Physica A 458 (2016) 303–312

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Correlated biased random walk with latency in one and two dimensions: Asserting patterned and unpredictable movement

E. Rodriguez-Horta [a,b], E. Estevez-Rams [a,b,*], R. Lora-Serrano [c], B. Aragón Fernández [d]

[a] Facultad de Física, University of Havana, San Lazaro y L. CP 10400, La Habana, Cuba
[b] Instituto de Ciencias y Tecnología de Materiales, University of Havana (IMRE), San Lazaro y L. CP 10400, La Habana, Cuba
[c] Universidade Federal de Uberlandia, AV. Joao Naves de Avila, 2121- Campus Santa Monica, CEP 38408-144, Minas Gerais, Brazil
[d] Universidad de las Ciencias Informáticas (UCI), Carretera a San Antonio, Boyeros, La Habana, Cuba

HIGHLIGHTS

- The balance between irreducible random movement and structured movement is reported for the correlated biased random walk model with latency (CBRWL).
- The CBRWL in two dimensions is constructed from the orthogonal extension of the one dimensional model.
- The relation between drift velocity and tortuosity as a function of entropy density and excess entropy.
- Entropic measures allows to characterize the dynamics independently of the particular control variables.

ARTICLE INFO

Article history:
Received 13 October 2015
Received in revised form 3 March 2016
Available online 12 April 2016

Keywords:
Random walk
Computational mechanics
SHANNON entropy

ABSTRACT

The correlated biased random walk with latency in one and two dimensions is discussed with regard to the portion of irreducible random movement and structured movement. It is shown how a quantitative analysis can be carried out by using computational mechanics. The stochastic matrix for both dynamics are reported. Latency introduces new states in the finite state machine description of the system in both dimensions, allowing for a full nearest neighbor coordination in the two dimensional case. Complexity analysis is used to characterize the movement, independently of the set of control parameters, making it suitable for the discussion of other random walk models. The complexity map of the system dynamics is reported for the two dimensional case.

Dynamical systems can be seen as generating and storing information. In this sense, they can be described as symbols generating computational machines. To know the extent that a given dynamical system is capable of such computing capacity can be important if, for example, one intends to practically tune the control parameters of the system to take advantage of its computing ability. Starting from Brownian motions, the random walk (RW) has been a much used model in statistical physics and related fields to simulate the behavior of different physical systems [1,2]. Its ubiquitous nature, has made it useful in the analysis of a broad number of situations beyond physics such as, biology, where it has been used to model the movement from micro-organisms and insects to mammals [3] or, the diffusion of charges in solids [4].

* Corresponding author at: Facultad de Física, University of Havana, San Lazaro y L. CP 10400, La Habana, Cuba.
E-mail address: estevez@imre.uh.cu (E. Estevez-Rams).

http://dx.doi.org/10.1016/j.physa.2016.03.017

The classical discrete RW involves a stochastic movement of a given length in some arbitrary direction in d-dimensional space. In what follows we will consider that the length of the step is constant over the whole walking process, and in consequence, a fixed time is taken for each step. We will also consider a lattice space, which implies a discrete space over which the walker movement is performed [5]. In its more simple case [5,6], at a given time, the choice of movement direction is a random variable following a given probability distribution with no memory whatsoever of the previous movements. Enlarging this model can be made in several ways. One choice is to introduce biased as a favored direction in space, while correlation is considered when the choice of movement takes into account the previous movements of the walker (the process has memory) [7]. Biased can be taken as a consequence of some driving force (e.g. a particular odor for a predator, an external field for a charge moving inside a solid, etc.) The most simple form of correlation, usually called persistence, is to consider, when choosing the next movement, the last step of the walker. Persistence then introduces a preference, that the walker follows the previous movement as some kind of inertial effect [8].

A further variable in the movement can be introduced if the possibility that, at a given time, the walker decides to stay at rest allowed [3]. The resulting RW will be said to have latency. Latency could be justified in a number of ways. One could consider it to be a transitional state between change of movement direction, due to the impossibility of instantaneous movement change. Yet, we will spare any rationale leading to latency and just consider it for its worth in enhancing the possibilities of RW models. In particular, it will be shown that introducing latency in one dimensional RW, allows to go to a two dimensional lattice RW in a simple manner while keeping full coordination of the movement.

In the analysis of RW, it has been usual to be interested in the statistical properties of the movement, such as the probability distribution over distance as a function of time, from where the diffusion coefficient can be calculated. Other questions have been the probability that the walker will return to a given point in space; the speed of the walker; the probability of escaping the origin; among others [5,6,9]. Closely related to these questions, is to assert the balance between unpredictable and structured movement in the RW, as a consequence of the competing effect of the control parameters. This analysis can be carried out by coding the RW as a symbol producing system and then quantify the production of patterns and irreducible randomness in the symbolic generator process. Viewed from this perspective, the RW can be considered as a computational machine that sequentially generates an infinite string of symbols with certain degree of predictability and certain degree of noise. Structure is then associated to the relation between some magnitude characterizing the predictable behavior and some magnitude characterizing the random part [10].

Computational mechanics is an approach to complexity based on reconstruction of the optimal computational machine, termed ϵ-machine, within a hierarchy of machines that allows to discover the nature of patterns and to quantify them [11,10]. It is rooted in information theory concepts, and has found applications in several areas [12–14]. Its use in statistical mechanics has allowed to define and calculate magnitudes that complement thermodynamic quantities. Optimality is the balance of having the best predictive power with less number of resources as reflected by the number of internal states [15].

For an ϵ-machine, the statistical complexity C_μ is defined as the Shannon entropy over the probability of the causal states. A causal state is taken in its more general meaning, as a set of pasts that determines (probabilistically) equivalent futures [16]. C_μ is a measure of the amount of knowledge or memory needed for optimal prediction. Another magnitude is the entropy density, which measures the irreducible randomness of the system and is defined as the asymptotic value of the Shannon block entropy over the generated sequence of symbols divided by the block length [17]. And finally, the excess entropy, defined as the mutual information between past and future, which, in the case of a first order Markov process, can be calculated by subtracting the entropy density from the statistical complexity. Excess entropy is the amount of memory needed to make optimal predictions without taking into account the irreducible randomness [18].

