Introduction to Linear Algebra (Gilbert Strang) : 5^{th} Ed

Problem set 1.3)

1) $3s_1 + 4s_2 + 5s_3 = b$.

$$\begin{bmatrix}3\\3\\3\end{bmatrix} + \begin{bmatrix}0\\4\\4\end{bmatrix} + \begin{bmatrix}0\\0\\5\end{bmatrix} = \begin{bmatrix}3\\7\\12\end{bmatrix} = b$$

$$\begin{bmatrix}1&0&0\\1&1&0\\1&1&1\end{bmatrix}\begin{bmatrix}3\\4\\5\end{bmatrix} = \begin{bmatrix}3\\7\\12\end{bmatrix}$$

2) $$\begin{bmatrix}1&0&0\\1&1&0\\1&1&1\end{bmatrix}\begin{bmatrix}y_1\\y_2\\y_3\end{bmatrix} = \begin{bmatrix}1\\1\\1\end{bmatrix},$$ $= y_1 = 1$, $y_1+y_2=1$ $y_2=0$, $y_1+y_2+y_3=1$ $y_3=0$

$$\begin{bmatrix}1&0&0\\1&1&0\\1&1&1\end{bmatrix}\begin{bmatrix}y_1\\y_2\\y_3\end{bmatrix} = \begin{bmatrix}1\\4\\9\end{bmatrix}$$ $= y_1=1$, $y_1+y_2=4$ $y_2=3$, $y_1+y_2+y_3=9$ $y_3=5$

4) $w_1 - 2w_2 + w_3 = 0$.

$$\begin{bmatrix}1\\2\\3\end{bmatrix} - 2\begin{bmatrix}4\\5\\6\end{bmatrix} + \begin{bmatrix}7\\8\\9\end{bmatrix}$$

$$= \begin{bmatrix}1\\2\\3\end{bmatrix} - \begin{bmatrix}8\\10\\12\end{bmatrix} + \begin{bmatrix}7\\8\\9\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}.$$

They are on the same plane.

$\therefore$ they are dependent

5) 1st set $= (y_1=1, y_2=-2, y_3=1)$

2nd set $= (y_1=2, y_2=-4, y_3=2)$

6) $$\begin{bmatrix}1&1&0\\3&2&1\\7&4&c\end{bmatrix} \rightarrow c=3 \quad \begin{bmatrix}1\\3\\7\end{bmatrix} - \begin{bmatrix}1\\2\\4\end{bmatrix} - \begin{bmatrix}0\\1\\3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}1&0&c\\1&1&0\\0&1&1\end{bmatrix} \rightarrow c=-1 \quad -\begin{bmatrix}1\\1\\0\end{bmatrix} + \begin{bmatrix}0\\1\\1\end{bmatrix} - \begin{bmatrix}-1\\0\\1\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}c&c&c\\2&1&5\\3&3&6\end{bmatrix} \rightarrow c=0. \quad \begin{bmatrix}0\\6\\9\end{bmatrix} - \begin{bmatrix}0\\1\\3\end{bmatrix} - \begin{bmatrix}0\\5\\0\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

7) $a_1 \cdot x_1 + a_2 \cdot x_2 + a_3 \cdot x_3 = 0$.

also, $r_1 \cdot x = 0$, $r_2 \cdot x = 0$, $r_3 \cdot x = 0$.

All three rows are perpendicular to x.

8) $$\begin{bmatrix}1&0&0&0\\-1&1&0&0\\0&-1&1&0\\0&0&-1&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = \begin{bmatrix}b_1\\b_2\\b_3\\b_4\end{bmatrix}$$

$x_1 = b_1$ | $x_1 = b_1$

$-x_1 + x_2 = b_2$ | $x_2 = b_1 + b_2$

$-x_2 + x_3 = b_3$ | $x_3 = b_2 + b_3$

$-x_3 + x_4 = b_4$ | $x_4 = b_3 + b_4$

$$A^{-1} = \begin{bmatrix}1&0&0&0\\1&1&0&0\\0&1&1&0\\0&0&1&1\end{bmatrix}\begin{bmatrix}b_1\\b_2\\b_3\\b_4\end{bmatrix} = A^{-1}b.$$

9) $$\begin{bmatrix}1&0&0&-1\\0&1&-1&0\\-1&0&1&0\\0&-1&0&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

$x_1 - x_4 = 0$

$x_2 - x_3 = 0$

$-x_1 + x_3 = 0$

$-x_2 + x_4 = 0$. $\rightarrow \therefore x_1 = x_2 = x_3 = x_4$.

$\therefore$ when $x = \begin{bmatrix}c\\c\\c\\c\end{bmatrix}$.

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10) $\begin{bmatrix} z_2 - z_1 \\ z_3 - z_2 \\ 0 - z_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}.$

Δ^{-1} in $z = \Delta^{-1} b$:

$= \begin{bmatrix} -1 & -1 & -1 \\ 0 & -1 & -1 \\ 0 & 0 & -1 \end{bmatrix}$

$z_2 - z_1 = b_1$

$z_3 - z_2 = b_2$

$0 - z_3 = b_3$

$z_3 = -b_3,\ z_2 = -b_2 - b_3,$

$,\ z_1 = -b_1 - b_2 - b_3.$

12) Invertible matrix $C =$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ -x_1 + x_3 \\ -x_2 + x_4 \\ -x_3 \end{bmatrix} = \begin{matrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{matrix}$$

$x_2 = b_1$

$x_3 = -b_4$

$x_1 = -b_2 - b_4$

$x_4 = b_1 + b_3$

$\rightarrow x = \begin{bmatrix} -b_2 - b_4 \\ b_1 \\ -b_4 \\ b_1 + b_3 \end{bmatrix}$