Proof 3.28

To prove that every group acts on itself we need show compatibility and faithfulness for a function $f : G \times G \to G : (g, g') \mapsto g'g$ with $G$ being a group.

Compatibility follows due to groups being commutative

Let $g_1, g_2 \in G$. Let $g g_1 \overset{(1)}{=} g g_2$ for all $g \in G$

assume $g_1 \neq g_2$ set $g$ to the neutral element $\varepsilon$ which is allowed since (1) holds true for all $g$ in $G$.

$\to \varepsilon g_1 = \varepsilon g_2 \Leftrightarrow g_1 = g_2 \; \lightning \; g_1 \neq g_2$

This conclude the proof. $\square$