## Student Information

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## Q1

Assume that there are positive integers smaller than 1 and let $n$ be the least of them. It can expressed as:

$$0 < n < 1$$

Multiply both sides of the inequality $n < 1$ by $n$:

$$n^2 < n$$

Since $n$ is a positive integer, it is known that $n^2$ is also a positive integer. This contradicts with our assumption that $n$ is the least positive integer between 0 and 1. Thus, there is no positive integer less than 1.

## Q2

Base case: $S(1,1) = \frac{(1+1-1)!}{1!0!} = 1$ and $x_1 = 1$ is the only solution. Therefore $S(1,1)$ is true.
Induction step for $m$: Assume $S(m,n)$ is true, then $S(m+1,n)$ should be true.

$$x_1 + x_2 + \cdots + x_m + x_{m+1} = n$$

For $x_{m+1} = 0$, number of possibilities are $S(m,n)$
For $x_{m+1} = 1$, number of possibilities are $S(m,n-1)$
$\vdots$
For $x_{m+1} = n-1$, number of possibilities are $S(m,1)$
For $x_{m+1} = n$, number of possibilities are 1

So, $S(m+1,n)$ can be represented as:

$$S(m+1,n) = S(m,n) + S(m,n-1) + S(m,n-2) + \cdots + S(m,1) + 1$$

## Q3

**a.** A unit square contains 4 congruent triangles. There are 21 unit squares in total. From squares, there are $21 \cdot 4 = 84$ congruent triangles. Apart from unit squares, dots in the figure that are on the hypotenuse, do not form a square. From the dots on hypotenuse, there are 7 congruent triangles. In total there are $84 + 7 = 91$ triangles.

**b.** Amount of onto functions can be found by subtracting non onto functions from all functions. There are $4^6$ functions in all possible ways. To find amount of non onto functions, a recursive function can be defined such that $F_n$ denotes the amount of onto functions from 6 elements to $n$ elements.

There must be at least one element in the codomain of a function which is not image of any element of domain, so that function is not onto. If an element is eliminated from codomain, number of onto functions will be $\binom{n}{1}F_{n-1}$. If each elimination considered a recursive function is obtained:

$$F_n = n^6 - \binom{n}{1}F_{n-1} - \binom{n}{2}F_{n-2} - \cdots - \binom{n}{n-1}F_1$$

It is obvious that $F_1 = 1$. Then $F_4$ can be calculated:

$$F_2 = 2^6 - \binom{2}{1} \cdot 1 = 62$$
$$F_3 = 3^6 - \binom{3}{1} \cdot 62 - \binom{3}{2} \cdot 1 = 540$$
$$F_4 = 4^6 - \binom{4}{1} \cdot 540 - \binom{4}{2} \cdot 62 - \binom{4}{3} \cdot 1 = 1560$$

# Q4

**a.** Let $a_n$ denote the number of possible strings. Then there are two disjoint possibilities for $a_n$:
1) Valid string with $n-1$ length + any number (0, 1, 2)
2) Invalid string with $n-1$ length + last digit of invalid string

For $n > 1$, $a_n = 3a_{n-1} + 3^{n-1} - a_{n-1} = 2a_{n-1} + 3^{n-1}$

**b.**

$$a_1 = 0$$

**c.** Homogeneous solution:

$$a_n^{(h)} = 2a_{n-1} = 2^n$$

Particular solution:

$$a_n^{(p)} = 2a_{n-1} + 3^{n-1}$$

Solution must be in form of $A \cdot 3^n$.

$$A \cdot 3^n = 2A \cdot 3^{n-1} + 3^{n-1}$$
$$A \cdot 3^{n-1} = 3^{n-1}$$
$$A = 1$$

General solution: $a_n = c \cdot 2^n + 3^n$, for $n = 1, a_1 = 0$

$$c \cdot 2^1 + 3^1 = 0$$
$$2c + 3 = 0$$
$$c = -3/2$$
$$a_n = -3 \cdot 2^{n-1} + 3^n$$