10

10.1

We prove that a connected transformation group $A = (Q, G)$ is primitive iff it is irreducable.
primiitive $\implies$ irreducable: A does not contain any primitive block $P \subset Q$ with $|P| \geq 2$ and $P \cap Pg \in \{P, \emptyset\}$ for all $g \in G$. Suppose A is reducable which means $|Q| \leq 1$ or there exists an admissable partition of Q that is not trivial Since G is transitive, Q has at least two states which contradicts $|Q| \leq 1$.
Suppose there exists a non trivial admissable partition $\pi = \{H_i\}_{i \in I}$. For all $i \in I$ and all $g \in G$ there exists $j \in I$ with $H_i g \subseteq H_j$. Notice, $i \neq j$ because otherwise $P = H_i$ and $P \cap Pg = P$. Thus, $H_i \cap H_j = \emptyset$ but then it follows that $P \subseteq H_i$ and $P \cap Pg = \emptyset$ - a contradiction.
irreducable $\implies$ primitive: A is irreducable which means $|Q| > 1$ and all admissable partitions are trivial. Suppose A is imprimitive. Thus, the group produces a primitive block system with some blocks $P \subset Q$ with $|P| \geq 2$ and $P \cap Pg \in \{P, \emptyset\}$ for all $g \in G$. The block system creates an admissable partition π of Q with for all $P \in \pi : 2 \leq |P| < |Q|$. This contradicts the assumption of A only containing trivial admissable partitions.

This concludes the proof. $\square$

10.2

10.3

10.4