Student Information

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Q. 1

$$\begin{aligned}
\sum_{n=1}^{\infty} a_n \cdot x^n &= \sum_{n=1}^{\infty} (a_{n-1} + 2^n) \cdot x^n \\
&= \sum_{n=1}^{\infty} a_{n-1} \cdot x^n + \sum_{n=1}^{\infty} 2^n \cdot x^n \\
&= \sum_{n=0}^{\infty} a_n \cdot x^{n+1} + \sum_{n=0}^{\infty} 2^{n+1} \cdot x^{n+1} \\
&= x \sum_{n=0}^{\infty} a_n \cdot x^n + 2x \sum_{n=0}^{\infty} 2^n \cdot x^n \\
&= x \sum_{n=0}^{\infty} a_n \cdot x^n + \frac{2x}{1-2x}
\end{aligned}$$

Let $F(x) = \sum_{n=0}^{\infty} a_n \cdot x^n$:

$$\begin{aligned}
F(x) - a_0 &= x \cdot F(x) + \frac{2x}{1-2x} \\
F(x)(1-x) &= \frac{2x}{1-2x} + a_0 \\
F(x) &= \frac{2x}{(1-2x)(1-x)} + \frac{a_0}{1-x} \\
F(x) &= \frac{2x}{(1-2x)(1-x)} + a_0 \sum_{n=0}^{\infty} x^n
\end{aligned}$$

Transform $\frac{2x}{(1-2x)(1-x)}$ into generating function:

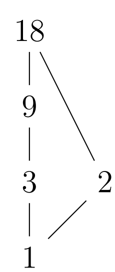
$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \; \frac{1}{1-2x} = \sum_{n=0}^{\infty} 2^n x^n$$
$$\frac{1}{1-x} - \frac{1}{1-2x} = \frac{-x}{(1-2x)(1-x)}$$
$$-2 \cdot (\frac{1}{1-x} - \frac{1}{1-2x}) = \frac{2x}{1-2x}$$
$$2\sum_{n=0}^{\infty} 2^n \cdot x^n - 2\sum_{n=0}^{\infty} x^n = \frac{2x}{1-2x}$$

$$F(x) = 2\sum_{n=0}^{\infty} 2^n \cdot x^n - 2\sum_{n=0}^{\infty} x^n + a_0 \sum_{n=0}^{\infty} x^n$$
$$F(x) = \sum_{n=0}^{\infty} (2^{n+1} - 2 + a_0) \cdot x^n$$
$$a_n = 2^{n+1} - 2 + a_0$$

When a_0 is substituted $a_n = 2^{n+1} - 1$.

Q. 2

a.



b.

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Q. 3

a.

Let A be a subset of natural numbers from 1 to n. Then all possible relations are:

$$\begin{aligned}
&(1,1),(1,2),(1,3),\ldots,(1,n)\\
&(2,1),(2,2),(2,3),\ldots,(2,n)\\
&(3,1),(3,2),(3,3),\ldots,(3,n)\\
&\vdots\\
&(n,1),(n,2),(n,3),\ldots,(n,n)
\end{aligned}$$

When relations (a,b) with different entries considered together, there are 3 options for them: (a,b) or (b,a) or non existent. There are $(n^2-n)/2$ of these relations.

When relations (a,a) with same entries considered, there are 2 options for them: existent or non existent. There are n of these relations.

Therefore there are $3^{\frac{n(n-1)}{2}} \cdot 2^n$ anti-symmetric relations on A.

b.

When reflexive relations on A are excluded from calculation, number of possible relations will be both reflexive and anti-symmetric.

$$\begin{aligned}
&\cancel{(1,1)},(1,2),(1,3),\ldots,(1,n)\\
&(2,1),\cancel{(2,2)},(2,3),\ldots,(2,n)\\
&(3,1),(3,2),\cancel{(3,3)},\ldots,(3,n)\\
&\vdots\\
&(n,1),(n,2),(n,3),\ldots,\cancel{(n,n)}
\end{aligned}$$

There are $3^{\frac{n(n-1)}{2}}$ both reflexive and anti-symmetric relations.