Introduction to Linear Algebra(Gilbert Strang): 5^{th} Ed.

Date

Problem set 2.2)

1) $l_{21}=5$. $\begin{cases} 5\cdot(2x+3y)=5\cdot 1 \\ 10x+9y=11 \end{cases}$

$\Rightarrow \begin{cases} 10x+3y=1 \\ -6y=6 \end{cases}$

$\Rightarrow \begin{bmatrix} 10 & 3 \\ 0 & -6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 1 \\ 6 \end{bmatrix}$

Pivots.

with backsub. $\rightarrow y=-1$

$x=\frac{2}{5}$

3) eqn.2 $-(-\frac{1}{2})$ eqn.1 $\therefore l_{21}=-\frac{1}{2}$

$\begin{cases} 2x-4y=6 \\ 3y=3 \end{cases} = \begin{bmatrix} 2 & -4 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 6 \\ 3 \end{bmatrix}$

$y=1$, $x=10$

if the right side changes to -6,

$\begin{cases} 2x-4y=6 \\ 3y=-3 \end{cases} = \begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 6 \\ -3 \end{bmatrix}$

$y=-1$, $x=5$

5) no solution = Singular case with parallel eqns.

$\rightarrow 3x+2y=10$
$6x+4y=22$

∞ solutions = Singular case on the same line.

$\rightarrow 3x+2y=10$
$6x+4y=20$

7) to make it break down permanently, $a=2$, to make the lines parallel (on the row picture).

to make it temporarily, $a=0$, so that we can P_{21}.

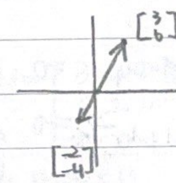
$\begin{bmatrix} 0 & 3 \\ 4 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} -3 \\ 6 \end{bmatrix}$

$\rightarrow \begin{bmatrix} 4 & 6 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 6 \\ -3 \end{bmatrix}$

$\therefore y=-1$, $x=3$

9) if you test if $b_1 \times 2=b_2$, you will be able to know if it is a singular case.

case#1 : $b=(1,2)$ $\begin{matrix} 3x-2y=1 \\ 6x-4y=2 \end{matrix} = x\begin{bmatrix} 3 \\ 6 \end{bmatrix}+y\begin{bmatrix} -2 \\ -4 \end{bmatrix}=\begin{bmatrix} 1 \\ 2 \end{bmatrix}$



case#2 : $b=(1,0)$ $\begin{matrix} 3x-2y=1 \\ 6x-4y=0 \end{matrix} = x\begin{bmatrix} 3 \\ 6 \end{bmatrix}+y\begin{bmatrix} -2 \\ -4 \end{bmatrix}=\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

same picture as case#1, but it does not combine to give $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

☆ 11) a) if (x,y,z) and (X,Y,Z) are two solutions, then

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13) $\begin{cases} 2x+3y+z=8 \\ 4x+7y+5z=20 \\ -2y+2z=0 \end{cases}$

$\rightarrow \begin{cases} 2x+3y+z=8 \\ y+3z=4 \\ -2y+2z=0 \end{cases}$

$\rightarrow \begin{cases} 2x+3y+z=8 \\ y+3z=4 \\ 8z=8 \end{cases}$

Pivots $\therefore z=1,\ y=1,\ x=2$

15) if $b=-2$, it leads to row exchange.

$\begin{cases} x-2y=0 \\ x-2y-z=0 \\ y+z=0 \end{cases} \Rightarrow \begin{cases} x-2y=0 \\ -z=0 \\ y+z=0 \end{cases}$ row exchange needed.

if $b=-1$, it leads to a missing pivot.