In this article we will be interested in quantifying the unpredictable and predictable movement in a biased correlated RW with latency (CBRWL) in both one- and two-dimensions.

The paper is organized as follows: in Section 1, notions of computational mechanics are introduced for completeness. In this section the mathematical definition of statistical entropy, entropy density and excess entropy are introduced. Section 2 describes the one dimensional RW as a symbol generating process. In Section 3 the two dimensional case is discussed as an extension of the one dimensional walk in two orthogonal directions, this is followed in Section 4 by discussion and conclusions.

1. Casual states, statistical complexity, entropy density and excess entropy

Computational mechanics relies on the concept of causal states [16]. Consider a process whose output is a bi-infinite string or sequence, the characters of which are drawn from an alphabet Σ. Consider, for any particular realization of the process, the output string ξ and partition the string in two half $\xi^- \equiv \xi(-\infty, -1)$ and $\xi^+ \equiv \xi(0, \infty)$ which are called past and future, respectively. If the output strings are considered to be drawn from a (in general unknown) distribution, then two ξ^- and ξ'^- that give the same probability $Pr(\xi^+|\xi^-) = Pr(\xi^+|\xi'^-)$ for all possible futures ξ^+, are said to belong to the same causal state C_p that it is written $\xi^- \sim \xi'^-(\xi^-, \ \xi'^- \in C_p)$.

The partition of the set of possible pasts (denoted by Ξ^-), in classes of causal states, is an equivalence relation complying with the transitivity condition (if $\xi_i^- \sim \xi_j^-$ and $\xi_j^- \sim \xi_k^-$ then $\xi_i^- \sim \xi_k^-$), symmetry (if $\xi_i^- \sim \xi_j^-$ then $\xi_j^- \sim \xi_i^-$) and reflectivity ($\xi_i^- \sim \xi_i^-$). The set of causal state (denoted by $\mathcal{C}$ with cardinality $|\mathcal{C}|$) uniquely determines the future of a sequence. Then, function ϵ can be defined over ξ^-, $\epsilon : \Xi^- \to \mathcal{C}$ which relates ξ^- with its causal states C,

$$\epsilon(\xi^-) \equiv C = \{\xi'^- | Pr(\xi^+|\xi^-) = Pr(\xi^+|\xi'^-) \ \forall \xi^+\}.$$

The statistical complexity is defined as the Shannon entropy over the causal states [15]

$$\begin{aligned} C_\mu &\equiv H[\mathcal{C}] \\ &= -\sum_{C_p \in \mathcal{C}} Pr(C_p) \log Pr(C_p) \end{aligned} \tag{1}$$

where the sum is over the set of causal states $\mathcal{C}$. The logarithm is usually taken in base two and the units are then bits. From the construction principle, as each causal state determines probabilistically a given future, then the set of causal states is related to the optimal memory required for prediction. More memory resources will not improve the predictive power of the process. Statistical complexity, being the Shannon entropy over the causal states, is therefore a measure of how much memory the system needs to optimally predict the future.

Consider a first order Markov process, where the occurrence of a given symbol $s_i (\in \Sigma)$ at step t, depends exclusively on the symbol $s_k (\in \Sigma)$ emitted at step $(t-1)$. A stochastic matrix P can then be defined whose entry $p_{ki} = Pr(s_i|s_k)$ is the probability of emitting a symbol s_i if in the previous step a symbol s_k was emitted. It is straightforward that if two rows j and m in the stochastic matrix are the same, then the corresponding symbols s_j and s_m belong to the same causal state. By using this property, causal state over the Markov process can be derived and the stochastic matrix over the causal state can be calculated.

If $\langle p^\infty|$ is the vector of probabilities over the causal states, then it is well known [19] that the stationary distribution is given by

$$\langle p^\infty| = \langle w_0| \tag{2}$$

where $\langle w_0|$ is the left dominant eigenvector of the stochastic matrix P

$$\langle w_0|P = \langle w_0|. \tag{3}$$

The vector $\langle p^\infty|$ allows to calculate the probability of a causal state $Pr(C_p)$ when the Markov process has been running for a sufficiently long time.

In order to account for the irreducible randomness, the entropy density, can be calculated as [18]

$$h_\mu = -\sum_{C_\alpha \in \mathcal{C}} Pr(C_\alpha) \sum_{s_k \in \Sigma} Pr(s_k|C_\alpha) \log Pr(s_k|C_\alpha). \tag{4}$$

Entropy density is a measure of the remaining unpredictability once infinite past has been observed. From there, the excess entropy can be calculated for a first order Markov process as

$$E_\mu = C_\mu - h_\mu. \tag{5}$$

Excess entropy is a measure of the resources needed once the irreducible randomness has been subtracted [20]. E is always non-negative, which implies

$$C_\mu \geq h_\mu.$$

If the system is perfectly periodic then $h_\mu = 0$ and

$$C_\mu = E_\mu.$$

As long as the cardinality of the alphabet Σ of the first order Markov process is finite, the number of causal states will be also finite, and the whole process can be described by the so called ϵ-machine description, which corresponds to the minimum deterministic finite state machine (FSM) able to optimally predict the dynamics of the process. The reader can refer to Refs. [16,18] for further discussion.

The ϵ-machine FSM can be represented by a digraph, where each node corresponds to a causal state and the directed transitions between nodes are labeled $s_i|Pr(C_m|C_p)$. s_i is the emitted symbol while making a transition $C_p \to C_m$, the arriving state is uniquely determined by the emitted symbol s_i, a property called unifiliarity.

2. One dimensional random walk as a symbol generating process

We first start by the one dimensional (1D) walk. The walker is allowed to move to the right $(\rightarrow)$ or to the left $(\leftarrow)$ a unit length in a unit time. Biased is quantified by a probability r that the walker chooses a right movement. If the walker performed a movement in some direction at time $t-1$, followed by the same movement at time t, it will be described by the probability p known as persistence. And finally, l, is the probability that the walker at a given time will make no movement. Then we define our RW by the set of control parameters (r, p, l). The control parameters will be taken fixed in time. Normalization condition will impose that the probability for a biased movement to the left will be given by $1-r-l$.

