$$X = \begin{pmatrix} 4 & 2 \\ \sqrt{2} & 2\sqrt{2} \\ -\sqrt{2} & -2\sqrt{2} \end{pmatrix}$$

$$\begin{array}{cc|ccc} & & 4 & \sqrt{2} & -\sqrt{2} \\ & & 2 & 2\sqrt{2} & -2\sqrt{2} \\ \hline 4 & 2 & 20 & 8\sqrt{2} & -8\sqrt{2} \\ \sqrt{2} & 2\sqrt{2} & 8\sqrt{2} & 10 & -10 \\ -\sqrt{2} & -2\sqrt{2} & -8\sqrt{2} & -10 & 10 \end{array}$$

$$X X^T = \begin{pmatrix} 20 & 8\sqrt{2} & -8\sqrt{2} \\ 8\sqrt{2} & 10 & -10 \\ -8\sqrt{2} & -10 & 10 \end{pmatrix}$$

eigenvalues:

$$\det(X X^T - \lambda I) = \begin{vmatrix} 20-\lambda & 8\sqrt{2} & -8\sqrt{2} \\ 8\sqrt{2} & 10-\lambda & -10 \\ -8\sqrt{2} & -10 & 10-\lambda \end{vmatrix} \begin{matrix} 20-\lambda & 8\sqrt{2} \\ 8\sqrt{2} & 10-\lambda \\ -8\sqrt{2} & -10 \end{matrix}$$

$$= (20-\lambda)\cdot(10-\lambda)\cdot(10-\lambda) + 8\sqrt{2}\cdot(-10)\cdot(-8\sqrt{2})$$

$$+ (-8\sqrt{2})\cdot(8\sqrt{2})\cdot(-10)$$

$$- (-8\sqrt{2})\cdot(10-\lambda)\cdot(-8\sqrt{2}) - (-10)\cdot(-10)\cdot(20-\lambda)$$

$$- (10-\lambda)\cdot 8\sqrt{2}\cdot 8\sqrt{2}$$

$$= -\lambda^3 + 40\lambda^2 - 500\lambda + \cancel{2000}$$

$$+ \cancel{1280} + \cancel{1280}$$

$$- \cancel{1280} + 128\lambda - \cancel{2000} + 100\lambda - \cancel{1280} + 128\lambda$$

$$= -\lambda^3 + 40\lambda^2 - 500\lambda + 128\lambda + 100\lambda + 128\lambda$$

$$= -\lambda^3 + 40\lambda^2 - 144\lambda$$

$$= -\lambda\,(\lambda^2 - 40\lambda + 144) = -\lambda\cdot(\lambda - 36)(\lambda - 4)$$

$\Rightarrow \quad \lambda_1 = 36 \qquad \lambda_2 = 4 \qquad \lambda_3 = 0$

eigenvectors:

λ_1:

$$\begin{bmatrix} 20-36 & 8\sqrt{2} & -8\sqrt{2} \\ 8\sqrt{2} & 10-36 & -10 \\ -8\sqrt{2} & -10 & 10-36 \end{bmatrix} = \begin{bmatrix} -16 & 8\sqrt{2} & -8\sqrt{2} \\ 8\sqrt{2} & -26 & -10 \\ -8\sqrt{2} & -10 & -26 \end{bmatrix} \qquad R1 \cdot \left(-\frac{1}{16}\right)$$

rref:

$$= \begin{bmatrix} 1 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 8\sqrt{2} & -26 & -10 \\ -8\sqrt{2} & -10 & -26 \end{bmatrix} \quad R2 = R2 - R1 \cdot 8\sqrt{2}$$

$$= \begin{bmatrix} 1 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 0 & -18 & -18 \\ -8\sqrt{2} & -10 & -26 \end{bmatrix} \quad R3 = R3 + R1 \cdot 8\sqrt{2}$$

$$= \begin{bmatrix} 1 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 0 & -18 & -18 \\ 0 & -18 & -18 \end{bmatrix} \quad \begin{aligned} R3 &= -\frac{R2}{18} \\ R3 &= -\frac{R3}{18} \end{aligned}$$

$$= \begin{bmatrix} 1 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \quad R3 = R3 - R2$$

$$= \begin{bmatrix} 1 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad R1 = R1 + \frac{\sqrt{2}}{2} \cdot R2$$

$$= \begin{bmatrix} 1 & 0 & \sqrt{2} \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$\vec{v}$:

$$\begin{bmatrix} 1 & 0 & \sqrt{2} \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_3 = t$ $\qquad$ $x_2 = -t$

