

Problem set 2.b)

$$\#3) l_{31}=1, l_{m}=2$$

$$\#5) E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} \quad U = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 4 & 2 \\ 0 & 0 & 5 \end{bmatrix}$$

$$A = LU = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 0 & 4 & 2 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 4 & 2 \\ 6 & 3 & 5 \end{bmatrix}$$

$$\#7) E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad E_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$$

$$E_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

$$E_{32} E_{31} E_{21} A = U$$

$$E = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & -2 & 1 \end{bmatrix} \quad L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 0 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

#9) ^(2x2) d has to be 0, which makes it impossible.

^(3x3)

$$d=1, e=1, g=1, l=1$$

$$el \neq 1, el=1, A=0?$$

makes it impossible

#11) A is already triangular, so

$$L = I.$$

$$D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

$$U = A,$$

$$\text{new } U = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\#13) U = \begin{bmatrix} a & a & a & a \\ 0 & b & a & b & a & b \\ 0 & 0 & b & c & b \\ 0 & 0 & 0 & d & c \end{bmatrix} \quad L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$a \neq 0, b \neq a, c \neq b, d \neq c.$$

$$\#15) \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 11 \end{bmatrix}$$

$$x=2$$

$$4x+y=11 \quad y=3.$$

$$C = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$A = LU = \begin{bmatrix} 2 & 4 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 11 \end{bmatrix}$$

$$2x+4y=2$$

$$8x+17y=11$$

$$b=3, z=-5, x = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$$

$$Ax=b \rightarrow \begin{bmatrix} 2 & 4 \\ 0 & 1 \end{bmatrix} x = \begin{bmatrix} 2 \\ 11 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 \\ 0 & 1 \end{bmatrix} x = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

\downarrow
C.

#17) a) L becomes an I.

$$E_2 E_1 E_{21} L = I$$

b) you get L^{-1} .

$$L = E_2^{-1} E_1^{-1} E_{21}^{-1} I$$

$$L^{-1} = E_2 E_1 E_{21} I$$

c) LU becomes U.

$$E_2 E_1 E_{21} L U$$

$$= I U$$

$$= U.$$

#19) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} a & a & 0 \\ a & a+b & b \\ 0 & b & b+c \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a & a & 0 \\ 0 & b & b \\ 0 & 0 & c \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

#21) For A, L keeps 3 zeros at the start of the row.

For B, L keeps 1 zero at the start of the row. U keeps 1 zero at the start of the column.

#23) 5 and 9.