The RW can now be described as a symbol generating process which takes values from the set $\Sigma = (\leftarrow, \circ, \rightarrow)$, the first representing a move to the left, the second no move, and the last a move to the right. The symbol generator outputs a symbol at each time step according to the control parameters, which describes the movement at that particular time. The sequence of symbols will describe a particular realization of the RW. The stochastic matrix for the process, which can be seen to comply with the definition of a first order Markov process, follows immediately

$$P = \begin{pmatrix} P(\rightarrow\mid\rightarrow) & P(\circ\mid\rightarrow) & P(\leftarrow\mid\rightarrow) \\ P(\rightarrow\mid\circ) & P(\circ\mid\circ) & P(\leftarrow\mid\circ) \\ P(\rightarrow\mid\leftarrow) & P(\circ\mid\leftarrow) & P(\leftarrow\mid\leftarrow) \end{pmatrix}$$
$$= \begin{pmatrix} \frac{pr}{l(1-p)+(-l-r+1)(1-p)+pr} & \frac{l(1-p)}{l(1-p)+(-l-r+1)(1-p)+pr} & \frac{(1-p)(-l-r+1)}{l(1-p)+(-l-r+1)(1-p)+pr} \\ \frac{(1-p)r}{lp+(1-p)(-l-r+1)+(1-p)r} & \frac{lp}{lp+(1-p)(-l-r+1)+(1-p)r} & \frac{(1-p)(-l-r+1)}{lp+(1-p)(-l-r+1)+(1-p)r} \\ \frac{(1-p)r}{l(1-p)+r(1-p)+p(-l-r+1)} & \frac{l(1-p)}{l(1-p)+r(1-p)+p(-l-r+1)} & \frac{p(-l-r+1)}{l(1-p)+r(1-p)+p(-l-r+1)} \end{pmatrix}. \quad (6)$$

$P(a|b)$ is the probability of performing a movement 'a' at time t, when at time $t-1$ the movement was 'b'. The most general finite state machine (FSM) describing this process is shown in Fig. 1(a). The current state of movement defines the state of the automaton. The FSM emits a symbol equal to the state he is arriving. If $r = 0$, then the RW can only move left or stay at rest at a given time, the FSM then collapse to a two state machine shown in Fig. 1(b), a topologically identical FSM is found for $r = 1$ and $l = 0$. If latency occurs with certainty, $l = 1$, then the generating machine is a one state FSM (Fig. 1(c)). Specific values of persistence lead to three different behaviors (see Fig. 1(d)). For $p = 1$, the initial state determines which fixed movement the RW will have, and therefore each of the three states are decoupled. At $p = 0$, the same movement cannot be observed at two consecutive times and therefore, the self-state transitions are lost. And finally, at $p = 1/2$ the stochastic matrix (6) reduces to

$$P = \begin{pmatrix} \frac{r}{2\left(\frac{l}{2}+\frac{1}{2}(-l-r+1)+\frac{r}{2}\right)} & \frac{l}{2\left(\frac{l}{2}+\frac{1}{2}(-l-r+1)+\frac{r}{2}\right)} & \frac{-l-r+1}{2\left(\frac{l}{2}+\frac{1}{2}(-l-r+1)+\frac{r}{2}\right)} \\ \frac{r}{2\left(\frac{l}{2}+\frac{1}{2}(-l-r+1)+\frac{r}{2}\right)} & \frac{l}{2\left(\frac{l}{2}+\frac{1}{2}(-l-r+1)+\frac{r}{2}\right)} & \frac{-l-r+1}{2\left(\frac{l}{2}+\frac{1}{2}(-l-r+1)+\frac{r}{2}\right)} \\ \frac{r}{2\left(\frac{l}{2}+\frac{1}{2}(-l-r+1)+\frac{r}{2}\right)} & \frac{l}{2\left(\frac{l}{2}+\frac{1}{2}(-l-r+1)+\frac{r}{2}\right)} & \frac{-l-r+1}{2\left(\frac{l}{2}+\frac{1}{2}(-l-r+1)+\frac{r}{2}\right)} \end{pmatrix}, \quad (7)$$

where all rows are identical showing that we have a collapse to a single causal state (the three initial states determine the same future) as shown in the right FSM of Fig. 1(d). In such case, the most unpredictable dynamics will be given by $r = l = 1/3$ ($P(\rightarrow) = P(\leftarrow) = P(\bullet) = 1/3$) with an entropy density of $h_\mu = \log_2 3 = 1.5849$ bits.

Consider the matrix formed by the sum of the powers of the stochastic matrix

$$Q^{(N)} = \sum_{i=0}^{N} P^i, \quad (8)$$

each entry $q_{ij}^{(N)}$ will be the average number of times, in N steps, the chain is in state j, given that it started in state i [21]. Then the expected number of times $E(j|i)$ the chain starting in state i will visit state j in the first N steps, is given by

$$E(j|i) = \sum_{(k=1)}^{N} q_{ij}^{(k)}. \quad (9)$$

The average value of the particle position $\langle x\rangle$ as a function of the number of steps $N(\gg 1)$ then follows:

$$\langle x\rangle(N) = \sum_{k=1}^{N}(q_{11}^{(k)} - q_{13}^{(k)}). \quad (10)$$

From (10), the asymptotic drift velocity v_d can be calculated as the slope of the linear dependence with N.

Fig. 2(a) plots the relation between drift velocity and excess entropy for a small value of biased probability to the right ($r = 0.01$). Large values of excess entropy implies small absolute values of velocity. This corresponds to a process with a small value of persistence where the ϵ-machine is described by a two state FSM of alternating directions similar to an antiferromagnetic order. As persistence increases while keeping small the latency, the system tends to a one state FSM with low memory, and therefore low excess entropy, but a high drift speed (the negative sign of the drift velocity comes from the movement tendency to the left, as the left bias is near one) corresponding to a ballistic movement, isomorphous with a ferromagnetic state.

