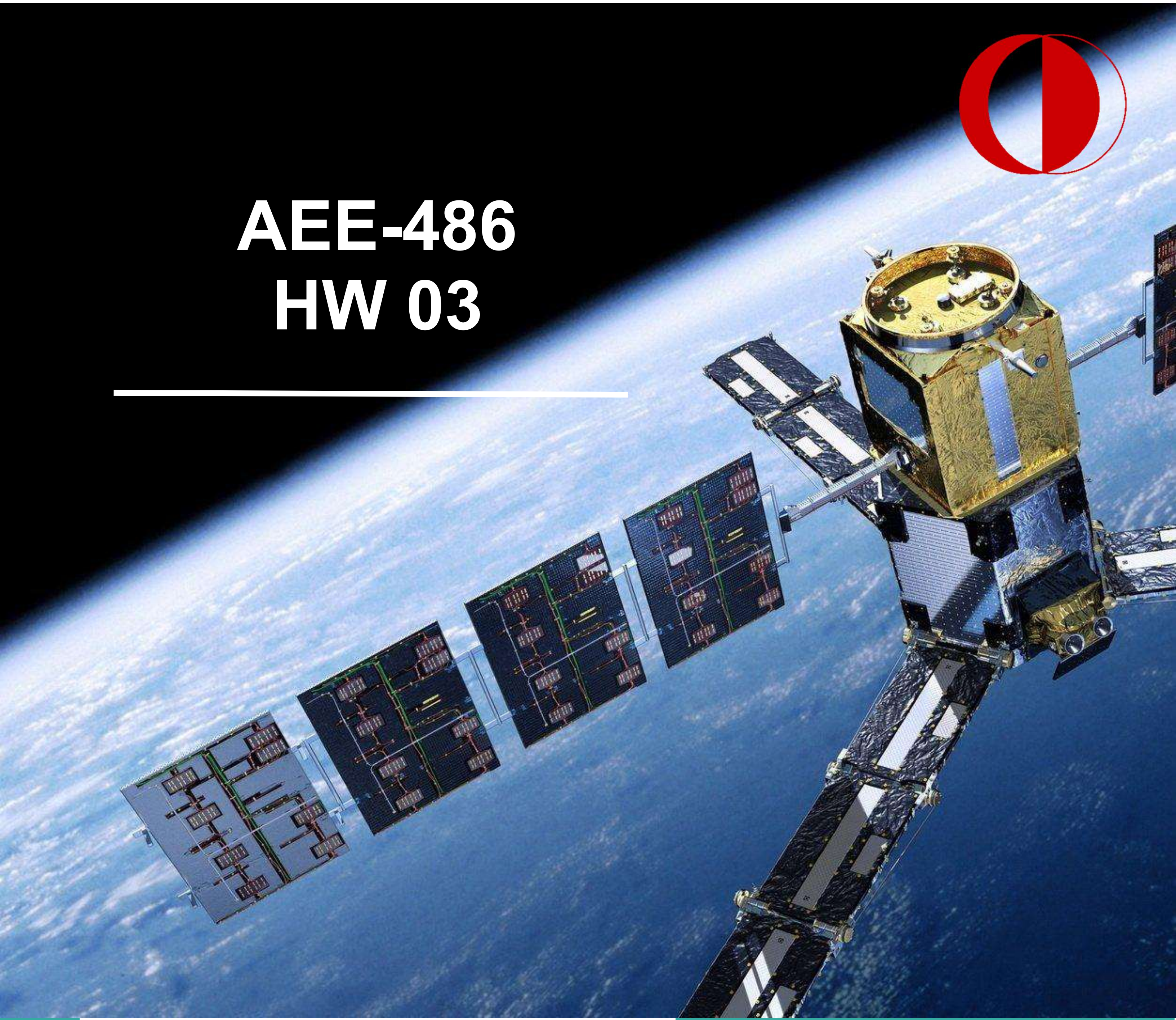
AEE-486
HW 03



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Q1

(a)

In this question the first task was to find the DCM matrix, A_{bi} , that transforms the given z initial vector in the ECI frame pointing towards the sun to the body frame of:

$$z_b = \begin{matrix} 0 \\ 0 \\ 1 \end{matrix}$$

First the z_i was found by equating it to the first S_i vector. Then the angles were found using the 3-1-3 rotation sequence and the last angle was set to zero. Then the angle2dcm function of matlab was used to find A_{bi}.

