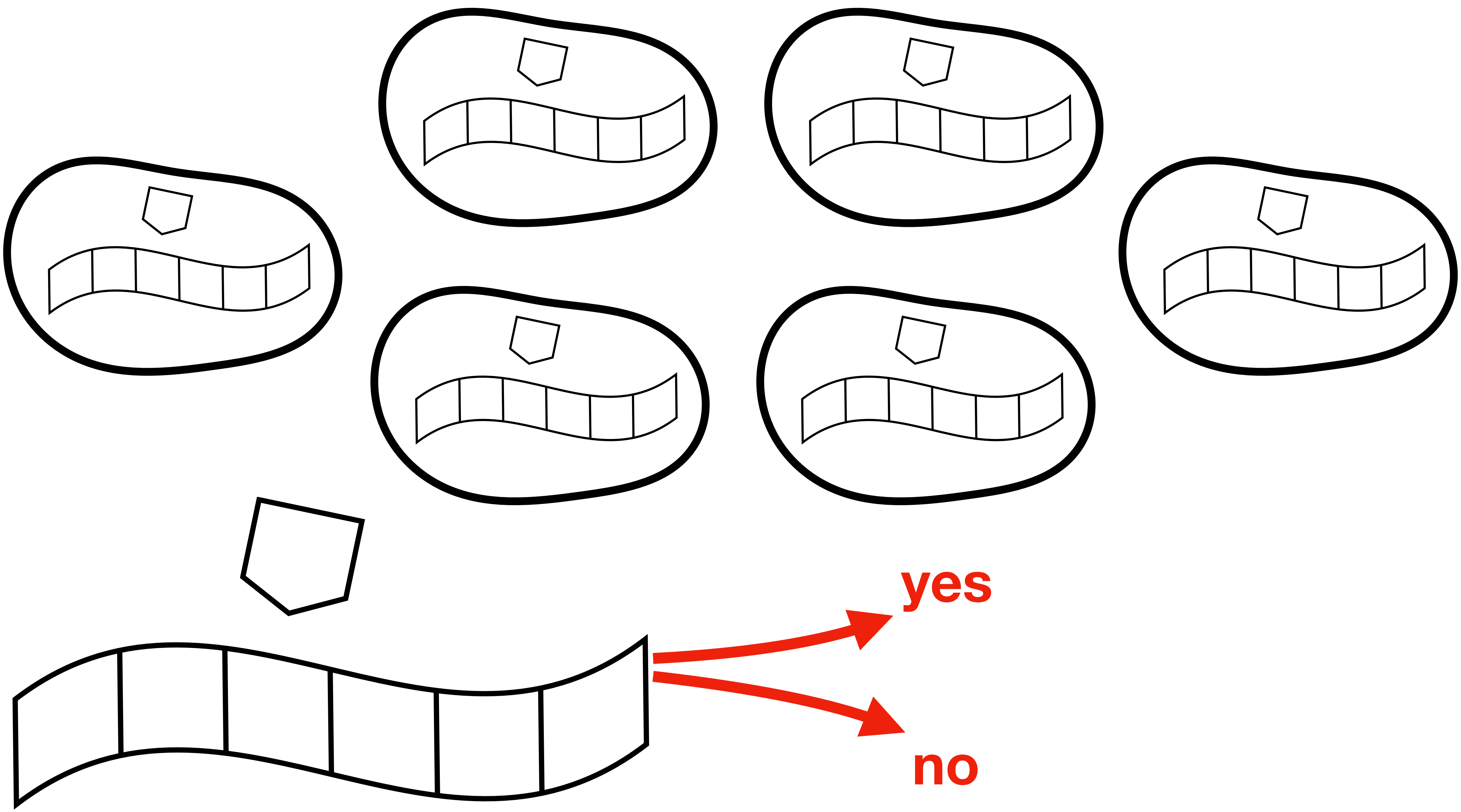
Answer on the tape



Resuming the simulation of the main TM



Final answer



Simulating (monodirectional, shallow) membrane systems is in $\mathbf{P}^{\#\mathbf{P}[1]}$

Counting the number of objects a sent out by a membrane at time t is in #P

- **for** i := 0 **to** t **do**
    - deterministically choose a maximal multisite of rules to apply inside the simulated membrane
    - apply all the rules except membrane division in deterministic polynomial time (“Milano Theorem”)
    - if applying membrane division, nondeterministically choose whether to simulate the left or the right resulting membrane
- if an object a was sent out in the last step then **accept**, otherwise **reject**