When latency probability is above zero, a third state appears in the FSM description. Again, high excess entropy corresponds to small values of persistence, but now the system alternates between three states and therefore the drift velocity is not reduced to zero. As persistence increases, with small latency, the excess entropy decreases as a result that the system has longer runs on the same movement state and this, in turn, implies a larger absolute value of the drift velocity. Latency

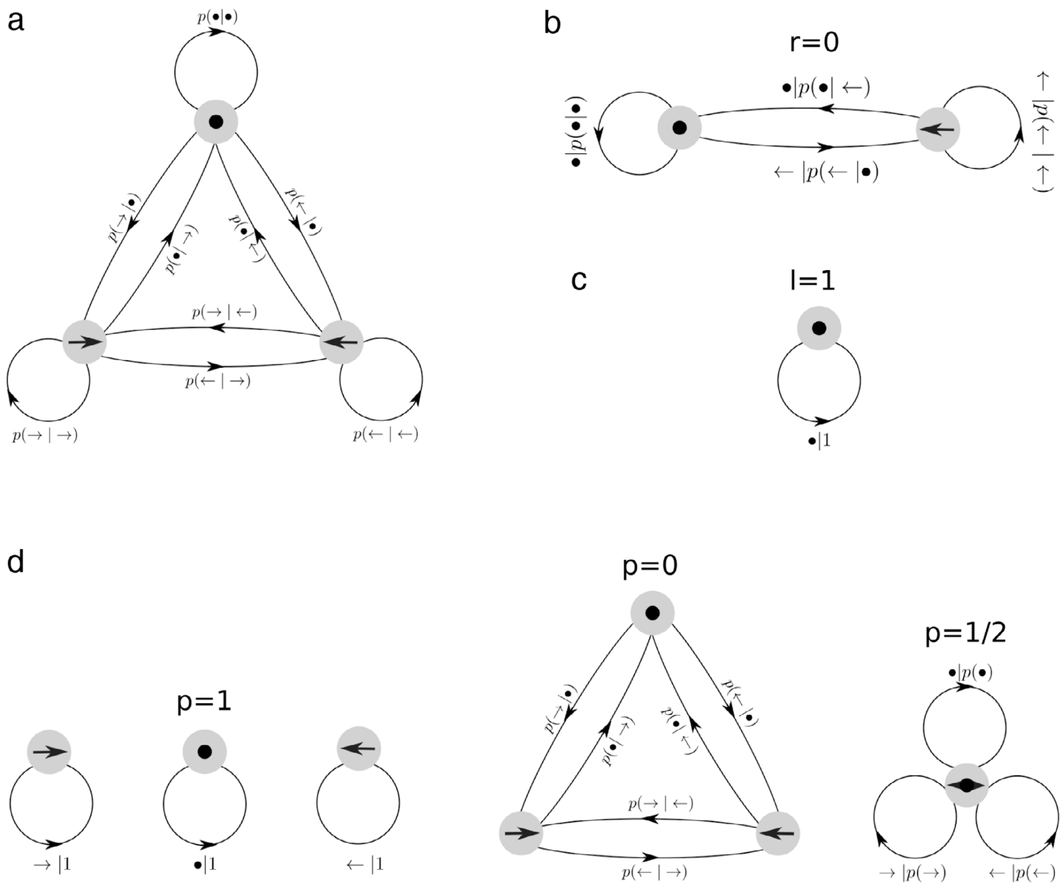

Fig. 1. Finite state machine (FSM) for the CBRWL in one dimension. (a) The general FSM, (b–d) FSM for different values of the control parameters. $s|p(r|m)$ is the probability of making the transition from state m to state r upon emitting a symbol s (When the complete notation clutters the diagram, the emitted symbol is skipped as no ambiguity arises). The reader should notice in (d) $p = 1/2$, that probabilities are not conditioned as all states collapse to just one state.

different from zero results in dynamics where, for each excess entropy value, two values of drift velocity could be associated. The two values are from equally structured movements with different dynamics, one with more alternating left and right movement, and the other with a larger value of persistence and more biased movement. As latency increases, the absolute value of the drift speed decreases as expected. Increasing biased r below $1/2$, shifts the drift speed dependency nearer zero as the left movement is less favored. Certain values of latency become unattainable due to the normalization condition of the probabilities.

The relation between drift velocity and entropy density is shown in Fig. 2(b). The increase of entropy density reduces the absolute value of the drift velocity in all cases. The two values functional dependence has a similar interpretation as the excess entropy behavior. More interesting is that in the 3D plot of Fig. 2(c), it can be seen that, for a given latency, the (E, h_μ) pair determines uniquely the drift velocity and, therefore, can serve as a fingerprint for the system dynamics. The trend to a ballistic movement follows from decreasing the excess entropy and the entropy density, approaching a single state FSM.

3. The two dimensional random walk

To extend the previous model to the two dimensional (2D) case, we consider two non-independent orthogonal RW, say in the x and y direction. For both directions the persistence probability p is the same, and so is the latency probability l. Yet the biased probability for each direction will now be given by $r = (b\cos\theta + 1)/2$ and $u = (b\sin\theta + 1)/2$ for each direction, x and y, respectively. Biased probability for a movement to the right is still represents r, while u represents the biased probability for a movement upwards. The stochastic matrix for the RW in the $\hat{x}$ direction is still given by Eq. (6). For the RW in the y direction the stochastic matrix is isomorphous with (6) just with a relabeling of the movements.

The new control parameter b ($\in[0, 1]$) represents the absolute strength of the biased, while θ gives how this biased is distributed between both orthogonal directions. The way biased is now introduced, intends to describe what should be expected for the presence of an external field pointing in some direction in the xy plane.

