9

9.1

We prove that if $M \leq N_1 \omega_1 N_2 ... N_{n-1} \omega_{n-1} N_n$ then $TS(M) \leq TS(N_1) \wr ... \wr TS(N_n)$. By 4.10

$$TS(M) \leq TS(N_1 \omega_1 N_2 ... N_{n-1} \omega_{n-1} N_n)$$

holds true. By 6.22 the cascade product is covered by the wreath product which results in

$$TS(N_1 \omega_1 N_2 ... N_{n-1} \omega_{n-1} N_n) \leq TS(N_1 \wr ... \wr N_n)$$

. This concludes the proof.

9.2

9.3

9.4

We can find admissable partitions for example:

$$\pi = \{\{q_0\}, \{q_1, q_2, q_3, q_4\}\}, \tau = \{\{q_1, q_3\}, \{q_0, q_2, q_4\}\}$$

However we never find an orthogonal partition with $\pi \cap \tau = id_Q$ because for an input of multiple $b \in \Sigma$ all transistions will eventually end in $q_1 \in Q$ where is no escape, thus $\pi \cap \tau = \emptyset$. Therefore the only partition with the id_Q function is the trivial partition of whole Q which is a contradiction. $\square$