Lemma: $\mathbf{P}^{\#\mathbf{P}[1]}$ = parallel $\mathbf{P}^{\#\mathbf{P}}$

- Trivially $\mathbf{P}^{\#\mathbf{P}[1]} \subseteq$ parallel $\mathbf{P}^{\#\mathbf{P}}$
- A polynomial number of queries $f(x_1), \ldots, f(x_m)$ can be replaced by a single query to

$$g(x_1\$x_2\$\cdots\$x_m) = \sum_{i=1}^{n} B^i \times f(x_i) \qquad \Rightarrow \qquad f(x_i) = \left\lfloor \frac{g(x_1\$x_2\$\cdots\$x_m)}{B^{i-1}} \right\rfloor$$

- The function g is also in **#P** because this class is closed under sums and products

Simulating (shallow, omnidirectional) membrane systems in $\mathbf{P}^{\#\mathbf{P}[1]}$

- for each membrane in the initial configuration, for each object type a and for each time step t, ask the oracle how many objects of type a are sent out by the membrane at time t
(note: polynomial number of parallel queries!)

- **while** the system has not produced the answer object **do**

    - simulate one step of the external environment deterministically (Milano Theorem)

    - add the objects sent out from the membranes (according to the queries asked) to the environment

- **accept** or **reject** according to the answer of the system simulated

Computational complexity of membrane systems

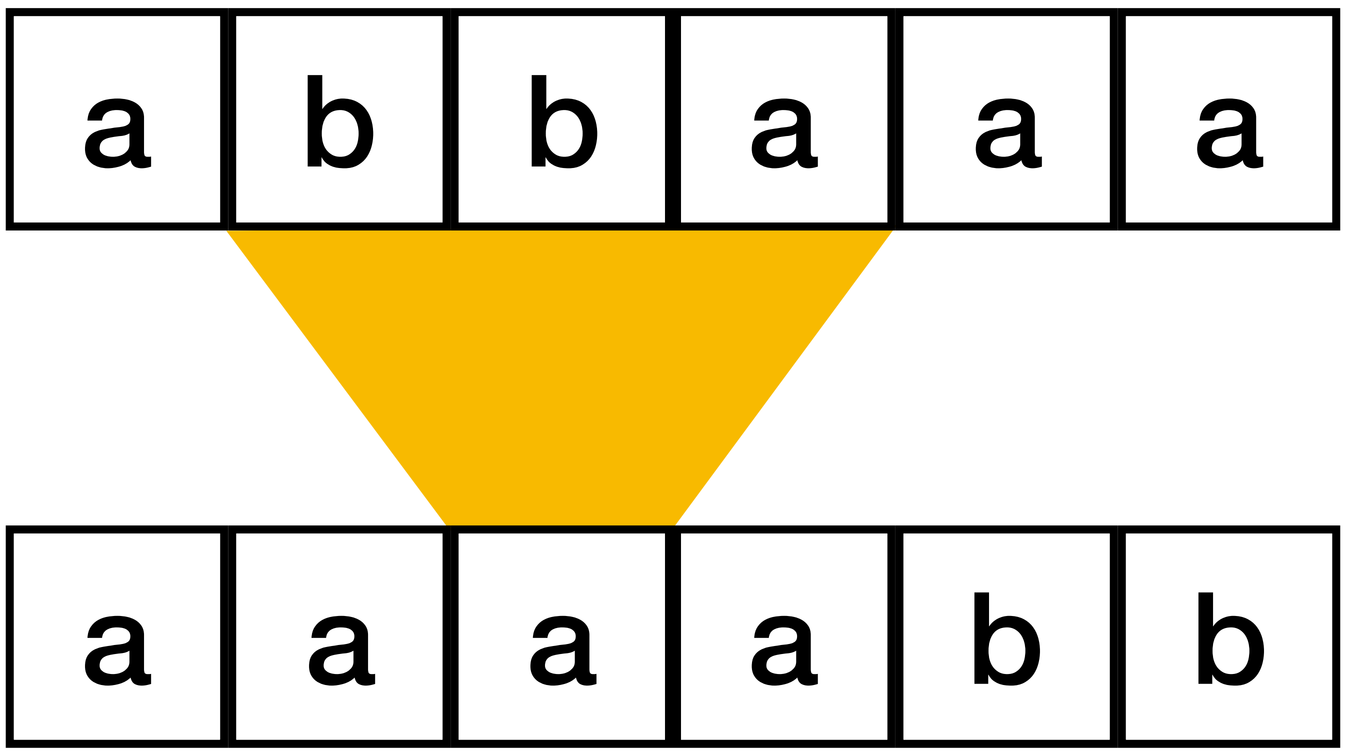
- No membranes (only environment) → **P**
- Shallow, monodirectonal → $\mathbf{P}^{\mathbf{\#P}[1]}$ = parallel $\mathbf{P}^{\mathbf{\#P}}$
- Shallow, bidirectional → $\mathbf{P}^{\mathbf{\#P}}$
- Constant depth k, bidirectional → $\mathbf{P}^{\mathbf{C}_k\mathbf{P}}$
  where $\mathbf{C}_0\mathbf{P} = \mathbf{P}$, $\mathbf{C}_1\mathbf{P} = \mathbf{PP}$, $\mathbf{C}_2\mathbf{P} = \mathbf{PP}^{\mathbf{PP}}$, $\mathbf{C}_k\mathbf{P} = \mathbf{PP}^{\mathbf{C}_{k-1}\mathbf{P}}$
  is the counting hierarchy [work in progress]
- Unbounded depth, bidirectional → **PSPACE**