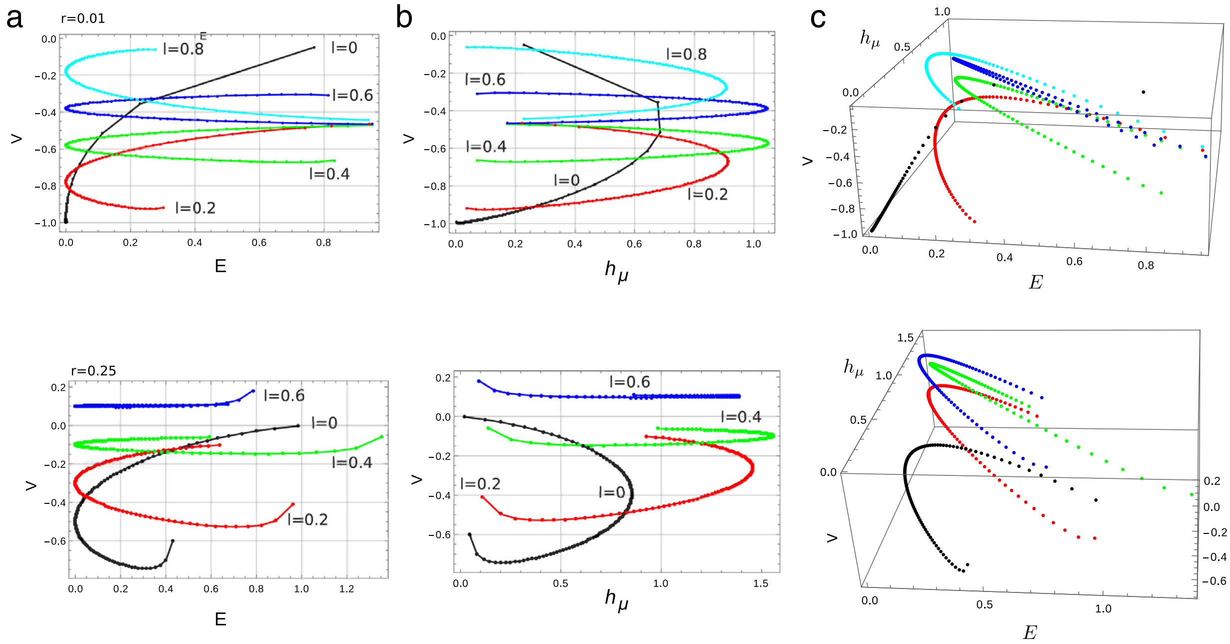

Fig. 2. Drift velocity as a function of (a) excess entropy E and (b) entropy density h_μ for different values of latency probability l, and for two bias probability r. The three dimensional (3D) plot of drift velocity as a function of both E and h_μ shows that for a fixed latency value, each attainable (E, h_μ) pair, determines a unique drift velocity.

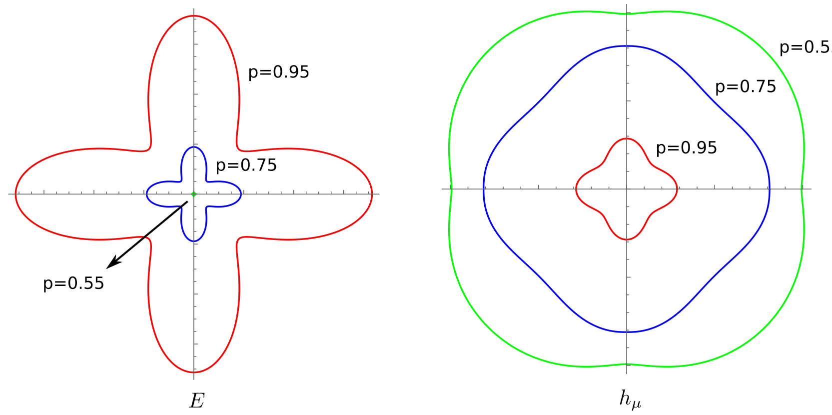

Fig. 3. Polar plot of the excess entropy E and the entropy density h_μ as function of the θ angle for different values of persistence probability. The distance of the plot from the origin is proportional to the value of the corresponding parameter for the given direction. The bias strength was taken as $b = 1/2$.

Without latency ($l = 0$), the allowed movements would be ($\nearrow, \nwarrow, \searrow, \swarrow$), which conform a four coordinated square lattice. The provision for latency adds the movements ($\rightarrow, \leftarrow, \uparrow, \downarrow, \circ$), keeping the square lattice, but with eight coordination. The FSM will now have, in the general case, nine causal states.

The stochastic matrix follows from

$$P = \begin{pmatrix}
P_{rr}P_{oo}^{y} & P_{lr}P_{oo}^{y} & P_{or}P_{uo} & P_{do}P_{or} & P_{rr}P_{uo} & P_{lr}P_{uo} & P_{do}P_{rr} & P_{do}P_{lr} & P_{or}P_{oo}^{y} \\
P_{rl}P_{oo}^{y} & P_{ll}P_{oo}^{y} & P_{ol}P_{uo} & P_{do}P_{ol} & P_{rl}P_{uo} & P_{ll}P_{uo} & P_{do}P_{rl} & P_{do}P_{ll} & P_{ol}P_{oo}^{y} \\
P_{ou}P_{ro} & P_{lo}P_{ou} & P_{uu}P_{oo}^{x} & P_{du}P_{oo}^{x} & P_{ro}P_{uu} & P_{lo}P_{uu} & P_{du}P_{ro} & P_{du}P_{lo} & P_{ou}P_{oo}^{x} \\
P_{od}P_{ro} & P_{lo}P_{od} & P_{ud}P_{oo}^{x} & P_{dd}P_{oo}^{x} & P_{ro}P_{ud} & P_{lo}P_{ud} & P_{dd}P_{ro} & P_{dd}P_{lo} & P_{od}P_{oo}^{x} \\
P_{ou}P_{rr} & P_{lr}P_{ou} & P_{or}P_{uu} & P_{du}P_{or} & P_{rr}P_{uu} & P_{lr}P_{uu} & P_{du}P_{rr} & P_{du}P_{lr} & P_{or}P_{ou} \\
P_{ou}P_{rl} & P_{ll}P_{ou} & P_{ol}P_{uu} & P_{du}P_{ol} & P_{rl}P_{uu} & P_{ll}P_{uu} & P_{du}P_{rl} & P_{du}P_{ll} & P_{ol}P_{ou} \\
P_{od}P_{rr} & P_{lr}P_{od} & P_{or}P_{ud} & P_{dd}P_{or} & P_{rr}P_{ud} & P_{lr}P_{ud} & P_{dd}P_{rr} & P_{dd}P_{lr} & P_{od}P_{or} \\
P_{od}P_{rl} & P_{ll}P_{od} & P_{ol}P_{ud} & P_{dd}P_{ol} & P_{rl}P_{ud} & P_{ll}P_{ud} & P_{dd}P_{rl} & P_{dd}P_{ll} & P_{od}P_{ol} \\
P_{ro}P_{oo}^{y} & P_{lo}P_{oo}^{y} & P_{uo}P_{oo}^{x} & P_{do}P_{oo}^{x} & P_{ro}P_{uo} & P_{lo}P_{uo} & P_{do}P_{ro} & P_{do}P_{lo} & P_{oo}^{x}P_{oo}^{y}
\end{pmatrix}. \tag{11}$$

