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To

Sima (F.) and Muriel (M.)

for their patience and forebearance

Foreword

The purpose of the IEEE Press Series on Electromagnetic Waves is to publish books of long-term archival significance in electromagnetics. Included are new titles, as well as reprints and revisions of recognized classics. *Radiation and Scattering of Waves*, by L. B. Felsen and N. Marcuvitz is recognized as a classic worldwide. I should like to take this opportunity to welcome the reprint of this book to the series. In one sense, we are responding to the wishes of the IEEE Antennas and Propagation Society membership. When polled concerning what books the membership would like to have reprinted, the Felsen-Marcuvitz book was widely mentioned in replies, both from the USA and abroad.

The book first appeared in 1973 and immediately became widely used by both researchers and graduate students, primarily in electromagnetics but also in acoustics. Soon thereafter, the book appeared in a two-volume Russian version. I find it a tribute to the authors that today, more than twenty years after its first appearance, the book abounds with timely material that is still difficult, or often impossible, to find anywhere else in the electromagnetic literature.

In my associations over the past twenty years with electromagnetic researchers and graduate students, I have found that there are many stories concerning sections of Felsen and Marcuvitz that have been singularly helpful in solving a particular electromagnetic problem. Chapter 4, Asymptotic Evaluation of Integrals, immediately comes to mind. It certainly is widely appreciated and cited. In addition, with the modern emphasis on numerical methods, the alternative representations and asymptotic expansions of fields that occur throughout the book are essential in obtaining convergence over a wide array of field points. Many of these alternative representations and asymptotics are difficult to derive and some are virtually impossible to find elsewhere. Also, there is material in the book on transients that is most timely, given recent emphasis on high-speed electronics and ultra-wideband radar.

It is a pleasure to welcome the book to the series. It has been too long out of print. I am pleased that it is once again available to challenge and reward another generation of electromagnetic, as well as acoustic, researchers and graduate students.

Donald G. Dudley
Series Editor
IEEE Press Series on Electromagnetic Waves

Perspectives on the Reissue

This reissue of *Radiation and Scattering of Waves* in its original form published two decades ago merits some perspectives pertaining to its relevance now. As stated in the original preface, we attempted to provide a comprehensive treatment of linear source-excited electromagnetic and acoustic fields, under time-harmonic and time-dependent conditions, in the presence of various types of "canonical" propagation and scattering environments that admit of rigorous solution by general eigenfunction expansion methods. Emphasis was placed on the construction of formal alternative representations of the time-harmonic and time-dependent fields, and also on the asymptotic reduction of these formal solutions at high frequencies for the purpose of highlighting the localization, as expressed in ray-optical terms, of the associated wave physics. The spectral and asymptotic methodologies developed in this context continue to provide the basis for exploring noncanonical extensions of the problems treated in the book, and this may explain its steady appeal as a reference volume for certain constituencies within the wave propagation and diffraction community. Most frequently cited is Chapter 4, Asymptotic Evaluation of Integrals, which is an entity by itself and still represents probably the most useful collection of asymptotic techniques and formulas for engineers and physicists who are not concerned primarily with rigorous mathematics. The general complex spectral methods in Chapter 3, Mode Functions in Closed and Open Waveguides, have likewise found increased application in the technical literature.

The pyramidal structure of the book was intended to provide a broad methodological base, which encompasses all of the specific applications. Accordingly, for each of the special scattering environments, detailed reference is made to earlier chapters which contain the required building blocks. While this format conveys the commonality of techniques for a broad class of problem conditions, it mitigates against a totally self-contained treatment of a particular problem. The global structuring has been well appreciated by experienced practitioners but it makes teaching at the first or second year graduate level more difficult. Moreover, most of the Problems sections at the end of various chapters are intended to show rather sophisticated extensions of the text instead of step-by-step approaches suited for the classroom.

Finally, accommodating the interest in network formulations of field problems in the 1950s and 1960s, the spectral theorems in Chapter 3 were phrased in generalized transmission line terminology which may not be familiar to those accustomed to Sturm-Liouville theory as such. These aspects would deserve attention in a reworked version of the original.

