Problem Set 2.5)

1) $\left[\begin{array}{cc|cc} 0 & 3 & 1 & 0 \\ 4 & 0 & 0 & 1 \end{array}\right]$

$\left[\begin{array}{cc|cc} 4 & 0 & 0 & 1 \\ 0 & 3 & 1 & 0 \end{array}\right]$

$A^{-1} = \begin{bmatrix} 0 & \frac{1}{4} \\ \frac{1}{3} & 0 \end{bmatrix}$

$B^{-1} = \begin{bmatrix} \frac{1}{2} & 0 \\ -1 & \frac{1}{2} \end{bmatrix}$

$C^{-1} = \begin{bmatrix} 7 & -4 \\ -5 & 3 \end{bmatrix}$

3) $10x + 20y = 1$
$20x + 50y = 0$

$2x + 4y = 2$
$-)\ 2x + 5y = 0$
$-y = 2$

$y = 2$, $x = 5$

$t + 2z = 0$
$2t + 5z = 1$

$2t + 4z = 0$
$-)\ 2t + 5z = 1$
$-z = -1$

$z = 1$, $t = -2$

$A^{-1} = \begin{bmatrix} 5 & -2 \\ 2 & 1 \end{bmatrix}$

5) $U = \begin{bmatrix} 1 & a \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & a \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

7) a) row1 + row2 = row3.
(row 1) $\cdot x$ and (row 2) $\cdot x$ is 0.
row3 is row1 + row2, and it can't add up to 1.

b) when $b_1 + b_2 = b_3$.

c) there is no 3rd pivot, so row 3 becomes a row of zeros.

9) It is invertible.

$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 8 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{cc|cc} 1 & 0 & 4 & -1 \\ 0 & 1 & -\frac{3}{2} & \frac{1}{2} \end{array}\right]$

$\left[\begin{array}{cc|cc} 3 & 8 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{cc|cc} 1 & 0 & -1 & 4 \\ 0 & 1 & \frac{1}{2} & -\frac{3}{2} \end{array}\right]$

Switch column 1 with column 2 in A^{-1} to find B^{-1}.

11) a) $\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}, \begin{bmatrix} -2 & -3 \\ -4 & -1 \end{bmatrix}$.

as above, if $B = -A$ $A + B$ is not invertible.

b) $c\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + c\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ is invertible

13) $M^{-1} = C^{-1}B^{-1}A^{-1}$.

$CM^{-1}A = B^{-1}$.

15) $\left[\begin{array}{cc|cc} 2 & 0 & 1 & 0 \\ 5 & 0 & 0 & 1 \end{array}\right]$

→ to be invertible, the matrix needs to have n pivots. but if you have a column of 0s, you don't have n pivots. you have less, which makes the matrix non invertible.

17) a) $E = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$

b) $L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} = E^{-1}$

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#21) $\begin{bmatrix}1&0\\0&1\end{bmatrix}, \begin{bmatrix}0&1\\1&0\end{bmatrix}, \begin{bmatrix}1&1\\1&0\end{bmatrix}, \begin{bmatrix}1&1\\0&1\end{bmatrix},$

$\begin{bmatrix}0&1\\1&1\end{bmatrix}, \begin{bmatrix}1&0\\1&1\end{bmatrix}$ 6 are invertible

#23)

$$[A\ I] = \begin{bmatrix}2&1&0&1&0&0\\1&2&1&0&1&0\\0&1&2&0&0&1\end{bmatrix}$$

$$\begin{bmatrix}2&1&0&1&0&0\\0&\frac{3}{2}&1&-\frac{1}{2}&1&0\\0&1&2&0&0&1\end{bmatrix}$$

$$\begin{bmatrix}2&1&0&1&0&0\\0&\frac{3}{2}&1&-\frac{1}{2}&1&0\\0&0&\frac{4}{3}&+\frac{1}{3}&-\frac{2}{3}&1\end{bmatrix}$$

