Date

Problem Set 2.4)

1) $A = 3\times5$ $B = 5\times3$ $C = 5\times1$ $D = 3\times1$

$B\cdot A$ is allowed. Result $= 5\times5$

$A\cdot B$ is allowed $= 3\times3$

$A\cdot B\cdot D$ is allowed $= 3\times1$

$D\cdot C$ is not allowed

$A(B+C)$ is not allowed

3) compare $AB+AC$ and $A(B+C)$

$AB = \begin{bmatrix}1&5\\2&3\end{bmatrix}\begin{bmatrix}0&2\\0&1\end{bmatrix} = \begin{bmatrix}0&7\\0&7\end{bmatrix}$

$AC = \begin{bmatrix}1&5\\2&3\end{bmatrix}\begin{bmatrix}3&1\\0&0\end{bmatrix} = \begin{bmatrix}3&1\\6&2\end{bmatrix}$

$AB+AC = \begin{bmatrix}3&8\\6&9\end{bmatrix}$

$B+C = \begin{bmatrix}3&3\\0&1\end{bmatrix}$

$A(B+C) = \begin{bmatrix}1&5\\2&3\end{bmatrix}\begin{bmatrix}3&3\\0&1\end{bmatrix} = \begin{bmatrix}3&8\\6&9\end{bmatrix}$

It is the same.

5) when $A = \begin{bmatrix}1&b\\0&1\end{bmatrix}$: $A^2 = \begin{bmatrix}1&b\\0&1\end{bmatrix}\begin{bmatrix}1&b\\0&1\end{bmatrix} = \begin{bmatrix}1&2b\\0&1\end{bmatrix}$

$A^3 = \begin{bmatrix}1&2b\\0&1\end{bmatrix}\begin{bmatrix}1&b\\0&1\end{bmatrix} = \begin{bmatrix}1&3b\\0&1\end{bmatrix}$

$A^5 = \begin{bmatrix}1&5b\\0&1\end{bmatrix}$, $A^n = \begin{bmatrix}1&nb\\0&1\end{bmatrix}$

when $A = \begin{bmatrix}2&2\\0&0\end{bmatrix}$: $A^2 = \begin{bmatrix}2&2\\0&0\end{bmatrix}\begin{bmatrix}2&2\\0&0\end{bmatrix} = \begin{bmatrix}4&4\\0&0\end{bmatrix}$

$A^3 = \begin{bmatrix}4&4\\0&0\end{bmatrix}\begin{bmatrix}2&2\\0&0\end{bmatrix} = \begin{bmatrix}8&8\\0&0\end{bmatrix}$

$A^5 = \begin{bmatrix}32&32\\0&0\end{bmatrix}$, $A^n = \begin{bmatrix}2^n&2^n\\0&0\end{bmatrix}$

7) a) True

$\begin{bmatrix}1&3\\4&2\end{bmatrix}\begin{bmatrix}1&2&1\\1&2&1\end{bmatrix} = \begin{bmatrix}4&8&4\\6&12&6\end{bmatrix}$

b) True

$\begin{bmatrix}1&2&3\\4&5&6\end{bmatrix}\begin{bmatrix}1&1\\2&2\\1&1\end{bmatrix} = \begin{bmatrix}8&8\\20&20\end{bmatrix}$

c) True

$\begin{bmatrix}1&1&1\\2&2&2\\1&1&1\end{bmatrix}\begin{bmatrix}1&2&3\\4&5&6\\7&8&9\end{bmatrix} = \begin{bmatrix}12&15&18\\24&30&36\\12&15&18\end{bmatrix}$

$\begin{bmatrix}12&15&18\\24&30&36\\12&15&18\end{bmatrix}\begin{bmatrix}1&1&2\\1&2&1\\2&1&1\end{bmatrix} = \begin{bmatrix}63&60&57\\126&120&114\\63&60&57\end{bmatrix}$

d) False

$A = \begin{bmatrix}1&2\\3&4\end{bmatrix}$ $B = \begin{bmatrix}5&6\\7&8\end{bmatrix}$

$(AB)^2 = ABAB$

$= \begin{bmatrix}19&22\\43&50\end{bmatrix}\begin{bmatrix}19&22\\43&50\end{bmatrix}$

$= \begin{bmatrix}1307&1518\\2967&3446\end{bmatrix}$

$A^2B^2 = AABB$

$= \begin{bmatrix}7&10\\15&22\end{bmatrix}\begin{bmatrix}67&78\\91&106\end{bmatrix}$

