Self-organization toward criticality in the Game of Life

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(Received November 12th, 1991)

Life seems to be at the border between order and chaos. The Game of Life, which is a cellular automaton to mimic life, also lies at the transition between ordered and chaotic structures. Kauffman recently suggested that the organizations at the edge of chaos may be the characteristic target of selection for systems able to coordinate complex tasks and adapt. In this paper, we present the idea of perpetual disequilibration proposed by Gunji and others as a general principle governing self-organization of complex systems towards the critical state lying at the border of order and chaos. The rule for the Game of Life has the minimum degree of perpetual disequilibrium among 2^{18} rules of the class to which it belongs.

Keywords: Life game; Cellular automata; Self-organization; Critical state; Evolution.

Phenomena governed by power laws are widely known in nature as $1/f$ noise and the fractal structure. Bak et al. (1988) proposed the idea of self-organized criticality as a general principle to explain such phenomena. A canonical example of self-organized criticality is a pile of sand. It was proved that self-organized criticality of the sand-pile model is a direct consequence of the local conservation law (Hwa and Karder, 1989). Bak et al. (1989) argued that the Game of Life (Gardner, 1970) is a new type of self-organized criticality, having no local conservation law. A question arises why the particular rule of the Game of Life is selected. It has been conjectured by several researchers that complex systems which are able to change their internal structure or relationship tend to self-organize toward criticality, and some supporting evidence has been presented (Kauffman, 1991; Kauffman and Johnsen, 1991; Langton, 1990; McIntosh, 1990). We need to find the general principle to explain why.

Gunji and Kon-no (1991) and Gunji and Nakamura (1991) consider that the behavior of

biological systems, in which the propagation velocity of information is slow (Matsuno, 1989), is undeterminable. Let us briefly explain this using the description of a cellular automaton system (Fig. 1). Suppose that the state of the cell is 0 when the cell does not contain material A or 1 when it contains A. The propagation velocity of information v_0 is one cell-distance per one time-step, while the moving velocity of material A v_p is two cell-distance per one time-step, being supposed to be faster than v_0. The central cell x can know the state of the nearest two cells only, and cannot know at time t whether material A is present in the next cells or not. Nevertheless, the next state of cell x is determined to be 0 if the system is in the state of Fig. 1a, or to be 1 if it is in the state of Fig. 1b. The next state is determined only a posteriori. When
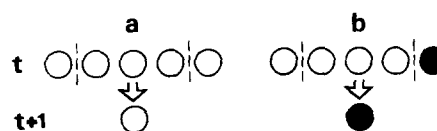

Fig. 1. A cellular automaton in which the observation velocity v_0 is less than the particle velocity v_p.

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we study this kind of system, we have been taking the probabilistic approach. We collect results, estimate the probability, and then predict the future using the probability. The valuable information that material A has come or not at the next time-step is not used directly in such a probabilistic approach. We present an alternative, stronger approach.

The time evolution of the above-mentioned system cannot be described by a deterministic equation. We assume a deterministic equation f as a tentative one for the time evolution from time t to $t + 1$. A set of possible equations which the system may obey are denoted by $\{f_{(i)};\ i = 0,1,...,n\}$. In order to use the information obtained at time $t + 1$, we define the time-reversed function of f in the following forms.

For the cellular automaton of Fig. 1, we have in general

$$s(x;t + 1) = f_{(i)}(s(x - 1;t),s(x;t),s(x + 1;t)) \quad (1)$$

where s is the state of cell. Two kinds of time-reversed function of $f_{(i)}$ are defined as

$$s(x;t) = g_{(i)0}(s(x;t + 1),s(x - 1;t),s(x + 1;t)) \quad (2a)$$

and

$$s(x + 1;t) = g_{(i)1}(s(x;t + 1),s(x - 1;t),s(x;t)) \quad (2b)$$

Since the system is irreversible, $g_{(i)0}$ and $g_{(i)1}$ are not unique but are one-to-many (m) type functions. Each of them is denoted by $g_{(i)k}{}^h$ ($i = 0,...,n;\ k = 0,1;\ h = 1,...,m$).

