Methods of Applied Mathematics – HW 1

Due: Tuesday, September 30

When a drop of liquid hits a wetted surface a crown formation appears, as shown in Figure (a).

Figure (a)

It has been found that the number of points N on the crown depends on the speed U at which the drop hits the surface, the radius r and density ρ of the drop, and the surface tension σ of the liquid making up the drop.

A. Use dimensional analysis to determine the functional dependence of N on U, r, ρ, and σ. Express your answer in terms of the Weber number $W = \rho U^2 r/\sigma$.
B. The value of N has been measured as a function of the initial height h of the drop and the results are shown in Figure (b). Express your answer in part A in terms of h by writing U in terms of h and g, the acceleration due to gravity. Assume the drop starts with zero velocity.

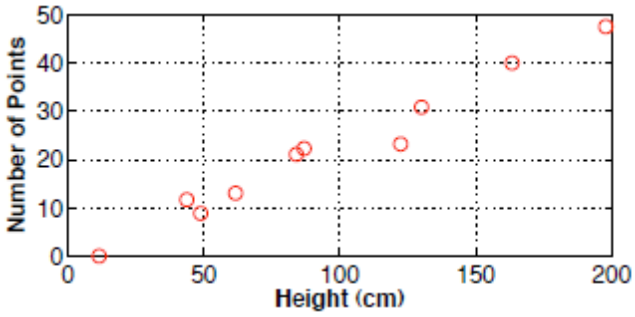

Figure (b)

C. The data in Figure (b) show a piecewise linear dependence on h, specifically, N can be described as a continuous function made up of two linear segments. Use this, and your result from part B, to find the unknown function in part A. In the experiments, r = 3.6 mm, $\rho = 1.1014$ gram/cm^3 and $\sigma = 50.5$ dyn/cm.

D. According to your result from part C, what must the initial height of the drop be to produce at least 80 points?
E. According to your result from part C, how many points are generated for a drop of mercury when h = 200 cm? Assume r = 3.6 mm, ρ = 13.5 gram/cm^3, and σ = 435 dyn/cm.