

# Chap 8 近似理论(噪声)

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## 离散最小二乘近似

- 线性最小二乘法

- $$a_0 m + a_1 \sum_{i=1}^m x_i = \sum_{i=1}^m y_i,$$
$$a_0 \sum_{i=1}^m x_i + a_1 \sum_{i=1}^m x_i^2 = \sum_{i=1}^m x_i y_i$$

- 多项式最小二乘法

- $$\begin{aligned} a_0 \sum_{i=1}^m x_i^0 + a_1 \sum_{i=1}^m x_i^1 + a_2 \sum_{i=1}^m x_i^2 + \cdots + a_n \sum_{i=1}^m x_i^n &= \sum_{i=1}^m y_i x_i^0, \\ a_0 \sum_{i=1}^m x_i^1 + a_1 \sum_{i=1}^m x_i^2 + a_2 \sum_{i=1}^m x_i^3 + \cdots + a_n \sum_{i=1}^m x_i^{n+1} &= \sum_{i=1}^m y_i x_i^1, \\ &\vdots \\ a_0 \sum_{i=1}^m x_i^n + a_1 \sum_{i=1}^m x_i^{n+1} + a_2 \sum_{i=1}^m x_i^{n+2} + \cdots + a_n \sum_{i=1}^m x_i^{2n} &= \sum_{i=1}^m y_i x_i^n. \end{aligned}$$

## 正交多项式的最小二乘法近似(对函数的近似)

- $$E = \int_a^b \left( f(x) - \sum_{i=0}^n a_i x^i \right)^2 dx$$
- $$a_j = \frac{\int_a^b w(x) f(x) \phi_j(x) dx}{\int_a^b w(x) \phi_j^2(x) dx}$$
- 勒让德多项式, 取  $w(x)=1$ .
- 缺陷是可能引发振荡, 近似误差决定了误差边界, 误差边界又反过来扩大近似误差

## 有理函数近似

- 帕德近似

- $$\left( \sum_{i=0}^k a_i q_{k-i} \right) = p_k, \quad k = 0, 1, \cdots, N$$
  - 近似误差传播得更加平稳

## 三角多项式近似