Introduction to Linear Algebra (Gilbert Strang) : 5^{th} Ed.

Problem set 2.1)

#2) The row picture is the same.
The solution in also same.
the column picture is changed.
the combination to give B is same.

#3) question1 + question2
$= (1x+0y+0z=2) + (2x+0y+0z=4)$
$(1x+1y+0z=5) + (0x+3y+0z=9)$
$(0x+0y+1z=4) + (0x+0y+4z=16)$

$= 3x+0y+0z=6 \quad \therefore x=2$
$1x+4y+0z=14 \quad \therefore y=3$
$0x+0y+5z=20 \quad \therefore z=4.$

$\therefore$ The solutions did not change;
but the row picture of row 2 is
changed, the colum picture is changed.

#5) if x, y, z satisfy the first two equations,
then the third equation is also satisfied
because the sum of the two equations
equals the third.
1) $y=1 \quad x=0 \quad z=1$
2) $y=1 \quad x=-1 \quad z=2$
3) $y=1 \quad x=1 \quad z=0$

#7) It is a singular, because the
first column and the third column are
the same.
(0, 1, 1), (1, 1, 0) are the combinations
of the columns that give $b=(2,3,5)$.
$c=10$ for solvability (same plane of the columns)

#9) a)
$$\begin{bmatrix} 1 & 2 & 4 \\ -2 & 3 & 1 \\ -4 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2+4+12 \\ -4+6+3 \\ -8+2+6 \end{bmatrix} = \begin{bmatrix} 18 \\ 5 \\ 0 \end{bmatrix}$$

b)
$$\begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2+1+0+0 \\ 1+2+1+0 \\ 0+1+2+2 \\ 0+0+1+4 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 5 \end{bmatrix}$$

#11)
$$\begin{bmatrix} 8+6 \\ 20+2 \end{bmatrix} = \begin{bmatrix} 14 \\ 22 \end{bmatrix}.$$

$$\begin{bmatrix} 6+(-6) \\ 12+(-12) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3+2+4 \\ 6+0+1 \end{bmatrix} = \begin{bmatrix} 9 \\ 7 \end{bmatrix}$$

#13) a) multiplies with n components
b) The planes from the m equations
are in n-d space. Combination of
the columns of A is in m dimensional
space.

Problem set 2.1)

#15) a) 2x2 $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

b) 2x2 $P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

#17) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} y \\ z \\ x \end{bmatrix}$

$\therefore P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} y \\ z \\ x \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

$\therefore Q = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

#19) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ x+z \end{bmatrix}$.

$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z-x \end{bmatrix}$

$E^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} \cdot E = \begin{bmatrix} 3 \\ 4 \\ 8 \end{bmatrix} \cdot E^{-1} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$

#21) $\begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\therefore R = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$

#25) $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad V = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

$A \cdot V = \begin{bmatrix} 4 \\ 4 \\ 4 \\ 4 \end{bmatrix}$

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#27) the row picture will show 2 planes in 3-D space.
the column picture is in 2-D space.
why 2-D? for column?

#29) $Au_1 = \begin{bmatrix} .8 & .3 \\ .2 & .7 \end{bmatrix} \begin{bmatrix} .8 \\ .2 \end{bmatrix}$

$= \begin{bmatrix} 0.64 + 0.06 \\ 0.16 + 0.14 \end{bmatrix} = \begin{bmatrix} 0.7 \\ 0.3 \end{bmatrix} = u_2$

$Au_2 = \begin{bmatrix} .8 & .3 \\ .2 & .7 \end{bmatrix} \begin{bmatrix} .7 \\ .3 \end{bmatrix}$

$= \begin{bmatrix} 0.56 & 0.09 \\ 0.14 & 0.21 \end{bmatrix} = \begin{bmatrix} 0.65 \\ 0.35 \end{bmatrix} = u_3$

The property for all u_0 to u_4 is that when you add the elements in the vector, it adds up to 1.