

9

9.1

We prove that if $M \leq N_1\omega_1N_2\dots N_{n-1}\omega_{n-1}N_n$ then $TS(M) \leq TS(N_1) \wr \dots \wr TS(N_n)$. By 4.10

$$TS(M) \leq TS(N_1\omega_1N_2\dots N_{n-1}\omega_{n-1}N_n)$$

holds true. By 6.22 the cascade product is covered by the wreath product which results in

$$TS(N_1\omega_1N_2\dots N_{n-1}\omega_{n-1}N_n) \leq TS(N_1 \wr \dots \wr N_n)$$

. This concludes the proof.

9.2

9.3

9.4

We can find admissable partitions for example:

$$\pi = \{\{q_0\}, \{q_1, q_2, q_3, q_4\}\}, \tau = \{\{q_1, q_3\}, \{q_0, q_2, q_4\}\}$$

However we never find an orthogonal partition with $\pi \cap \tau = id_Q$ because $q_2, q_4 \in Q$ are tightly coupled such that in each partition q_2, q_4 are in the same block. Therefore we never find π, τ with $\pi \cap \tau = id_Q$ because q_2, q_4 is always a subset of the intersection. \square