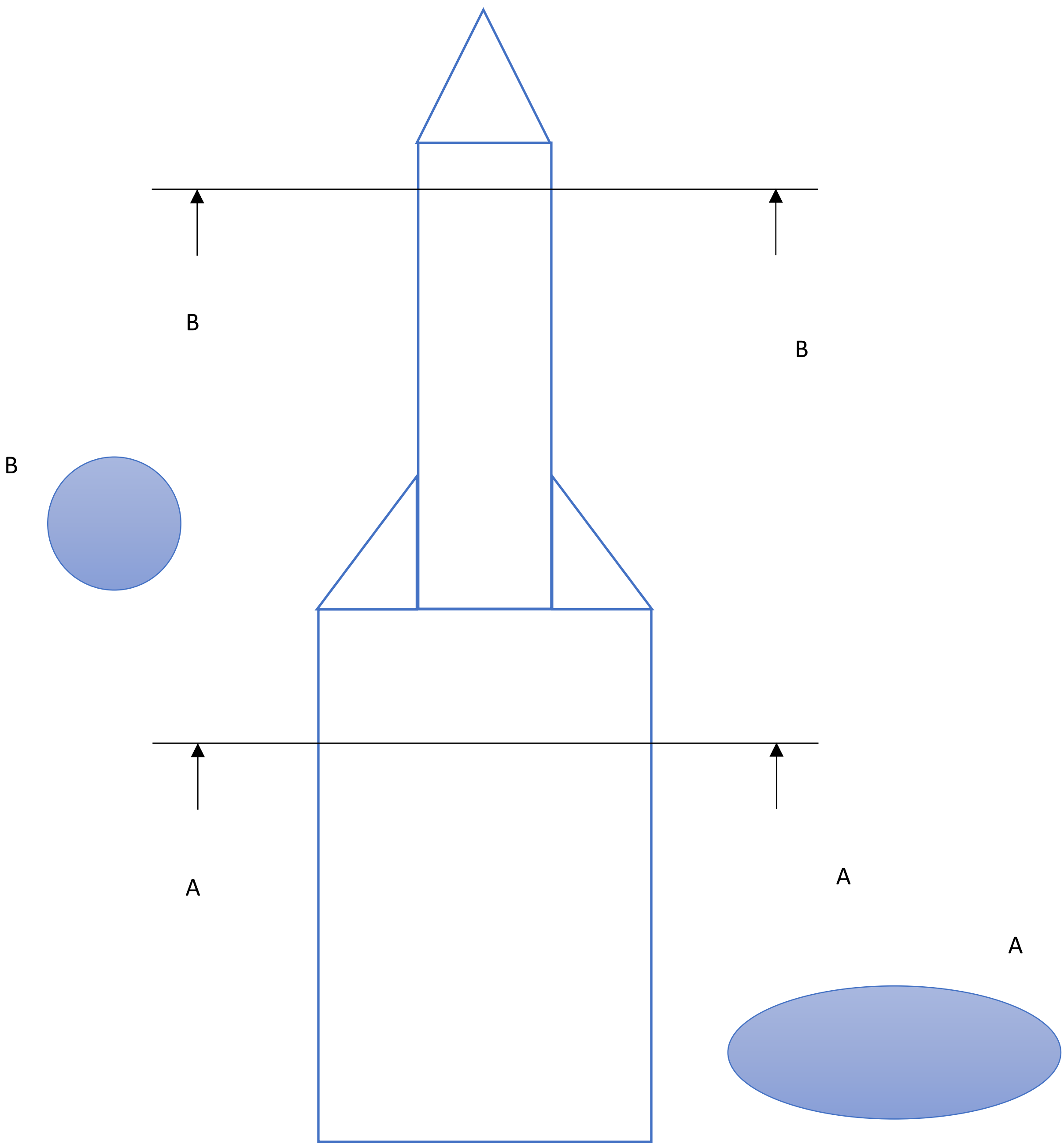
AERODYNAMICS

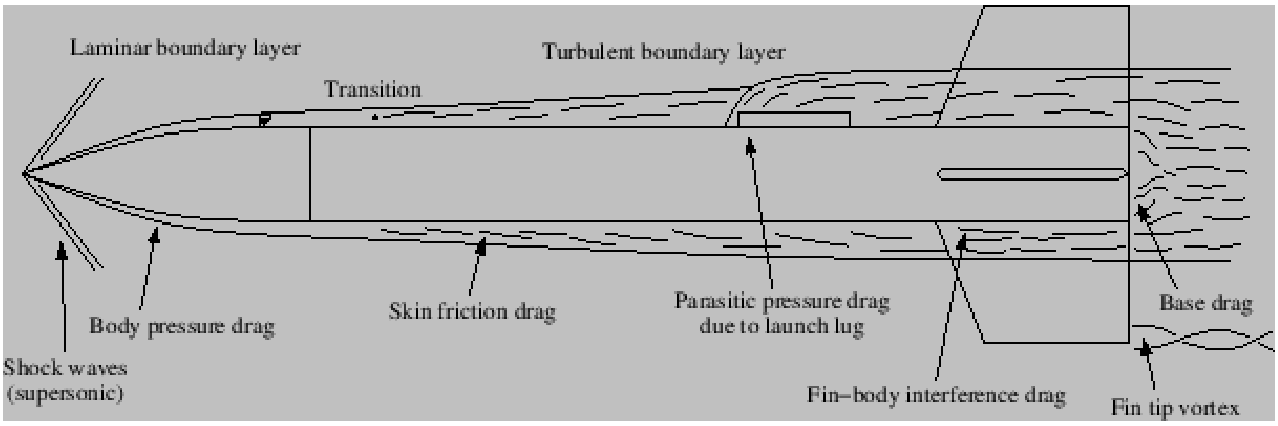
In this chapter we will analyse the Ariane 5 ECA rocket from the aerodynamics point of view. Aerodynamics is a very complex aspect to model during a design phase. The behaviour of the flow around a rocket is very complicated and difficult to foresee. For these reasons we will analyse the rocket in a simplistic way, since it is a conceptual design. First, we need to model the rocket in a way simple to manage. In Figure 1 is represented the model we will adopt.