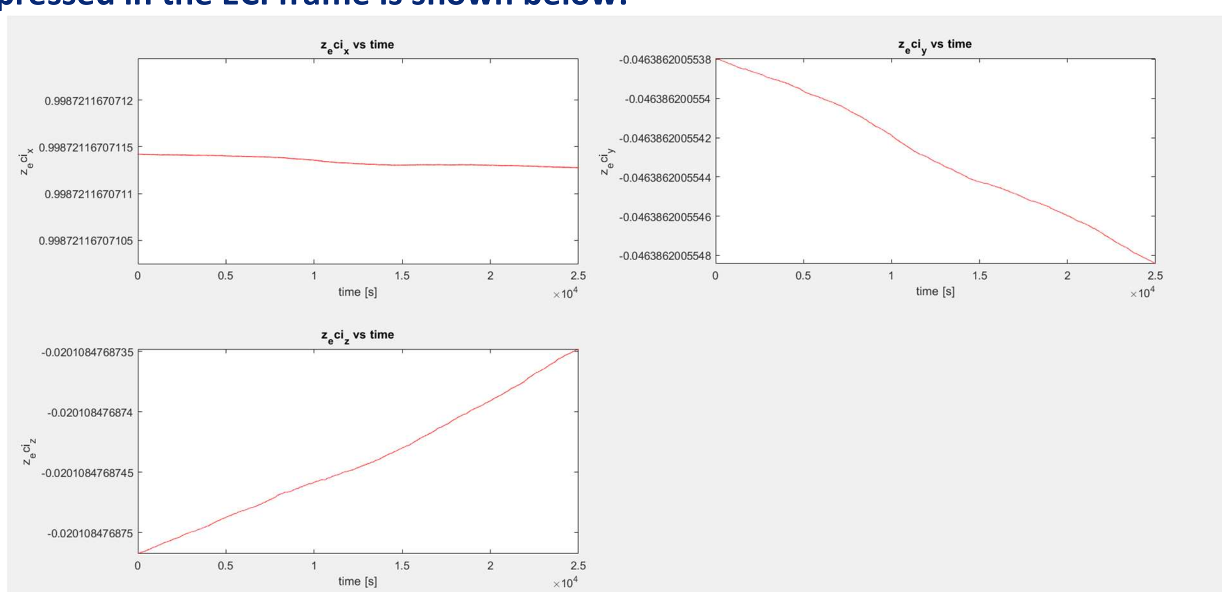
(b)

In this part we were tasked with propagating the quaternion obtained by the DCM in the previous part.

The angular velocity vector was found to be a constant as the following expression resulted in 0:

$$\frac{d\boldsymbol{\omega}_{bi}^{b}}{dt} = J^{-1}\left[\boldsymbol{N} - \boldsymbol{\omega}_{bi}^{b} \times \left(J\boldsymbol{\omega}_{bi}^{b}\right)\right]$$

Hence the quaternion was propagated in a similar method as used in Homework 1. Then the obtained quaternion at each time step was used to update the A_{bi} matrix, which was then used to find the variation of the z component in the body frame expressed in the ECI frame is shown below:



Q2

In this question we were asked to include torque disturbances.

