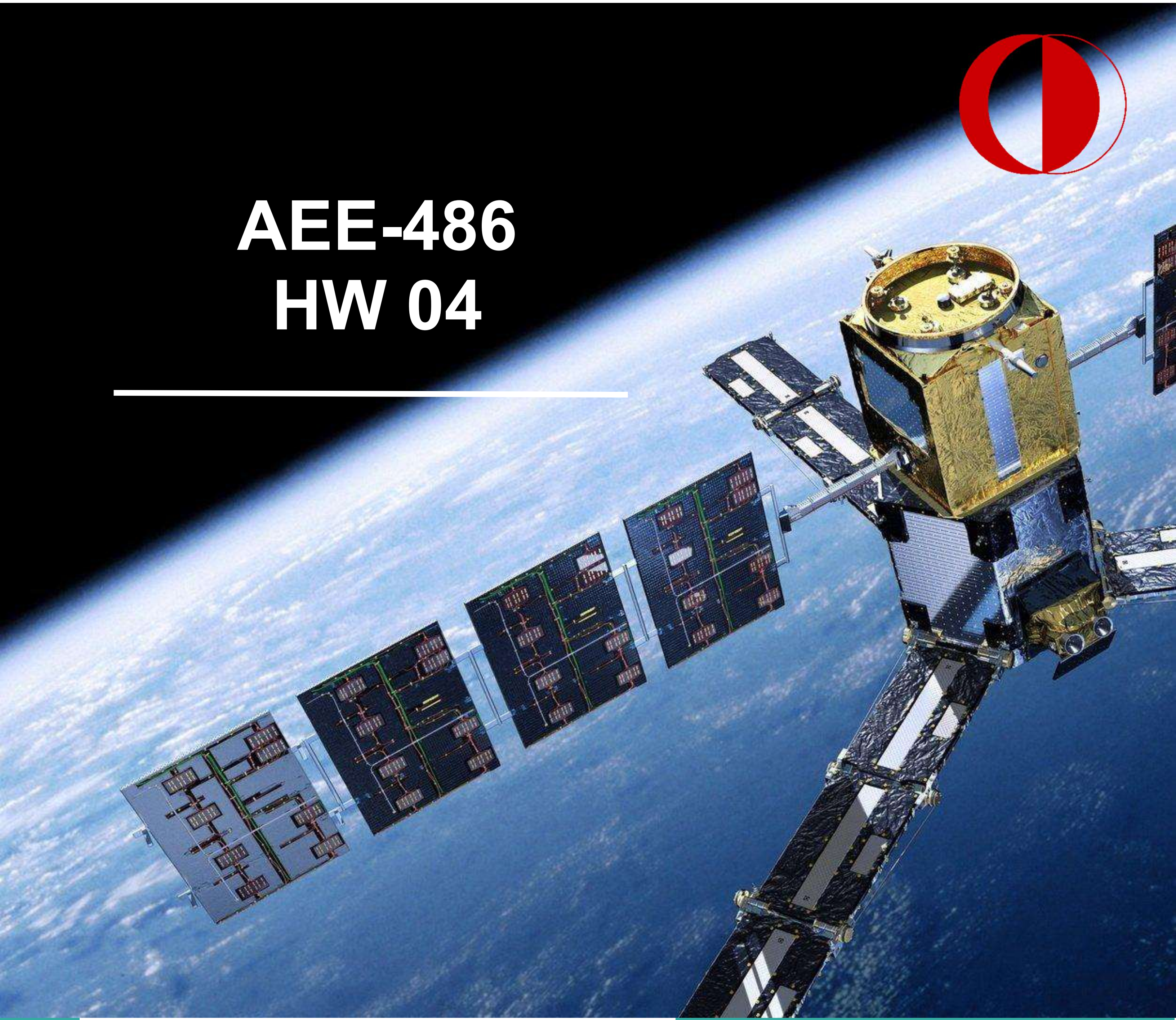

AEE-486
HW 04

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Q1

In this question we were asked to detumble a spacecraft that has some magnetic interference from the magnetic field of the earth which was given to us in the ECI frame.

The initial quaternion was given, which was then converted to find the initial A_{bi} matrix. This was done as for this problem attitude kinematics was used rather than quaternion kinematics, due to the lesser complexity of propagating attitude kinematics and finding the new magnetic vector in body frame.

The constant of radius of the orbit and the inclination angle were taken from Homework-02, which were used to find the orbital period of the spacecraft. This was then used to find the value of the constant k in the formula of the magnetic dipole moment.