Expanding cellular automata (XCA)

Expanding CA

a	b	b	a	a	a

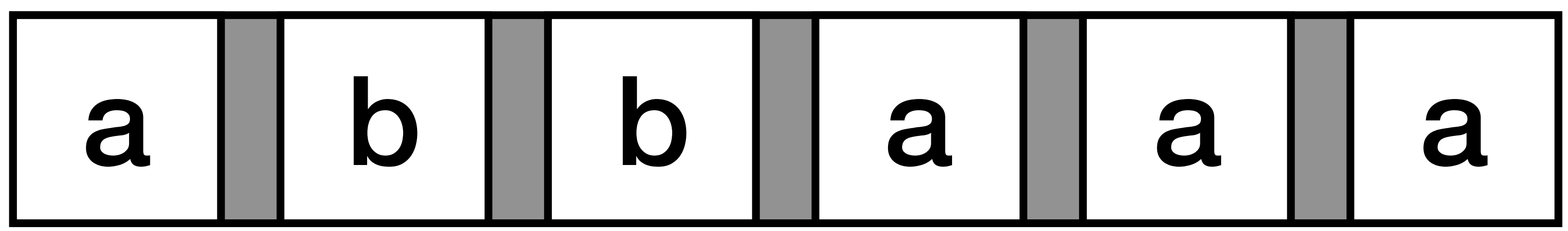
Expanding CA



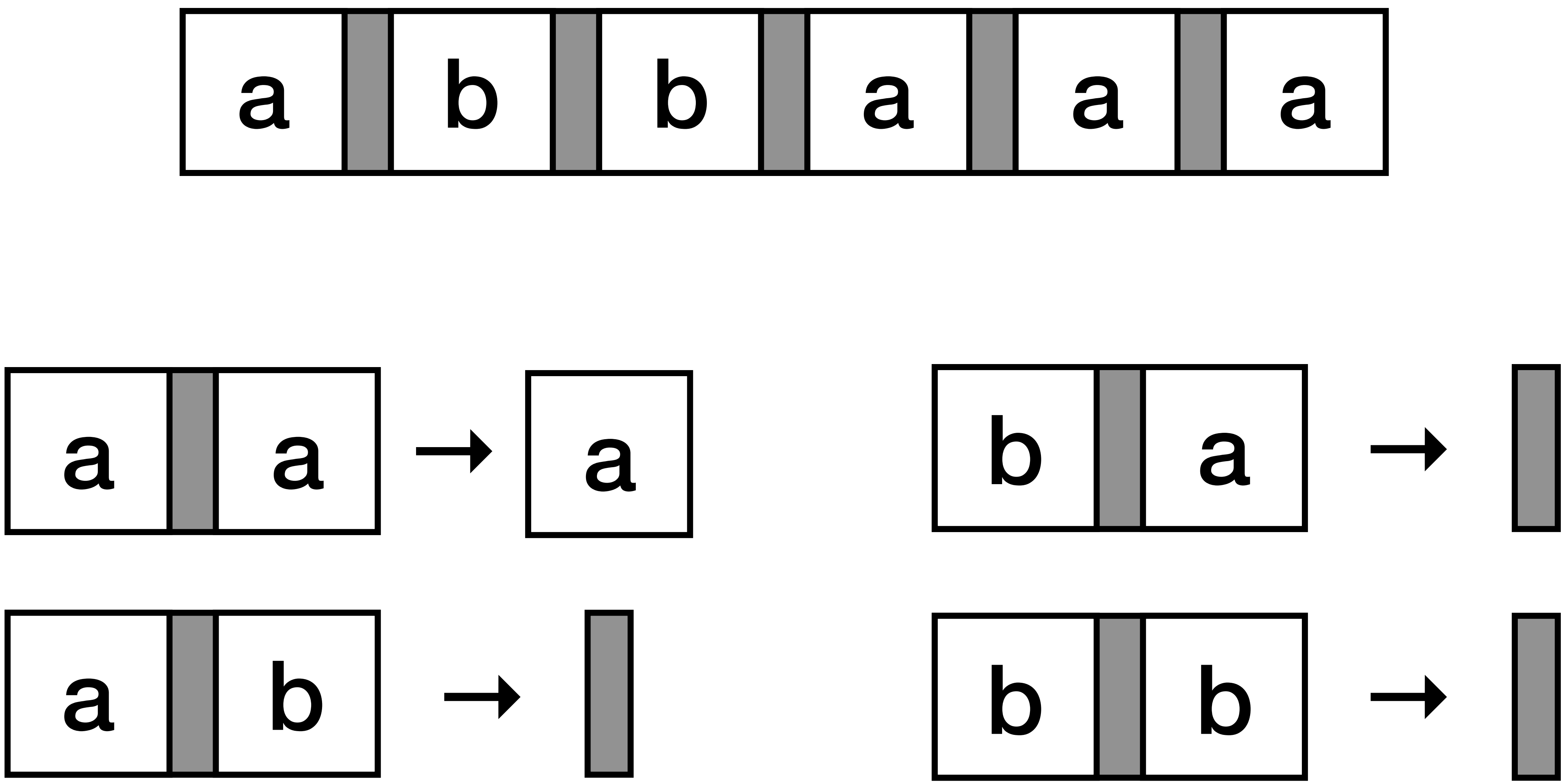
Expanding CA

a	b	b	a	a	a

Expanding CA



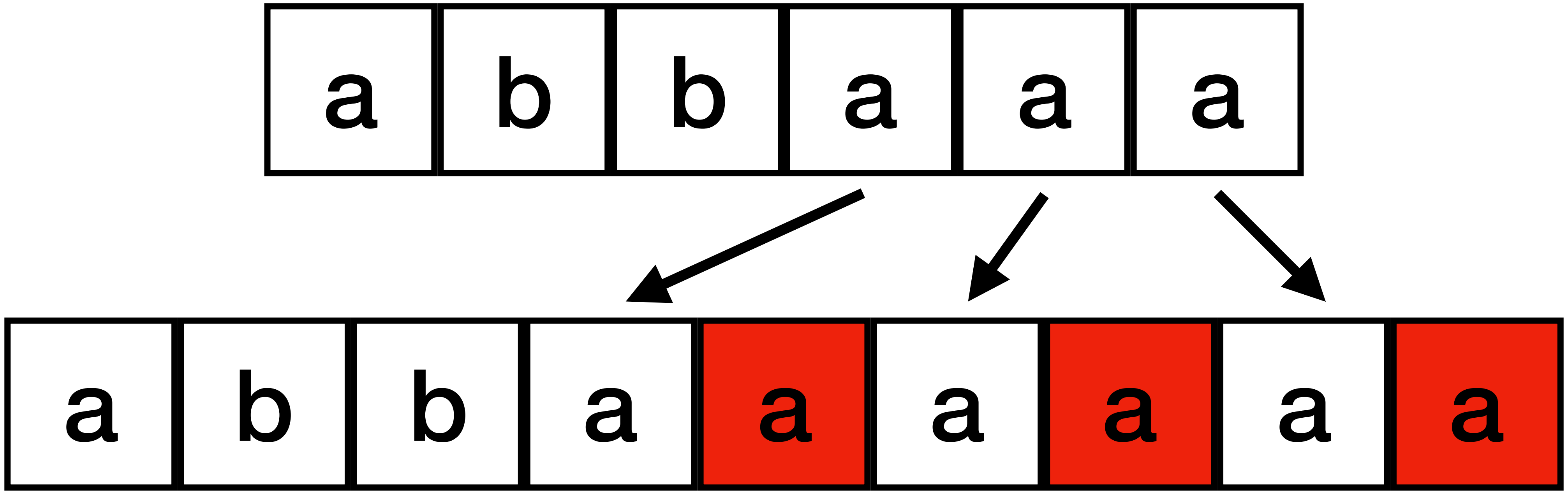
Expanding CA



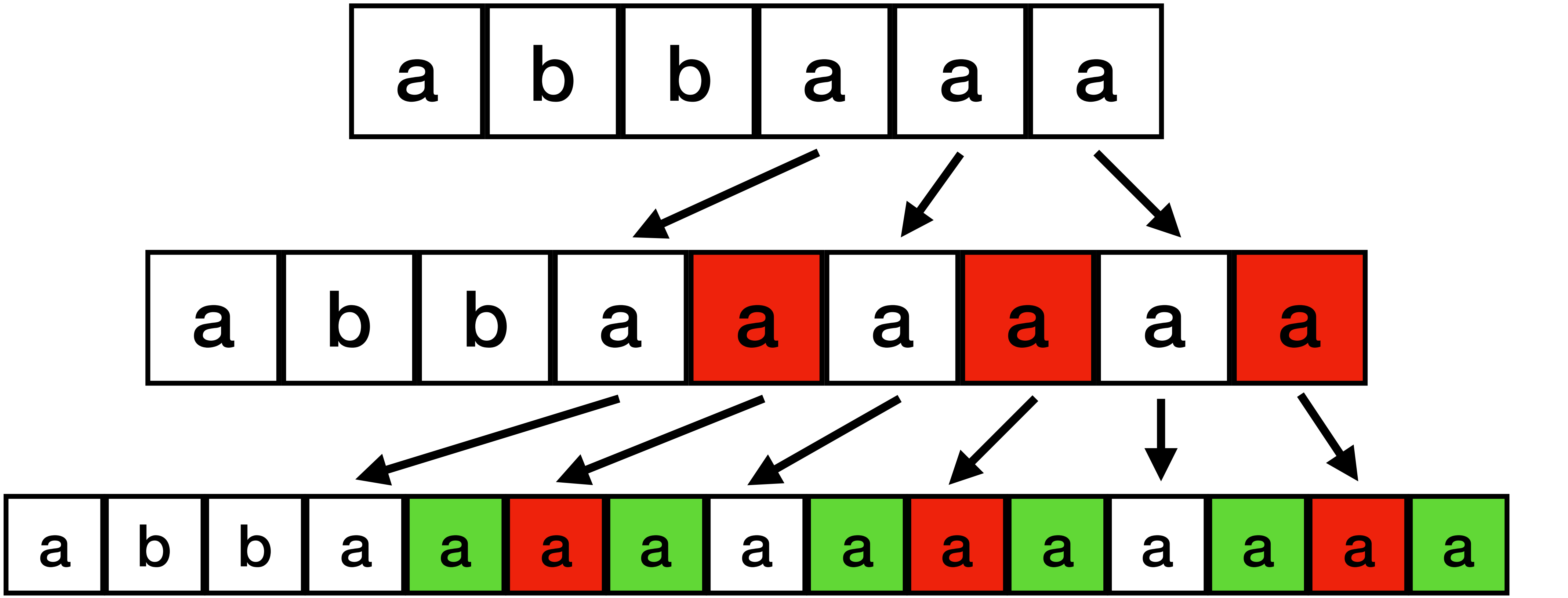
Expanding CA

a	b	b	a	a	a

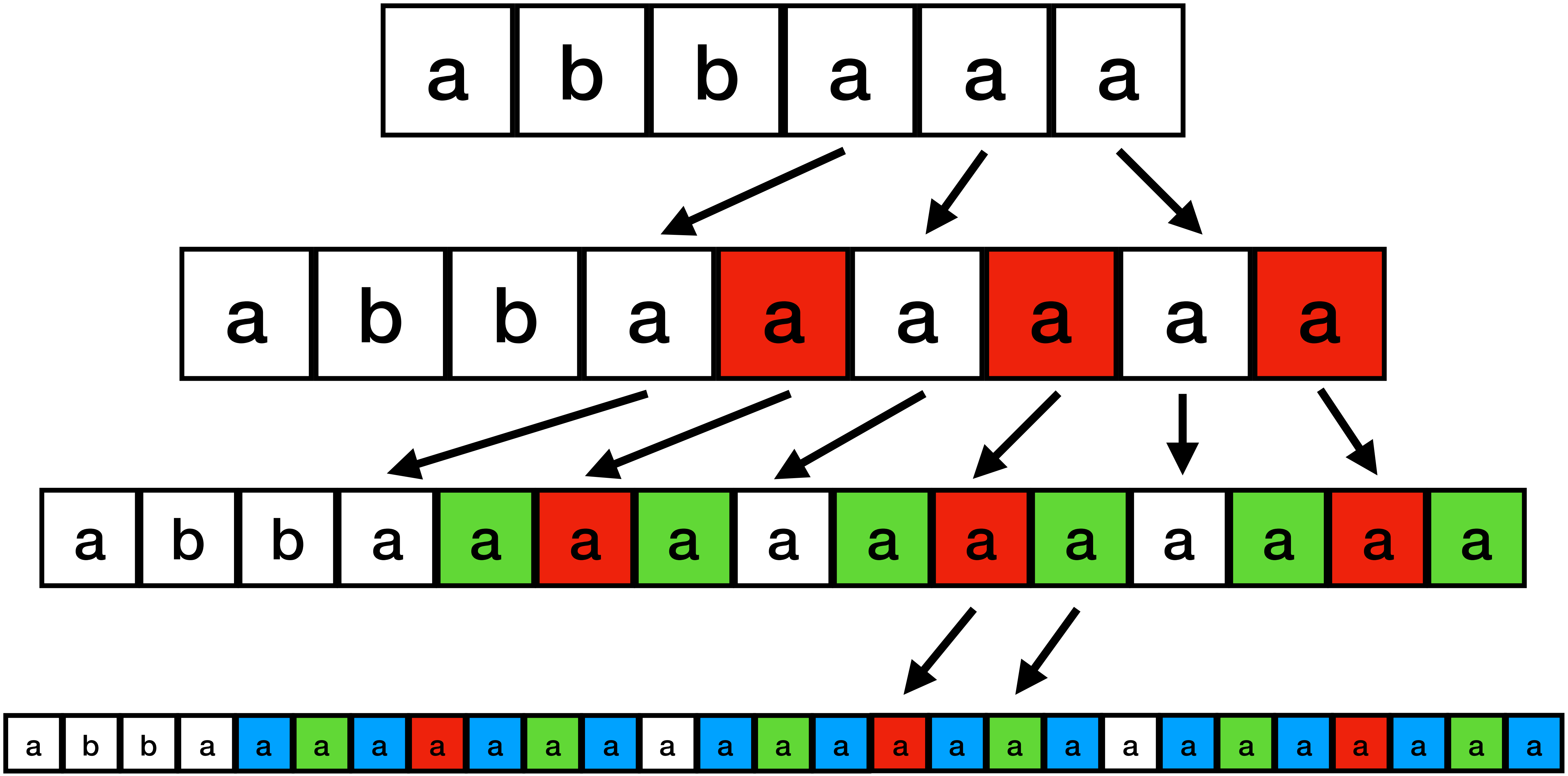
Expanding CA



Expanding CA



Expanding CA



Complexity results on XCA

- The class of problems solved in polynomial time by XCA is exactly the class of problems truth-table reducible to **NP**