We have refrained from appending a list of corrections to the original text. Apart from occasional typographical errors or fairly obvious errors of omissions of symbols in equations, we have not kept track–nor have we been advised by users–of substantive mistakes or discrepancies. We hope that the reissued volume will continue to fill a need within the wave radiation, propagation, and scattering community, and we express our appreciation to Professor Donald G. Dudley, the IEEE PRESS Electromagnetic Wave Series Editor, and to Mr. Dudley R. Kay, Director of Book Publishing of IEEE PRESS, for having taken the initiative in this effort.

L. B. Felsen
N. Marcuvitz

Preface

Classical field theory is concerned with the space-time behavior of physical variables describing field phenomena excited by prescribed sources. In the linear regime the methodology of description is to a large extent independent of the nature of the field and equally applicable to acoustic, electromagnetic, plasma and other fields. Within a stated space-time domain, the general linear field requires a specification of the field variables and prescribed sources, usually in terms of partial differential equations, with uniqueness of solution following from a statement of boundary and initial conditions. Solution of the so specified field problem can be effected by formal field representations whose reduction to rapidly convergent forms in appropriate space-time domains poses problems of special interest.

The general field problem is a scattering or diffraction problem distinguished by excitation from sources located either at finite distances or at infinity, and by spatial and (or) temporal complexities in the scattering region. Equivalence concepts permit replacement of the scatterers by (initially unknown) "induced currents;" they reduce the overall problem to that of finding fields radiated by prescribed and induced sources in domains of relatively simple geometrical shape. It is this latter radiation problem of determining fields excited in relatively simple regions by arbitrary sources, and the concomitant propagation of these fields, with which this book is primarily concerned. The determination of induced currents is regarded as a distinctly separate problem; it frequently poses analytical questions of considerable difficulty and usually requires integral equation techniques or the treatment of infinite sets of simultaneous equations.†

For linear fields, wherein the superposition principle is applicable, the basic radiation problem is that of determining the field excited by a point source. This is the so-called Green's function problem. Green's functions are scalar for the simple acoustic field, dyadic functions for the vector electromagnetic field, and $N \times N$ matrix functions for more complex fields. For a general linear field the components

†For an account of original pioneering waveguide applications, see Julian Schwinger and David Saxon, *Discontinuities in Waveguides*, Gordon and Breach, New York (1969); L. A. Weinstein, *The Theory of Diffraction and the Factorization Method*, Golem Press, Boulder, Colorado (1969).

of a dyadic or matrix Green's function are not usually independent, but for "separable" regions the overall Green's function may be decomposed (scalarized) into a number of independent scalar Green's functions. Thus, in the case of "separable" regions, dyadic electromagnetic Green's functions are reducible to scalar acoustic-type Green's functions, an observation that implies the direct applicability of results from one field to that of another. The central theme of this book revolves essentially about the evaluation of Green's functions in homogeneous and inhomogeneous regions of planar, cylindrical, spherical, etc., symmetry.

A Green's function may be represented in various ways as a superposition of wave functions that display the symmetries of a field region. Thus, in a linear, homogeneous, stationary, unbounded region, the plane wave functions $\exp[i(\mathbf{k}\cdot\mathbf{r} - \omega t)]$ constitute a convenient set capable of representing completely a relatively arbitrary space-time dependent field; $\mathbf{k}$ and ω denote, respectively, the wave vector and radian frequency, with $(\mathbf{k}, \omega) = (k_x, k_y, k_z, \omega)$ spanning an appropriate spectrum. In the so-called Fourier-Laplace representation the $(\mathbf{k}, \omega)$ spectrum is continuous over (almost) all real values from $-\infty$ to $+\infty$, and the resulting plane waves comprise a complete orthogonal set in space-time; a characteristic feature of such a representation is that the plane-wave field amplitudes are determined by a simple algebraic analysis. In what shall be called space- or time-guided wave representations, only three of the $(\mathbf{k}, \omega)$ periodicities are employed to define the wave spectrum, the remaining one being determined by a so-called dispersion equation; the resulting plane wave set has orthogonality and completeness properties on a three-dimensional hypersurface of space-time, and the field amplitudes are determined by solutions of ordinary differential equations. These alternative plane wave representations are typical of similar representations that obtain for bounded and anisotropic regions; each representation is characterized by convergence properties and ranges of applicability that are useful in the solution of different types of field problems.

