Linear Algebra (Gilbert Strang) : 04. Factorization into A=LU

□ Inverse of AB

↳

$(AB)(B^{-1}A^{-1}) = I$ $\quad AA^{-1} = I$

$(B^{-1}A^{-1})(AB) = I$ $\quad \underline{(A^{-1})^T} A^T = I$

↳ this is $(A^T)^{-1}$. Inverse of A^T

□ A = LU

↳

$$\underset{E_{21}}{\begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix}} \underset{A}{\begin{bmatrix} 2 & 1 \\ 8 & 7 \end{bmatrix}} = \underset{U}{\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}} \qquad \underset{A}{\begin{bmatrix} 2 & 1 \\ 8 & 7 \end{bmatrix}} = \underset{L}{\begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}} \underset{U}{\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}}$$

Inverse of E_{21}

Lower Triangular

$E_{32}E_{31}E_{21}A = U$ (no row exchange)

↳ $A = \underline{E_{21}^{-1}E_{31}^{-1}E_{32}^{-1}}\,U$

$= LU$

$$\underset{E_{32}}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -5 & 1 \end{bmatrix}} \underset{E_{21}}{\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 10 & -5 & 1 \end{bmatrix} = E \qquad EA = U$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 5 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 5 & 1 \end{bmatrix} = L \qquad A = LU$$

A = LU ↳ if no row exchanges,
multipliers go directly into L.

How many <u>operations</u> on $n \times n$ matrix A? ↳ count $n^2 + (n-1)^2 + \cdots + 2^2 + 1^2$.

↳ = (multiply + subtract)

$\approx \boxed{\frac{1}{3}n^3}$

↳ on A.

cost of $b \longrightarrow n^2$

□ Permutations

↳

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$ ↳ 6 of these). $P^{-1} = P^T$.