Signals and Systems – Spring 2023

Lab Assignment #2

Issued: May 14, 2023 Due: Jun. 14, 2023

You should hand in the Matlab code (.m files), and a brief description of your reasoning as well as comments if any. Please make sure that your Matlab code can be run on Matlab R2012b or higher version. You should pack all of your files into a .rar or .zip file, titled as xxxxxxx(your student ID) xxxx(your name) Lab 2, and then submit it by sending an email to 12131095@zju.edu.cn before 11:59pm of the due day.

Problem 1

Illustrate a signal or a system in your daily life and apply what you have learned to analyze it.

Problem 2

Echo Cancellation via Inverse Filtering

In this exercise, you will consider the problem of removing an echo from a recording of a speech signal. This project will use the audio capabilities of MATLAB to play recordings of both the original speech and the result of your processing. To begin this exercise you will need to load the speech file `lineup.mat`, by typing

```
>> load lineup.mat
```

Once you have loaded the data into MATLAB, the speech waveform will be stored in the variable `y`. Since the speech was recorded with a sampling rate of 8192 Hz, you can hear the speech by typing

```
>> sound(y,8192)
```

You should hear the phrase "line up" with an echo. The signal $y[n]$, represented by the vector `y`, is of the form

$$y[n] = x[n] + \alpha x[n-N], \tag{1.1}$$

where $x[n]$ is the uncorrupted speech signal, which has been delayed by N samples and added back in with its amplitude decreased by $\alpha < 1$. This is a reasonable model for an echo resulting from the signal reflecting off of an absorbing surface like a wall. If a microphone is placed in the center of a room, and a person is speaking at one end of the room, the recording will contain the speech which travels directly to the microphone, as well as an echo which traveled across the room, reflected off of the far wall, and then into the microphone. Since the echo must travel further, it will be delayed in time. Also, since the speech is partially absorbed by the wall, it will be decreased in amplitude. For simplicity ignore any further reflections or other sources of echo.

For the problems in this exercise, you will use the value of the echo time, $N = 1000$, and the echo amplitude, $\alpha = 0.5$.

Basic Problems

(a). In this exercise you will remove the echo by linear filtering. Since the echo can be represented by a linear system of the form Eq. (1.1), determine and plot the impulse response of the echo system Eq. (1.1). Store this impulse response in the vector `he` for $0 \leq n \leq 1000$.

(b). Consider an echo removal system described by the difference equation

$$z[n] + \alpha z[n - N] = y[n], \tag{1.2}$$

where $y[n]$ is the input and $z[n]$ is the output which has the echo removed. Show that Eq. (1.2) is indeed an inverse of Eq. (1.1) by deriving the overall difference equation relating $z[n]$ to $x[n]$. Is $z[n] = x[n]$ a valid solution to the overall difference equation?

Intermediate Problems

(c). The echo removal system Eq. (1.2) will have an infinite-length impulse response. Assuming that $N = 1000$, and $\alpha = 0.5$, compute the impulse response using `filter` with an input that is an impulse given by `d=[1 zeros(1,4000)]`. Store this 4001 sample approximation to the impulse response in the vector `her`.

(d). Implement the echo removal system using `z=filter(1,a,y)`, where `a` is the appropriate coefficient vector derived from Eq. (1.2). Plot the output using `plot`. Also, listen to the output using `sound`. You should no longer hear the echo.

(e). Calculate the overall impulse response of the cascaded echo system, Eq. (1.1), and echo removal system, Eq. (1.2), by convolving `he` with `her` and store the result in `hoa`. Plot the overall impulse response. You should notice that the result is not a unit impulse. Given that you have computed `her` to be the inverse of `he`, why is this the case?

Advanced Problem

(f). Suppose that you were given $y[n]$ but did not know the value of the echo time, N, or the amplitude of the echo, α. Based on Eq. (1.1), can you determine a method of estimating these values? Hint: Consider the output `y` of the echo system to be of the form

$$y[n] = x[n] * (\delta[n] + \alpha\delta[n - N])$$

and consider the signal

$$R_{yy}[n] = y[n] * y[-n].$$

This is called the autocorrelation of the signal $y[n]$ and is often used in applications of echo-time estimation. Write $R_{yy}[n]$ in terms of $R_{xx}[n]$ and also plot $R_{yy}[n]$. Also try experimenting with simple echo problems such as

```
>> NX=100;
>> x=randn(1,NX);
>> N=50;
>> alpha=0.9;
>> y=filter([1 zeros(1,N-1) alpha],1,x);
>> Ryy=conv(y,fliplr(y));
>> plot([-NX+1:NX-1],Ryy)
```

by varying `N,alpha`, and `NX`. Also, when you loaded `lineup.mat`, you loaded in two additional vectors. The vector y2 contains the phrase "line up" with a different echo time N and different echo amplitude α. The vector y3 contains the same phrase with two echoes, each with different times and amplitudes. Can you estimate N and α for y2, and N_1, α_1, N_2, and α_2 for y3? Note that getting accurate answers for this problem is very difficult.

