Linear Algebra (Gilbert Strang) : 06. column space and nullspace

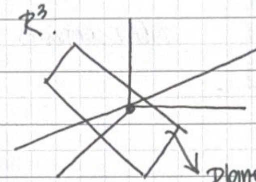
☐ Vector Spaces and Subspaces

↳ Vector Space Requirements: { $v+w$ and cv are in the space
all combs $cv+dw$ are in the space

! Subspaces muts go through an origin.

→ must solve $Ax=0$.

ex) R^3.



(L) line is a subspace.

plane through $\begin{bmatrix}0\\0\\0\end{bmatrix}$ is a subspace.
(P)

2 Subspaces : P and L.

$P \cup L$ = all vectors in P or L or both.

↳ This is not a subspace, because if you add a vector from P to a vector from L, it will go outside of the union.

$P \cap L$ = all vectors in both P and L

↳ This is a subspace.

↳ Column Space of a Matrix A: ↳ is a subspace of R^4

$$A = \begin{bmatrix}1&1&2\\2&1&3\\3&1&4\\4&1&5\end{bmatrix}$$ = all linear combinations of columns

Does $Ax=b$ always have a solution for every b?

↳ No. ex) 4 equations but only 3 unknowns.

$$Ax = \begin{bmatrix}1&1&2\\2&1&3\\3&1&4\\4&1&5\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}b_1\\b_2\\b_3\\b_4\end{bmatrix}$$

→ You can't solve 4 with only 3 unknowns.
However, there are b's that allow this equation to be solved.

→ $\begin{bmatrix}1\\2\\3\\4\end{bmatrix}$ when x is $\begin{bmatrix}1\\0\\0\end{bmatrix}$, it solves it.

Same with $\begin{bmatrix}1\\1\\1\\1\end{bmatrix}$ and $\begin{bmatrix}2\\3\\4\\5\end{bmatrix}$

∴ You can solve $Ax=b$ when the right hand side b is a vector in the column space ($C(A)$).

even if you throw away column 1 or column 3, you get the same column space.

column1 + column2 = column3.

→ 2 dimensional subspace of R^4.

↳ Nullspace of A :

↳ all solutions $x=\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$ to $Ax=0$.

$$Ax=\begin{bmatrix}1&1&2\\2&1&3\\3&1&4\\4&1&5\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\end{bmatrix}.$$

$N(A)$ contains $\begin{bmatrix}0\\0\\0\end{bmatrix}, \begin{bmatrix}1\\1\\-1\end{bmatrix} \cdots \quad \therefore c\begin{bmatrix}1\\1\\-1\end{bmatrix}$

$\rightarrow$ a line in R^3.

↳ to use the word space (for null space):

If $Ax=0$ and $Aw=0$, then $A(x+w)=0$.

$$\begin{bmatrix}1&1&2\\2&1&3\\3&1&4\\4&1&5\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}1\\2\\3\\4\end{bmatrix}$$

↓ solutions are a subspace? NO.

no zero vector.

$\begin{bmatrix}1\\0\\0\end{bmatrix}\begin{bmatrix}0\\-1\\1\end{bmatrix}\cdots$

↳ it is a plane or a line that doesn't go through the origin.