$= \begin{bmatrix}1379&1606\\3007&3502\end{bmatrix}$

9) a) $AF = \begin{bmatrix}a&b\\c&d\end{bmatrix}\begin{bmatrix}1&1\\0&1\end{bmatrix} = \begin{bmatrix}a&a+b\\c&c+d\end{bmatrix}$

$E(AF) = \begin{bmatrix}1&0\\1&1\end{bmatrix}\begin{bmatrix}a&a+b\\c&c+d\end{bmatrix} = \begin{bmatrix}a&a+b\\a+c&a+b+c+d\end{bmatrix}$

b) $E(AF)$ equals $(EA)F$.

associative is obeyed by matrix multiplication

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13) $AB = BA$, $AC = CA$

$AB = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} a & 0 \\ c & 0 \end{bmatrix}$

$BA = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}$

$\begin{bmatrix} a & 0 \\ c & 0 \end{bmatrix} = \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}$. $\therefore b=0, c=0$

$AC = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & a \\ 0 & c \end{bmatrix}$

$CA = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} c & d \\ 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 0 & a \\ 0 & c \end{bmatrix} = \begin{bmatrix} c & d \\ 0 & 0 \end{bmatrix}$ $\therefore c=0$, $\underline{a=d}$

$A = d\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

15) a) True. $3\times\boxed{3} \cdot \boxed{3}\times 3$

b) False $4\times 3 \cdot 3\times 4$, $4\times 3 \cdot 3\times 4$

c) True "

d) False. if $B=0$, A can be anything.

17) a) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$ row2 $= \begin{bmatrix} 0 & 1 \end{bmatrix}$

D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$ row2 $= \begin{bmatrix} 3 & -2 \end{bmatrix}$

19) a) $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \rightarrow$ Diagonal Matrix

b) $\begin{bmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{bmatrix} \rightarrow$ Lower Triangular Matrix

c) $\begin{bmatrix} a & d & e \\ d & b & f \\ e & f & c \end{bmatrix} \rightarrow$ symmetric

d) $\begin{bmatrix} a & b & c \\ a & b & c \\ a & b & c \end{bmatrix} \rightarrow$ all rows are equal.

21) $A^2 = \begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$A^3 = \begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$A^4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$AV = \begin{bmatrix} 2y \\ 2z \\ 2t \\ 0 \end{bmatrix}$ $A^2V = \begin{bmatrix} 4z \\ 4t \\ 0 \\ 0 \end{bmatrix}$

$A^3V = \begin{bmatrix} 8t \\ 0 \\ 0 \\ 0 \end{bmatrix}$ $A^4V = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

23) nonzero matrix. $A^2=0$

$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = 0.$

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25) $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} a \\ d \\ g \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} b \\ e \\ h \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \end{bmatrix} + \begin{bmatrix} c \\ f \\ i \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} a & 0 & 0 \\ d & 0 & 0 \\ g & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & b & 0 \\ 0 & e & 0 \\ 0 & h & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & c \\ 0 & 0 & f \\ 0 & 0 & i \end{bmatrix}$

$= \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

27) (row 3 of A) · (column 1 of B),
(row 3 of A) · (column 2 of B).
gives zeros

$\begin{bmatrix} x \\ x \\ 0 \end{bmatrix}\begin{bmatrix} 0 & x & x \end{bmatrix} = \begin{bmatrix} 0 & x & x \\ 0 & x & x \\ 0 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} x \\ x \\ x \end{bmatrix}\begin{bmatrix} 0 & 0 & x \end{bmatrix} = \begin{bmatrix} 0 & 0 & x \\ 0 & 0 & x \\ 0 & 0 & x \end{bmatrix}$

29) $E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $E_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix}$

$E = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix}$

$EA = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 0 \\ -2 & 0 & 1 \\ 8 & 5 & 3 \end{bmatrix}$

$= \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 3 \end{bmatrix}$

31) $\begin{bmatrix} A & -B \\ B & A \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} Ax - By \\ Bx + Ay \end{bmatrix}\begin{matrix} \text{real} \\ \text{imaginary} \end{matrix}$.

32) A times X would be

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

33) $A\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $A\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $A\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$A\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$

$b = \begin{bmatrix} 3 \\ 5 \\ 8 \end{bmatrix}$ $x = 3x_1 + 5x_2 + 8x_3$.

$= \begin{bmatrix} 3 \\ 8 \\ 16 \end{bmatrix}$