Homework 3

Instructions

1) You are free to discuss the questions with your classmates, but if I notice the same answer is copied all the copiers will get "0" for that question.
2) I prefer typed reports (MS Word, LaTeX etc.) Please be consistent in notations. You can use the following notations:
   - lower case bold for vector ($\boldsymbol{q} = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T$)
   - lower case regular for scalar (q_1),
   - capital letter for matrix (C)
   - lower case right subscript/superscript for the reference frame notations (C_{ab}, $\boldsymbol{\omega}_{ab}^{b}$).
3) Provide the algorithms that you coded in the submitted package.
4) Due date is 3rd December (Sunday) until the midnight (23:59).
5) Submit your reports on ODTUClass.
6) Title of your report file (or zipped package) should be: AE486_2023_Name_Surname_HW3

Preliminaries

The "hw2_data.mat" file shared within the HW package includes:

pos_eci: Position of the spacecraft in ECI frame

vel_eci: Velocity of the spacecraft in ECI frame

sun_eci: Sun direction vector in ECI frame

mag_eci: Magnetic field direction vector in ECI frame

Each of these data is sampled at every second starting from 18 March 2019 00:00:00 (UT) and there is a total of 25000s data.

Questions

1) (**40pts**) Use the initial Sun orientation information in HW2 package. For initial three-axis attitude of the spacecraft ($\boldsymbol{A}_{bi}$) assume that the initial Z axis of the spacecraft is looking through the Sun in the Earth centred inertial (ECI) frame. For Y and X axis you can assume any orientation (Easiest way for obtaining the initial attitude is to use 3-1-3 Euler angle rotation sequence. First two rotations can align the Z axis with $\boldsymbol{s}_i$. Third angle can be assumed 0 – Page 80 from Week 1 Slides can help you for visualizing the problem.)

   This time together with the satellite kinematics (what you used in HW2), you need to propagate the attitude dynamics (Euler's rotational equation).

$$\frac{d\boldsymbol{\omega}_{bi}^{b}}{dt} = J^{-1}\left[\boldsymbol{N} - \boldsymbol{\omega}_{bi}^{b} \times \left(J\boldsymbol{\omega}_{bi}^{b}\right)\right]$$

You will be doing this analyses first of all for nanosatellite which has moment of inertia tensor of

$$J = \begin{bmatrix} 0.018 & 0 & 0 \\ 0 & 0.018 & 0 \\ 0 & 0 & 0.065 \end{bmatrix} \mathrm{kgm}^2$$

Use **quaternion kinematics** equation. Assume that the satellite is spinning about body Z axis with a rate of 4rpm. So your initial angular velocity to start integrating the dynamics equation is $\boldsymbol{\omega}_{bi}^{b} = \begin{bmatrix} 0 & 0 & 4\mathrm{rpm} \end{bmatrix}^T$. Do not forget to change the rpm value into **rad/s** first!

Assume there is no torque acting on the spacecraft. Do not forget to normalize the quaternions and use a time step small enough ($\Delta t = 0.1\mathrm{s}$) for numerical integration since we have pretty quick dynamics here. If you use a relatively coarse method such as Euler's method you may need even smaller time step. Plot the unit direction vector components of the spin axis in the inertial frame.

2) (**40pts**) Include magnetic disturbance and gravity gradient torques into your analyses (one by one). The gravity gradient torque can be calculated in the satellite body frame as

$$\boldsymbol{N}_{gg} = \frac{3\mu}{|\boldsymbol{r}|^3} \boldsymbol{n} \times (J\boldsymbol{n})$$

Here $\mu = 3.986004418 \times 10^{14} \mathrm{m}^3/\mathrm{s}^2$ is the gravitational parameter for Earth, r is the distance from the center of the Earth in ECI (pos_eci in shared data) and n is the nadir pointing unit vector in the body frame. Note that to calculate this vector you need also the attitude of the spacecraft at that instant.

The magnetic disturbance torque is calculated in satellite body frame as

$$\boldsymbol{N}_{md} = \boldsymbol{M} \times \boldsymbol{B}_b$$

Here $\boldsymbol{M} = \begin{bmatrix} -0.09 & 0.01 & 0.11 \end{bmatrix}^T \mathrm{Am}^2$ is the constant residual magnetic moment vector and $\boldsymbol{B}_b$ is the magnetic field vector in the body frame. Note that again you need to use the instantenous attitude information to transform the magnetic field in ECI (given in the data) to calculate this vector.

One important point that you need to be careful in this Q2, the position vector and magnetic field vector in ECI are sampled at each second. So you need to interpolate these values depending on your integration time step.

Include the torques into the dynamics individually (first graivty gradient and then the magnetic disturbance). Repeat the integration and plot the satellite Z axis in ECI frame again. What do you observe? Which torque effects more?

3) (**20 pts**) Repeat Q2 this time with an MOI tensor of

$$J = \begin{bmatrix} 6.9 & 0 & 0 \\ 0 & 7.5 & 0 \\ 0 & 0 & 8.4 \end{bmatrix} \mathrm{kgm}^2$$

which is for a satellite appoximately the size of RASAT.

Again plot the Z axis in ECI frame. What happened this time? Briefly discuss.

4) Bonus Question (20pts)

a) What happens if you change the MOI tensor values for Z and Y axis in Q3 and try spinning the spacecraft still about the body Z axis?

b) What happens when you keep the MOI tensor values same but introduce small non-diagonal terms such as $0.1\mathrm{kgm}^2$? What is different in the attitude motion of the spacecraft?

Good luck!
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