3

3.1

3.2

3.3

x	q_0	q_1	
a	q_0	q_1	
b	q_1	$\perp$	
c	$\perp$	q_0	
aa	q_0	q_1	same as a
ab	q_1	$\perp$	same as b
ac	$\perp$	q_0	same as c
ba	q_1	$\perp$	same as b
bb	$\perp$	$\perp$	
bc	q_0	$\perp$	
ca	$\perp$	q_0	same as c
cb	$\perp$	q_1	
cc	$\perp$	$\perp$	same as bb
$\vdots$	$\vdots$	$\vdots$	

From the table we compute $S(M) = \{[a], [b], [c], [b^2], [bc], [cb]\}$. Suppose $S(M)$ is a group with $u, v \in \Sigma^*, \delta_u, \delta_v \in \langle \mathcal{F}(M) \rangle$ then there exists for an arbitrary but fixed δ_u another δ_v with $\delta_u \delta_v = id_Q$. Notice we can choose $q_1 \delta_b = \perp$ and we cannot find any δ_v with $\perp \delta_v = q_1$. Therefore $S(M)$ is not a group.

Now assume $S(M)$ is a monoid. Hence, there exists $u \in \Sigma^*, \delta_u \in \langle \mathcal{F}(M) \rangle$ with $\delta_u = id_Q$. Notice δ_a satisfies this condition and acts a neutral element. Thus, $S(M)$ is a monoid. $\square$

3.4