Problem 3

Amplitude Modulation and the Continuous-Time Fourier Transform

This exercise will explore amplitude modulation of Morse code messages. A simple amplitude modulation system can be described by

$$x(t) = m(t)\cos(2\pi f_0 t), \tag{2.1}$$

where $m(t)$ is called the message waveform and f_0 is the modulation frequency. The continuous-time Fourier transform (CTFT) of a cosine of frequency f_0 is

$$C(j\omega) = \pi\delta(\omega - 2\pi f_0) + \pi\delta(\omega + 2\pi f_0). \tag{2.2}$$

Using $C(j\omega)$ and the multiplication property of the CTFT, you can obtain the CTFT of $x(t)$, namely,

$$X(j\omega) = \frac{1}{2}M\big(j(\omega - 2\pi f_0)\big) + \frac{1}{2}M\big(j(\omega + 2\pi f_0)\big), \tag{2.3}$$

where $M(j\omega)$ is the CTFT of $m(t)$. Since the CTFT of a sinusoid can be expressed in terms of impulses in the frequency domain, multiplying the signal $m(t)$ by a cosine places copies of $M(j\omega)$ at the modulation frequency.
The remainder of this exercise will involve the signal,

$$x(t) = m_1(t)\cos(2\pi f_1 t) + m_2(t)\sin(2\pi f_2 t) + m_3(t)\sin(2\pi f_1 t), \qquad (2.4)$$

and several parameters that can be loaded into MATLAB from the file `ctftmod.mat`.
If the file has been successfully loaded, then typing `who` should produce the following result:

```
>> who

Your variables are:

af          dash        f1          t
bf          dot         f2          x
```

In addition to the signal $x(t)$, you also have loaded

- a lowpass filter, whose frequency response can be plotted by `freqs(bf,af)`,
- modulation frequencies `f1` and `f2`,
- two prototype signals `dot` and `dash`,
- a sequence of time samples `t`.

To make this exercise interesting, the signal $x(t)$ contains a simple message. When loading the file, you should have noticed that you have been transformed into Agent 008, the code-breaking sleuth. The last words of the aging Agent 007 were "The future of technology lies in … " at which point Agent 007 produced a floppy disk and keeled over. The floppy disk contained the MATLAB file `ctftmod.mat`. Your job is to decipher the message encoded in $x(t)$ and complete Agent 007's prediction.
Here is what is known. The signal $x(t)$ is of the form of Eq. (2.4), where f_1 and f_2 are given by the variables `f1` and `f2`, respectively. It is also known that each of the signals $m_1(t)$, $m_2(t)$, and $m_3(t)$ correspond to a single letter of the alphabet which has been encoded using International Morse Code, as shown in the following table:

A	· -	H	· · · ·	O	- - -	V	· · · -
B	- · · ·	I	· ·	P	· - - ·	W	· - -
C	- · - ·	J	· - - -	Q	- - · -	X	- · · -
D	- · ·	K	- · -	R	· - ·	Y	- · - -
E	·	L	· - · ·	S	· · ·	Z	- - · ·
F	· · - ·	M	- -	T	-		
G	- - ·	N	- ·	U	· · -		

Basic Problems

(a). Using the signals `dot` and `dash`, construct the signal that corresponds to the letter 'Z' in Morse code, and plot it against `t`. As an example, the letter C is constructed by typing `c = [dash dot dash dot]`. Store your signal $z(t)$ in the vector `z`.

(b). Plot the frequency response of the filter using `freqs(bf,af)`.

(c). The signals `dot` and `dash` are each composed of low frequency components such that their Fourier transforms lie roughly within the passband of the lowpass filter. Demonstrate this by filtering each of the two signals, using

```
>> ydash=lsim(bf,af,dash,t(1:length(dash)));
>> ydot=lsim(bf,af,dot,t(1:length(dot)));
```

Plot the outputs `ydash` and `ydot` along with the original signals `dash` and `dot`.

(d). When the signal `dash` is modulated by $\cos(2\pi f_1 t)$, most of the energy in the Fourier transform will move outside the passband of the filter. Create the signal $y(t)$ by executing `y=dash.*cos(2*pi*f1*t(1:length(dash)))`. Plot the signal $y(t)$. Also plot the output `yo=lsim(bf,af,y,t)`. Do you get a result that you would have expected?

Intermediate Problems

(e). Determine analytically the Fourier transform of each of the signals

$$m(t)\cos(2\pi f_1 t)\cos(2\pi f_1 t),$$

$$m(t)\cos(2\pi f_1 t)\sin(2\pi f_1 t),$$

and

$$m(t)\cos(2\pi f_1 t)\cos(2\pi f_2 t)$$

in terms of $M(j\omega)$, the Fourier transform of $m(t)$.

(f). Using your results from Part (e) and by examining the frequency response of the filter as plotted in Part (b), devise a plan for extracting the signal $m_1(t)$ from $x(t)$. Plot the signal $m_1(t)$ and determine which letter is represented in Morse code by the signal.

(g). Repeat Part (f) for the signals $m_2(t)$ and $m_3(t)$. Agent 008, where does the future of technology lie?