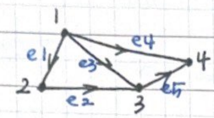
Linear Algebra(Gilbert Strang) : 12. Graphs, Networks, Incidence Matrices

□ Graph

↳ composed of Nodes, Edges.

ex)



$n = 4$ nodes

$m = 5$ edges

Tree : graph with no loops.

□ Incidence Matrix

↳

$$A = \begin{array}{c|cccc|c} & \text{node } 1 & 2 & 3 & 4 & \\ \hline & -1 & 1 & 0 & 0 & \text{edge } 1 \\ & 0 & -1 & 1 & 0 & 2 \\ & -1 & 0 & 1 & 0 & 3 \\ & -1 & 0 & 0 & 1 & 4 \\ & 0 & 0 & -1 & 1 & 5 \end{array}$$

$\left.\begin{matrix}1\\2\\3\end{matrix}\right\}$ loop. → corresponds to dependent rows (linearly).

$$Ax = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = 0.$$

rank = 3.

$$= \begin{bmatrix} x_2 - x_1 \\ x_3 - x_2 \\ x_3 - x_1 \\ x_4 - x_1 \\ x_4 - x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$x = x_1, x_2, x_3, x_4$

Potentials at nodes

$A \downarrow$

$x_2 - x_1$, etc. $\xrightarrow[\text{OHM's Law}]{C}$ currence y_1, y_2, y_3, y_4, y_5 on the edges

potential differences.

$\uparrow A^T$

$A^T y = 0$. Kirchoff's Current Law (KCL)

↓

'current on the edge is some number times the potential drop'.

$$x = c \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \qquad \dim N(A) = 1.$$

∴ it is change in potential that make current happen.
it is OHM's Law that says how much current happens.

$$A^T y = 0 = \begin{bmatrix} -1 & 0 & -1 & -1 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\dim N(A^T) = m - r$

↳ # of loops = # of edges − (# of nodes − 1) ☆.

# of nodes − # of edges + # of loops = 1

→ Euler's formula.

$= -y_1 - y_3 - y_4 = 0$ ↳ we can tell that it is all going out from node 1.

$y_1 - y_2 = 0$ node 2.

$y_2 + y_3 - y_5 = 0$ ↳ the currences have the balance!

$y_4 + y_5 = 0$

$$\text{Basis for } N(A^T) = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$