

Problem Set 2.2)

(1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 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799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 979, 980, 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1136, 1137, 1138, 1139, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1198, 1199, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 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1583, 1584, 1585, 1586, 1587, 1588, 1589, 1589, 1590, 1591, 1592, 1593, 1594, 1595, 1596, 1597, 1598, 1598, 1599, 1599, 1600, 1601, 1602, 1603, 1604, 1605, 1606, 1607, 1608, 1609, 1609, 1610, 1611, 1612, 1613, 1614, 1615, 1616, 1617, 1618, 1618, 1619, 1619, 1620, 1621, 1622, 1623, 1624, 1625, 1626, 1627, 1628, 1629, 1629, 1630, 1631, 1632, 1633, 1634, 1635, 1636, 1637, 1638, 1639, 1639, 1640, 1641, 1642, 1643, 1644, 1645, 1646, 1647, 1648, 1649, 1649, 1650, 1651, 1652, 1653, 1654, 1655, 1656, 1657, 1658, 1659, 1659, 1660, 1661, 1662, 1663, 1664, 1665, 1666, 1667, 1668, 1669, 1669, 1670, 1671, 1672, 1673, 1674, 1675, 1676, 1677, 1678, 1679, 1679, 1680, 1681, 1682, 1683, 1684, 1685, 1686, 1687, 1688, 1689, 1689, 1690, 1691, 1692, 1693, 1694, 1695, 1696, 1697, 1698, 1698, 1699, 1699, 1700, 1701, 1702, 1703, 1704, 1705, 1706, 1707, 1708, 1709, 1709, 1710, 1711, 1712, 1713, 1714, 1715, 1716, 1717, 1718, 1718, 1719, 1719, 1720, 1721, 1722, 1723, 1724, 1725, 1726, 1727, 1728, 1729, 1729, 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1879, 1880, 1881, 1882, 1883, 1884, 1885, 1886, 1887, 1888, 1889, 1889, 1890, 1891, 1892, 1893, 1894, 1895, 1896, 1897, 1898, 1898, 1899, 1899, 1900, 1901, 1902, 1903, 1904, 1905, 1906, 1907, 1908, 1909, 1909, 1910, 1911, 1912, 1913, 1914, 1915, 1916, 191

Problem set 2.2)

$$13) \begin{cases} 2x+3y+z=8 \\ 4x+7y+5z=20 \\ -2y+2z=0 \end{cases}$$

$$\rightarrow \begin{cases} 2x+3y+z=8 \\ y+3z=4 \\ -2y+2z=0 \end{cases}$$

$$\rightarrow \begin{cases} 2x+3y+z=8 \\ y+3z=4 \\ 8z=0 \end{cases}$$

Pivots $\therefore z=1, y=1, x=2$

15) If $b=-2$, it leads to row exchange.

$$\begin{cases} x-2y=0 \\ x-2y-z=0 \\ y+z=0 \end{cases} \rightarrow \begin{cases} x-2y=0 \\ -z=0 \\ y+z=0 \end{cases}$$

row exchange needed.

$$\begin{cases} x-y=0 \\ x-2y-z=0 \\ y+z=0 \end{cases} \rightarrow \begin{cases} x-y=0 \\ 0-y-z=0 \\ 0+0+0=0 \end{cases}$$

$$x-y=0 \rightarrow x=y \quad \therefore x=y=-z$$

$$-y-z=0 \rightarrow -z=y \quad x=4, y=4, z=-4$$

(when row1= row2)

$$17) \begin{cases} 2x-y+z=0 \\ 2x-y+z=0 \\ 4x+y+z=0 \end{cases} \rightarrow \begin{cases} 2x-y+z=0 \\ 0+0+0=0 \\ 4x+y+z=0 \end{cases}$$

$$\rightarrow \begin{cases} 2x-y+z=0 \\ 3y-z=0 \\ 0+0+0=0 \end{cases}$$

(when col1= col2)

$$\begin{cases} 2x+2y+z=0 \\ 4x+4y+z=0 \\ 6x+6y+z=2 \end{cases} \rightarrow \begin{cases} 2x+2y+z=0 \\ 0+0+0=0 \\ 0+0-2z=2 \end{cases}$$

\rightarrow the y pivot (second pivot) is missing.

19) to make the system singular and infinitely many solutions:

$$\begin{cases} x+4y-2z=1 \\ x+4y-2z=1 \\ x+7y-6z=6 \end{cases} \rightarrow \begin{cases} x+4y-2z=1 \\ 3y-4z=5 \\ 3y+9z=t \end{cases}$$

\rightarrow when $g=-4, t=5$
no third pivot. $0=0$. infinite solutions.

$(AX=B)$

$$\begin{cases} 2x+y=0 \\ x+2y+z=0 \\ y+2z+t=0 \\ z+2t=5 \end{cases}$$

$$\rightarrow \begin{cases} 2x+y=0 \rightarrow 2x+y=0 \\ \frac{3}{2}y+z=0 \\ y+2z+t=0 \\ z+2t=5 \end{cases}$$

$$\therefore x=4, y=-3, z=2, t=-1$$

Pivots

Problem set 2.2)

(1st = b)

$$21) \begin{cases} 2a - y = 0 \\ -y + 2z - t = 0 \\ -y + 2z - t = 0 \\ -z + 2t = 5 \end{cases}$$

$$\Rightarrow \begin{cases} 2a - y = 0 \\ \frac{2}{3}y - z = 0 \\ -4 + 2z - t = 0 \\ -z + 2t = 5 \end{cases} \Rightarrow \begin{cases} 2a - y = 0 \\ \frac{2}{3}y - z = 0 \\ \frac{4}{3}z - t = 0 \\ -z + 2t = 5 \end{cases}$$

$$\therefore t = 4, z = 3, y = 2, a = 1$$

$$25) A = \begin{bmatrix} a & 2 & 3 \\ a & a & 4 \\ a & a & a \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a & 2 & 3 \\ 0 & a-2 & 1 \\ a-2 & a-3 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} a & 2 & 3 \\ 0 & a-2 & 1 \\ 0 & 0 & a-4 \end{bmatrix}$$

when $a=0, a=2, a=4$, elimination will fail to give three pivots.

22) if you follow the 1,2,1 pattern of question 21, 27)

5th pivot = 5, nth pivot - if odd = -n

if even = n

$$\begin{cases} 3a = 3 \\ 6a + 2y = 8 \\ 9a - 2y + z = 9 \end{cases}$$

if you follow the -1,2,-1 pattern of question 21,

5th pivot = 5, nth pivot = n.

$$\Rightarrow \begin{cases} 3a = 3 \\ 2y = 2 \\ -2y + z = 0 \end{cases} \Rightarrow \begin{cases} 3a = 3 \\ 2y = 2 \\ z = 2 \end{cases}$$

23)

$$\begin{cases} a + y = 1 \\ 0 + 2y = 3 \end{cases} \rightarrow \text{what can be the original?}$$

$$1) \begin{cases} a + y = 1 \\ 2a + 4y = 5 \end{cases} \quad 2) \begin{cases} a + y = 1 \\ 2a + 5y = 6 \end{cases}$$

$$3) \begin{cases} a + y = 1 \\ 4a + 6y = 7 \end{cases}$$

$$U = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \text{ solution} = (1, 1, 2)$$