Student Information

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Q. 1

$$\begin{aligned}
\sum_{n=1}^{\infty} a_n \cdot x^n &= \sum_{n=1}^{\infty} (a_{n-1} + 2^n) \cdot x^n \\
&= \sum_{n=1}^{\infty} a_{n-1} \cdot x^n + \sum_{n=1}^{\infty} 2^n \cdot x^n \\
&= \sum_{n=0}^{\infty} a_n \cdot x^{n+1} + \sum_{n=0}^{\infty} 2^{n+1} \cdot x^{n+1} \\
&= x \sum_{n=0}^{\infty} a_n \cdot x^n + 2x \sum_{n=0}^{\infty} 2^n \cdot x^n \\
&= x \sum_{n=0}^{\infty} a_n \cdot x^n + \frac{2x}{1-2x}
\end{aligned}$$

Let $F(x) = \sum_{n=0}^{\infty} a_n \cdot x^n$:

$$\begin{aligned}
F(x) - a_0 &= x \cdot F(x) + \frac{2x}{1-2x} \\
F(x)(1-x) &= \frac{2x}{1-2x} + a_0 \\
F(x) &= \frac{2x}{(1-2x)(1-x)} + \frac{a_0}{1-x} \\
F(x) &= \frac{2x}{(1-2x)(1-x)} + a_0 \sum_{n=0}^{\infty} x^n
\end{aligned}$$

Transform $\frac{2x}{(1-2x)(1-x)}$ into generating function:

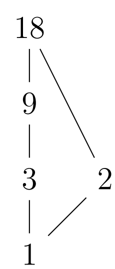
$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \ \frac{1}{1-2x} = \sum_{n=0}^{\infty} 2^n x^n$$
$$\frac{1}{1-x} - \frac{1}{1-2x} = \frac{-x}{(1-2x)(1-x)}$$
$$-2 \cdot (\frac{1}{1-x} - \frac{1}{1-2x}) = \frac{2x}{1-2x}$$
$$2\sum_{n=0}^{\infty} 2^n \cdot x^n - 2\sum_{n=0}^{\infty} x^n = \frac{2x}{1-2x}$$

$$F(x) = 2\sum_{n=0}^{\infty} 2^n \cdot x^n - 2\sum_{n=0}^{\infty} x^n + a_0 \sum_{n=0}^{\infty} x^n$$
$$F(x) = \sum_{n=0}^{\infty} (2^{n+1} - 2 + a_0) \cdot x^n$$
$$a_n = 2^{n+1} - 2 + a_0$$

When a_0 is substituted $a_n = 2^{n+1} - 1$.

Q. 2

a.



b.

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

c.

Q. 3

a.

Let A be a subset of natural numbers from 1 to n. Then all possible anti-symmetric relations are:

$$\begin{array}{ll}
(1,1),(1,2),(1,3),\ldots,(1,n) & n \text{ options} \\
\cancel{(2,1)},(2,2),(2,3),\ldots,(2,n) & n-1 \text{ options} \\
\cancel{(3,1)},\cancel{(3,2)},(3,3),\ldots,(3,n) & n-2 \text{ options} \\
\vdots & \\
\cancel{(n,1)},\cancel{(n,2)},\ldots,\cancel{(n,n-1)},(n,n) & 1 \text{ option}
\end{array}$$

In total, there are $1+2+\cdots+n = \frac{n(n+1)}{2}$ options to choose from. Therefore, number of all possible combinations is $2^{\frac{n(n+1)}{2}}$.

b.

When reflexive relations on A are excluded too, number of possible relations will be both reflexive and anti-symmetric.

$$\begin{array}{ll}
\cancel{(1,1)},(1,2),(1,3),\ldots,(1,n) & n-1 \text{ options} \\
\cancel{(2,1)},\cancel{(2,2)},(2,3),\ldots,(2,n) & n-2 \text{ options} \\
\cancel{(3,1)},\cancel{(3,2)},\cancel{(3,3)},\ldots,(3,n) & n-3 \text{ options} \\
\vdots & \\
\cancel{(n,1)},\cancel{(n,2)},\ldots,\cancel{(n,n-1)},\cancel{(n,n)} & 0 \text{ option}
\end{array}$$

In total, there are $0+1+\cdots+(n-1) = \frac{n(n-1)}{2}$ options to choose from. Therefore, number of all possible combinations is $2^{\frac{n(n-1)}{2}}$.