$\Rightarrow x_2 + t = 0$ $\qquad$ $\Rightarrow x_1 + 0x_2 \sqrt{2}t = 0$ $\qquad$ $\Rightarrow \begin{pmatrix} -\sqrt{2}t \\ -t \\ 1 \end{pmatrix} = \begin{pmatrix} -\sqrt{2} \\ -1 \\ 1 \end{pmatrix} t$

$\Rightarrow x_2 = -t$ $\qquad$ $\Rightarrow x_1 = -\sqrt{2}t$

$= \begin{pmatrix} -\sqrt{2} \\ -1 \\ 1 \end{pmatrix} t$ $\qquad$ Einheitenvektor: $\begin{pmatrix} -\frac{\sqrt{2}}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} = v_1$

eigenvector v_2:

$\lambda_2 = 4$

$$\begin{bmatrix} 20-4 & 8\sqrt{2} & -8\sqrt{2} \\ 8\sqrt{2} & 10-4 & -10 \\ -8\sqrt{2} & -10 & 10-4 \end{bmatrix} = \begin{bmatrix} 16 & 8\sqrt{2} & -8\sqrt{2} \\ 8\sqrt{2} & 6 & -10 \\ -8\sqrt{2} & -10 & 6 \end{bmatrix}$$

Stufenform:

$$\begin{bmatrix} 16 & 8\sqrt{2} & -8\sqrt{2} \\ 8\sqrt{2} & 6 & -10 \\ -8\sqrt{2} & -10 & 6 \end{bmatrix} \qquad R_3 = R_3 + R_2$$

$$\begin{array}{ccc} 16 & 8\sqrt{2} & -8\sqrt{2} \\ 8\sqrt{2} & 6 & -10 \\ 0 & -4 & -4 \end{array} \qquad \begin{array}{l} R2 = \sqrt{2}\cdot R_2 - R3 \\ R3 = \frac{R3}{-4} \end{array}$$

$$\begin{array}{ccc} 16 & 8\sqrt{2} & -8\sqrt{2} \\ 0 & -2\sqrt{2} & -2\sqrt{2} \\ 0 & 1 & 1 \end{array} \qquad \begin{array}{l} R1 = \frac{R1}{16} \\ R2 = \frac{R2}{-2\sqrt{2}} \end{array}$$

$$\begin{array}{ccc} 1 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{array} \qquad \begin{array}{l} R1 = R1 - \frac{\sqrt{2}}{2}\cdot R2 \\ \\ R3 = R3 - R2 \end{array}$$

$$\begin{bmatrix} 1 & 0 & -\sqrt{2} \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -\sqrt{2} \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_3 = t$ $\qquad$ $x_2 = -t$

$\Rightarrow x_2 + t = 0$ $\qquad \Rightarrow x_1 - \sqrt{2}t = 0$

$x_2 = -t$ $\qquad \Rightarrow x_1 = \sqrt{2}t$

$$\Rightarrow \begin{pmatrix} \sqrt{2}t \\ -t \\ t \end{pmatrix} = \begin{pmatrix} \sqrt{2} \\ -1 \\ 1 \end{pmatrix} t \qquad \text{Einheitsvektor}: \begin{pmatrix} \frac{\sqrt{2}}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} = V_2$$

eigenvector V_3:

$$\begin{array}{ccc} 20 & 8\sqrt{2} & -8\sqrt{2} \\ 8\sqrt{2} & 10 & -10 \\ -8\sqrt{2} & -10 & 10 \end{array} \qquad \begin{array}{l} R1 = \frac{R1}{20} \\ \\ R_3 = R_3 + R_2 \end{array}$$

$$= \begin{matrix} 1 & \frac{2\sqrt{2}}{5} & -\frac{2\sqrt{2}}{5} \\ 8\sqrt{2} & 10 & -10 \\ 0 & 0 & 0 \end{matrix} \qquad R2 = R2 - 8\sqrt{2}\,R1$$

$$= \begin{matrix} 1 & \frac{2\sqrt{2}}{5} & -\frac{2\sqrt{2}}{5} \\ 0 & \frac{18}{5} & -\frac{18}{5} \\ 0 & 0 & 0 \end{matrix} \qquad R2 \Rightarrow \frac{5}{18} R2$$

$$= \begin{matrix} 1 & \frac{2\sqrt{2}}{5} & -\frac{2\sqrt{2}}{5} \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{matrix} \qquad R1 = R1 - \frac{2\sqrt{2}}{5} R2$$

$$= \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{matrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_3 = t$ $\qquad x_2 = t$

$$\Rightarrow \quad x_2 - t = 0 \quad \Rightarrow \quad x_3 = 0 \quad \Rightarrow \begin{pmatrix} 0 \\ t \\ t \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} t$$