The procedure of perpetual disequilibration is explained in Fig. 2. Starting from the configuration $A(t)$ of the system at time t, the configuration $A(t + 1)$ is obtained through the operation of $f_{(0)}$. Interesting results arise from the following backward operation. All kinds of time-reversed functions $g_{(i)k}{}^h$ operate on the configuration $A(t + 1)$ and corresponding configurations $B_{(i)k}{}^h$ are obtained. In order to most effectively use the information obtained at $t + 1$ a posteriori, we take the following procedure. Select $g_{(i)k}{}^h$ that minimizes the Hamming distance between $A(t)$ and $B_{(i)k}{}^h$, and denote
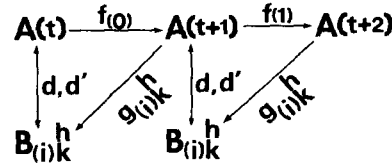

Fig. 2. The procedure of perpetual disequilibration. Subscripts: $i = 0, 1,...,n$ for all possible dynamical equations, and $k = 0,1$ for all possible time-reversed functions. Superscripts: $h = 1,...,m$ for all possible values of one-to-many type functions.

it by $g_{(1)}$, and let $f_{(1)}$ be the time-forward function corresponding to $g_{(1)}$. The system will be best described, when $f_{(1)}$ is used at the next time-step so that the discrepancy caused by the temporal use of $f_{(0)}$ at time t may be minimized. As long as the system is not reversible, the discrepancy persists perpetually and the Hamming distance remains positive. When the system has the capability to evolve like living organisms, the process of minimizing the discrepancy by changing the operational function $f_{(i)}$ is nothing but the evolution dynamics of the system to modify its structure and function. Accordingly, the principle of perpetual disequilibration is the base of the rule dynamics of evolution. We apply the principle to explain why the system evolves to select the rule of the Game of Life among many other possible ones.

The Game of Life is a kind of two-dimensional cellular automaton. The rule is expressed by

$$s(i,j;t + 1) = f(s(i,j;t),\ \mathrm{sum}(i,j;\ t)) \quad (3)$$

where s is the state of cell being either 0 or 1, and sum is the total sum of the state values of 8-nearest neighbors of the cell at (i,j). The rule of the Game of Life is one of the possible 2^{18} rules, and is described particularly as

$f(0,\mathrm{sum}) = 1$, only when sum = 3, otherwise $f(0,\mathrm{sum}) = 0$

$f(1,\mathrm{sum}) = 1$, only when sum = 2 or 3, otherwise $f(1,\mathrm{sum}) = 0$

Two types of the time-reversed function of f are defined as

$$s(i,j;t) = g_0{}^h(s(i,j;t + 1), \text{sum}(i,j;t)) \qquad (4a)$$

$$\text{sum}(i,j;t) = g_1{}^h(s(i,j;t + 1), s(i,j;t)) \qquad (4b)$$

Both g_0 and g_1 are one-to-many (m) type functions, and each is denoted by $g_0{}^h$ and $g_1{}^h$ $(h = 1,...,m)$. Let the configuration of the system at t be $S(t)$. We get $S(t + 1)$, operating f on $S(t)$. We then reverse the operation, operating $g_k{}^h(k = 0,1)$ on $S(t + 1)$, and get $S_k{}^h$. The measure of perpetual disequilibrium (PD) is a function of the Hamming distance between $S(t)$ and $S_k{}^h$. The zeroth order measure at time t is defined by the minimum of them as

$$d_0(S(t)) = \min\left(\sum_{i,j}|s(i,j;t) - s_0{}^h(i,j;t)|\right) \qquad (5a)$$

$$d_1(S(t)) = \min\left(\sum_{i,j}|\text{sum}(i,j;t) - \text{sum}_1{}^h(i,j;t)|\right) \qquad (5b)$$

