Linear Algebra (Gilbert Strang) : 02. Elimination with Matrices.

☐ Elimination — Success / Failure

↳
$$\begin{aligned} x + 2y + z &= 2 \\ 3x + 8y + z &= 12 \\ 4y + z &= 2 \end{aligned} \rightarrow \underbrace{\begin{bmatrix} \boxed{1} & 2 & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{bmatrix}}_{A} \rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 0 & \boxed{2} & -2 \\ 0 & 4 & 1 \end{bmatrix} \rightarrow \begin{matrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & \boxed{5} \end{matrix}$$

$Ax = b$

1st Pivot. 2nd Pivot. 3rd Pivot

U. (upper triangular)

The whole point of elimination is to get to U from A.

! 0 cannot be in the position of pivot.

↳ if $A(2,2)$ was 6, or $A(3,3)$ was -4 it would have failed.

→ would have to switch. (row exchange).

☐ Back-Substitution

↳
$$\underbrace{\begin{matrix} 1 & 2 & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{matrix}}_{A} \left| \underbrace{\begin{matrix} 2 \\ 12 \\ 2 \end{matrix}}_{b} \right. \rightarrow \begin{matrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 4 & 1 \end{matrix} \left| \begin{matrix} 2 \\ 6 \\ 2 \end{matrix} \right. \rightarrow \underbrace{\begin{matrix} 1 & 2 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 5 \end{matrix}}_{U} \left| \underbrace{\begin{matrix} 2 \\ 6 \\ -10 \end{matrix}}_{c} \right. = \begin{aligned} x+2y+z&=2 \\ 2y+2z&=6 \\ 5z&=-10. \end{aligned} \quad \begin{aligned} x&=2 \\ y&=1 \\ z&=-2 \end{aligned}$$

U = what happens to A.

c = what happens to b.

Back-Substitution.

'simple step solving the equations in reverse order, because the system is triangular'.

☐ Matrices

↳
$$\begin{bmatrix} - & - & - \\ - & - & - \\ - & - & - \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \left. \begin{matrix} 3 \times col1 \\ 4 \times col2 \\ 5 \times col3 \end{matrix} \right) + . \qquad [1\ 2\ 7] \begin{bmatrix} - & - & - \\ - & - & - \\ - & - & - \end{bmatrix} = \left. \begin{matrix} 1 \times row1 \\ 2 \times row2 \\ 7 \times row3 \end{matrix} \right) +$$

matrix × column = column.

↳ linear combination of the rows.

Step1:
$$\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 4 & 1 \end{bmatrix}$$

E_{21}: used to get this (2,1) position

Step2:
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & \underline{0} & 5 \end{bmatrix}$$

E_{32}

→ $E_{32}(E_{21}A) = U$

→ $(E_{32}E_{21})A = U$. ↳ You 'can' move the parenthesis.

□ Permutation

↳ exchange 'rows' 1 and 2

$$\underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}_{P}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} c & d \\ a & b \end{bmatrix}$$

⇒ Permutation on the left

exchange 'columns'

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}_{P} = \begin{bmatrix} b & a \\ d & c \end{bmatrix}$$

⇒ Permutation on the right.

□ Inverses.

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{E^{-1}}\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{E} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{I}$$