We can notice that the forward part has been maintain as in the reality, removing all the minor parts which characterize the real rocket. For the second part is more difficult to model. For this reason, a simple approach is to consider the body as a lifting body with dimension *a* and *b*: in this way we have considered the first liquid stage and the two boosters as a unique body. We have removed all the complexities related to the strap on boosters (e.g. flares, connection with the main body, …).

1.1 Drag coefficient

At this point is important to evaluate the drag coefficient of all the rocket in the first part of its mission, when the boosters are in operation. The main contributions to the overall drag coefficient are [Figure 2]: wave drag, skin friction and base drag. We neglect all the minor contributions, such as the parasitic drag.



1.1.1 Wave drag

The wave drag is related to the shock wave generated in front of a body, which is moving at supersonic speed (M > 1): we consider only the contribution related to the supersonic flight range, neglecting the effect in the transonic region. We adopt an empirical relation given by Bonney, valid for a sharp nose:

$$c_{D0}^{body\ wave} = \left(1.586 + \frac{1.834}{M^2}\right)\left[\tan^{-1}\left(\frac{0.5}{\frac{l_N}{d_B}}\right)\right]^{1.69} \qquad M > 1$$

Since the nose of the rocket is rounded we can do something more. We can consider a circle that approximates at best the roundness and we can compute two different drag coefficients, and at the end we will make a weighted average. First, we consider a that nose as sharp and we evaluate the respective coefficient with the Bonney relation:

$$\left(c_{D0\ s}^{body\ wave}\right)_{mb} = \left(1.586 + \frac{1.834}{M^2}\right)\left[\tan^{-1}\left(\frac{0.5}{\frac{l_N}{d_B}}\right)\right]^{1.69}$$

Then we use the same relation to evaluate the drag coefficient for a fictitious hemispherical nose with a fineness ratio equal to 0.5, obtaining:

$$\left(c_{D0\,h}^{body\,wave}\right)_{mb} = \left(1.586 + \frac{1.834}{M^2}\right) * 0.665$$

Finally, we can evaluate the resulting wave drag coefficient:

$$\left(c_{D0}^{body\,wave}\right)_{mb} = \left(c_{D0\,s}^{body\,wave}\right)_{mb} * \frac{A_{ref} - A_{nose}}{A_{ref}} + \left(c_{D0\,h}^{body\,wave}\right)_{mb} * \frac{A_{nose}}{A_{ref}}$$

Also on the boosters there is the generation of a shock wave, but is much more complicated to evaluate. However, we can assume that their noses can be approximated as sharp, and we also assume to adopt the Bonney relation. In this way, each booster contributes to the overall wave drag coefficient as:

$$\left(c_{D0}^{body\,wave}\right)_{b} = \left(1.586 + \frac{1.834}{M^2}\right)\left[\tan^{-1}\left(\frac{0.5}{\frac{l_N}{d_B}}\right)\right]^{1.69} * \frac{A_{SRM}}{A_{ref}}$$

That coefficient has been weighted on the reference area. The overall wave drag coefficient is given by:

$$c_{D0}^{body\,wave} = \left(c_{D0}^{body\,wave}\right)_{mb} + 2 * \left(c_{D0}^{body\,wave}\right)_{b}$$

1.1.2 Skin friction

The skin friction drag is the major component for subsonic drag considering slender bodies. In this case it is easier to consider separately the main body and the two boosters. In practice, we evaluate the skin drag coefficient for the two and then we sum them. In this case we use the following empirical relation:

$$c_{D0}^{body\,friction} = 0.053 * \left(\frac{l}{d}\right) * \left(\frac{M}{q * l}\right)^{0.2} \qquad q\;[fps] \quad l\;[ft]$$

Applying that relation, we obtain:

$$\left(c_{D0}^{body\ friction}\right)_{mb} = 0.053 * \left(\frac{l}{d}\right) * \left(\frac{M}{q * l}\right)^{0.2}$$

$$\left(c_{D0}^{body\ friction}\right)_{b} = 0.053 * \left(\frac{l}{d}\right) * \left(\frac{M}{q * l}\right)^{0.2} * \frac{A_{SRM}}{A_{ref}}$$

$$c_{D0}^{body\ friction} = \left(c_{D0}^{body\ friction}\right)_{mb} + 2 * \left(c_{D0}^{body\ friction}\right)_{b}$$

This coefficient depends also on the dynamic pressure, so on the precise flight condition.

