10

10.1

We prove that a connected transformation group $A = (Q, G)$ is primitive iff it is irreducable.
primiitive $\implies$ irreducable: A does not contain any primitive block $P \subset Q$ with $|P| \geq 2$ and $P \cap Pg \in \{P, \emptyset\}$ for all $g \in G$. Suppose A is reducable which means $|Q| \leq 1$ or there exists an admissable partition of Q that is not trivial Since G is transitive, Q has at least two states which contradicts $|Q| \leq 1$.
Suppose there exists a non trivial admissable partition $\pi = \{H_i\}_{i \in I}$. For all $i \in I$ and all $g \in G$ there exists $j \in I$ with $H_i g \subseteq H_j$. Notice, $i \neq j$ because otherwise $P = H_i$ and $P \cap Pg = P$. Thus, $H_i \cap H_j = \emptyset$ but then it follows that $P \subseteq H_i$ and $P \cap Pg = \emptyset$ - a contradiction.
irreducable $\implies$ primitive: A is irreducable which means $|Q| > 1$ and all admissable partitions are trivial. Suppose A is imprimitive. Thus, the group produces a block system with primitive blocks $P \subset Q$ with $|P| \geq 2$ and $P \cap Pg \in \{P, \emptyset\}$ for all $g \in G$.
A primitive block produces a partition by the right coset $\pi = \{Pg | g \in G\}$. We prove that by showing $Pg_1 \cap Pg_2 = \emptyset$. Suppose there exists an element $x \in Pg_1 \cap Pg_2$. Then $x = p_1 g_1 = p_2 g_2$ and $p_1 = p_2 g_2 g_1^{-1}$. Therefore $p_1 \in P$ and also $p_1 \in P g_2 g_1^{-1}$ following

$$
\begin{aligned}
P &= P g_2 g_1^{-1} \\
P g_1 &= P g_2
\end{aligned}
$$

shows that pg_1 and pg_2 are in the same primitive block.
Now we show $\pi = \{Pg | g \in G\}$ is an admissable partition.
Finally this contradicts the assumption of A only containing trivial admissable partitions.

□

10.2

10.3

10.4