□ Computing the Nullspace ($AX=0$)

↳

$$A=\begin{bmatrix}1&2&2&2\\2&4&6&8\\3&6&8&10\end{bmatrix}$$ 3rd row is dependant.

in the same direction. dependant.

① You do elimination

② find the r. and the number of free variables

$$\begin{bmatrix}1&2&2&2\\0&0&2&4\\0&0&2&4\end{bmatrix} \rightarrow \begin{bmatrix}1&2&2&2\\0&0&2&4\\0&0&0&0\end{bmatrix}=U.$$

2 pivot columns. → you can solve the equations with x_1, x_3.

2 free columns. ↳ you can assign any number to x_2, x_4.

Rank of A = # of pivots $=2$.

$$X=c\begin{bmatrix}\square\\1\\\square\\0\end{bmatrix}$$

$x_1+2x_2+2x_3+2x_4=0 \rightarrow x_1=-2$

$2x_3+4x_4=0$ ↳ $x_3=0$

↳ $\boxed{UX=0}$

임의로 지정. free column이니까. you can give any variable.

any multiple of this is in the null space.

"The null space contains exactly all the combinations of the special solutions".

□ Reduced Row Echelon Form

↳ has zeros above and also under pivots.

$$\begin{bmatrix}1&2&2&2\\0&0&2&4\\0&0&0&0\end{bmatrix} ↳ \begin{bmatrix}1&2&0&-2\\0&0&2&4\\0&0&0&0\end{bmatrix} \rightarrow \begin{bmatrix}\boxed{1}&2&0&-2\\0&0&\boxed{1}&2\\0&0&0&0\end{bmatrix}=R.=\mathrm{rref}(A)$$

multiplying α won't change the solutions

notice $I=\begin{bmatrix}1&0\\0&1\end{bmatrix}$ in pivot rows/columns

↳ $\boxed{RX=0}$

$\begin{bmatrix}2&-2\\0&2\end{bmatrix}$ ↳ free cols. F.

rref form

$$R=\begin{bmatrix}I&F\\0&0\end{bmatrix}$$ r pivot rows

r pivot cols, n−r free cols

$RN=0$ ↳ $N=\begin{bmatrix}-F\\I\end{bmatrix}$

nullspace matrix (columns = special sols).

$\therefore RX=0$

$$\begin{bmatrix}I&F\end{bmatrix}\begin{bmatrix}X_{pivot}\\X_{free}\end{bmatrix}=0$$

$X_{pivot}=-FX_{free}$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 2 & 6 & 8 \\ 2 & 8 & 10 \end{bmatrix}$$

→ we can expect that we will only have 2 pivot columns, because col1 + col2 = col3.

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 4 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} \boxed{1} & 2 & 3 \\ 0 & \boxed{2} & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = U.$$

↳ rank is 2.

pivot cols. free col.

$Ux = 0$ (free variable = 1) $\rightarrow x = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$

whole nullspace $= c\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} = c\begin{bmatrix} F \\ I \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = R$$

I

↳ F

zeros.