

Linear Algebra (Gilbert Strang): 10. The four fundamental subspaces

□ 4 Fundamental Subspaces

↳

1) column space $C(A)$ in \mathbb{R}^m

2) null space $N(A)$ in \mathbb{R}^n

3) row space = all combs. of rows of A .

= all combs. of columns of A^T

= $C(A^T)$ in \mathbb{R}^n

4) null space of A^T = $N(A^T)$ $\pi =$ left nullspace of A .

in \mathbb{R}^m

\mathbb{R}^n

row
space

Column
space

\mathbb{R}^m

null
space

$N(A^T)$

$C(A)$, $C(A^T)$

$N(A)$

$N(A^T)$

pivot cols.

special
solution

dimension?

r

$n-r$

$m-r$

$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 2 & 3 & 1 \end{bmatrix}$$

row operations

$$\rightarrow \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} I & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = R$$

zeros.

$C(R) \neq C(A)$

different
column spaces,
but same row space.

Basis of Row Space for both R and A
is first r rows of R .

□ $N(A^T)$

$\rightarrow A^T y = 0$

$$y^T A = 0^T$$

$$\begin{bmatrix} & & \end{bmatrix} \begin{bmatrix} & & \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix} \quad \begin{bmatrix} y^T \end{bmatrix} \begin{bmatrix} A \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

'E' for this operation

$$= \begin{bmatrix} -1 & 2 & 0 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$E[A_{mn} \ I_{mn}] \rightarrow [R_{mn} \ E_{mn}]$$

$$\rightarrow EA = R$$

Basis for the left nullspace \rightarrow look for the combs. of rows that give the zero row.

A vector that produces zero row.

! You have to keep track of E to find out
the left nullspace.

□ new Vector Space M .

↳ All " 3×3 matrices" !! ($A+B$, $C \cdot A$. not AB)
for now.

Subspaces of $M \rightarrow$ upper triangular

symmetric matrices

diagonal matrices

↳ dim: 3.

ex) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 7 \end{bmatrix}$