Linear Algebra (Gilbert Strang) : 14. Orthogonal Vectors and Subspaces

□ Orthogonal Vectors

↳ means : n-dimensional space the angle between those vectors is 90 degrees.

test for orthogonality : $\boxed{x^T \cdot y = 0}$

→ $\|x\|^2 + \|y\|^2 = \|x+y\|^2$ (not always true. True only if orthogonal)

$$\cancel{x^Tx} + \cancel{y^Ty} = (x+y)^T(x+y)$$

$$= \cancel{x^Tx} + \cancel{y^Ty} + x^Ty + y^Tx$$

x^Ty in the same as y^Tx

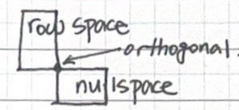
$$0 = 2x^Ty$$

□ Orthogonal Subspaces

↳ 'Subset S in orthogonal to subspace T'

means : every vector in S in orthogonal to every vector in T.

row space is orthogonal to nullspace.



→ why? : The nullspace has vectors that solve

$Ax = 0$ (x in nullspace). →
$$\begin{bmatrix} \text{row 1 of } A \\ \text{row 2 of } A \\ \vdots \\ \text{row m of } A \end{bmatrix} \begin{bmatrix} \\ x \\ \\ \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$+\begin{cases} c_1(\text{row1})^T x = 0 \\ c_2(\text{row2})^T x = 0 \end{cases}$$

↓

$$(c_1\text{row}_1 + c_2\text{row}_2 \cdots)^T x = 0$$

the combinations of rows are also in row space.

nullspace and row space are orthogonal Complements in R^n

↓

Nullspace contains all vectors perpendicular to row space.

□ Coming : $Ax = b$.

↳ "solve" $Ax = b$, when there is no solution.

$N(A^TA) = N(A)$

rank of A^TA = rank of A.

A^TA in invertible if $N(A)$ only has zero-vectors.

which means, the columns of A are independant.