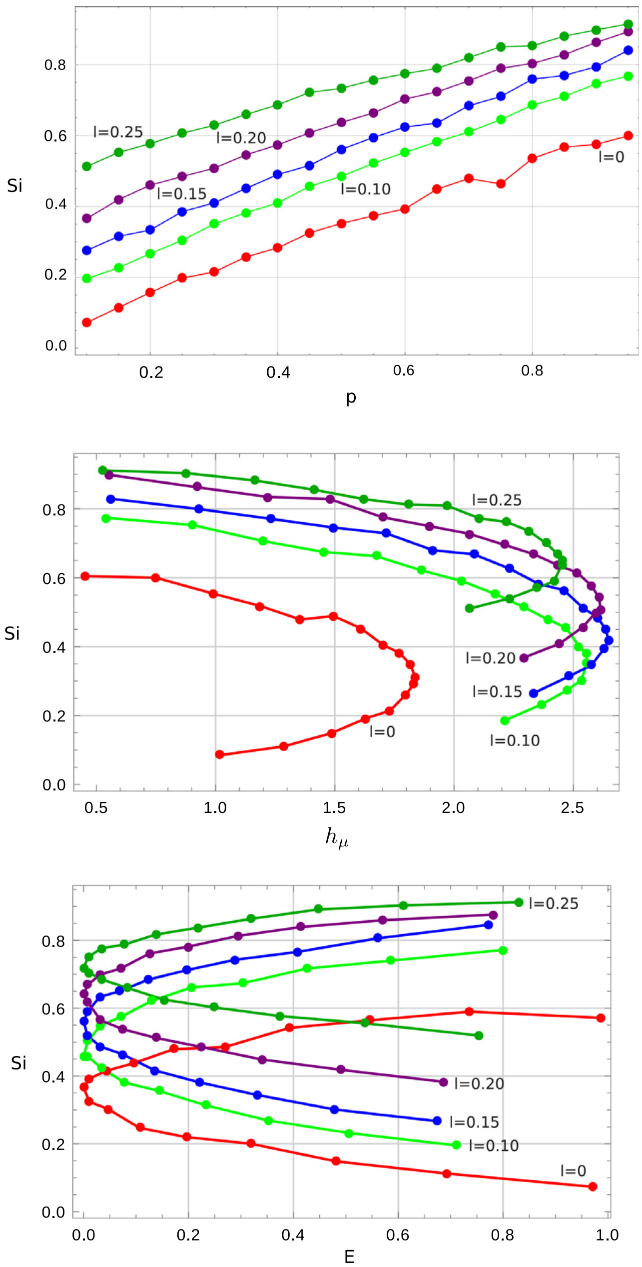

Fig. 4. Straight index as a function of persistence probability p, excess entropy E and entropy density h_μ, for different values of latency.

$$
\begin{array}{lll}
P_{rr} = P(\rightarrow \mid \rightarrow) & P_{lr} = P(\leftarrow \mid \rightarrow) & P_{or} = P(\circ \mid \rightarrow) \\
P_{rl} = P(\rightarrow \mid \leftarrow) & P_{ll} = P(\leftarrow \mid \leftarrow) & P_{ol} = P(\circ \mid \leftarrow) \\
P_{ro} = P(\rightarrow \mid \circ) & P_{lo} = P(\leftarrow \mid \circ) & P^{x}_{oo} = P^{x}(\circ \mid \circ) \\
P_{dd} = P(\downarrow \mid \downarrow) & P_{ud} = P(\uparrow \mid \downarrow) & P_{od} = P(\circ \mid \downarrow) \\
P_{du} = P(\downarrow \mid \uparrow) & P_{uu} = P(\uparrow \mid \uparrow) & P_{ou} = P(\circ \mid \uparrow) \\
P_{do} = P(\downarrow \mid \circ) & P_{uo} = P(\uparrow \mid \circ) & P^{y}_{oo} = P^{y}(\circ \mid \circ).
\end{array}
\qquad (12)
$$

The expressions (1) and (4) for the statistical complexity and the entropy density remain valid. Fig. 3 shows the polar plot for the excess entropy and the entropy density at zero latency ($l = 0$) for different values of persistence. Observe that at $p = 0.55$, entropy density is large for all θ values, while the system shows no structure as can be observed for the excess entropy. As persistence increases, entropy density decreases and excess entropy increases. Bias along one axis ($\theta = 0, \pi/2, \pi, 3\pi/2$) shows a maximum in the excess entropy, this is the result of that the movement along the

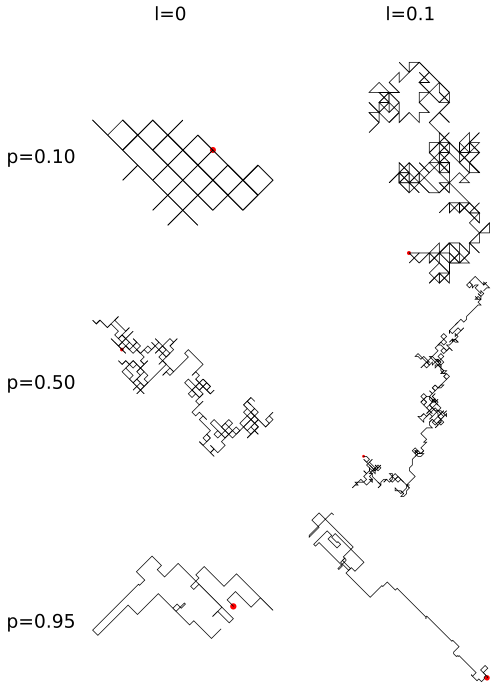

Fig. 5. Random walk for the two dimensional CBRWL for three values of persistence p and two values of latency l. In every case, no biased was considered ($b = 0$).

perpendicular axis is anti persistent (=1/2) even if the movement along the biased axis tends to be ballistic. The same argument explains the minimum of the excess entropy at $\theta = \pi/4$ and equivalent directions.