- If shrinking (deleting cells) is also allowed, then the class becomes **PSPACE**

Conclusions and future work

Summary of results

- A lot of parallels computing models characterise either **P** or **PSPACE** when working in polynomial time
- Some variants of membrane systems characterise more “exotic” complexity classes with oracles, like $\mathbf{P}^{\#\mathbf{P}[1]}$, $\mathbf{P}^{\#\mathbf{P}}$, $\mathbf{P}^{\mathbf{NP}}$
- Expanding CA characterise the class of problems truth-table reducible to **NP**, which is somehow similar to oracle complexity classes

Conjectures and future work

- Find out why these models happen to characterise these exotic complexity classes

- Find out how the topology of the parallel computing units influences the efficiency:

  - Trees or stars for membrane systems

  - Linear or Euclidean grid for CA

  - Linear but expanding for XCA

References

- Sosík, P. and Rodríguez-Patón, A., 2007. Membrane computing and complexity theory: A characterization of PSPACE. Journal of Computer and System Sciences, 73(1), pp. 137–152
- Leporati, A., Manzoni, L., Mauri, G., Porreca, A.E. and Zandron, C., 2017. Characterising the complexity of tissue P systems with fission rules. Journal of Computer and System Sciences, 90, pp. 115–128
- Leporati, A., Manzoni, L., Mauri, G., Porreca, A.E. and Zandron, C., 2016. Monodirectional P systems. *Natural Computing*, *15*(4), pp. 551–564
- Modanese, A., 2019. Complexity-theoretic aspects of expanding cellular automata. arXiv preprint arXiv:1902.05487 (accepted at AUTOMATA 2019)

Thanks for your attention!

Merci de votre attention !