

Linear Algebra (Gilbert Strang): 03. Multiplication and Inverse Matrices

Matrix Multiplication

\rightarrow row 3 of A $\begin{bmatrix} a_{31} & a_{32} & \dots \end{bmatrix}$ $\begin{matrix} \text{column 4} \\ \begin{bmatrix} b_{14} \\ b_{24} \\ \vdots \end{bmatrix} \end{matrix} = \begin{bmatrix} \cdot \end{bmatrix} \rightarrow C_{34}$

$A_{m \times n} \quad B_{n \times p} = C_{m \times p}$

$C_{34} = (\text{row 3 of } A) \cdot (\text{column 4 of } B)$

$a_{31}b_{14} + a_{32}b_{24} + \dots = \sum_{k=1}^n a_{3k}b_{k4}$

$\begin{bmatrix} \quad \end{bmatrix}_{A_{m \times n}} \begin{bmatrix} \quad \quad \end{bmatrix}_{B_{n \times p}} = \begin{bmatrix} \quad \quad \end{bmatrix}_{C_{m \times p}}$

$\rightarrow A \cdot \text{col 1}$
 $\rightarrow A \cdot \text{col 2}$

\rightarrow columns of C are combinations of columns of A .

$\begin{bmatrix} \quad \quad \end{bmatrix}_{A_{m \times n}} \begin{bmatrix} \quad \quad \end{bmatrix}_{B_{n \times p}} = \begin{bmatrix} \quad \quad \end{bmatrix}_{C_{m \times p}}$

$\rightarrow \text{row } A \cdot B$

\rightarrow rows of C are combinations of rows of B .

Column of $A \times$ row of B
 $m \times 1 \quad 1 \times p$

$\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \begin{bmatrix} 1 & 6 \end{bmatrix} = \begin{bmatrix} 2 & 12 \\ 3 & 18 \\ 4 & 24 \end{bmatrix}$

\rightarrow all multiples of $\begin{bmatrix} 1 & 6 \end{bmatrix}$



$AB = \text{sum of (cols of } A) \times (\text{rows of } B)$

$\begin{bmatrix} 2 & 7 \\ 3 & 8 \\ 4 & 9 \end{bmatrix} \begin{bmatrix} 1 & 6 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \begin{bmatrix} 1 & 6 \end{bmatrix} + \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix} \begin{bmatrix} 0 & 0 \end{bmatrix}$

Block

$\rightarrow \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \begin{bmatrix} B_1 & B_2 \\ B_3 & B_4 \end{bmatrix} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$

$A \quad B$

$\rightarrow A_1B_1 + A_2B_3$

□ Inverses (Square Matrices)

$$\hookrightarrow A^{-1}A = I = AA^{-1}$$

↑

'if' it exists.

↳ invertible, non-singular } 'good' ones.

Singular case. No Inverse.

↳ ex) $\begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$ ↳ both of these are on the same line, so we can't get (1,0).

+ You can find a vector x with $Ax = 0$.

$$Ax = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow \text{You can never escape from 0 when you once get 0.}$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \begin{matrix} A \\ A^{-1} \end{matrix} \quad \rightarrow Ax \text{ column } j \text{ of } A^{-1} = \text{column } j \text{ of } I.$$

□ Gauss-Jordan (solve 2 equations at once)

↳

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 2 & 7 & 0 & 1 \end{array} \right] \rightarrow \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{array} \right] \rightarrow \left[\begin{array}{cc|cc} 1 & 0 & 7 & -3 \\ 0 & 1 & -2 & 1 \end{array} \right]$$

↑ we are going to make this an identity then the inverse will show up here.

↑ I

↑ A^{-1}

$$E[AI] = [IA^{-1}]$$

$$\hookrightarrow EA = I \text{ tells us } E = A^{-1}$$