After that the loop was created which implemented the following torque formula suing the B-dot control law:

$$\mathbf{m} = -\frac{k}{\|\mathbf{B}\|}\dot{\mathbf{B}}$$

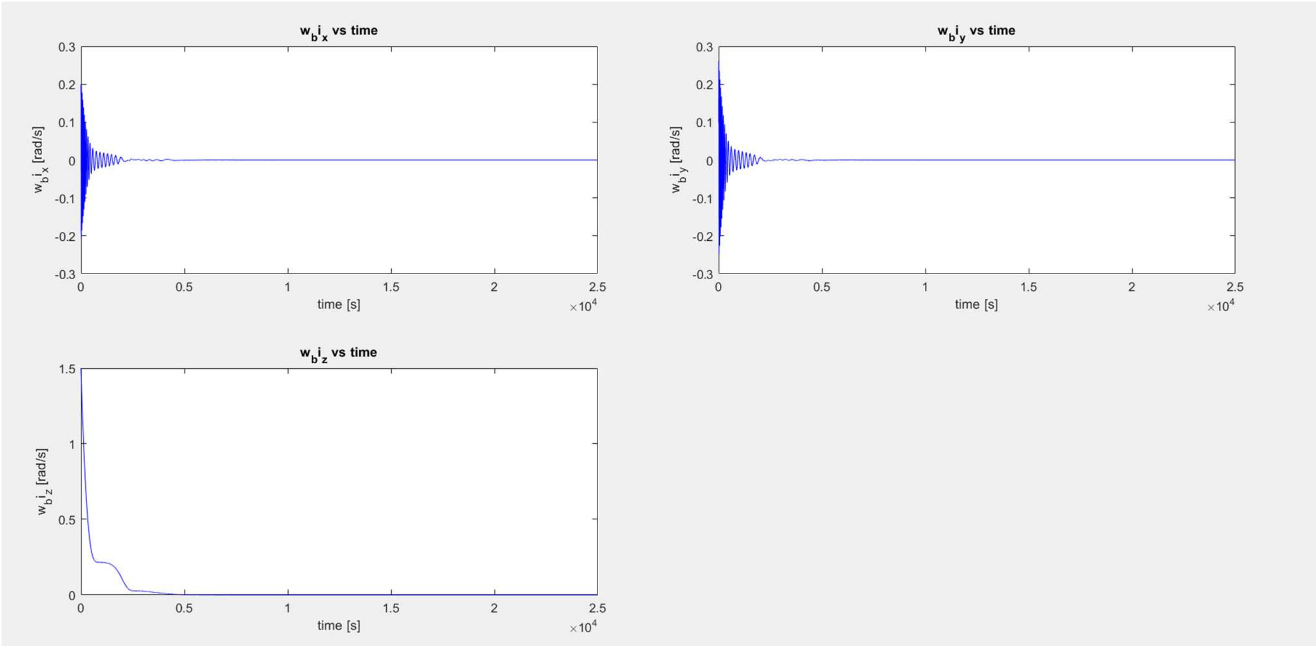
$$\mathbf{L} = \mathbf{m} \times \mathbf{B}$$

