Linear Algebra (Gilbert Strang) : 11. Matrix spaces ; rank 1 ; small world graphs

□ Bases of new Vector spaces

↳ M = all 3 by 3 matrices

Subspace of symmetric matrices } ↱ when you add to these matrices,
upper triangular } they are still symmetric or upper triangular

Basis for M = all 3×3's.

$\rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \dots, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \dim M = 9.$

dim Symmetric Matrices = 6.

(diagonal + above.
if you know the numbers above the diagonal, you automatically know below the diagonal.)

dim Upper Triangular Matrices = 6

$S \cap U$ = symmetric and uppertriangular

= diagonal 3×3s. $\dim(S\cap U) = 3$.

why not interested in $S \cup U$?

↳ it's not a subspace!

both 6-dimensional subspaces are heading in different directions,
so we can't just put them together.
we must fill in.

⇒ $S+U$ = combinations of things in S and U.

= any element of S + any element of U

= all 3×3s. $\dim(S+U) = 9$.

⇒ $\dim S + \dim U = \dim(S\cap U) + \dim(S+U)$

$\frac{d^2y}{dx^2} + y = 0$

→ $y = \underline{\cos x, \sin x}$. Complete Solution = $y = c_1\cos x + c_2 \sin x$

↳ Basis. dim(Solution space) = 2.

□ Rank one matrices

↳ $\underset{2\times3}{A} = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 8 & 10 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 4 & 5 \end{bmatrix}$

↳ rank one matrix.
dependent!

→ basis : $[1\ 4\ 5]$,
dim : $r = 1$.

Rank 1 Matrix :

$A = u v^T$

column vector

column vector transposed. (row)

□ Small world graphs

↳ Graph = {nodes, edges}

with a few shortcuts, the distances come down dramatically.