Problem set 2.7)

#1) $A = \begin{bmatrix} 1 & 0 \\ 9 & 3 \end{bmatrix}$ #9) $P_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ $P_2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$A^T = \begin{bmatrix} 1 & 9 \\ 0 & 3 \end{bmatrix}$ $A^{-1} = \begin{bmatrix} 1 & 0 \\ -3 & \frac{1}{3} \end{bmatrix}$

$(A^{-1})^T = \begin{bmatrix} 1 & -3 \\ 0 & \frac{1}{3} \end{bmatrix}$ $(A^T)^{-1} = \begin{bmatrix} 1 & -3 \\ 0 & \frac{1}{3} \end{bmatrix}$ $P_1 P_2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $P_2 P_1 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$A = \begin{bmatrix} 1 & C \\ C & 0 \end{bmatrix}$ $P_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ $P_4 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$A^T = \begin{bmatrix} 1 & C \\ C & 0 \end{bmatrix}$ $A^{-1} = \begin{bmatrix} 0 & \frac{1}{C} \\ \frac{1}{C} & -\frac{1}{C^2} \end{bmatrix}$

$(A^T)^{-1} = \begin{bmatrix} 0 & \frac{1}{C} \\ \frac{1}{C} & -\frac{1}{C^2} \end{bmatrix}$ $(A^{-1})^T = \begin{bmatrix} 0 & \frac{1}{C} \\ \frac{1}{C} & -\frac{1}{C^2} \end{bmatrix}$ $P_{34} = P_{43} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

#3) a) $((AB)^{-1})^T = (B^{-1}A^{-1})^T$ #11) $P_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$\qquad = (A^{-1})^T (B^{-1})^T$

b) U^{-1} is upper triangular,   #13) a) $P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$
so if you transpose it,
it will become lower triangular   b) $\hat{P} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

#5) a) $\begin{bmatrix} 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = 5$

#15) a) row 4 to row 1

b) $\begin{bmatrix} 4 & 5 & 6 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$

#7) a) True   #17) a) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$
b) False
c) True   b) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
D) True.

c) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

#19) a) $(n \times m) \cdot (m \times m) \cdot (m \times n)$

$$= n \times n$$

b) $(A^T A)_{jj} = A_j \cdot A_j = \text{Same number}^2.$

#33)

$$\begin{bmatrix} 1 & 2 \\ 4 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$$

↑ elementary   ↑ symmetric.

#21)

$$\begin{array}{ccc} 2 & 4 & 8 \\ 0 & -5 & -7 \\ 0 & -7 & -32 \end{array}$$

$$\begin{array}{ccc} 1 & b & c \\ 0 & d+b^2 & e-bc \\ 0 & e-bc & f-c^2 \end{array}$$

#23) $A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, $P = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

$L = I$ $U = I$

#25)

#27)

$$\begin{array}{cccc} 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 0 \\ 2 & 3 & 0 & 2 \\ 3 & 0 & 2 & 1 \end{array} \quad \text{Symmetric.}$$

#29) a) $A^T y = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} y_{BC} \\ y_{CS} \\ y_{BS} \end{bmatrix}$

$$= \begin{bmatrix} y_{BC} + y_{BS} \\ -y_{BC} + y_{CS} \\ -y_{CS} - y_{BS} \end{bmatrix}$$

b) $(Ax)^T y = y_{BC}(x_B - x_C) + y_{CS}(x_C - x_S) + y_{BS}(x_B - x_S)$

$x^T(A^T y) = x_B(y_{BC} + y_{BS}) + x_C(-y_{BC} + y_{CS}) + x_S(-y_{CS} - y_{BS})$

Both b.