Student Information

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Answer 1

a)

Confidence intervals for mean of a sample with known standart deviation can be calculated as:

$$\overline{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Mean of sample above,$\overline{x}$ equals to 16.96, sample size n equals to 10 and standart devation of sample σ equals to 3.
For %90 confidence interval $z_{0.05} = 1.645$.

$$(16.96 - z_{0.05} \cdot \frac{\sigma}{\sqrt{n}},\ 16.96 + z_{0.05} \cdot \frac{\sigma}{\sqrt{n}})$$
$$(15.38,\ 18.52)$$

For %99 confidence interval $z_{0.005} = 2.57$.

$$(16.96 - z_{0.005} \cdot \frac{\sigma}{\sqrt{n}},\ 16.96 + z_{0.005} \cdot \frac{\sigma}{\sqrt{n}})$$
$$(14.52,\ 19.40)$$

b)

For %98 confidence interval margin is $z_{0.01} \cdot \frac{\sigma}{\sqrt{n}}$ and $z_{0.01}$ equals to 2.33.

$$z_{0.01} \cdot \frac{3}{\sqrt{n}} = 1.55$$
$$\sqrt{n} = 4.51$$
$$n = 20.34$$

n must be at least 21.

Answer 2

a)

No, they are not enough. Standart deviation is also required to conduct a hypothesis about a restaurant.

b)

$H_0 : \mu = 7.5$
$H_A : \mu < 7.5$
$\alpha = 0.05$
$\overline{x} = 7.4$
$s = 0.8$
$n = 256$

Since customer ratings are independent and identically distributed, sample mean $\overline{x}$ has a normal distribution. When null hypothesis H_0 is assumed to be true, t value must be greater than $-t_\alpha$ with the degree of freedom 255.

$$t = \frac{\overline{x} - \mu}{s/\sqrt{n}} = \frac{7.4 - 7.5}{0.8/\sqrt{256}} = -2$$
$$-t_\alpha = -t_{0.05} = -1.645$$

Since -2 is not greater than -1.645, null hypothesis is rejected. Thus, this restaurant would not be in my list.

c)

When t value in previous part recalculated with $s = 1.0$:

$$t = \frac{\overline{x} - \mu}{s/\sqrt{n}} = \frac{7.4 - 7.5}{1/\sqrt{256}} = -1.6$$

Since -1.6 is greater than -1.645, null hypothesis is accepted. Thus this restaurant would be in my list.

d)

Since 7.6 is greater than 7.5, t value will always be a positive number. Because of that, t value will never be less than $-t_\alpha = -t_{0.05} = -1.645$. That is why, there is no need for a hypothesis test.

Answer 3

a)

$H_0 : \mu_A - \mu_B = 90$
$H_A : \mu_A - \mu_B < 90$
$\alpha = 0.01$
$\overline{X}_A = 211$
$\overline{X}_A = 133$
$s_A = 5.2$

$s_B = 22.8$
$n_A = 20$
$n_B = 32$

When null hypothesis H_0 is assumed to be true, t value must be greater than $-t_\alpha$ with the degree of freedom 50 $(n_A + n_B - 2)$. The t value when variances of A and B are assumed to be same, is:

$$t = \frac{\overline{X}_A - \overline{X}_B - 90}{s_p\sqrt{1/n_A + 1/n_B}}$$

Where s_p, the pooled standart deviation is:

$$s_p = \sqrt{\frac{(n_A - 1)s_A^2 + (n_B - 1)s_B^2}{n_A + n_B - 2}} = \sqrt{\frac{(19)5.2^2 + (31)22.8^2}{50}} = 18.237$$

Then t value is:

$$t = \frac{211 - 133 - 90}{18.237\sqrt{1/20 + 1/32}} = -2.308$$

$-t_{0.01}$ with the degree of freedom 50 approximately equals to -2.4. Since t value (-2.308) is greater than -2.4, null hypothesis is accepted. Researcher can claim that computer B provides a 90-minute or better performance when population variances are assumed to be same.

b)

When population variances are not assumed to be same, t value equals to:

$$t = \frac{\overline{X}_A - \overline{X}_B - 90}{\sqrt{s_A^2/n_A + s_B^2/n_B}} = \frac{211 - 113 - 90}{\sqrt{5.2^2/20 + 22.8^2/32}} = -2.86$$

Degree of freedom for the t_α value is estimated by the Satterthwaite approximation:

$$\nu = \frac{\left(\dfrac{s_A^2}{n_A} + \dfrac{s_B^2}{n_B}\right)^2}{\dfrac{s_A^4}{n_A^2(n_A - 1)} + \dfrac{s_B^4}{n_B^2(n_B - 1)}}$$

When all values subsituted v is found to be approximately 36. Thus $-t_{0.01}$ equals to approximately -2.44. Since t value (-2.86) is not greater than $t_{0.01}$ (-2.44), null hypothesis is rejected. Hence researcher can not claim that computer B provides a 90-minute or better performance when population variances are not assumed to be same.