Linear Algebra (Gilbert Strang) : 01. The geometry of Linear Equations

☐ n linear equations, n unknowns

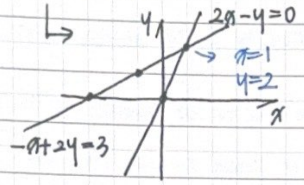
→ (when $n=2$)

$2x - y = 0$
$-x + 2y = 3$

$$\underbrace{\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 0 \\ 3 \end{bmatrix}}_{b}$$

Row Picture

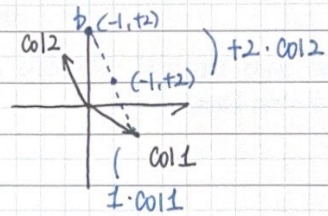
↳



Column Picture

↳ $x\begin{bmatrix} 2 \\ -1 \end{bmatrix} + y\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$ called as 'Linear Combination'

if we put $x=1$, $y=2$...



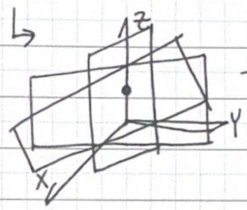
(when $n=3$)

$2x - y = 0$
$-x + 2y - z = -1$
$-3y + 4z = 4$

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -3 & 4 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}$$

Row Picture

↳



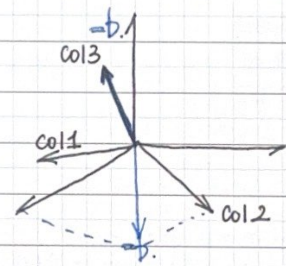
→ Consists of 3 planes.

Column Picture

↳ $x\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + y\begin{bmatrix} -1 \\ 2 \\ -3 \end{bmatrix} + z\begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}$

↓ $x=0$ $y=0$ $z=1$

if ans is $x=1$, $y=1$, $z=0$



☐ multiplying matrix by a vector

↳ $Ax = b$

$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$= 1\begin{bmatrix} 2 \\ 1 \end{bmatrix} + 2\begin{bmatrix} 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 12 \\ 7 \end{bmatrix}$

You can also use dot product.

! Ax is a combination of columns of A.

Q. Is there a solution for $Ax=b$ for every b?

= Do the linear combinations of the columns fill the 3-D space?

For the matrix above, the answer is Yes.
However, if all the column vectors are on the same plane, it would be a singular case.
= no solution.

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