

Problem set 2.b)

Date _____

$$\#3) b_{31}=1, b_{22}=2$$

#11) A is already triangular, so
 $L = I$.

$$\#5) E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} \quad U = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 4 & 2 \\ 0 & 0 & 5 \end{bmatrix}$$

$$D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

$$A = LU = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 0 & 4 & 2 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 4 & 2 \\ 6 & 3 & 5 \end{bmatrix}$$

$$U = A, \text{ new } U = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\#7) E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad E_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$$

$$\#13) U = \begin{bmatrix} a & a & a & a \\ 0 & b & b & b \\ 0 & 0 & c & c \\ 0 & 0 & 0 & d \end{bmatrix} L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$E_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$a \neq 0, b \neq a, c \neq b, d \neq c$.

$$E_{32} E_{21} E_{31} A = U$$

$$E = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} \quad L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

#9) $\begin{array}{l} (2 \times 2) \\ d \text{ has to be 0, which makes it} \\ \text{impossible.} \\ (3 \times 3) \end{array}$

$$d=1, e=1, f=1, g=1, h=1.$$

$$el+hf=1, eh=1, f=0?$$

makes it impossible

$$\#15) \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 11 & 1 \end{bmatrix}$$

$$a=2$$

$$4a+4c=11 \Rightarrow c=3.$$

$$c = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$A = LU = \begin{bmatrix} 2 & 4 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} z & t \\ c & d \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 11 & 1 \end{bmatrix}$$

$$2z+4t=2$$

$$8z+17t=11$$

$$t=3, z=-5, \alpha = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$$

$$A\alpha = b \rightarrow \begin{bmatrix} 2 & 4 \\ 0 & 1 \end{bmatrix} \alpha = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 \\ 0 & 1 \end{bmatrix} \alpha = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

↓
 c .

(1.2 Triangular)

#17) a) L becomes an I.

$$E_2 E_3 E_2^{-1} L = I$$

b) you get L^{-1} .

$$L = E_2^{-1} E_3^{-1} E_2 I$$

$$L^{-1} = E_2 E_3 E_2^{-1} I$$

c) LU becomes U.

#21) For A, L keeps 3 zeros at the start of the row.

For B, L keeps 1 zero at the start of the row. U keeps 1 zero at the start of the column.

#23). 5 and 9.

$$E_2 E_3 E_2^{-1} L U$$

$$= I U$$

$$= U.$$

$$\#19) A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} a & a & 0 \\ a & a+b & b \\ 0 & b & bc \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} a & a & 0 \\ 0 & b & b \\ 0 & 0 & c \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$