$\begin{cases} x-y=0 \\ x-2y-z=0 \\ y+z=0 \end{cases} \rightarrow \begin{cases} x-y\ 0=0 \\ 0-y-z=0 \\ 0+0+0=0 \end{cases}$

$x-y=0 \rightarrow x=y$

$-y-z=0 \rightarrow -z=y$

$\therefore x=y=-z$

$x=4,\ y=4,\ z=-4$

17) (when row1=row2)

$\begin{cases} 2x-y+z=0 \\ 2x-y+z=0 \\ 4x+y+z=0 \end{cases} \rightarrow \begin{cases} 2x-y+z=0 \\ 0+0+0=0 \\ 3y-z=0 \end{cases} \rightarrow \begin{cases} 2x-y+z=0 \\ 3y-z=0 \\ 0+0+0=0 \end{cases}$

(when col1=col2)

$\begin{cases} 2x+2y+z=0 \\ 4x+4y+z=0 \\ 6x+6y+z=2 \end{cases} \rightarrow \begin{cases} 2x+2y+z=0 \\ 0+0+-z=0 \\ 0+0-2z=2 \end{cases}$

→ the y pivot (second pivot) is missing.

19) to make the system singular and infinitely many solutions:

$\begin{cases} x+4y-2z=1 \\ x+7y-6z=6 \\ 3y+qz=t \end{cases} \rightarrow \begin{cases} x+4y-2z=1 \\ 3y-4z=5 \\ 3y+qz=t \end{cases}$

→ when $q=-4$, $t=5$.

no third pivot. $0=0$. infinite solutions.

21) $(Ax=b)$

$\begin{cases} 2x+y=0 \\ x+2y+z=0 \\ y+2z+t=0 \\ z+2t=5 \end{cases}$

$\rightarrow \begin{cases} 2x+y=0 \\ \frac{3}{2}y+z=0 \\ y+2z+t=0 \\ z+2t=5 \end{cases} \rightarrow \begin{cases} 2x+y=0 \\ \frac{3}{2}y+z=0 \\ \frac{4}{3}z+t=0 \\ \frac{5}{4}t=5 \end{cases}$

Pivots.

$\therefore t=4,\ z=-3,\ y=2,\ x=-1$

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21) $(Ax=b)\downarrow$

$$\begin{cases} 2x-y = 0 \\ -x+2y-z = 0 \\ -y+2z-t=0 \\ -z+2t=5 \end{cases}$$

$$\Rightarrow \begin{cases} 2x-y = 0 \\ \frac{3}{2}y-z = 0 \\ -y+2z-t=0 \\ -z+2t=5 \end{cases} \Rightarrow \begin{matrix} 2x-y = 0 \\ \frac{3}{2}y-z = 0 \\ \frac{4}{3}z-t=0 \\ \frac{5}{4}t=5 \end{matrix}$$

$\therefore t=4,\ z=3,\ y=2,\ x=1$

22) if you follow the 1,2,1 pattern of question 21,

fifth pivot $= -5$, nth pivot - if odd $= -n$

if even $= n$.

if you follow the -1,2,-1 pattern of question 21,

fifth pivot $= 5$, n^{th} pivot $= n$.

23)

$$\begin{cases} x+y=1 \\ 0+2y=3 \end{cases}$$ → what can be the original?

1) $$\begin{cases} x+y=1 \\ 2x+4y=5 \end{cases}$$

2) $$\begin{cases} x+y=1 \\ 3x+5y=6 \end{cases}$$

3) $$\begin{cases} x+y=1 \\ 4x+6y=7 \end{cases}$$

25) $A = \begin{bmatrix} a & 2 & 3 \\ a & a & 4 \\ a & a & a \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} a & 2 & 3 \\ 0 & a-2 & 1 \\ & a-2 & a-3 \end{bmatrix} \Rightarrow \begin{bmatrix} a & 2 & 3 \\ 0 & a-2 & 1 \\ 0 & 0 & a-4 \end{bmatrix}$$

$\therefore$ when $a=0$, $a=2$, $a=4$, elimination will fail to give three pivots.

27)

$$\begin{cases} 3x = 3 \\ 6x+2y = 8 \\ 9x-2y+z=9 \end{cases}$$

$$\Rightarrow \begin{cases} 3x = 3 \\ 2y = 2 \\ -2y+z=0 \end{cases} \Rightarrow \begin{matrix} 3x = 3 \\ 2y = 2 \\ z=2 \end{matrix}$$

$U = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, solution $=(1,1,2)$