5

5.1

We have $M = (Q, \Sigma, \delta)$ and $\pi = \{H_i\}_{i \in I}$ a admissible partion of Q. If M is complete then for all $i \in I$ and for all $a \in \Sigma$ there exists **exactly one** partion such that for $j \in I$ $H_i\delta_a \subseteq H_j$. By definition of π (Lemma 3.48) there exists **at least one** $j \in I$ with $H_i\delta_a \subseteq H_j$. Notice $H_i\delta_a \neq \emptyset$ because M is complete.
We show that there only exists **exactly one** $j \in I$. Suppose there exists $j, k \in I$ with $H_i\delta_a \subseteq H_{j,k}$ for all $a \in \Sigma$ and $j \neq k$. We choose an arbitray $q \in H_i$ then the following must hold:

$$q\delta_a = q_j \in H_j$$
$$q\delta_a = q_k \in H_k$$
$$q_j \neq q_k$$

This is a contraction because $q\delta_a$ is not right unique anymore and therefore $q_j = q_k$ and $H_j = H_k$.

5.2

- cases for only one admissible partition and for $|Q|$ admissible partitions
- one partition $\implies |qS| = 1$:
  - all states are in R
  - $H_i s = H_i$ with $i \in [1]$
  - for all $q \in Q$: $q \in H_i$ and therefore $Q = H_i$
  - for all $q, q' \in Q$: $qs = q'$ with $q, q' \in H_i$ because $q \sim_R q'$
  - proof of contradiction with $q' \notin H_i \implies (q, q') \notin R$ which contradicts first statement.
- $|Q|$ partitions $\implies qS = Q$:
  - $|Q|$ partions implies each state own partition, $R = \{(q_i, q_i) | i \in [|Q|\}$
  - for all $q \in Q$ and $s \in S$: $qs = q$ with $q \in H_i, i \in [|Q|]$

- suppose $qs = q'$ with $q \neq q'$ then $(q, q') \in R$ which contradicts first statement

5.3

5.4