$$\begin{bmatrix}2&1&0&1&0&0\\0&3&2&-1&2&0\\0&0&4&+1&-2&3\end{bmatrix}$$

$$\begin{bmatrix}2&1&0&1&0&0\\0&3&0&-\frac{3}{2}&3&-\frac{3}{2}\\0&0&4&1&-2&3\end{bmatrix}$$

$$\begin{bmatrix}2&0&0&\frac{3}{2}&-1&\frac{1}{2}\\0&3&0&-\frac{3}{2}&3&-\frac{3}{2}\\0&0&4&1&-2&3\end{bmatrix}$$

$$[I\ A^{-1}] = \begin{bmatrix}1&0&0&\frac{3}{4}&-\frac{1}{2}&\frac{1}{4}\\0&1&0&-\frac{1}{2}&1&-\frac{1}{2}\\0&0&1&\frac{1}{4}&-\frac{1}{2}&\frac{3}{4}\end{bmatrix}$$

#25)

$$[A\ I] = \begin{bmatrix}2&1&1&1&0&0\\1&2&1&0&1&0\\1&1&2&0&0&1\end{bmatrix}$$

$$\begin{bmatrix}2&1&1&1&0&0\\0&\frac{3}{2}&\frac{1}{2}&-\frac{1}{2}&1&0\\0&\frac{1}{2}&\frac{3}{2}&-\frac{1}{2}&0&1\end{bmatrix}$$

$$\begin{bmatrix}2&1&1&1&0&0\\0&\frac{3}{2}&\frac{1}{2}&-\frac{1}{2}&1&0\\0&0&\frac{4}{3}&-\frac{1}{3}&-\frac{1}{3}&1\end{bmatrix}$$

$$\begin{bmatrix}2&1&1&1&0&0\\0&3&1&-1&2&0\\0&0&4&-1&-1&3\end{bmatrix}$$

$$\begin{bmatrix}2&1&0&\frac{5}{4}&\frac{1}{4}&-\frac{3}{4}\\0&3&0&-\frac{3}{4}&\frac{9}{4}&-\frac{3}{4}\\0&0&4&-1&-1&3\end{bmatrix}$$

$$\begin{bmatrix}2&0&0&\frac{6}{4}&-\frac{2}{4}&-\frac{2}{4}\\0&3&0&-\frac{3}{4}&\frac{9}{4}&-\frac{3}{4}\\0&0&4&-1&-1&3\end{bmatrix}$$

$$[I\ A^{-1}] \begin{bmatrix}1&0&0&\frac{3}{4}&-\frac{1}{4}&-\frac{1}{4}\\0&1&0&-\frac{1}{4}&\frac{3}{4}&-\frac{1}{4}\\0&0&1&-\frac{1}{4}&-\frac{1}{4}&\frac{3}{4}\end{bmatrix}$$

If there is x other than zero vector that makes matrix 0, then it is not invertible so B is not invertible

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#27) $A=\begin{bmatrix}1&0&0\\2&1&3\\0&0&1\end{bmatrix}$

$$[I\ A^{-1}]=\left[\begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix}\ \ \begin{matrix}1&0&0\\-2&1&-3\\0&0&1\end{matrix}\right]$$

$A=\begin{bmatrix}1&1&1\\1&2&2\\1&2&3\end{bmatrix}$

$$[I\ A^{-1}]=\left[\begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix}\ \ \begin{matrix}2&-1&0\\-1&2&-1\\0&-1&1\end{matrix}\right]$$

#29) a) True. you must have n pivots to be invertible, but if you have a row of 0s, that would not be possible.

b) False. $\begin{bmatrix}1&0&0\\0&1&1\\0&1&1\end{bmatrix}$

c) True. Inverse of A is A^{-1}, of A^2 is $(A^{-1})^2$

#31) $A^{-1}=\begin{bmatrix}1&1&0&0\\0&1&1&0\\0&0&1&1\\0&0&0&1\end{bmatrix}$