Although the stated field representations are formally exact field solutions, the integrations occurring therein must be performed explicitly so as to yield rapidly convergent or closed form expressions for the field. This process can be effected exactly in a number of cases but generally requires approximations. A powerful and physically significant approximation procedure is provided by the function-theoretic method of "saddle point integration." This method yields asymptotic "quasi-optic" field approximations in regions illuminated by field sources, and can be modified to apply as well to "shadow" and "transition" (penumbra) regions. Such methods are related intimately to ray theories,† wavepacket propagation, WKB procedures, etc., employed in the solution of systems of partial differential equations.

With the knowledge of a proper set of modes or waves as a base, modal representation of solutions to field problems requires a two-fold procedure: (1) an analysis or transform process to determine the dependence of modal amplitudes on sources, and (2) a modal synthesis or inverse transform for the evaluation of the space-time dependence of the desired fields. The various chapters of this book develop and illustrate these modal analysis and synthesis procedures in a wide range of radiation and wave-scattering applications. Although the table of contents provides a detailed indication of subject organization, it may be desirable to elucidate and place in

†J. B. Keller, "A Geometric Theory of Diffraction," Calculus of Variations and its Applications, *Symposia Appl. Math.*, McGraw-Hill, New York, **8** (1958), pp. 27–52.

proper perspective interrelationships among the topics covered. With this intent, we sketch below some of the guiding themes that underly the organization of the chapters.

Chapter 1 is devoted both to the formulation of linear field problems and to an indication of methods of their solution. Features, properties, and methodology common to linear acoustic, electromagnetic, and plasma fields are emphasized within the context of a first order (partial differential equation) field theory; "reduced" (second and higher order) field formulations are considered subsequently. Green's functions for the above fields are introduced in comparative form highlighting their similarities and interrelationships. Exact modal representations of these Green's functions are presented in alternative ways and evaluated in closed form for simple unbounded homogeneous regions. Approximate evaluations, equally valid for inhomogeneous, anisotropic, and dispersive regions, are considered in some detail, firstly via saddle point integration and then by ray-optic and transport equation techniques. This first chapter ought not to be neglected in a first reading; it is intended to knit together with a unified viewpoint, and to anticipate with simple illustrations, many of the applications in subsequent chapters. The introductory comments to the various sections should be helpful in providing a quick overall perspective.

Chapters 2 and 3 are concerned with the modal analysis of fields in regions that generally are bounded and inhomogeneous. In Chapter 2, eigenvalue problems that provide a modal basis for transformation of vector electromagnetic problems into transmission-line (ordinary differential equation) problems are deduced for electromagentic fields in uniform and spherical waveguide regions. Techniques for solution of transmission-line equations, to which field equations are reduced in waveguide regions, are reviewed via network-theoretic and one-dimensional scalar Green's function methods. Chapter 3 contains explicit expressions for vector and scalar mode functions and their orthogonality properties for a variety of waveguide cross-sections. By classical methods, characteristic Green's function (resolvent) methods, and the method of comparison equations, these results are derived from both exact and approximate treatment of one-dimensional Sturm-Liouville type problems appropriate to homogeneous or inhomogeneously filled cross-sections. Apart from their relevance to applications treated subsequently in this book, Chapters 2 and 3 can form the basis for a course dealing with transmission-line and related eigenvalue problems.

Chapter 4 contains an extensive discussion of saddle point methods of integration necessary for approximate closed form synthesis of modal representations. An account of steepest descent integration is included, with particular attention to mathematically uniform descriptions of effects arising from the presence of different types of singularities near saddle points and from the confluence of several saddle points. These effects relate physically to field descriptions within so-called transition or penumbra regions separating "light" and "shadow" areas—or more generally, different propagation modes—in a field. Although the physical significance of various integral representations is emphasized, this chapter is self-contained and may serve as a reference to the theory of asymptotic evaluation of integrals.

