Student Information

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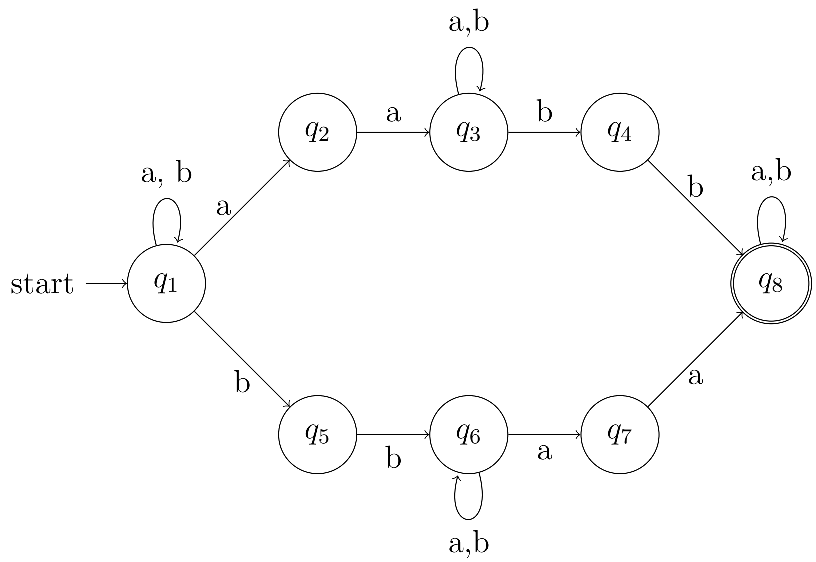
Q1

a)

$$L = (a \cup b)^*(aa(a \cup b)^*bb \cup bb(a \cup b)^*aa)(a \cup b)^*$$

b)



c)

Let $M = (K, \Sigma, \Delta, s, F)$ be the nondeterministic finite automaton in the previous question and $M' = (K', \Sigma, \delta, s', F')$ be the deterministic equivalent of M. Then,

$$K' = 2^K$$

$$s' = E(s)$$

$$F' = \{Q \subseteq K \mid Q \cap F \neq \emptyset\}$$

$$\delta(Q, a) = \bigcup \{E(p) \mid p \in K \wedge (q, a, p) \in \Delta\}$$

$$
\begin{aligned}
s' = E(s) = \{q_1\} \\
For\{q_1\}: \\
(q_1,a,q_1),(q_1,a,q_2) \in \Delta,\ \delta(\{q_1\},a) = E(q_1) \cup E(q_2) = \{q_1,q_2\} \\
(q_1,b,q_1),(q_1,b,q_5) \in \Delta,\ \delta(\{q_1\},b) = E(q_1) \cup E(q_5) = \{q_1,q_5\} \\
For\{q_1,q_2\}: \\
(q_1,a,q_1),(q_1,a,q_2),(q_2,a,q_3) \in \Delta, \\
\delta(\{q_1,q_2\},a) = E(q_1) \cup E(q_2) \cup E(q_3) = \{q_1,q_2,q_3\} \\
(q_1,b,q_1),(q_1,b,q_5) \in \Delta,\ \delta(\{q_1,q_2\},b) = E(q_1) \cup E(q_5) = \{q_1,q_5\} \\
For\{q_1,q_5\}: \\
(q_1,a,q_1),(q_1,a,q_2) \in \Delta,\ \delta(\{q_1,q_5\},a) = E(q_1) \cup E(q_2) = \{q_1,q_2\} \\
(q_1,b,q_1),(q_1,b,q_5),(q_5,b,q_6) \in \Delta,\ \delta(\{q_1,q_5\},b) \\
= E(q_1) \cup E(q_5) \cup E(q_6) = \{q_1,q_5,q_6\} \\
For\{q_1,q_2,q_3\}: \\
(q_1,a,q_1),(q_1,a,q_2),(q_2,a,q_3),(q_3,a,q_3) \in \Delta, \\
\delta(\{q_1,q_2,q_3\},a) = E(q_1) \cup E(q_2) \cup E(q_3) = \{q_1,q_2,q_3\} \\
(q_1,b,q_1),(q_1,b,q_5),(q_3,b,q_3),(q_3,b,q_4) \in \Delta, \\
\delta(\{q_1,q_2,q_3\},b) = E(q_1) \cup E(q_3) \cup E(q_4) \cup E(q_5) = \{q_1,q_3,q_4,q_5\} \\
For\{q_1,q_5,q_6\}: \\
(q_1,a,q_1),(q_1,a,q_2),(q_6,a,q_6),(q_6,a,q_7) \in \Delta, \\
\delta(\{q_1,q_5,q_6\},a) = E(q_1) \cup E(q_2) \cup E(q_6) \cup E(q_7) = \{q_1,q_2,q_6,q_7\} \\
(q_1,b,q_1),(q_1,b,q_5),(q_5,b,q_6),(q_6,b,q_6) \in \Delta, \\
\delta(\{q_1,q_5,q_6\},b) = E(q_1) \cup E(q_5) \cup E(q_6) = \{q_1,q_5,q_6\}
\end{aligned}
$$