Instead of using the average speed to describe overall movement of the walker, the straightness index is introduced [22]. Straightness index (Si) has been used as a measure of tortuosity in RW [22], which describes the amount of turning in a RW realization. For our purposes, the Si for a RW of N steps will be defined as the ratio between the maximum net displacement in a RW with the total path length,

$$Si(N) = \frac{Max(|(x, y)|)}{N}. \tag{13}$$

Si increases with persistence probability p for all latency values as shown in Fig. 4(a). The plot was taken with $\theta = \pi/4$ and $b = 1/2$. The relation of the straightness index with excess entropy and entropy density resembles that of the drift velocity for the one dimensional RW and the discussion made there is also valid for this case. Again, the pair (E, h_μ) uniquely determines the Si value, similar to the drift velocity for a fixed latency (not shown).

To better understand the dynamics of the RW, Fig. 5 shows several realizations of the RW for zero bias ($b = 0$) and different values of persistence and latency. We simulated RW with given control parameters 10^2 times to minimize fluctuations and the graphs are just a representative of such walks. Increasing persistence allows the walker reach further away from the starting point. Interestingly, when going from zero latency to a positive latency, the reach of the walker increases significantly. As soon as bias settles (Fig. 6), the walk looses "randomness" and approaches a ballistic movement.

4. Discussion and conclusion

Biased and correlated random walk with latency allows to describe a large number of situations, as three (four) independent control parameters can be tuned in the one (two) dimensional movement. Simulations of models for real life phenomena then reduce to find appropriate relations for the control parameters, as a function of the physical variables controlling the dynamics of the system. Latency can then be see, for example, as the result of the frequency of hoping in a charge movement within a solid, or, describing transient rest states in a living being movement.

For the two dimensional RW, the change of direction has been often characterized by defining a turn as the angle difference between two consecutive movements. The model here described, considers discrete values of turn as integer multiples of $\pi/4$. The probability of a given turn can be readily calculated from the stochastic matrix (11). For example, the probability of a $\pi/4$ turn would be given by

$$P(\pi/4) = P(\uparrow \mid \nearrow)P(\nearrow) + P(\nwarrow \mid \uparrow)P(\uparrow) + P(\leftarrow \mid \nwarrow)P(\nwarrow) + \cdots. \tag{14}$$

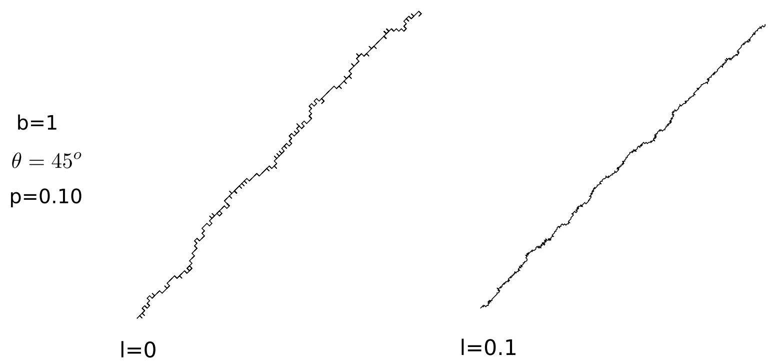

Fig. 6. Random walk for the two dimensional CBRWL for two values of latency l. In every case, the biased strength was taken as $b = 0$ and the biased direction as $\theta = \pi/4$, persistence probability was $p = 0.10$.

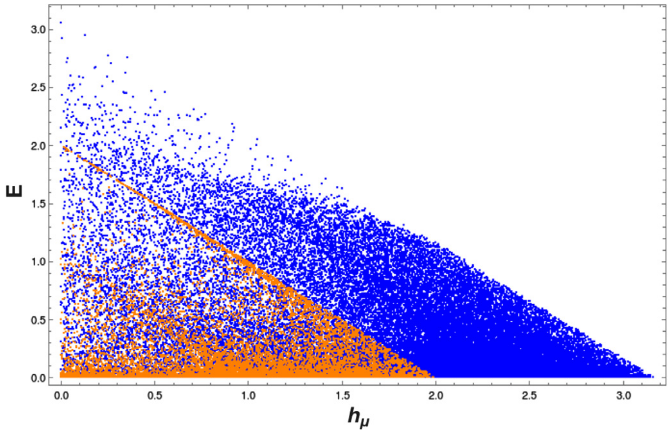

Fig. 7. Complexity map for the two dimensional CBRWL. The map was built from 5×10^4 random values of the control parameters (b, θ, p, l). The yellow portion corresponds to zero latency $l = 0$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

In the context of animal movement, it has been argued that correlated biased random walk can model such behavior, even with constant length step size [23]. In such cases, turn probability is considered a physical parameter. In this type of analysis bias and persistence are usually related to taxi orientation mechanism or differential-klino (DK) kinesis movement. Taxi has to do with the animals settling their movement direction of motion at a given time with respect to a global goal direction (in our case given by the parameter θ). The strength of the goal direction attraction is characterized by the control parameter b. DK-kinesis movement is driven by turn concentration or turn spatial frequency. Latency can then be seen as the animal rest state due to some external or internal factor e.g. bees spending time at a given flower, rodents feeding at some point.

