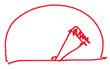
Notes I (mid-term)

2022年3月27日 15:34

1. $a_n = \omega^2 R = \omega \times (\omega \times R)$.

2. $r_c = \dfrac{\int r\,dm}{\int dm}$.

3. 面元 $dm = \sigma r\,d\theta\,dr$. 

4. 密舍尔斯基方程(求推力) : $m\dfrac{d\vec{v}}{dt} = \vec{F} + \vec{v}\dfrac{dm}{dt}$.

证: $\vec{F} = \dfrac{(m+dm)(v+dv) - (mv + u\,dm)}{dt} = \dfrac{m\,d\vec{v} + dm\,\vec{v} - \vec{u}\,dm}{dt}$ $-\vec{v}$(流体时)

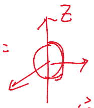
注: $\dfrac{dm}{dt}$ 可负.

$= m\dfrac{d\vec{v}}{dt} + (\vec{v}-\vec{u})\dfrac{dm}{dt}$

5. 保守力: $\oint F\,dr = 0$

6. $L \overset{def}{=} \vec{r}\times\vec{p}$ ($\vec{r}$在前), 故 $M = \vec{r}\times\vec{F}$.

7. $J = \int r^2\,dm$

7.(1) 垂直轴. $J_z = J_x + J_y$ (在xoy平面). $\rightarrow$ eg:  圆环 $J_x = J_z + J_y$

$\therefore mR^2 = 2J_z$

$\therefore J_z = \frac{1}{2}mR^2$

8. 回转半径: $R_G \overset{def}{=} \sqrt{\frac{J}{m}}$, 等效成薄圆环. 已知 $R_G \Rightarrow J$.

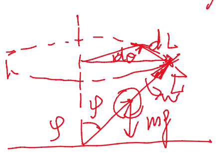
9. 力矩的功: $A = \int_{\theta_0}^{\theta} M\,d\theta$, $P = \dfrac{dA}{dt} = M\dfrac{d\theta}{dt} = M\omega$

10. 刚体定轴转动动能: $E_k = \frac{1}{2}J\omega^2 = \frac{1}{2}(J_c + Mr_c^2)\omega^2 = \frac{1}{2}J_c\omega^2 + \frac{1}{2}Mv_c^2$.

刚体动轴转动动能: $E_k = \frac{1}{2}J_c\omega^2 + \frac{1}{2}Mv_c^2$

 v_c. 定轴转动和动轴转动可以相互转化, 动轴转动就是没找到瞬时轴的一般情况, 通常取质心轴.

10. 回转.



看出 $L\sin\varphi\,d\theta = dL$, 同除 dt

$L\sin\varphi\,\Omega = M$

$\Rightarrow \Omega = \dfrac{M}{L\sin\varphi}$.

11. 伯努利方程 $p + \frac{1}{2}\rho v^2 + \rho g h = \text{cons}$.

$pV + \frac{1}{2}mv^2 + mgh = \text{cons} \rightarrow pV$:压力能

12. 变换方程: $x' = \dfrac{x-ut}{\sqrt{1-u^2/c^2}}$, $t' = \dfrac{t - ux/c^2}{\sqrt{1-u^2/c^2}}$

$v_x' = \dfrac{v_x - u}{1 - v_x u/c^2}$, $v_y' = \dfrac{v_y\sqrt{1-u^2/c^2}}{1 - v_x u/c^2}$, $v_z' = \dfrac{v_z\sqrt{1-u^2/c^2}}{1 - v_x u/c^2}$ (用 $v_x = \dfrac{dx}{dt}$ 易推)

13. 长度的相对性: 同时测量两端 $l = l_0\sqrt{1-v^2/c^2}$

14. 时间的相对性: 同地发生事件 $\Delta t = \dfrac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}}$

$\Big\}$ 不区分K、K'系

15. $E_k = (m-m_0)c^2$. 地参考系

16. 三角等式 $(pc)^2 + (m_0c^2)^2 = E^2 = (mc^2)^2$.