For $\{q_1,q_3,q_4,q_5\}$:

$$(q_1,a,q_1),(q_1,a,q_2),(q_3,a,q_3) \in \Delta$$

$$\delta(\{q_1,q_3,q_4,q_5\},a) = E(q_1) \cup E(q_2) \cup E(q_3) = \{q_1,q_2,q_3\}$$

$$(q_1,b,q_1),(q_1,b,q_5),(q_3,b,q_3),(q_3,b,q_4),(q_4,b,q_8),(q_5,b,q_6) \in \Delta,$$

$$\delta(\{q_1,q_3,q_4,q_5\},b) = E(q_1) \cup E(q_3), E(q_4) \cup E(q_5) \cup E(q_6) \cup E(q_8) = \{q_1,q_3,q_4,q_5,q_6,q_8\}$$

For $\{q_1,q_2,q_6,q_7\}$

$$(q_1,a,q_1),(q_1,a,q_2),(q_2,a,q_3),(q_6,a,q_6),(q_6,a,q_7),(q_7,a,q_8) \in \Delta$$

$$\delta(\{q_1,q_2,q_6,q_7\},a) =$$

$$E(q_1) \cup E(q_2) \cup E(q_3) \cup E(q_6) \cup E(q_7) \cup E(q_8) = \{q_1,q_2,q_3,q_6,q_7,q_8\}$$

$$(q_1, b, q_1), (q_1, b, q_5), (q_6, b, q_6) \in \Delta$$

$$\delta(\{q_1, q_2, q_6, q_7\}, b) = E(q_1) \cup E(q_5) \cup E(q_6) = \{q_1, q_5, q_6\}$$

For $\{q_1, q_3, q_4, q_5, q_6, q_8\}$:

$$(q_1, a, q_1), (q_1, a, q_2), (q_3, a, q_3), (q_6, a, q_6), (q_6, a, q_7), (q_8, a, q_8) \in \Delta$$

$$\delta(\{q_1, q_3, q_4, q_5, q_6, q_8\}, a) =$$

$$E(q_1) \cup E(q_2) \cup E(q_3) \cup E(q_6) \cup E(q_7) \cup E(q_8) = \{q_1, q_2, q_3, q_6, q_7, q_8\}$$

$$(q_1, b, q_1), (q_1, b, q_5), (q_3, b, q_3), (q_3, b, q_4), (q_4, b, q_8), (q_6, b, q_6), (q_8, a, q_8) \in \Delta$$

$$\delta(\{q_1, q_3, q_4, q_5, q_6, q_8\}, b) =$$

$$E(q_1) \cup E(q_3), E(q_4) \cup E(q_5) \cup E(q_6) \cup E(q_8) = \{q_1, q_3, q_4, q_5, q_6, q_8\}$$