where $s_k{}^h$ and $\text{sum}_k{}^h$ are s and sum, respectively, in the configuration $S_k{}^h$. The minimum is taken over all possible h of $g_k{}^h$ which is one-to-many type. The zeroth order measure of PD d_k for a pair of functions f and g_k is then defined by the minimum of $d_k(S(t))$ over an infinitely long time. Finally, the zeroth order measure d of PD for the transition rule f is defined as the sum of d_0 and d_1 as

$$d = d_0 + w_{01}d_1 \qquad (6)$$

Here, w_{01} is taken to be 1/8, because the maximum of $|s(i,j;t) - s_0{}^h(i,j;t)|$ is 1, while the maximum of $|\text{sum}(i,j;t) - \text{sum}_1{}^h(i,j;t)|$ is 8. Furthermore, we define the first order measure of PD by the average of the Hamming distance as

$$d_0{}' = \langle|s(i,j;t) - s_0{}^h(i,j;t)|\rangle \qquad (7a)$$

$$d_1{}' = \langle|\text{sum}(i,j;t) - \text{sum}_1{}^h(i,j;t)|\rangle \qquad (7b)$$

and

$$d' = d_0{}' + w_{01}d_1{}' \qquad (7c)$$

The average is taken for all steady-state configurations.

The measures of PD for the set of rules, including the Game of Life, are computed, using a two-dimensional lattice of 16 × 16. The rule number RN is given by

$$RN = \sum\{2^9 \times s(i,j) + 2^{\text{sum}(i,j)}\} \times f(s,\text{sum}) \qquad (8)$$

Starting from a random configuration with an equal proportion of 0 and 1, we iterate using rule f over 200 steps to obtain a steady state, then compute $d_0(S(t))$ and $d_1(S(t))$ operating f and $g_k{}^h$ repeatedly. We repeat this procedure 50 times, starting from different initial configurations. We then obtain d_0, $d_0{}'$, d_1, and $d_1{}'$, taking the minimum or the average of $d_0(S(t))$ and $d_1(S(t))$, respectively.

Because it is hard to compute for all 2^{18} rules, we computed only for $2^8 = 256$ rules which are close to the rule of the Game of Life. Namely, $f(s,\text{sum})$ is fixed to be zero when sum is 0 or greater than 4. The results are shown in Fig. 3. The 256 rules are arranged in increasing order of the zeroth order measure d of PD. For rules of which d is equal to zero, the rules are arranged in decreasing order of the first order measure d' of PD. In general, rules with $d > 0$ belong to the class III cellular automata by Wolfram's classification (Wolfram, 1984) with a few excep-
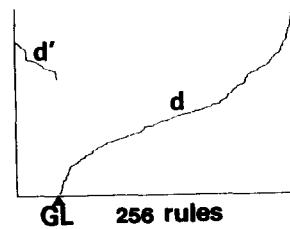

Fig. 3. The zeroth order and the first order measures of perpetual disequilibrium in 256 sets of cellular automata rules including the Game of Life. GL denotes the rule of the Game of Life.

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tions of class II, and rules with $d = 0$ belong to class I or II. In this sense, the zeroth order measure of PD is analogous to the entropy. The most important feature obtained in Fig. 3 is that the rule of the Game of Life lies just at the border at which not only the zeroth order measure of PD is minimum but also the first order measure of PD is also minimum among rules of which the zeroth order measure is zero.

We propose the principle of minimum perpetual disequilibrium (PD) in evolutionary dynamics as follows:

(1) Systems with the capability to change the structure and internal dynamics tend to self-organize so that the zeroth order measure of PD may become minimum.

(2) When the zeroth order measure of PD is equal to zero, systems tend to evolve so that the first order measure of PD may become minimum.

The first principle leads to antichaos and the second one causes prochaos. Eventually, systems optimize at the border between order and chaos. The state at the border is critical in the sense of phase transitions, thus the principle is the base of Bak's self-organized criticality without local conservation laws. The principle of minimum perpetual disequilibrium can be applied not only to cellular automaton dynamics but also to self-organization of neural networks, and other continuous dynamical systems in general.

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