Example of MHE in casadi for a mass-spring system

In the MHE example, we consider a simple mass-spring system. Measurements of the position of the mass are available. We would like to solve the following MHE-problem:

$$\begin{aligned}
\underset{\mathbf{x}(\cdot),\mathbf{w}(\cdot)}{\text{minimize}} \quad & \int_{\tau=t-T}^{t} \| \mathbf{w}(\tau) \|^2_{Q_\mathrm{c}} + \| \mathbf{y}(\tau) - h(\mathbf{x}) \|^2_{R_\mathrm{c}} \, d\tau + \| x(t-T) - \hat{x}(t-T) \|^2_S \\
\text{subject to} \quad & \dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}, \mathbf{w}, \tau), \qquad \tau \in [t-T, t],
\end{aligned}$$

where:

- $\vec{x}$ is the system state,
- $\vec{y}$ are the measurements,
- $\vec{u}$ is the control input,
- $\vec{w}$ is a noise term introduced to model the model uncertainty (i.e. the process noise),
- t is the current time,
- $\hat{x}(t-T)$ is an estimate for the state at the beginning of the horizon
- Q_c and R_c S are the weighting matrices for the model and measurement noise and for the arrival cost respectively,
- T is the considered time horizon,
- f is a model of the considered system, and
- h is the measurement function that maps the system state to the measurements.

In the example, this example is discretized using multiple shooting, which results in the following problem:

$$\begin{aligned}
\underset{\mathbf{x}_k,\mathbf{w}_k}{\text{minimize}} \quad & \sum_{k=i-N}^{i-1} \| \mathbf{w}_k \|^2_{Q_{\mathrm{d}_k}} + \sum_{k=i-N}^{i} \| \mathbf{y}_k - h(\mathbf{x}_k) \|^2_{R_{\mathrm{d}_k}} + \| x_{i-N} - \hat{x}_{i-N} \|^2_S \\
\text{subject to} \quad & \mathbf{x}_{k+1} = \phi(\mathbf{x}_k, \mathbf{u}_k, \mathbf{w}_k), \; k = i-N, \ldots, i-1
\end{aligned}$$

where

- ϕ integrates our model f, given the state $\vec{x}_k$, control input $\vec{u}_k$ and process noise $\vec{w}_k$, over one sampling interval,
- N is the number of measurements in the considered time horizon,
- i is the current sampling index, and
- Q_{d} and R_{d} are the discretized versions of covariance matrices Q_{c} and R_{c}, which might vary over the time horizon.

This is the setup of the MHE that is implemented in this example. Note that in this example, Structures are used. If you are unfamiliar with this, a tutorial can be found here: http://docs.casadi.org/tutorials/tools/structure.pdf The arrival cost is updated using the smoothed EKF update.