For $\{q_1, q_2, q_3, q_6, q_7, q_8\}$

$$(q_1, a, q_1), (q_1, a, q_2), (q_2, a, q_3), (q_3, a, q_3), (q_6, a, q_6),$$

$$(q_6, a, q_7), (q_7, q, q_8), (q_8, a, q_8) \in \Delta$$

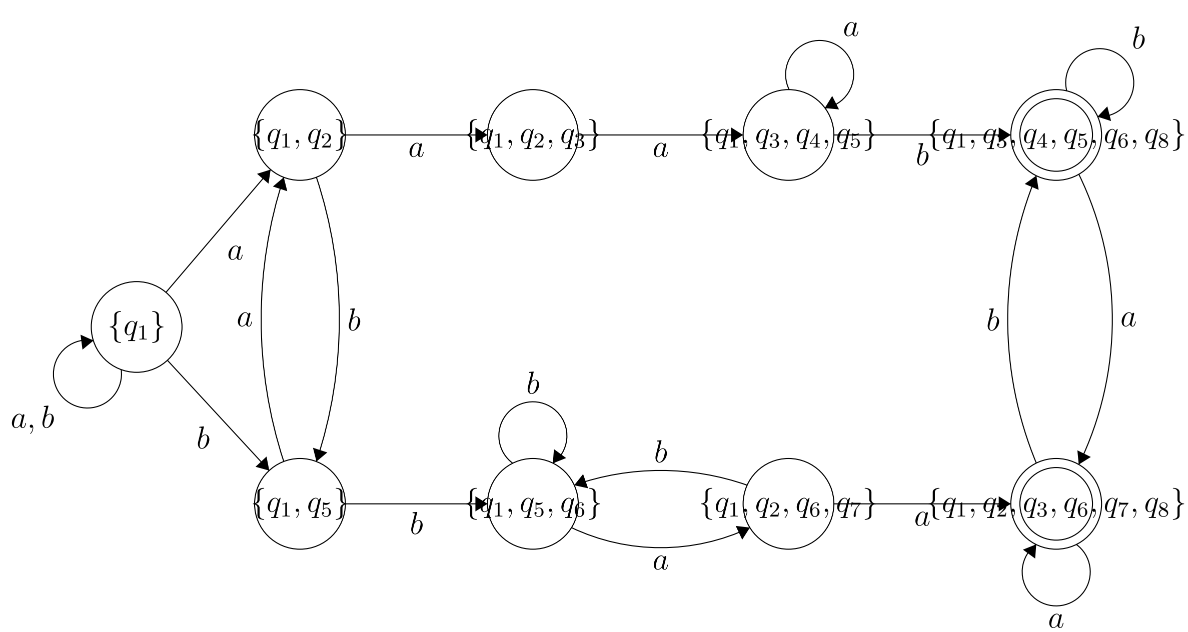
$$\delta(\{q_1, q_2, q_3, q_6, q_7, q_8\}, a) =$$

$$E(q_1) \cup E(q_2) \cup E(q_3) \cup E(q_6) \cup E(q_7) \cup E(q_8) = \{q_1, q_2, q_3, q_6, q_7, q_8\}$$

$$(q_1, b, q_1), (q_1, b, q_5), (q_3, b, q_3), (q_3, b, q_4)(q_6, b, q_6), (q_8, b, q_8) \in \Delta$$

$$\delta(\{q_1, q_2, q_3, q_6, q_7, q_8\}, b) =$$

$$E(q_1) \cup E(q_3), E(q_4) \cup E(q_5) \cup E(q_6) \cup E(q_8) = \{q_1, q_3, q_4, q_5, q_6, q_8\}$$



d)

For NFA:

$$(q_1, bbabb) \vdash_M (q_5, babb) \vdash_M (q_6, abb) \vdash_M (q_6, bb) \vdash_M (q_6, b) \vdash_M (q_6, e)$$

For DFA:

$$(\{q_1\}, bbabb) \vdash_{M'} (\{q_1, q_5\}, babb) \vdash_{M'} (\{q_1, q_5, q_6\}abb) \vdash_{M'}$$
$$(\{q_1, q_2, q_6, q_7\}, bb) \vdash_{M'} (\{q_1, q_5, q_6\}, b) \vdash_{M'} (\{q_1, q_5, q_6\}, e)$$

Since string "$bbabb$" does not lead to any final state, it is not accepted by the automatons.

Q2

a)

Assume L_1 is a regular languge. Then by the pumping lemma for any string $w \in L_1$ with $\mid w \mid \geq n$ can be written as $w = xyz$ such that $y \neq e$, $\mid xy \mid \leq n$ and $xy^iz \in L_1$ for each $i \geq 0$. Let's choose string $w = a^k b^l$ where $k > l$. Let $w = xyz$ since $|w| \geq k$ then $|xy|$ must be less or equal to k. y can be represented as a^s where $s \leq k$. Then for $i = 0$, string $x(a^s)^i z = xz = a^{k-s}b^l \notin L_1$ contradicts the theorem, therefore L_1 is not regular. Since L_1 is not regular, because of the closure properties of regular languages L_2 is not also a regular language.

b)

L_5 includes all strings starting with any number of a's followed by any number of b's. On the other hand L_4 only includes strings starting with any number of a's followed by same number of b's. Therefore L_4 is a subset of L_5. Then, $L_4 \cup L_5 = L_5$. L_5 can also be written as $L_5 = a^*b^*$. Since L_5 can be written in form of regular expressions it is a regular language. L_6 is written in form of regular expressions, therefore L_6 is also a regular language.

$$L_4 \cup L_5 \cup L_6 = L_5 \cup L_6$$

By the closure properties of regular languages, if L_5 and L_6 is regular then $L_5 \cup L_6$ must be regular.