10

10.1

We prove that a connected transformation group $A = (Q, G)$ is primitive iff it is irreducable.
primiitive $\implies$ irreducable: A does not contain any primitive block $P \subset Q$ with $|P| \geq 2$ and $P \cap Pg \in \{P, \emptyset\}$ for all $g \in G$. Suppose A is reducable which means $|Q| \leq 1$ or there exists an admissable partition of Q that is not trivial. Since G is transitive, Q has at least two states which contradicts $|Q| \leq 1$.
Suppose there exists a non trivial admissable partition $\pi = \{H_i\}_{i \in I}$. For all $i \in I$ and all $g \in G$ there exists $j \in I$ with $H_i g \subseteq H_j$. Notice, $i \neq j$ because otherwise $P = H_i$ and $P \cap Pg = P$. Thus, $H_i \cap H_j = \emptyset$ but then it follows that $P \subseteq H_i$ and $P \cap Pg = \emptyset$ - a contradiction.
irreducable $\implies$ primitive: A is irreducable which means $|Q| > 1$ and all admissable partitions are trivial. Suppose A is imprimitive. Thus, the group produces a block system with primitive blocks $P \subset Q$ with $|P| \geq 2$ and $P \cap Pg \in \{P, \emptyset\}$ for all $g \in G$.
A primitive block produces a partition by the right coset $\pi = \{Pg | g \in G\}$. We prove that by showing $Pg_1 \cap Pg_2 = \emptyset$. Suppose there exists an element $x \in Pg_1 \cap Pg_2$. Then $x = p_1 g_1 = p_2 g_2$ and $p_1 = p_2 g_2 g_1^{-1}$. Therefore $p_1 \in P$ and also $p_1 \in Pg_2 g_1^{-1}$ following

$$\begin{aligned} P &= Pg_2 g_1^{-1} \\ Pg_1 &= Pg_2 \end{aligned}$$

shows that pg_1 and pg_2 are in the same primitive block.
By Lemma 8.12 any subgroup and its right coset builds also an admissable partition which is not trivial due to $2 \leq |P| < |Q|$. This contradicts the assumption A being irreducable.
We showed that both directions hold true which concludes the proof. $\square$

10.2

We want to show the following: Let G be a finite group, then $\mathrm{TS}(SM(\mathcal{G})) = (G, \mathsf{G})$ with $\mathsf{G} = \{[g] | g \in G\}$.

For this lets have a look at the definitions:

$$\begin{aligned} TS(SM(\mathcal{G})) &= TS((G, G, \delta)) && \text{(Def. 8.10)} \\ &= (G, S(\mathcal{G})) && (3.18) \\ &= (G, \mathsf{G}) && ((1), 3.10) \end{aligned}$$

Regarding (1), as per definition in the script $S(M) = \Sigma^+ \backslash \sim_M$ which describes the equivalence classes over all words of M defined by the congruence relation $\sim_M$, thus, it is equivalent to $\mathsf{G} = \{[g] | g \in G\}$. $\square$

10.3

10.4

We begin by constructing the permutation machine as depicted in 10.1. Next we define

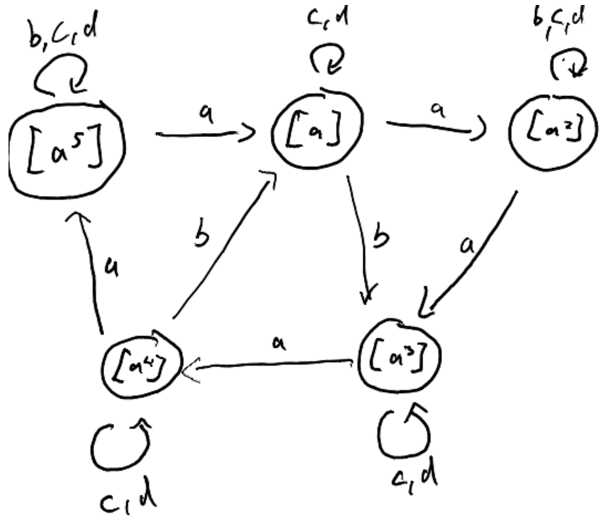

Abbildung 10.1: Permutation machine

the reset machine as follows: $N = (Q, (\mathbb{Z}_5 \times \Sigma) \cup \{\lambda\}, \delta')$ with

	q_0	q_1	q_2	q_3	q_4
λ	q_0	q_1	q_2	q_3	q_4
$([a], a)$	q_1	q_2	q_3	q_4	q_0
$([a], c)$	$\perp$	q_0	$\perp$	q_0	q_0
$([a], d)$	$\perp$	$\perp$	q_3	q_3	$\perp$
$([a], b)$	q_2	q_1	q_3	q_0	q_4
$([a^2], a)$	q_1	q_2	q_3	q_4	q_0
$([a^2], c)$	q_4	$\perp$	q_4	q_4	$\perp$
$([a^2], d)$	$\perp$	q_2	q_2	$\perp$	$\perp$
$([a^2], b)$	q_0	q_2	q_4	q_3	q_1
$([a^3], a)$	q_1	q_2	q_3	q_4	q_0
$([a^3], c)$	$\perp$	q_3	q_3	$\perp$	q_3
$([a^3], d)$	q_1	q_1	$\perp$	$\perp$	$\perp$
$([a^3], b)$	q_1	q_3	q_2	q_0	q_4
$([a^4], a)$	q_1	q_2	q_3	q_4	q_0
$([a^4], c)$	q_2	q_2	$\perp$	q_2	$\perp$
$([a^4], d)$	q_0	$\perp$	$\perp$	$\perp$	q_0
$([a^4], b)$	q_2	q_1	q_4	q_3	q_0
$([a^5], a)$	q_1	q_2	q_3	q_4	q_0
$([a^5], c)$	q_1	$\perp$	q_1	$\perp$	q_1
$([a^5], d)$	$\perp$	$\perp$	$\perp$	q_4	q_4
$([a^5], b)$	q_0	q_3	q_2	q_4	q_1

Then we get $TS(M) \leq N\omega P$. $\square$