## □ Linear Independence

↳ Suppose $A$ is $m$ by $n$ with $m<n$ (more columns than rows).

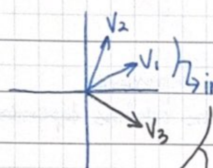
Then there are non-zero solutions to $Ax=0$. (more unknowns than eqns)

Reason: There will be free variables!

→ $n$ variables, at most $m$ pivots
→ at least one free variable.

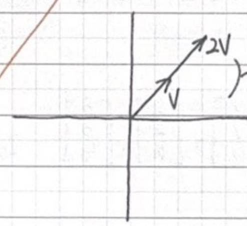
'Vectors $x_1, x_2, \cdots x_n$ are independant if no combination gives zero-vector (except for the zero comb)'

$$c_1x_1 + c_2x_2 + \cdots c_nx_n \neq 0.$$

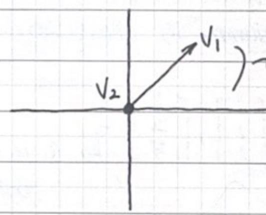



↳ independant.

when there is a 3rd equation in a 2-D space, it is no longer independant.




→ dependant. ↳ $2V_1 - V_2 = 0$. Comb that gives zero.




→ dependant. ↳ $0V_1 + 6V_2 = 0$ Comb that gives zero.

ex) $A = \begin{bmatrix} 2 & 1 & 2.5 \\ 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$

The columns are dependant if there is something in the null space.

Repeat: When $V_1, \cdots, V_n$ are columns of $A$.

They are independant if nullspace of $A$ is only the zero-vector.) : rank $= m$. no free variable

They are dependant if $A \cdot C = 0$ for some non-zero $C$. ) : rank $< n$ yes free variable.

## □ Span

↳ Vectors $V_1, \cdots, V_n$ span a space means :

→ The space consists of all combs of those vectors.

↳ "take all linear combinations and put them in a space"

□ Basis

↳ Basis for a space is a sequence of vectors
$v_1, v_2, \cdots, v_d$ with 2 properties.

1) they are independant
2) they span the space.

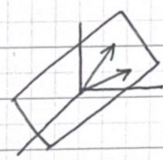
ex) Space is $\mathbb{R}^3$.

↳ one basis is $\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}$ → you can't add these up to make 0.
(only 0 vectors)

Another basis? $\begin{bmatrix}1\\1\\2\end{bmatrix}, \begin{bmatrix}2\\2\\5\end{bmatrix}, \begin{bmatrix}3\\3\\8\end{bmatrix}$ ↳ no! the 'rows' are dependant!
→ which makes the columns dependant.

$\mathbb{R}^n$ → n vectors give basis if the n×n matrix with those columns is invertible.

$\begin{bmatrix}1\\1\\2\end{bmatrix}, \begin{bmatrix}2\\2\\5\end{bmatrix}$ ↳ Q. Is there a space for which this is a basis?



A. Yes. ↳ 1) They are independant.
2) They are a plane. (span).

* Given a space:

Every basis for the space has the same number of vectors.
→ which tells you how big the space is.
how many vectors ~~do~~ you have to have to have a basis.
= Dimension of the space.

Space is C(A)  N(A)

$\begin{bmatrix}1&2&3&1\\1&1&2&1\\1&2&3&1\end{bmatrix}$  $\begin{bmatrix}-1\\-1\\1\\0\end{bmatrix}$ → 2 pivots (2 ranks).

span(o). not independant.

rank(A) = # of pivot columns
= dimension of the column space.

$\begin{bmatrix}1\\1\\2\end{bmatrix}, \begin{bmatrix}3\\4\\3\end{bmatrix}$
→ span이 된다.

N(A) = $\begin{bmatrix}-1\\-1\\1\\0\end{bmatrix}, \begin{bmatrix}-1\\0\\0\\1\end{bmatrix}$ # of free variables
= # of dimension of null space.