Homework 2

Instructions

1) You are free to discuss the questions with your classmates, but if I notice the same answer is copied all the copiers will get “0” for that question.
2) I prefer typed reports (MS Word, LaTeX etc.) Please be consistent in notations. You can use the following notations:
   - lower case bold for vector ($\boldsymbol{q} = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T$)
   - lower case regular for scalar (q_1),
   - capital letter for matrix (C)
   - lower case right subscript/superscript for the reference frame notations (C_{ab}, $\boldsymbol{\omega}_{ab}^{b}$).
3) Provide the algorithms that you coded in the submitted package.
4) Due date is 19th November (Sunday) until the midnight (23:59).
5) Submit your reports on ODTUClass.
6) Title of your report file (or zipped package) should be: AE486_2023_Name_Surname_HW2

Preliminaries

The “hw1_data.mat” file shared within the HW package includes:

pos_eci: Position of the spacecraft in ECI frame

vel_eci: Velocity of the spacecraft in ECI frame

sun_eci: Sun direction vector in ECI frame

mag_eci: Magnetic field direction vector in ECI frame

Each of these data is sampled at every second starting from 18 March 2019 00:00:00 (UT) and there is a total of 25000s data.

Questions

1) **(20pts)** First obtain the magnetic field vector in orbit frame (local vertical/local horizontal frame) using the provided magnetic field vector in the ECI frame (this is the IGRF outout). Then calculate the same vector using the dipole model given in Week 4 Lecture slides and compare your results. Take orbit inclination as $i = 98.1245\,\mathrm{deg}$. The satellite is at a circular orbit, so the distance is constant and $r = 7056198.5\ \mathrm{m}$. For a circular orbit, the orbital angular velocity can be calculated as $\omega_o = \sqrt{\dfrac{\mu}{r^3}}$, where μ is Earth gravitational paramater and $\mu = 3.986004418 \times 10^{14} m^3 s^{-2}$.

   Note: Time in dipole model can be started from 0. You can also play with time a little bit to see how good you can match two different results.

2) **(5 pts)** Obtain the Sun direction vector in orbit frame.

3) **(20 pts)** Assume that the spacecraft's attitude is controlled and it is required to be nadir pointing. Thus it is +Z axis would be pointing to the Earth center. The remaining body axes (X and Y) would be matching with the axes of the orbit frame. As we discussed in the lecture, this is a usual configuration for Earth imaging satellites, such as Rasat. Thus the quaternion vector showing the orientation of the body frame with respect to the orbit frame is a unit quaternion in perfect case, e.g. $\boldsymbol{q} = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T$.

   However, of course, nothing is perfect in practice and the spacecraft is making periodic oscillations about one of the axes (roll, pitch or yaw axes). You are free about the period but the maximum magnitude should be 10 deg. How would you model the attitude of the spacecraft with respect to the orbit frame? Plot the Euler angles.

4) **(20pts)** Using the modelled attitude in Q3, model your magnetometer and Sun sensor measurements in body frame. To do that you need to use the measurement model given as

$$\boldsymbol{S}_b = A\boldsymbol{S}_o + v$$

   Here $\boldsymbol{S}_o$ is for the reference direction vectors (for magnetic field and Sun direction) in orbit frame. For the magnetic field, you can use either the ones from the dipole model or the IGRF model.

   While modeling the measurements assume that the standard deviation of magnetometer measurement Gaussian white noise is $300\text{nT}\ (1\sigma)$ and the standard deviation for Sun sensor Gaussian white noise is $0.002\ (1\sigma)$. Note that the standard deviation value is same for all three axes of measurements. Sun sensor noise does not have a unit since it is for unit vector.

5) **(35 pts)** Write a code for QUEST algorithm and estimate the attitude of the spacecraft using the magnetometer and Sun sensor measurements that you generated. The HW package includes an example. But I want you to write your own code. The code must be:

   - Taking measurements, their weights and reference vectors as inputs at each instant (so you must be calling QUEST function sequentially at every second.
   - Output the estimated quaternion vector.

   Compare the estimated attitude and the actual attitude (the one you modelled in Q3) in terms of Euler angles. Euler angles must be calculated according to 3-2-1 Euler angle sequence.

   Note: You may need to normalize magnetometer measurements before using in the QUEST algorithm.

Good luck!
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