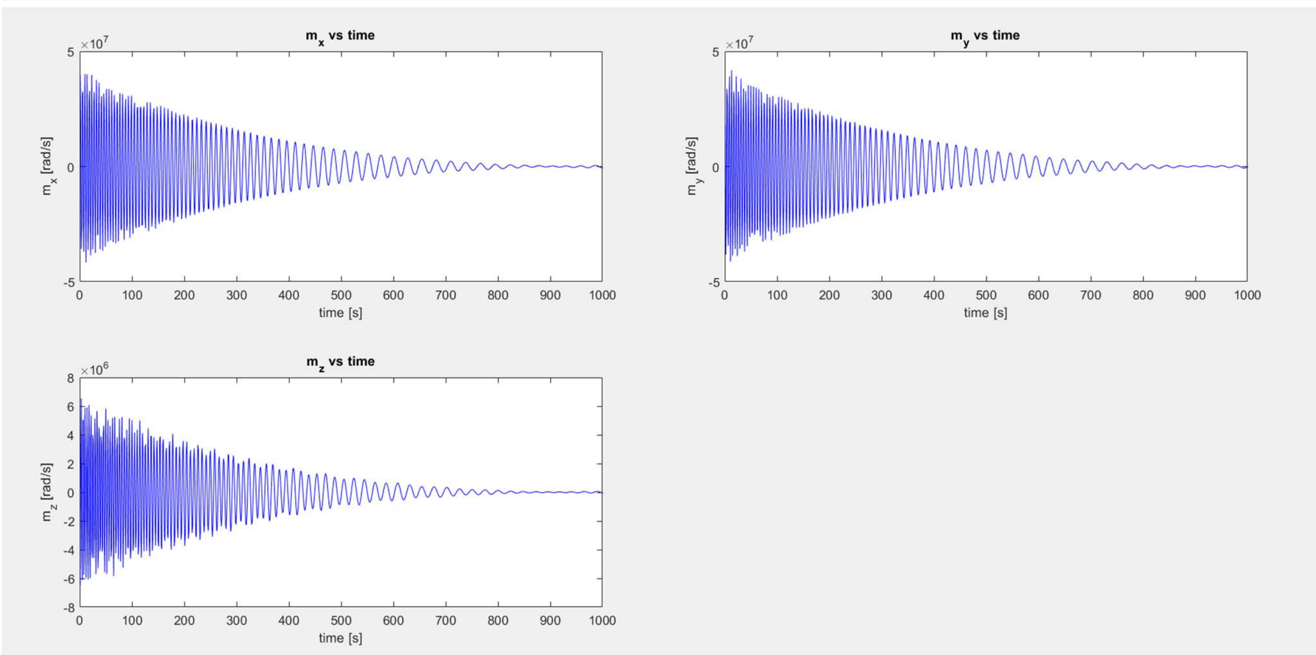
Where m is the magnetic dipole moment, L is the torque generated by the magnetic torquers. $\dot{B}$ is the derivative of the magnetic field in the body frame given by the formula:

$$\dot{B} = -\omega_{bi} \times B$$

After implementing these formulas, the ω_{bi} vector was propagated using the classical RK-4 method. Then the A_{bi} matrix was also propagated to find the new B vector for the next iteration.

The variation of the angular velocity and the magnetic dipole is given below:





As we can see the magnetic torquer is successful in making both the angular velocities and the dipole moments in all three axis converge to zero. This means that the spacecraft has successfully detumbled.

If were to limit the magnetic dipole moment by $\pm 3\ Am^2$ the spacecraft will take a lesser time to make the dipole moment converge to 0.

Q2

In this question the spacecraft was desired for it's z-axis in the body frame to be aligned with the sun vector in the ECI frame. The initial quaternion and the angular velocity of the spacecraft was given.
A set of 4 reaction wheels was used to control and make the spacecraft align to the desired attitude. Hence first the desired quaternion was calculated using a similar method the one used in Homework-03. This became our q_c.
The control law gave us then the following torque equation:

$$\bar{\mathbf{L}} = -k_p \operatorname{sign}(\delta q_4)\boldsymbol{\delta}\mathbf{q}_{1:3} - k_d\, \boldsymbol{\omega}$$

The values of the constants k_p and k_d are set to 0.1 and 0.15 respectively. Over here the δq vector is the error vector calculated by:

$$\boldsymbol{\delta}\mathbf{q}_{1:3} = \Xi^T(\mathbf{q}_c)\mathbf{q}$$

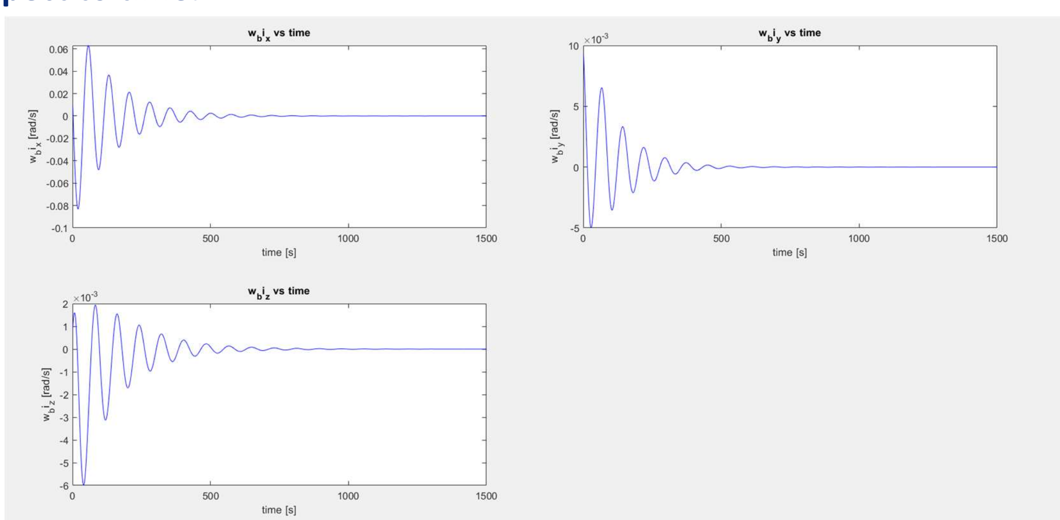
$$\delta q_4 = \mathbf{q}^T\mathbf{q}_c$$

