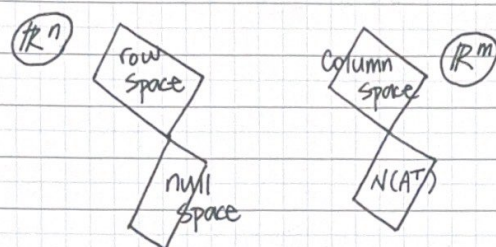


Linear Algebra (Gilbert Strang): 10. The four fundamental subspaces

4 Fundamental Subspaces

- 1) Column Space $C(A)$ in \mathbb{R}^m
 2) null space $N(A)$ in \mathbb{R}^n
 3) Row Space = all combs. of rows of A .
 = all combs. of columns of A^T
 = $C(A^T)$ in \mathbb{R}^n
 4) null space of $A^T = N(A^T)$ \neq left nullspace of A .
 in \mathbb{R}^m



	$C(A), C(A^T)$	$N(A)$	$N(A^T)$
basis?	pivot cols.	special solution	
dimension?	r	$n-r$	$m-r$

$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 2 & 3 & 1 \end{bmatrix} \xrightarrow{\text{row operations}} \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = R.$$

$C(R) \neq C(A)$ different column spaces, but same row space.

Basis of Row Space for both R and A is first r rows of R .

$N(A^T)$

$$A^T y = 0$$

$$y^T A = 0^T$$

$$\begin{bmatrix} \end{bmatrix} \begin{bmatrix} \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix} \quad \begin{bmatrix} y^T \end{bmatrix} \begin{bmatrix} A \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

$$E[A_{m \times n} \quad I_{m \times m}] \rightarrow [R_{m \times n} \quad E_{m \times m}]$$

$$\rightarrow EA = R$$

'E' for this operation

$$= \begin{bmatrix} -1 & 2 & 0 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Basis for the left nullspace \rightarrow look for the combs of rows that give the zero row.

! You have to keep track of E to find out the left nullspace.

a vector that produces zero row.

□ new Vector Space M .

↳ All " $n \times n$ matrices" !! ($A+B$, $C \cdot A$ not AB)
form now.

Subspaces of $M \rightarrow$ upper triangular

symmetric matrices

diagonal matrices

↳ dim: 3.

ex) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 7 \end{bmatrix}$