8

8.1

8.2

Let $M = (Q, \Sigma, \delta)$, $P \subseteq Q$, $M|_P = (P, \Sigma, \delta|_P)$ and $\delta|_P : P \times \Sigma \to \Sigma$ is undefinded for all $q \in Q \setminus P$. We define $S(M|_P) = \Sigma^+ / \sim_{M|_P}$ with $u \sim_{M|_P} v$ iff $\delta_u|_P = \delta_v|_P$. To obtain $S(M|_P)$ you can build an operation table with $\delta|_P$ and all $p \in P$ for all $a \in \Sigma^+$.
Finally, we construct $TS(M|_P) = (P, S(M|_P))$. □

8.3

8.4

We prove for a given function $g \in S^T$, $(S^T \times T, \wr)$ is a semigroup. Notice $\wr$ is a closed binary operation by definition as it is a function and is closed:

$$\wr : (S^T \times T) \times (S^T \times T) \to (S^T \times T)$$

So we just have to show $\wr$ is associative:

$$\begin{aligned}((f,t) \wr (g,t')) \wr (h,t'') &= (fg_t, tt') \wr (h, t'') \\ &= (fg_{t'}h_{tt'}, tt't''') \\ (f,t) \wr ((g,t') \wr (h,t'')) &= (f,t) \wr (gh_{t'}, t't'') \\ &= (fg_t h_{tt'}, tt't'')\end{aligned}$$

Notice $gh_{t'} = x \mapsto g(x)h(xt')$ and $(gh_{t'})_t = x \mapsto g(xt)h(xt't) = g_t h_{tt'}$. □