1. INDIVIDUAL AND COLLECTIVE RISK MODELS

1.1) Install `Julia` and `Visual Studio Code`

1.2) Consider a 1-year term life insurance policy j with insured amount $\$c_j$ and mortality rate $0 < q_j << 1$ with an additional coverage of paying twice the insured amount in case of accidental death, considering a probability of death in an accident of $0 < \kappa_j << 1$. If X_j is a random variable of the claim amount from such policy j then:

a) Calculate $\mathbb{E}(X_j)$ using formula (4) from class notes.

b) Obtain an explicit formula for F_{X_j} using formula (3) from class notes.

c) Using b) calculate again $\mathbb{E}(X_j)$ a compare with the result in a).

1.3) Consider a random vector (X, Y) with joint probability density function:

$$f_{XY}(x, y) \;=\; e^{-y}\mathbf{1}_{\{0<x<y\}}$$

Calculate $\mathrm{VaR}_{0.995}$ of $X, Y, X+Y$, and $\mathrm{VaR}_{0.995}(X) + \mathrm{VaR}_{0.995}(Y)$.

1.4) The same as in 1.3, but considering X and Y independent.

1.5) Let S be a Pareto$(1, \beta)$ random variable, that is with probability density function:

$$f(s \,|\, \beta) \;=\; \frac{\beta}{s^{\beta+1}}\mathbf{1}_{\{s>1\}}\,, \qquad \beta > 0$$

Prove that for any given $0 < \alpha < 1$ there exists a $\beta > 1$ such that $\mathbb{E}(S) > \mathrm{VaR}_\alpha(S)$.

1.6) Prove Proposition 8 from class notes.

1.7) Consider a portfolio of 1-year term life insurance independent policies from the file `LIFEinsurance.csv` that specifies age and insured amount for each policy, with the additional benefit of twice the insured amount in case of accidental death, assuming that 1 out of 10 deaths is accidental (regardless of the age). Use the mortality table in the file `mortality.csv` and do the following:

a) Make use of the Julia packages `CSV.jl` and `DataFrames.jl` to read the data, and the package `Plots.jl` for the histogram.

b) Calculate the theoretical mean and variance for total claims.

c) Through 1 million simulations of the portfolio in Julia, estimate mean and variance of total claims, and compare to the theoretical values obtained in the previous item.

d) Also from c) estimate a non-parametric *Value at Risk* (VaR) of level 99.5% for total claims.

e) Using item b) results, calculate normal approximation of VaR of level 99.5% for total claims, and compare to the VaR obtained in item d).

f) Graph a histogram of the simulations of total claims from item c) and add to the same graph the approximated normal density.

1.8) Build a collective risk model that approximates the results of the individual risk model from Exercise 1.7

1.9) Let X be a *Pareto* random variable (rv) with parameters $\beta > 0$ and $\theta > 0$ and therefore with probability density function (pdf) as follows:

$$f_X(x \,|\, \beta, \theta) \;=\; \frac{\beta\theta^\beta}{x^{\beta+1}}\mathbf{1}_{\{x \geq \theta\}}$$

Consider a portfolio of 1-year term property and casualty insurance, under the collective risk model:

$$S \;=\; Y_1 \,+\, \cdots \,+\, Y_N$$

where the *frequency* is a rv $N := \max\{n \in \{0, 1, \ldots\} : n \leq X - \theta\}$ and the conditional *severity* per claim is given by $Y \,|\, N = n \sim \text{Pareto}(2 + \frac{1}{n}\,, \,\delta)$ for $n \geq 1$. Calculate or estimate the expected value, variance, median, and $\text{VaR}_{0.995}$ of S and $S \,|\, S > 0$, with parameter values $\beta = 3, \theta = 1 = \delta$.

1.10 Consider the data in the file `CRMdata.txt` which represents weekly claims (in million pesos) in the last 20 years for a certain insurance product, one week per row. It is only data for those weeks were claims were filed. Fit a collective risk model and estimate the expected value, variance, median, and $\text{VaR}_{0.995}$ of the total claims per week.

1.11 Make some personal research about the following formulas:

a) De Piril's formula in the context of individual risk models.

b) Panjer's formula in the context of collective risk models.