(a)

First we added the gravity gradient torque:

$$\boldsymbol{N}_{gg} = \frac{3\mu}{|\boldsymbol{r}|^3}\boldsymbol{n} \times (J\boldsymbol{n})$$

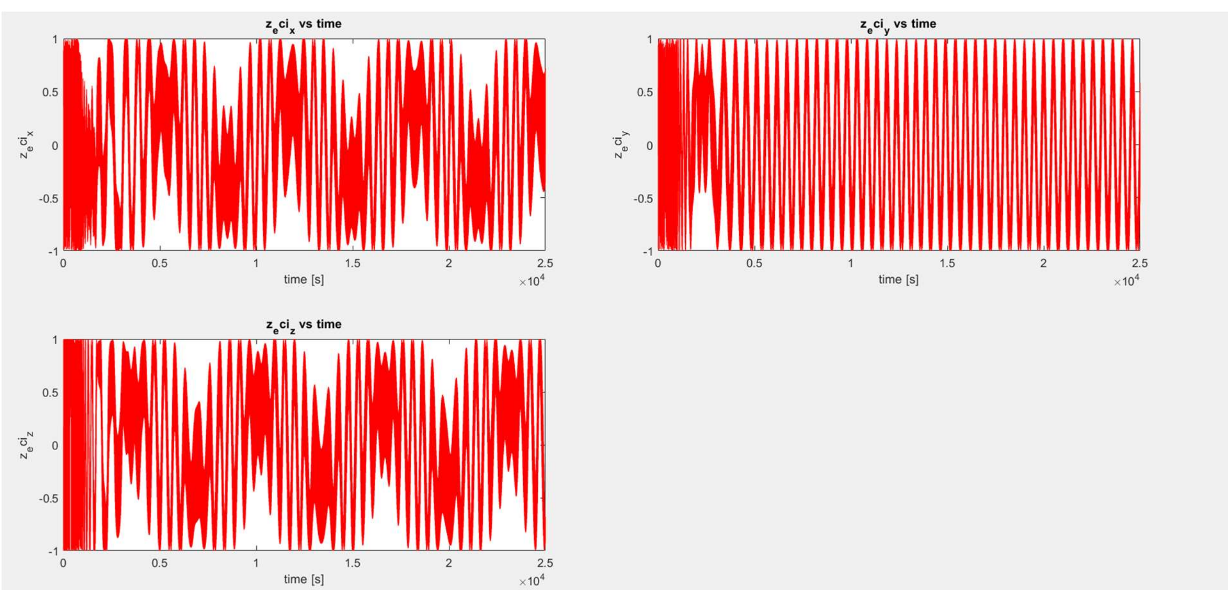
The value of the r vector was interpolated from the pos_eci vectors given at each one second and our time step size was 1s. Then the value of the nadir vector was calculated using the following formula:

$$\vec{\boldsymbol{n}} = \boldsymbol{A_{bi}}\left(-\frac{\vec{\boldsymbol{r}}}{\|\boldsymbol{r}\|}\right)$$

The value of N_{gg} was then inserted into the equation for the time derivative of the angular velocity.

Then classical 4th order Runge-Kutta method was used to calculate the value of the angular velocity, which was then used to propagate the quaternion vector similar to question 1.