The transport of ions in solids can be interpreted as a hoping movement of the defect-ions within the lattice [24]. If the mobile defects have some king of mutual interaction (e.g. Coulomb repulsion) or correlated structural defects, then the movement differs from a classical uncorrelated RW. Latency in this case can result from a relaxation time depending on the "defect cloud" surrounding the moving defect-charge, the lattice state (e.g. thermal motion) and the actual energy landscape at a given time. Considering latency a fixed value is a kind of mean approximation. Biased is related to the strength and direction of an applied field.

In any case, the random walk with latency introduces, in a natural way, a full coordinated movement for the two dimensional lattice as an orthogonal extension of the one dimensional case. As a consequence, the introduction of latency results in a loitering movement that adds a new quality for the walk, as witness by some of its realization shown in Fig. 6.

Complementary with the usual statistical analysis, the competing effect of unpredictable movement and structured movement is not usually asserted when analyzing random walks. Yet, it allows to characterize the movement independently of the set of control parameters used. For such analysis, computational mechanics, as exemplified here, has proven already to give an adequate framework to model statistically the dynamics of the system in its most optimal achievable way [10]. Furthermore, computational mechanics allows to compute the excess entropy and entropy density and, therefore, asserts the irreducible randomness of a movement against the structured movement. Such relations could be, for example, related to energy dissipation as result of random ("thermal") movement, or energy transport in the short-, medium- and long-term. Or, in the case of animal movement modeling, question as how much of the animal movement is related to futile wandering, and how much to purposed movement, can be given a quantitative answer. Fig. 7 shows a (complexity) map of excess entropy and entropy density for the two dimensional CBRWl model. It is clear that there is no functional relation between both parameters, yet as entropy density increases (unpredictable random movement increases) the system attainable memory (structural movement) decreases. At the maximum h_μ (apex of the map), no patterned movement can be accommodated in the dynamics of the system independently of how the RW control parameters are tuned. The complexity map also shows that for zero latency the reachable portion of the complexity map reduces, and the largest possible excess entropy, attainable at zero entropy density, lowers to 1 bit/step. This further emphasizes that latency changes the nature of the dynamics, a result that ultimately is consequence of the change of the coordination number in the lattice movement.

Acknowledgments

This work was partially financed by FAPEMIG under the project BPV-00047-13. EER wish to thank PVE/CAPES for financial support under the grant 1149-14-8. Infrastructure support was given under project FAPEMIG APQ-02256-12.

The authors would like to thank an anonymous referee for several useful suggestions that improved significantly the article quality.

References

[1] G.I. Taylor, Diffusion by continuous movements, Proc. Lond. Math. Soc. 20 (1922) 196–212.
[2] E.W. Montroll, Markoff chains and excluded volume effect in polymer chains, J. Chem. Phys. 18 (1950) 734–743.
[3] E.A. Codling, M.J. Plank, S. Benhamou, Random walk models in biology, J. R. Soc. Interface 5 (2008) 813–834.
[4] D.J. Jacobs, H. Nakanishi, A persistent random walk model for the frequency-dependent electrical conductivity, Physica A 197 (1993) 204–222.
[5] J.W. Haus, K.W. Kehr, Diffusion in regular and disordered lattices, Phys. Rep. 150 (1987) 263–406.
[6] J.W. Hanneken, D.R. Franceschetti, Exact distribution function for discrete time correlated random walks in one dimension, J. Chem. Phys. 109 (1998) 6533–6539.
[7] R. Garcia-Pelayo, Solution of the persistent, biased random walk, Physica A 384 (2007) 143–149.
[8] N. Pottier, Analytic study of the effect of persistence on a one-dimensioanl biased random walk, Phys. A 230 (1996) 563–576.
[9] R. Gracia-Pelayo, The random flight and the persistent random walk, in: A. Skogseid, V. Fasano (Eds.), Statistical Mechanics and Random Walk, Nova Sciebe Pub. Inc., 2012.
[10] J.P. Crutchfield, Between order and chaos, Nature 8 (2012) 17–24.
[11] J.P. Crutchfield, Knowledge and meaning …chaos and complexity, in: L. Lam, V. Narodditsty (Eds.), Modeling Complex Phenomena, Springer, Berlin, 1992, pp. 66–101.
[12] D.P. Varn, G.S. Canright, J.P. Crutchfield, ϵ-machine spectral reconstruction theory: a direct method for inferring planar disorder and structure from X-ray diffraction, Acta Crystallogr. Sect. A 69 (2013) 197–206.
[13] V. Ryabov, D. Neroth, Computational mechanics of molecular systems: quantifying high-dimensional dynamics by distribution of poincare recurrence times, Chaos 21 (2011) 037113–037122.
[14] R. Haslinger, K.L. Klinker, C.R. Shalizi, The computational structure of spike trains, Neural Comput. 22 (2010) 121–157.
[15] J.P. Crutchfield, The calculi of emergence: computation, dynamics and induction, Physica D 75 (1994) 11–54.
[16] C.R. Shalizi, J.P. Crutchfield, Computational mechanics: pattern and prediction, J. Stat. Phys. 104 (2001) 817–879.
[17] J. Crutchfield, D.P. Feldman, Regularities unseen, randomness observed: Levels of entropy convergence, Chaos 13 (2003) 25–54.
[18] D.P. Feldman, Computational mechanics of classical spin systems, 1998.
[19] E. Behrends, Introduction to Markov Chain, Springer, Heidelberg, 2000.
[20] D.P. Feldman, C.S. McTague, J.P. Crutchfield, The organization of intrinsic computation: complexity-entropy diagrams and the diversity of natural information processing, Chaos 18 (2008) 043106–043121.
[21] C.M. Grinstead, J.L. Snell, Introduction to Probability, https://www.dartmouth.edu/~chance/teaching_aids/books_articles/probability_book/amsbook.mac.pdf.
[22] S. Benhamou, How to reliably estimate the tortuosity of an animal's path: straightness, sinousity, or fractal dimension? J. Theoret. Biol. 229 (2004) 209–220.
[23] S. Benhamou, Detecting and orientation component in animal paths when the prefered direction is individual dependent, Ecology 87 (2006) 518–528.
[24] K. Funke, Jump relaxation in solid state ionic conductors, Solid State Ion. 28–30 (1988) 100–107.