1.1.3 Base drag

The base drag is a pressure-based contribution and depends on the interaction between the flow and the nozzle section of the launcher. First, we consider the following quantities:

$$\bar{A}_{ref} = A_{ref} + 2 * A_{SRM}$$

$$\bar{A}_{e} = A_{e\ EPC} + 2 * A_{e\ SRM}$$

We have concentrated all the reference areas and nozzle exit areas in two values respectively. Then we adopt the following empirical model, considering that during all the first phase all the motors are working:

$$c_{D0}^{base} = (0.12 + 0.13 * M^2) * \left(1 - \frac{A_e}{A_{ref}}\right) \qquad M < 1$$

$$c_{D0}^{base} = \frac{0.25}{M} * \left(1 - \frac{A_e}{A_{ref}}\right) \qquad M > 1$$

Finally, we apply the two relations considering our reference area, and adopting the bar areas. Applying those relations, we obtain:

$$c_{D0}^{base} = (0.12 + 0.13 * M^2) * \left(1 - \frac{\bar{A}_e}{\bar{A}_{ref}}\right) * \frac{\bar{A}_{ref}}{A_{ref}}$$

$$c_{D0}^{base} = \frac{0.25}{M} * \left(1 - \frac{\bar{A}_e}{\bar{A}_{ref}}\right) * \frac{\bar{A}_{ref}}{A_{ref}}$$

1.2 Body normal force

The normal force coefficient is an important parameter, especially for the evaluation of the normal force effect on an aerodynamic model. We must evaluate it since it will be important; moreover, its derivative with respect to the angle of attack is important for the study of the cross-wind effect. From the model we have described previously, we notice that the cross-section changes along the rocket. What we can do, to obtain a result in a simple way, is to consider the coefficient of the two sections separately, and then we make a weighted average with the lengths. The empirical relation we adopt is the following:

$$|c_N| = \left[\frac{a}{b} * \cos^2\phi + \frac{b}{a} * \sin^2\phi\right]\left[\left|\sin(2\alpha) * \cos\left(\frac{\alpha}{2}\right)\right| + 1.3 * \frac{l}{d} * \sin^2\alpha\right]$$

At this point we have to distinguish among the two sections. For the main body we obtain:

$$(c_N)_{mb} = \left[\left|\sin(2\alpha) * \cos\left(\frac{\alpha}{2}\right)\right| + 1.3 * \frac{l}{d} * \sin^2\alpha\right]$$

While for the lifting part of the body:

$$(c_N)_l = \left[\frac{a}{b} * \cos^2\phi + \frac{b}{a} * \sin^2\phi\right]\left[\left|\sin(2\alpha) * \cos\left(\frac{\alpha}{2}\right)\right| + 1.3 * \frac{l}{d} * \sin^2\alpha\right]$$

Finally, we obtain:

$$|c_N| = \frac{(c_N)_{mb} * l_B + (c_N)_l * l_{SRM}}{l_B + l_{SRM}}$$

Since we are considering a rocket, we can adopt a simple relation to evaluate the derivative of the normal force coefficient with respect to the angle of attack, given by:

$$c_{N/\alpha} = 2 * \frac{A_b}{A_{ref}}$$

1.3 Aerodynamic efficiency

The aerodynamic efficiency is not an important parameter for a launcher, since it typically works away from its maximum value, due to the low angle of attack. In order to predict E, we adopt the following relation:

$$E = \frac{L}{D} = \frac{c_L}{c_D} = \frac{c_N * \cos\alpha - c_{D0} * \sin\alpha}{c_N * \sin\alpha + c_{D0} * \cos\alpha}$$

1.4 Stability

Considering the mass of all the stages concentrated in the middle, we can evaluate the position of the centre of mass of the rocket:

$$x_{CG} = \frac{x_F * M_F + x_{ESC-A} * M_{ESC-A} + x_{EPC} * M_{EPC} + 2 * x_{SRM} * M_{SRM}}{M_F + M_{ESC-A} + M_{EPC} + 2 * M_{SRM}} = 34.20\ [m]$$

For the evaluation of the position of the centre of pressure we adopt a relation coming from the slender body theory (sufficient for our rocket working at low angle of attack):

$$\frac{x_{CP}}{d} = \frac{2}{3} * \frac{l_N}{d} * \frac{A_{ref}}{A_b} + \left(1 - \frac{A_{ref}}{A_b}\right) * \frac{l_B}{d} \qquad \longrightarrow \qquad x_{CP} = 44.03\ [m]$$

Finally, we can evaluate the margin of stability:

$$\frac{x_{CG} - x_{CP}}{d} = -1.82 < 0$$

So, the rocket is statically stable at the launch pad.