

Linear Algebra (Gilbert Strang) : 03. Multiplication and Inverse Matrices

□ Matrix Multiplication

$$\hookrightarrow \text{row 3} \begin{bmatrix} a_{31} a_{32} \dots \end{bmatrix} \begin{bmatrix} \text{Column 4} \\ b_{14} \\ b_{24} \\ \vdots \\ b_{n4} \end{bmatrix} = \begin{bmatrix} \cdot \end{bmatrix} \xrightarrow{\text{C34}} C_{34} \\ A_{m \times n} \quad B_{n \times p} = C_{m \times p} \quad C_{34} = (\text{row 3 of } A) \cdot (\text{Column 4 of } B) \\ a_{31}b_{14} + a_{32}b_{24} + \dots = \sum_{k=1}^n a_{3k}b_{k4}$$

$$\begin{bmatrix} \cdot \end{bmatrix} \begin{bmatrix} 00 \end{bmatrix} = \begin{bmatrix} 00 \end{bmatrix} \xrightarrow{\text{A} \cdot \text{col 1}} \xrightarrow{\text{A} \cdot \text{col 2}}$$

$$\begin{bmatrix} \cdot \end{bmatrix} \begin{bmatrix} \cdot \end{bmatrix} = \begin{bmatrix} \cdot \end{bmatrix} \xrightarrow{\text{row } A \cdot B}$$

↳ Columns of C are combinations of Columns of A.

↳ rows of C are combinations of rows of B.

Column of A \times row of B
 $m \times 1 \quad 1 \times p$

$$\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \begin{bmatrix} 1 & 6 \end{bmatrix} = \begin{bmatrix} 2 & 12 \\ 3 & 18 \\ 4 & 24 \end{bmatrix} \quad \text{↳ all multiples of } [1 \ 6]$$

\downarrow
 $AB = \text{sum of (cols of } A) \times (\text{rows of } B)$

$$\begin{bmatrix} 2 & 7 \\ 3 & 8 \\ 4 & 9 \end{bmatrix} \begin{bmatrix} 1 & 6 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \begin{bmatrix} 1 & 6 \end{bmatrix} + \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix} \begin{bmatrix} 0 & 0 \end{bmatrix}$$

□ Block

$$\hookrightarrow \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \begin{bmatrix} B_1 & B_2 \\ B_3 & B_4 \end{bmatrix} = \begin{bmatrix} \cdot \\ \cdot \end{bmatrix} \quad \text{A}_1\text{B}_1 + \text{A}_2\text{B}_3$$

□ Inverses (Square Matrices)

$$\hookrightarrow A^{-1}A = I = AA^{-1}$$

↑
'if' it exists.

↪ invertible, non-singular → 'good' ones.

'singular case'. No inverse.

↪ ex) $\begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$ ↳ both of these are on the same line,
so we can't get (1,0).

+ You can find a vector X with $AX = 0$.

$$AX = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow \text{You can never escape from 0 when you once get 0.}$$

$$A \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} \begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow AX \text{ column } j \text{ of } A^{-1} = \text{column } j \text{ of } I.$$

□ Gauss-Jordan (solve 2 equations at once)

$$\hookrightarrow \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} \left| \begin{array}{c|cc} 1 & 0 \\ 0 & 1 \end{array} \right. \rightarrow \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

↑↑
we are going to make this an identity
then the inverse will show up here.

$$E[A I] = [I A^{-1}]$$

$$\hookrightarrow EA = I \text{ tells us } E = A^{-1}$$