6

6.1

6.2

6.3

6.4

We show $M \leq M'$ by using the following covering (η, ξ):

$$\eta : Q' \hookrightarrow Q; q' \mapsto \begin{cases} q_0, & \text{for } q' = r_0, \\ q_1, & \text{for } q' = r_2, \end{cases}$$

$$\xi : \Sigma \to \Sigma'; u \mapsto \begin{cases} b', & \text{for } u = a, \\ a', & \text{for } u = b, \end{cases}$$

Moreover we prove η, ξ is well defined. First, ξ is a function because all $dom(\xi) = \Sigma$ and it is right unique. η is surjective as its range hits all states of Q. Also it is right unique and therefore a partial function. Finally, we show $\eta(q')\delta_w \subseteq \eta(q'\delta'_{\xi(w)})$ for all $q' \in Q'$ and $w \in \Sigma^*$. We build an operation table to get $\Sigma^+/\sim_M$ to get all relevant w.

x	q_0	q_1	
a	q_1	$\perp$	
b	q_0	q_1	
aa	$\perp$	$\perp$	
ab	q_1	$\perp$	same as a
bb	q_0	q_1	same as b
ba	q_1	$\perp$	same as a
aaa	$\perp$	$\perp$	same as aa
aab	$\perp$	$\perp$	same as aa
aba	$\perp$	$\perp$	same as aa
abb	q_1	$\perp$	same as a
bba	q_1	$\perp$	same as a
baa	q_1	$\perp$	same as aa
bab	q_1	$\perp$	same as a

We get $\Sigma^+/\sim_M = \{[a],[b],[aa]\}$. Since $\eta(q') = \emptyset | q' \notin \mathcal{D}(\eta)$ we only observe for r_0, r_2 as states from M'. We check for all relevant $q' \in Q'$ and $win\Sigma$:

$$\eta(r_0)\delta_a = q_0\delta_a = q_1 \subseteq \eta(r_0\delta'_{\xi(a)}) = \eta(r_0\delta'_{b'}) = \eta(r_2) = q_1$$
$$\eta(r_0)\delta_b = q_0\delta_b = q_0 \subseteq \eta(r_0\delta'_{\xi(b)}) = \eta(r_0\delta'_{a'}) = \eta(r_0) = q_0$$
$$\eta(r_0)\delta_{aa} = q_0\delta_{aa} = \emptyset \subseteq \eta(r_0\delta'_{\xi(aa)}) = \eta(r_2\delta'_{b'b'}) = \eta(\emptyset) = \emptyset$$
$$\eta(r_2)\delta_a = q_1\delta_a = \emptyset \subseteq \eta(r_2\delta'_{\xi(a)}) = \eta(r_2\delta'_{b'}) = \eta(r_3) = \emptyset$$
$$\eta(r_2)\delta_b = q_1\delta_b = q_1 \subseteq \eta(r_2\delta'_{\xi(b)}) = \eta(r_2\delta'_{a'}) = \eta(r_2) = q_1$$
$$\eta(r_2)\delta_{aa} = q_1\delta_{aa} = \emptyset \subseteq \eta(r_2\delta'_{\xi(aa)}) = \eta(r_2\delta'_{b'b'}) = \eta(\emptyset) = \emptyset$$

For each combination $\eta(q')\delta_w \subseteq \eta(q'\delta'_{\xi(w)})$ holds which concludes the proof.