Applications of the preceding theory to the explicit determination of fields radiated by sources in isotropically stratified planar, cylindrical and spherical regions are presented in Chapters 5 and 6. Although examples relate primarily to the electromagnetic fields, scalarization is frequently permissible (either directly or by decomposition), in which event the results then apply as well to acoustic and other scalar problems. Because of the complexity of several of the calculations, an attempt has been made to standardize the format for presentation of many of the results. After statement of the problem, a summary and physical interpretation of the calculated results are first presented in their various ranges of applicability; this is followed, under the heading of *Discussion*, by a more detailed indication of the function-theoretic analysis and limitations, if any. This separation of theory and results is intended to appeal both to the application- and theory-oriented reader; it should provide a type of handbook listing of the problems solved, as is evident from the table of contents for these chapters.

Chapters 7 and 8 are concerned with extensions and applications to fields in anisotropic regions. Uniaxial media are considered in Chapter 7, while gyrotropic and somewhat more general anisotropic media are treated in Chapter 8. The anisotropic regions under consideration are intended to apply to crystalline, plasma and ferromagnetic type media and, in a "reduced" electromagnetic formulation, are characterized by dyadic (tensor) permittivity and permeability parameters. This view of such media ignores certain non-electromagnetic effects, but when applicable, does provide a quantitative indication of many of the dispersive wave phenomena to be expected.

Concerning overall philosophy of subject presentation, much effort in this book has been expended on developing and applying a unified formalism for systematized eigenmode and transmission-line (network) analysis of linear field problems. Whether such systematization is justified in the solution of one or two individual problems is debatable. However, for analysis of classes of field problems having similar but not identical features, elimination of redundant aspects becomes almost essential. While the treatment thus emphasizes techniques applicable to broad classes of problems, an attempt has been made, by self-contained problem statement and frequent cross referencing, also to serve the reader interested in only a particular case. The guided-wave approach alluded to above has been found successful for many electromagnetic and acoustic field problems, and it may prove to be equally valid in similar applications for plasma, solid state, and other fields.

A note of apology ought to be sounded because of a possible unevenness in portions of this book resulting from the chronology of its preparation. Much of the material has been presented in lectures by the authors over the past 15-20 years, mostly at the Polytechnic Institute of Brooklyn and partially, by one of the authors, at New York University; in fact, a series of widely distributed reports issued some time ago by the Microwave Research Institute of the Polytechnic under the title "Modal Analysis and Synthesis of Electromagnetic Fields" constituted a first draft of portions of this manuscript. For initial support in the preparation of these reports, the authors express their appreciation to the Air Force Cambridge Research Laboratories, Bedford, Massachusetts; they also gratefully acknowledge the sponsorship by the Air Force Cambridge Research Laboratories and by the Joint Services Electronics Program of research, the results of which are included in this book. Although a good deal of effort has been expended in continually revising the text material, the authors do not feel that the presentation has been optimized in all respects. One feature of subject treatment should be mentioned in this context. The imaginary unit $\sqrt{-1}$ is designated throughout the book as i or $-j$, depending on whether the subject matter relates primarily to mathematical physicists or engineers. Usage of exp $(-i\omega t)$ and exp $(+j\omega t)$ in these respective disciplines has been fairly customary in the treatment of time-harmonic fields. Evidently, the division is not unambiguous, but engineers have traditionally been concerned more with transmission-line and network aspects of the overall field problem and less with scattering and diffraction. To minimize confusion, the time dependence is stated whenever relevant, and the facility in switching from one dependence to another is often useful when comparing various results published in the technical literature. Finally, it should be mentioned that no attempt has been made to include a comprehensive bibliography; however, the references cited provide adequate background information.

A number of individuals have contributed to the preparation of this book. We have benefited from comments and criticisms by colleagues and students. With special gratitude, we would like to acknowledge the efforts of Mrs. Margaret Bartoli who did much of the painstaking work in the typing and organization of the final manuscript. For providing necessary services and facilities, thanks in large measure are due to the Electrophysics Department of the Polytechnic, and in the final stages also to the School of Engineering and Science, New York University. Finally, we gratefully note the continued encouragement and patience of our respective families, which made completion of this effort possible.

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