$x_2 = t$

Einheitenvektor : $\begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = v_3$

$$v_1 = \begin{pmatrix} -\frac{\sqrt{2}}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \quad v_2 = \begin{pmatrix} \frac{\sqrt{2}}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \quad v_3 = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

$\sigma = \sqrt{36} = 6 \qquad \sigma_2 = \sqrt{4} = 2$

$$\Sigma = \begin{bmatrix} 6 & 0 \\ 0 & 2 \\ 0 & 0 \end{bmatrix}$$

$$V = \begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{\sqrt{2}}{2} & -\frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \quad X = \begin{bmatrix} 4 & 2 \\ \sqrt{2} & 2\sqrt{2} \\ -\sqrt{2} & -2\sqrt{2} \end{bmatrix}$$

$$u_1 = \frac{1}{\sigma_1} \cdot X^T \cdot v_1 = \frac{1}{6} \cdot \begin{bmatrix} 4 & \sqrt{2} & -\sqrt{2} \\ 2 & 2\sqrt{2} & -2\sqrt{2} \end{bmatrix} \cdot \begin{bmatrix} -\frac{\sqrt{2}}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \end{bmatrix} = \frac{1}{6} \cdot \begin{bmatrix} -\frac{6\sqrt{2}}{2} \\ -\frac{6\sqrt{2}}{2} \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \end{bmatrix}$$

$$u_2 = \frac{1}{\sigma_2} \cdot X^T \cdot V_2 = \frac{1}{2} \cdot \begin{bmatrix} 4 & \sqrt{2} & -\sqrt{2} \\ 2 & 2\sqrt{2} & -2\sqrt{2} \end{bmatrix} \cdot \begin{bmatrix} \frac{\sqrt{2}}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \end{bmatrix} = \frac{1}{2} \cdot \begin{bmatrix} \frac{2\sqrt{2}}{2} \\ -\frac{2\sqrt{2}}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \end{bmatrix}$$

$$X = U \Sigma V^T = \begin{bmatrix} -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix} \cdot \begin{bmatrix} 6 & 0 \\ 0 & 2 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{\sqrt{2}}{2} & -\frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

2) Notizen

D

d

ε

3	1	0	0,17	0,73	0,37	5,83	1,28	-0,36
1	0	3	0,15	0,52	0,47	1,81	-0,91	5,53
0	3	5	0,12	0,45	0,23	-0,10	5,57	9,78

$E_{00} = 5{,}83 \cdot 0{,}45 + 1{,}28 \cdot 0{,}78 - 0{,}36 \cdot 0{,}37 =$

$E_{01} = 5{,}83 \cdot 0{,}02 + 1{,}28 \cdot 0{,}67 - 3{,}36 \cdot 0{,}35 =$

$\underset{kj}{A_{00}} = 5{,}66 \cdot 0{,}34 + 1{,}62 \cdot 0{,}38 + -0{,}20 \cdot 0{,}211 = 1{,}985$

$\underset{ij}{D_{00}} \cdot \underset{ik}{E_{00}} \quad \underset{ij}{D_{10}} \quad \underset{ik}{E_{10}} \quad \underset{ij}{D_{20}} \quad \underset{ik}{E_{20}}$

$\to$ 0,45 0,78 0,37

$\to$ 0,02 0,67 0,35

0,17

$\underset{kj}{A_{10}} = 5{,}66 \cdot 0{,}67 + 1{,}62 \cdot 0{,}93 - 0{,}2 \cdot 0{,}39 = 4{,}04$

$\underset{ij}{D_{00}} \cdot \underset{ik}{E_{01}} \quad \underset{ij}{D_{10}} \quad \underset{ik}{E_{11}} \quad \underset{ij}{D_{20}}$

0,34 0,67

0,38 0,93

0,211 0,39

0,34	0,67
0,38	0,93
0,211	0,39

5,66	0,518	-0,72
1,62	-1,83	5,06
-0,2	5,14	9,57
A00	A01	

0,34 0,38 0,21 A00

0,67 0,93, 0,39 A10

5,66	0,57	-0,72	A_{00}	A_{10}
1,62	-1,83	5.0678	A01	
-0,20	5,14	9,57		

$\underset{kj}{A_{01}} = 0{,}57 \cdot 0{,}34 - 1{,}83 \cdot 0{,}38 + 5{,}14 \cdot 0{,}21 = 0{,}55$

$\underset{ij}{D_{01}} \quad \underset{ik}{E_{00}} \quad D_{11} \quad E_{10} \quad D_{21} \quad E_{20}$