Linear Algebra (Gilbert Strang) : 05. Transposes, Permutations, spaces R^n

☐ Permutations P :

↳ execute row exchanges.

$A = LU$

becomes $PA = LU$. // any invertible A.

↳ P = identity Matrix
with reordered rows.

→ possibilities = $n!$
counts possible reorderings
Counts all the $n \times n$ permutations.

$P^{-1} = P^T \quad P^{-1}P^T = I$.

☐ Transpose

↳ $\begin{bmatrix} 1 & 3 \\ 2 & 3 \\ 4 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 3 & 1 \end{bmatrix}$ ← R

R^T

$(A^T)_{ij} = A_{ji}$

Symmetric Matrices → $A^T = A$. ex) $\begin{bmatrix} 3 & 1 & 7 \\ 1 & 2 & 9 \\ 7 & 9 & 4 \end{bmatrix}$

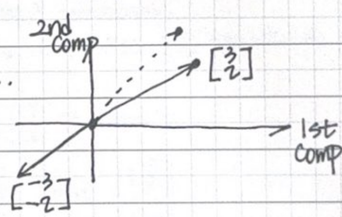
R^TR is always symmetric.

$\begin{bmatrix} 1 & 3 \\ 2 & 3 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 4 \\ 3 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 10 & 11 & 7 \\ 11 & & \\ 7 & & \end{bmatrix}$ → why? ↳ Take Transpose!

$(R^TR)^T = R^T(R^T)^T$
$= R^TR$

☐ Vector Spaces

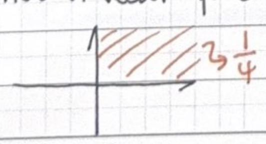
↳ Examples : R^2 = all 2-dim real vectors. $\begin{bmatrix} 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \pi \\ e \end{bmatrix}, \ldots$
= "x-y plane"



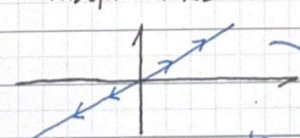
R^3 = all vectors with 3 components.

R^n = all column vectors with n real components

· not a Vector Space



· Subspace of R_2 : a vector space inside R_2.



you need this whole line.
multiplication · adding on the line.

Line in R_2 must go through a 0 vector

· Subspaces of R^2

→ 1) all of R^2

2) any line through $\begin{bmatrix}0\\0\end{bmatrix}$

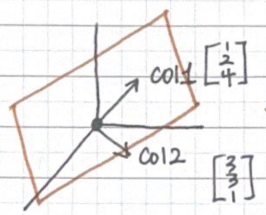
3) Zero vector only.

↳ because zero vector + zero vector = zero vector.
same with multiplication.

$A = \begin{bmatrix}1 & 3\\2 & 3\\4 & 1\end{bmatrix}$ → columns in R^3

all their linear combinations form a subspace.

↳ called Column Space $C(A)$



↳ when you take all their combinations you fill out the whole plane.