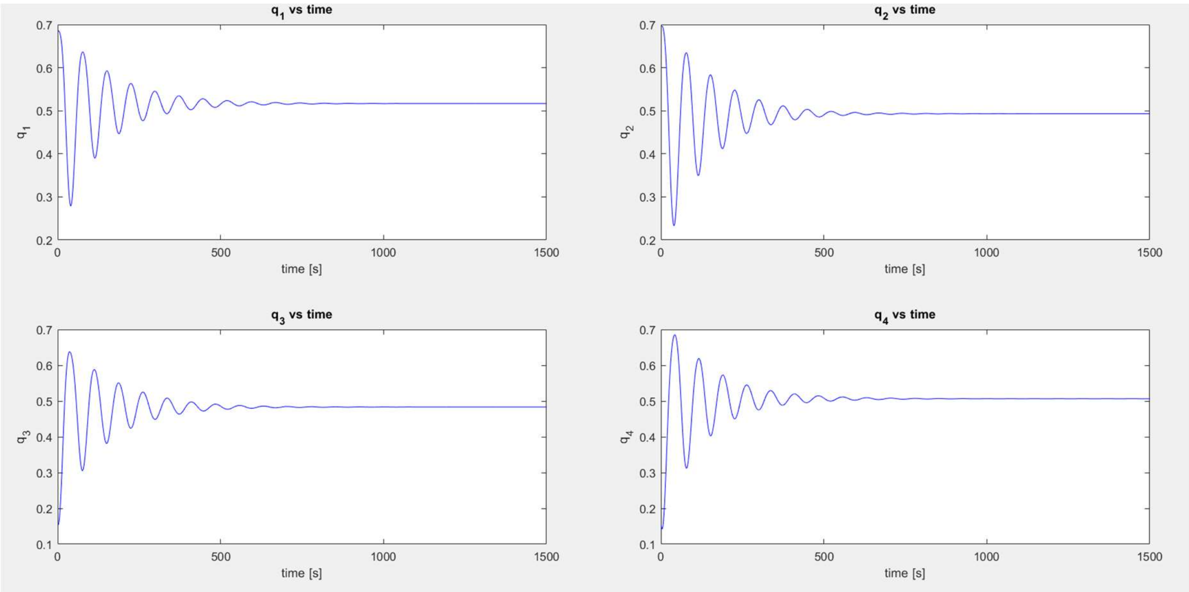
Where q is the quaternion at a certain time step and q_c is the desired quaternion. The idea is to make the error go to $\delta q = [0, 0, 0, 1]^T$. This formula was again implemented in the loop. The ω_{bi} and the wheel angular momentum in body frame vectors were propagated by the following equations by using classical RK-4 methods:

$$J\dot{\omega} = -[\omega\times]J\omega + \bar{\mathbf{L}}$$

$$\dot{\mathbf{h}} = -[\omega\times]\mathbf{h} - \bar{\mathbf{L}}$$

The results show the variation of the angular velocities and the quaternion terms with respect to time:





The wheel angular momentum vector was then used to find the individual angular momentums of the individual wheels using the pseudoinverse law:

$$\mathbf{H}_W^w = \mathscr{W}_n^{\dagger}\mathbf{H}_B^w$$

Where the pseudoinverse matrix is:

$$\mathscr{W}_4^{\dagger} = \frac{1}{2}\begin{bmatrix} 1/a & b/(b^2+c^2) & 0 \\ -1/a & b/(b^2+c^2) & 0 \\ 0 & c/(b^2+c^2) & 1/d \\ 0 & c/(b^2+c^2) & -1/d \end{bmatrix}$$

Where $a = b = c = d = \frac{1}{\sqrt{2}}$, as given in the problem. The resulting moments are plotted below: