B

Source-Field Relationships for Cylinders Illuminated by an Obliquely Incident Field

The following is a compilation of formulas for the various field components produced by a single strip cell of constant-current density radiating in space. The expressions are used when calculating the moment-method matrix elements for cylindrical scattering problems under the condition that the z-dependence of the excitation is

$$e^{j\gamma z} \tag{B.1}$$

Thus, the incident field may be a plane wave impinging on the scatterer from an oblique angle (not perpendicular to the cylinder axis). This particular z-dependence also arises if a Fourier transformation in z is used to replace a three-dimensional problem involving an infinite cylinder by the superposition of two-dimensional problems.

Figure B.1 illustrates the geometry under consideration. The strip cell of unit current density is centered at the origin, is of cross-sectional length W, and is oriented so that its outward normal vector makes a polar angle ϕ with $\hat{x}$-axis (outward must be defined in the context of a closed cylinder with an inside and outside; our strip is considered to be one of a number modeling such a cylinder). The field components of interest are the $\hat{z}$- and $\hat{T}$-components of the electric ($\bar{E}$) and magnetic ($\bar{H}$) field at some observation point (x, y), where $\hat{T}$ is the tangent vector to a similar strip, with outward normal vector given by the polar angle ψ. The source may be the $\hat{z}$- or $\hat{t}$-component of magnetic or electric current density. We consider only the case where the current density is constant on the strip.

The notation employed will identify both the source component and the field component. For example, $H_z \cdot J_t$ denotes the $\hat{z}$-component of the $\bar{H}$-field produced by the $\hat{t}$-component of electric current density. Appropriate expressions for the fields produced by

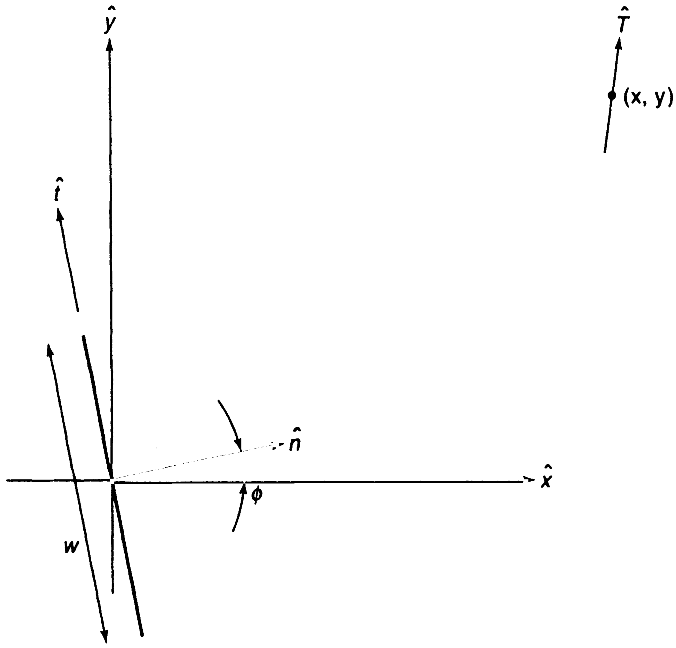

Figure B.1 Geometry of source strip.

sources are found in terms of the vector potentials $\bar{A}$ and $\bar{F}$:

$$\bar{H} = \text{curl } \bar{A} + \frac{\text{grad div} + k^2}{jk\eta} \bar{F} \tag{B.2}$$

$$\bar{E} = \eta \frac{\text{grad div} + k^2}{jk} \bar{A} - \text{curl } \bar{F} \tag{B.3}$$

where the vector potentials are defined as

$$\bar{A}(x, y) = \int_{s=-W/2}^{W/2} [\hat{z} J_z(s) + \hat{t} J_t(s)] \tilde{G}(R; k, \gamma) \, ds \tag{B.4}$$

$$\bar{F}(x, y) = \int_{s=-W/2}^{W/2} [\hat{z} K_z(s) + \hat{t} K_t(s)] \tilde{G}(R; k, \gamma) \, ds \tag{B.5}$$

and

$$\tilde{G}(R; k, \gamma) = \begin{cases} \dfrac{1}{4j} H_0^{(2)}(R\sqrt{k^2 - \gamma^2}) & k^2 > \gamma^2 \\[2ex] \dfrac{1}{2\pi} K_0(R\sqrt{\gamma^2 - k^2}) & \gamma^2 > k^2 \end{cases} \tag{B.6}$$

$$R = \sqrt{(x + s\sin\phi)^2 + (y - s\cos\phi)^2} \tag{B.7}$$

In Equation (B.6), H refers to the Hankel function and K the modified Bessel function of the second kind.

Because of the assumed $e^{j\gamma z}$-dependence, any derivatives with respect to z in the curl, grad, and div operations are replaced by multiplications with $j\gamma$. In some cases, derivatives will be transferred to the Green's function in Equation (B.6), and we note that

$$\tilde{G}'(R;k,\gamma) = \begin{cases} -\dfrac{\sqrt{k^2-\gamma^2}}{4j} H_1^{(2)}(R\sqrt{k^2-\gamma^2}) & k^2 > \gamma^2 \\ -\dfrac{\sqrt{\gamma^2-k^2}}{2\pi} K_1(R\sqrt{\gamma^2-k^2}) & \gamma^2 > k^2 \end{cases} \tag{B.8}$$

For explicit calculations, the vectors $\hat{t}$ and $\hat{T}$ are defined as

$$\hat{t} = -\hat{x}\sin\phi + \hat{y}\cos\phi \tag{B.9}$$

$$\hat{T} = -\hat{x}\sin\psi + \hat{y}\cos\psi \tag{B.10}$$

The $\hat{z}$-component of the $\bar{H}$-field produced at (x, y) by a $\hat{z}$-component of magnetic current density on the strip of Figure B.1 is given by

$$H_z \cdot K_z(x,y) = \frac{k^2-\gamma^2}{jk\eta} \int_{s=-W/2}^{W/2} \tilde{G}(R;k,\gamma)\, ds \tag{B.11}$$

where $\tilde{G}$ is defined in Equation (B.6). A closed-form expression for Equation (B.11) is not available, and in general it must be evaluated numerically. However, in many cases it can be approximated by

$$H_z \cdot K_z(x,y) \simeq \frac{k^2-\gamma^2}{jk\eta} W\tilde{G}(\rho;k,\gamma) \qquad \rho \neq 0 \tag{B.12}$$

where

$$\rho = \sqrt{x^2+y^2} \tag{B.13}$$

and

$$H_z \cdot K_z(0,0) \simeq -\frac{k^2-\gamma^2}{4k\eta} W \begin{cases} 1 - j\dfrac{2}{\pi}\ln\left(\dfrac{W\sqrt{k^2-\gamma^2}}{6.10482}\right) & k^2 > \gamma^2 \\ -j\dfrac{2}{\pi}\ln\left(\dfrac{W\sqrt{\gamma^2-k^2}}{6.10482}\right) & \gamma^2 > k^2 \end{cases} \tag{B.14}$$

Equation (B.14) is obtained by integrating a small-argument form of the Hankel or modified Bessel function of Equation (B.6). The type of approximation employed here is accurate within a few percent as long as the strip size does not exceed about a tenth of a wavelength.

The $\hat{z}$-component of the $\bar{H}$-field produced by a $\hat{t}$-component of unit magnetic current density on the strip of Figure B.1 may be obtained from the expression

$$H_z \cdot K_t(x,y) = \frac{1}{jk\eta}\hat{z} \cdot \text{grad}\,\text{div}\bar{F}_t \tag{B.15}$$

which reduces to

$$H_z \cdot K_t(x,y) = \frac{\gamma}{k\eta}\frac{\partial}{\partial t} \int_{s=-W/2}^{W/2} \tilde{G}(R;k,\gamma)\, ds \tag{B.16}$$

and finally to the closed-form expression

$$H_z \cdot K_t(x,y) = \frac{\gamma}{k\eta}[\tilde{G}(R_1;k,\gamma) - \tilde{G}(R_2;k,\gamma)] \tag{B.17}$$

where

$$R_1 = \sqrt{(x - \tfrac{1}{2}W\sin\phi)^2 + (y + \tfrac{1}{2}W\cos\phi)^2} \tag{B.18}$$

$$R_2 = \sqrt{(x + \tfrac{1}{2}W\sin\phi)^2 + (y - \tfrac{1}{2}W\cos\phi)^2} \tag{B.19}$$

No $\hat{z}$-component of an $\bar{H}$-field is generated by a $\hat{z}$-component of electric current density; thus

$$H_z \cdot J_z(x, y) = 0 \tag{B.20}$$

A $\hat{t}$-component of $\bar{J}$ does produce a $\hat{z}$-component of $\bar{H}$, according to

$$H_z \cdot J_t(x, y) = \left(\cos\phi\frac{\partial}{\partial x} + \sin\phi\frac{\partial}{\partial y}\right)\int_{s=-W/2}^{W/2} \tilde{G}(R; k, \gamma)\, ds \tag{B.21}$$

For a point (x, y) away from the strip, this expression becomes

$$H_z \cdot J_t(x, y) = \int_{s=-W/2}^{W/2} \left(\cos\phi\frac{\Delta x}{R} + \sin\phi\frac{\Delta y}{R}\right)\tilde{G}'(R; k, \gamma)\, ds \tag{B.22}$$

where

$$\Delta x = x + s\sin\phi \tag{B.23}$$

$$\Delta y = y - s\cos\phi \tag{B.24}$$

and

$$R = \sqrt{\Delta x^2 + \Delta y^2} \tag{B.25}$$

The G' function is defined in Equation (B.8). In general, Equation (B.22) must be evaluated numerically.

As the observation point (x, y) approaches the strip from the outward side (as defined by ϕ), a limiting procedure can be used to compute

$$H_z \cdot J_t(0, 0)\,|_{\text{outside}} = -\tfrac{1}{2} \tag{B.26}$$

If (x, y) approached the strip from the inside, a similar procedure produces

$$H_z \cdot J_t(0, 0)\,|_{\text{inside}} = \tfrac{1}{2} \tag{B.27}$$

The transverse component of the $\bar{H}$-field produced by the $\hat{z}$-component of a magnetic current density may be obtained from the expression

$$H_t \cdot K_z(x, y) = \frac{\gamma}{k\eta}\hat{T} \cdot \text{grad } F_z \tag{B.28}$$

This reduces to

$$H_t \cdot K_z(x, y) = \frac{\gamma}{k\eta}\int_{s=-W/2}^{W/2} \left(\frac{-\Delta x}{R}\sin\psi + \frac{\Delta y}{R}\cos\psi\right)\tilde{G}'(R; k, \gamma)\, ds \tag{B.29}$$

where Δx, Δy, and R are defined in Equations (B.23)–(B.25), and $\tilde{G}'$ is defined in Equation (B.8). In general, Equation (B.29) must be evaluated numerically. For the special case when the observation point (x, y) happens to lie on the source strip, the field vanishes and

$$H_t \cdot K_z(0, 0) = 0 \tag{B.30}$$

The transverse H-field produced by a transverse magnetic current is given by the expression

$$H_t \cdot K_t(x, y) = \frac{1}{jk\eta}\hat{T} \cdot [\text{grad div} + k^2]\bar{F}_t \tag{B.31}$$

which can be expanded to produce

$$\begin{aligned} H_t \cdot K_t(x, y) = \frac{1}{jk\eta}\Bigg[&\left(-\sin\psi\frac{\Delta x_1}{R_1} + \cos\psi\frac{\Delta y_1}{R_1}\right)\tilde{G}'(R_1; k, \gamma) \\ &-\left(-\sin\psi\frac{\Delta x_2}{R_2} + \cos\psi\frac{\Delta y_2}{R_2}\right)\tilde{G}'(R_2; k, \gamma)\Bigg] \\ &+\frac{k}{j\eta}\cos(\psi - \phi)\int_{s=-W/2}^{W/2} \tilde{G}(R; k, \gamma)\, ds \end{aligned} \tag{B.32}$$

where

$$\Delta x_1 = x - \tfrac{1}{2}W\sin\phi \tag{B.33}$$

$$\Delta y_1 = y + \tfrac{1}{2}W\cos\phi \tag{B.34}$$

$$R_1 = \sqrt{\Delta x_1^2 + \Delta y_1^2} \tag{B.35}$$

$$\Delta x_2 = x + \tfrac{1}{2}W\sin\phi \tag{B.36}$$

$$\Delta y_2 = y - \tfrac{1}{2}W\cos\phi \tag{B.37}$$

$$R_2 = \sqrt{\Delta x_2^2 + \Delta y_2^2} \tag{B.38}$$

Although the remaining integral in Equation (B.32) cannot be reduced to a closed-form expression, the approximation employed previously to convert Equation (B.11) to Equations (B.12) and (B.13) may be used for computational purposes.

The transverse $\bar{H}$-field produced by a $\hat{z}$-component of electric current density is given by

$$H_t \cdot J_z(x, y) = \hat{T} \cdot \left(\hat{x}\frac{\partial A_z}{\partial y} - \hat{y}\frac{\partial A_z}{\partial x}\right) \tag{B.39}$$

which reduces to

$$H_t \cdot J_z(x, y) = -\int_{s=-W/2}^{W/2} \left(\sin\psi\frac{\Delta y}{R} + \cos\psi\frac{\Delta x}{R}\right)\tilde{G}'(R; k, \gamma)\, ds \tag{B.40}$$

where Δx, Δy, and R are defined in Equations (B.23) and (B.25). Again, numerical integration must be used to accurately evaluate Equation (B.40). When the observation point lies on the strip, a limiting argument similar to that employed in Equations (B.26) and (B.27) can be used to show that the transverse $\bar{H}$-field an infinitesimal distance outside the strip is given by

$$H_t \cdot J_z(0, 0)\,|_{\text{outside}} = \tfrac{1}{2} \tag{B.41}$$

The transverse $\bar{H}$-field an infinitesimal distance inside the strip is

$$H_t \cdot J_z(0, 0)\,|_{\text{inside}} = -\tfrac{1}{2} \tag{B.42}$$

The transverse $\bar{H}$-field produced by $\hat{t}$-component of electric current density can be found from the expression

$$H_t \cdot J_t(x, y) = -j\gamma \hat{T} \cdot (\hat{x} \cos\phi + \hat{y} \sin\phi) A_t \tag{B.43}$$

which reduces to

$$H_t \cdot J_t(x, y) = j\gamma \sin(\psi - \phi) \int_{s=-W/2}^{W/2} \tilde{G}(R; k, \gamma)\, ds \tag{B.44}$$

The integral can also be approximated according to the procedure outlined in Equations (B.12) and (B.13), if desired. When the observation point approaches the strip, the expression vanishes. Therefore,

$$H_t \cdot J_t(0, 0) = 0 \tag{B.45}$$

The above equations describe the magnetic field produced by a constant electric or magnetic current density. Expressions for the electric field produced by the same sources can be found directly from the above expressions using the principle of duality. These formulas are given as follows:

$$E_z \cdot J_z(x, y) = \eta^2 H_z \cdot K_z(x, y) \tag{B.46}$$

$$E_z \cdot J_t(x, y) = \eta^2 H_z \cdot K_t(x, y) \tag{B.47}$$

$$E_z \cdot K_z(x, y) = 0 \tag{B.48}$$

$$E_z \cdot K_t(x, y) = -H_z \cdot J_t(x, y) \tag{B.49}$$

$$E_t \cdot J_z(x, y) = \eta^2 H_t \cdot K_z(x, y) \tag{B.50}$$

$$E_t \cdot J_t(x, y) = \eta^2 H_t \cdot K_t(x, y) \tag{B.51}$$

$$E_t \cdot K_z(x, y) = -H_t \cdot J_z(x, y) \tag{B.52}$$

$$E_t \cdot K_t(x, y) = -H_t \cdot J_t(x, y) \tag{B.53}$$