Student Information

Full Name : Ahmet Eren Çolak
Id Number : 2587921

Example ND Proof

Assume that you are asked to prove the following statement using natural deduction for propositional logic.

$$p \wedge \neg q \to r, \neg r, p \vdash q.$$

1.	$p \wedge \neg q \to r$	premise
2.	$\neg r$	premise
3.	p	premise
4.	$\neg q$	assumption
5.	$p \wedge \neg q$	$\wedge i$ 3,4
6.	r	$\to e$ 1,5
7.	$\perp$	$\neg e$ 6,2
8.	$\neg\neg q$	$\neg i$ 4–7
9.	q	$\neg\neg e$ 8

Q. 1

$$\begin{aligned}
\neg(p \wedge q) \leftrightarrow (\neg q \to p) &\equiv (\neg(p \wedge q) \to (\neg q \to p)) \wedge ((\neg q \to p) \to \neg(p \wedge q)) && \text{by logical equivalency for } \leftrightarrow \\
&\equiv (\neg\neg(p \wedge q) \vee (\neg\neg q \vee p)) \wedge (\neg(\neg\neg q \vee p) \vee \neg(p \wedge q)) && \text{by logical equivalency for } \to \\
&\equiv ((p \wedge q) \vee (q \vee p)) \wedge (\neg(q \vee p) \vee \neg(p \wedge q)) && \text{by double negation law} \\
&\equiv ((p \wedge q) \vee (q \vee p)) \wedge \neg((q \vee p) \wedge (p \wedge q)) && \text{by the first De Morgan law} \\
&\equiv ((q \vee p \vee p) \wedge (q \vee p \vee q)) \wedge \neg((p \wedge q \wedge q) \vee (p \wedge q \wedge p)) && \text{by distributive law} \\
&\equiv ((q \vee (p \vee p)) \wedge ((q \vee q) \vee p)) \wedge \neg((p \wedge (q \wedge q)) \vee ((p \wedge p) \wedge q)) && \text{by associative and commutative laws} \\
&\equiv ((q \vee p)) \wedge (q \vee p)) \wedge \neg((p \wedge q) \vee (p \wedge q)) && \text{by idempotent law} \\
&\equiv (q \vee p) \wedge \neg(p \wedge q) && \text{by idempotent law} \\
&\equiv (p \vee q) \wedge (\neg p \vee \neg q) && \text{by commutative law}
\end{aligned}$$

Q. 2

a. $\forall x \forall y \forall f((I(x,f) \wedge I(y,f)) \to \neg\exists z(E(x,z) \wedge E(y,z)))$
b. $\forall f \exists x(I(x,f) \wedge S(x,x) \wedge \neg\exists y(S(x,y) \wedge y \neq x))$
c. $\forall j(J(j,m) \to \exists x \exists y((A(x,j) \wedge A(y,j) \wedge x \neq y) \to \neg\exists z(A(z,j) \wedge z \neq x \wedge z \neq y)))$
Note: m is constant and means medicine faculty

Q. 3

a.

1.	$p \vee \neg q$	premise
2.	$p \vee r$	premise
3.	$r \to q$	assumption
4.	$\neg p$	assumption
5.	p	assumption
6.	$\bot$	$\neg e$ 4,5
7.	r	$\bot e$ 4
8.	$\neg q$	$\bot e$ 4
9.	r	assumption
10.	$\neg q$	assumption
11.	r	$\vee e$ 5–7, 9
12.	$\neg q$	$\vee e$ 5–8, 10
13.	q	$\to e$ 3,11
14.	$\bot$	$\neg e$ 12,13
15.	$\neg\neg p$	$\neg i$ 4–14
16.	p	$\neg\neg e$ 15
17.	$(r \to q) \to p$	$\to i$ 3–16

b.

1.	$((q \to p) \to q)$	assumption
2.	$\neg q$	assumption
3.	$q \to p$	assumption
4.	q	$\to e$ 1,3
5.	$\bot$	$\neg e$ 2,4
6.	$\neg(q \to p)$	$\neg i$ 3–5
7.	$\neg(\neg q \vee p)$	derived proof (3)
8.	$\neg\neg q \wedge \neg p$	derived proof (2)
9.	$q \wedge \neg p$	$\neg\neg e$ 8
10.	q	$\wedge e$ 9
11.	$\bot$	$\neg e$ 2,10
12.	$\neg\neg q$	$\neg i$ 2–11
13.	q	$\neg\neg e$ 12
14.	$((q \to p) \to q) \to q$	$\to i$ 1–13

Q. 4

a.

1.	$\neg\forall x(P(x) \to Q(x))$	premise
2.	$\neg(P(a) \to Q(x))$	$\forall e$ 1
3.	$\neg(\neg P(a) \vee Q(a))$	derived proof (3)
4.	$\neg\neg P(a) \wedge \neg Q(a)$	derived proof (2)
5.	$P(a) \wedge \neg Q(a)$	$\neg\neg e$ 4
6.	$\exists x(P(x) \wedge \neg Q(x))$	$\exists i$ 1–5

Derived Proofs

$$\neg(p \land q) \vdash \neg p \lor \neg q. \tag{1}$$

1. $\neg(p \land q)$ premise
2. $\neg(\neg p \lor \neg q)$ assumption
3. $\neg p$ assumption
4. $\neg p \lor \neg q$ $\lor i$ 3
5. $\bot$ $\neg e$ 2,4
6. $\neg\neg p$ $\neg i$ 3–5
7. p $\neg\neg e$ 6
8. $\neg q$ assumption
9. $\neg p \lor \neg q$ $\lor i$ 8
10. $\bot$ $\neg e$ 2,9
11. $\neg\neg q$ $\neg i$ 8–10
12. q $\neg\neg i$ 11
13. $p \land q$ $\land i$ 7,12
14. $\bot$ $\neg e$ 1,13
15. $\neg p \lor \neg q$ $\neg i$ 1,14

$$\neg(p \lor q) \vdash \neg p \land \neg q. \tag{2}$$

1. $\neg(p \lor q)$ premise
2. $\neg(\neg p \land \neg q)$ assumption
3. $\neg\neg p \lor \neg\neg q$ derived proof (1)
4. $p \lor q$ $\neg\neg e$ 3
5. $\bot$ $\neg e$ 1,4
6. $\neg p \land \neg q$ $\neg i$ 2–5

$$p \to q \vdash \neg p \lor q. \tag{3}$$

1. $p \to q$ premise
2. $\neg(\neg p \lor q)$ assumption
3. $\neg\neg p \land \neg q$ derived proof (2)
4. $p \land \neg q$ $\neg\neg e$ 3
5. p $\land e$ 4
6. $\neg q$ $\land e$ 4
7. q $\to e$ 1,5
8. $\bot$ $\neg e$ 6,7
9. $\neg\neg(\neg p \lor q)$ $\neg e$ 2–8
10. $\neg p \lor q$ $\neg\neg e$ 9