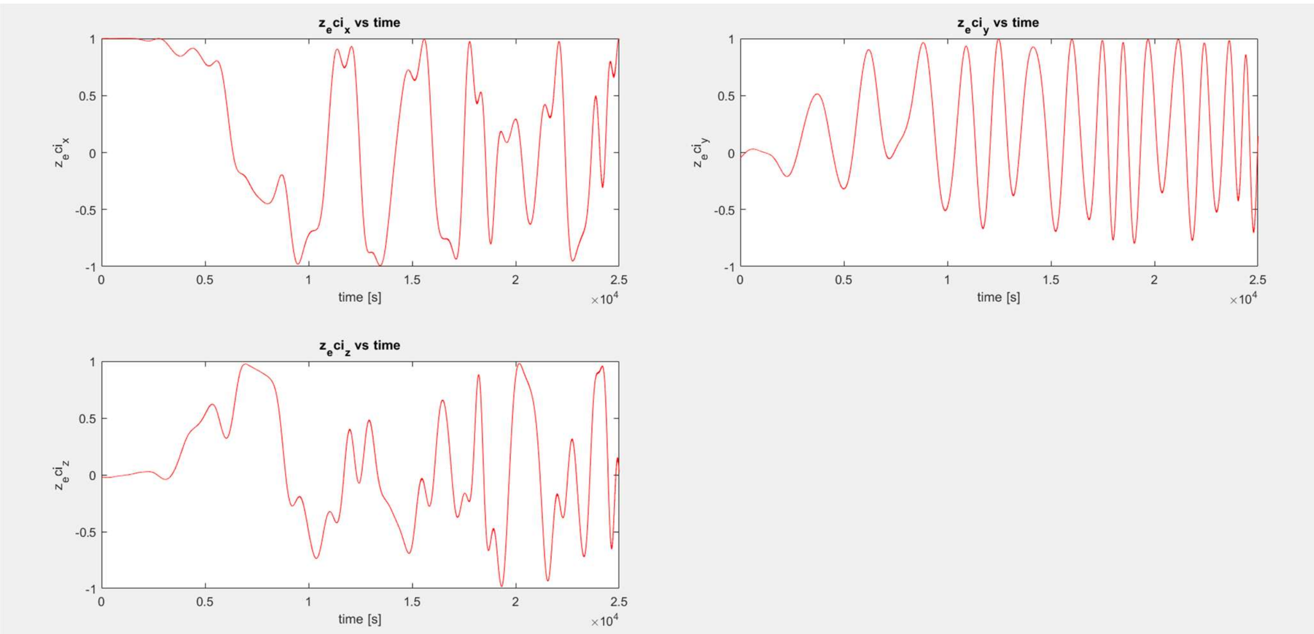
Then the obtained quaternion at each time step was used to update the A_{bi} matrix, which was then used to find the variation of the z component in the body frame expressed in the ECI frame is shown below:



(b)

For second part we were required to put magnetic torque disturbances instead of gravitational gradient torque:

$$\boldsymbol{N}_{md} = \boldsymbol{M} \times \boldsymbol{B}_b$$



As we can see, the effect of the gravitational gradients is more rapid than the effect of the magnetic torque, however the magnitude of the of the maximum and minimum deflection is the same.

