CENG 384 - Signals and Systems for Computer Engineers
Spring 2023
Homework 4

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1. (a) $H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{j\omega-1}{j\omega+1}$

   $Y(j\omega)(j\omega+1) = X(j\omega)(j\omega-1)$

   $j\omega Y(j\omega) + Y(j\omega) = j\omega X(j\omega) - X(j\omega)$

   $\frac{dy(t)}{dt} + y(t) = \frac{dx(t)}{dt} - x(t)$

   (b) $H(j\omega) = \frac{j\omega}{j\omega+1} - \frac{1}{j\omega+1} = j\omega\frac{1}{j\omega+1} - \frac{1}{j\omega+1}$

   From linearity and differentiation in time properties,

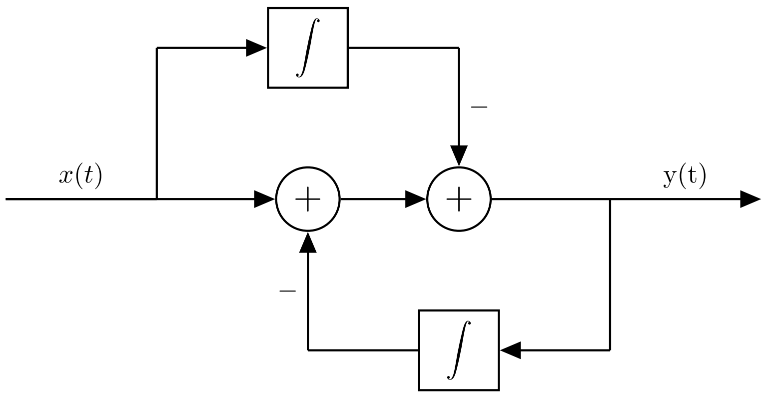
   $h(t) = \frac{d(e^{-t}u(t))}{dt} - e^{-t}u(t) = -2e^{-t}u(t)$

   (c) $x(t) = e^{-2t}u(t) \longleftrightarrow X(j\omega) = \frac{1}{j\omega+2}$

   $Y(j\omega) = X(j\omega)H(j\omega) = \frac{j\omega-1}{(j\omega+1)(j\omega+2)} = \frac{3}{j\omega+2} - \frac{2}{j\omega+1}$

   $y(t) = 3e^{-2t}u(t) - 2e^{-t}u(t)$

   (d)



2. (a) $Y(e^{j\omega})(e^{j\omega} - \frac{1}{2}) = X(e^{j\omega})e^{j\omega}$

   $x[n] = \delta[n] \longleftrightarrow X(e^{j\omega}) = 1$

   $H(e^{j\omega})(e^{j\omega} - \frac{1}{2}) = e^{j\omega}$

   $H(e^{j\omega}) = \frac{e^{j\omega}}{e^{j\omega}-\frac{1}{2}}$

   (b) $H(e^{j\omega}) = \frac{e^{j\omega}}{e^{j\omega}-\frac{1}{2}} = \frac{e^{-j\omega}(e^{j\omega})}{e^{-j\omega}(e^{j\omega}-\frac{1}{2})} = \frac{1}{1-\frac{1}{2}e^{-j\omega}}$

   From table, $h[n] = (\frac{1}{2})^n u[n]$

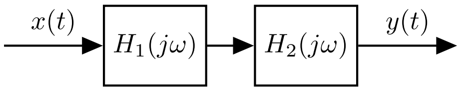
(c) $x[n] = (\frac{3}{4})^n u[n] \longleftrightarrow X(e^{j\omega}) = \frac{1}{1-\frac{3}{4}e^{-j\omega}}$

$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}) = \frac{1}{1-\frac{1}{2}e^{-j\omega}} \cdot \frac{1}{1-\frac{3}{4}e^{-j\omega}}$

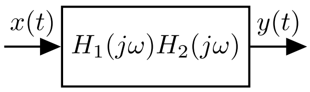
$= \frac{-2}{1-\frac{1}{2}e^{-j\omega}} + \frac{3}{1-\frac{3}{4}e^{-j\omega}}$

From table and linearity property, $y[n] = (3(\frac{3}{4})^n - 2(\frac{1}{2})^n)u[n]$

3. (a)



Means,



$H(j\omega) = H_1(j\omega)H_2(j\omega) = \frac{1}{(j\omega+1)(j\omega+2)} = \frac{1}{(j\omega)^2+3j\omega+2} = \frac{Y(j\omega)}{X(j\omega)}$

$Y(j\omega)((j\omega)^2 + 3j\omega + 2) = X(j\omega)$

$\frac{d^2y(t)}{dt} + 3\frac{dy(t)}{dt} + 2y(t) = x(t)$

(b) $H(j\omega) = H_1(j\omega)H_2(j\omega) = \frac{1}{(j\omega+1)(j\omega+2)} = \frac{1}{j\omega+1} - \frac{1}{j\omega+2}$

From table, $h(t) = (e^{-t} - e^{-2t})u(t)$

(c) $Y(j\omega) = X(j\omega)H(j\omega)$

$= j\omega\frac{1}{j\omega+1} - j\omega\frac{1}{j\omega+2}$

From linearity and differentiation in time properties,

$y(t) = \frac{d(e^{-t}u(t))}{dt} - \frac{d(e^{-2t}u(t))}{dt}$

$y(t) = (-e^{-t} + 2e^{-2t})u(t)$

4. (a)

$$H(e^{jw}) = H_1(e^{jw}) + H_2(e^{jw})$$

$$H(e^{jw}) = \frac{6 + 4.5e^{-jw}}{3 + 4.5e^{-jw} + 1.5e^{-jw}}$$

$$\sum_k a_k Y(e^{jw})e^{-jwk} = \sum_k b_k X(e^{jw})e^{-jwk}$$

$$H(e^{jw}) = \frac{Y(e^{jw})}{X(e^{jw})} = \frac{\sum_k a_k e^{-jwk}}{\sum_k b_k e^{-jwk}}$$

$$3y[n] + 4.5y[n-1] + 1.5y[n-2] = 6x[n] + 4.5x[n-1]$$

(b)

$$H(e^{jw}) = H_1(e^{jw}) + H_2(e^{jw})$$

$$H(e^{jw}) = \frac{6 + 4.5e^{-jw}}{3 + 4.5e^{-jw} + 1.5e^{-jw}}$$

(c)

$$h[n] = \mathcal{F}^{-1}\{H_1(e^{jw})\} + \mathcal{F}^{-1}\{H_2(e^{jw})\}$$

Using the look-up tables:

$$h[n] = \left(\frac{-1}{3}\right)^n u[n] + \left(\frac{-1}{2}\right)^n u[n]$$

5.