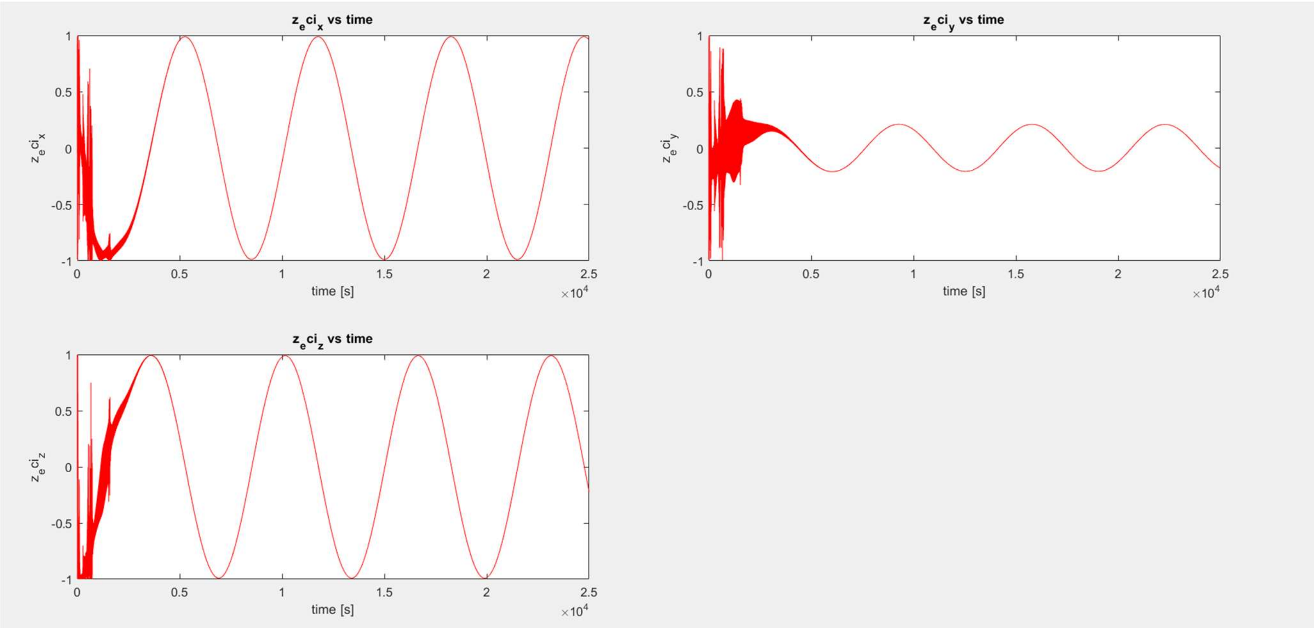
Q3

In this question we were asked to replace the J matrix with the new J matrix and asked to repeat analysis in Q2:

$$J = \begin{bmatrix} 6.9 & 0 & 0 \\ 0 & 7.5 & 0 \\ 0 & 0 & 8.4 \end{bmatrix} \text{kgm}^2$$

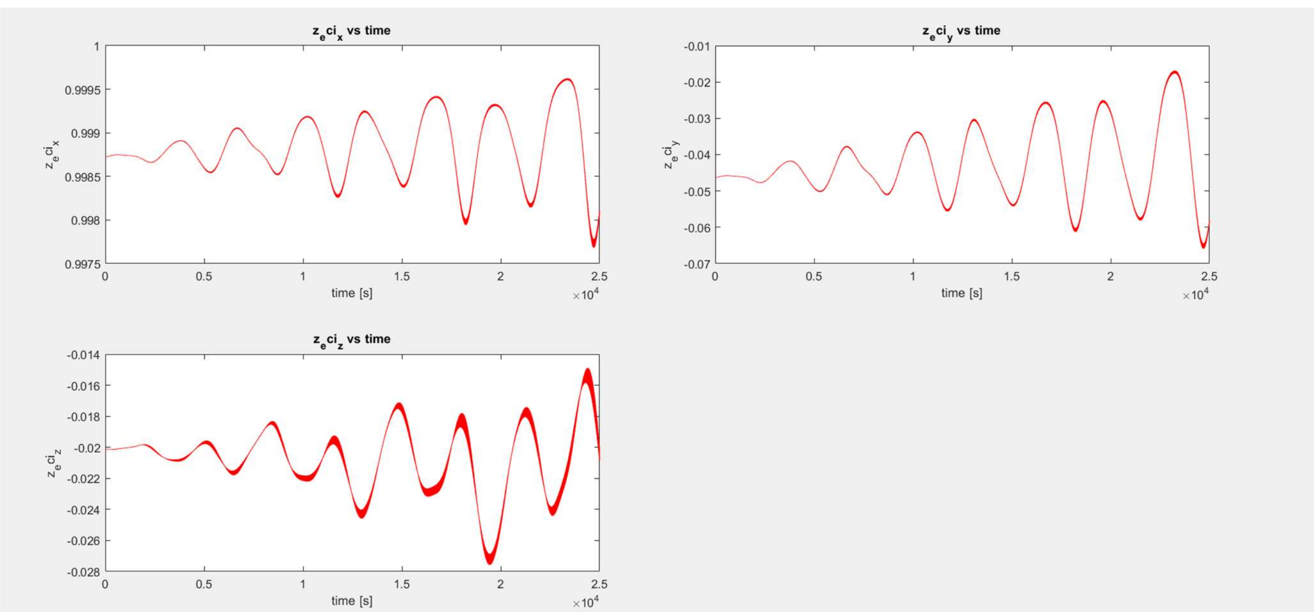
(a)

The following result was obtained:



(b)

The following result was obtained:



Again the effect of the gravity gradient torque is more than the magnetic gradient torques.