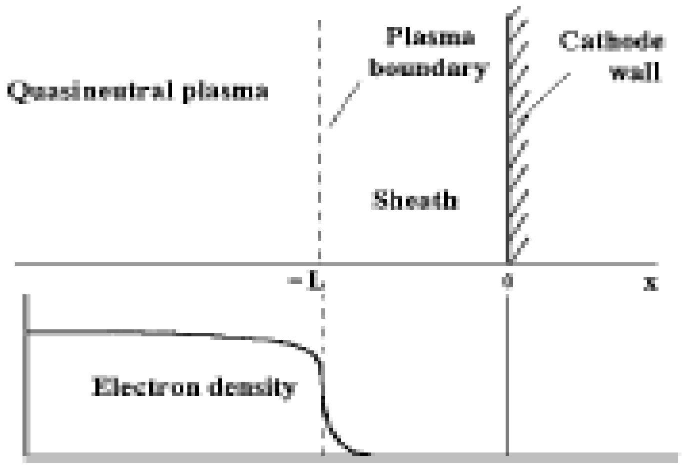
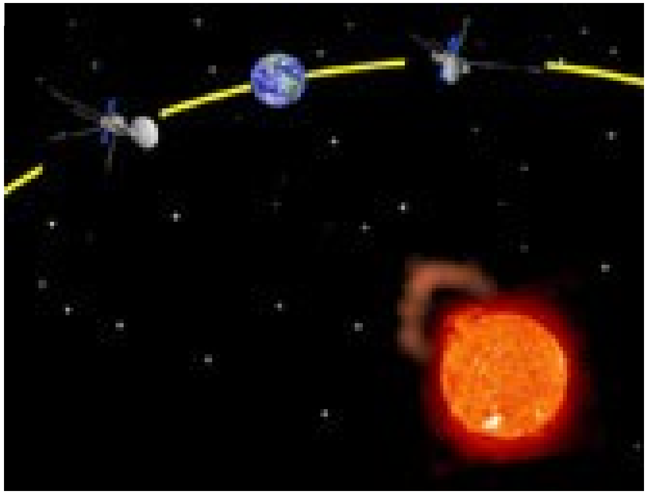
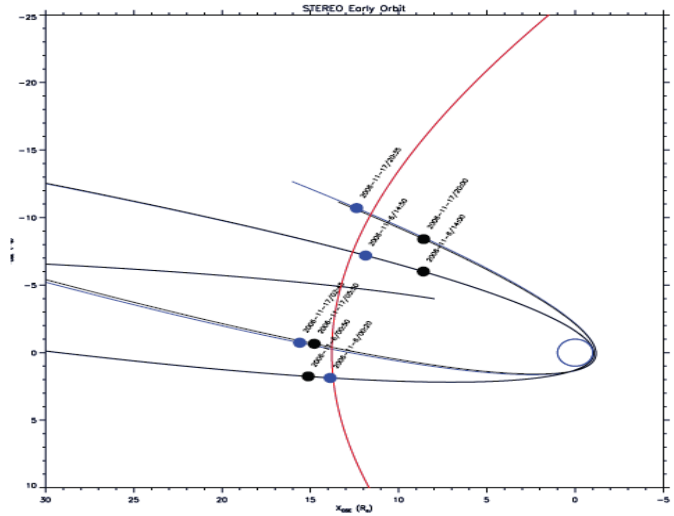
The Physics of the plasma sheath







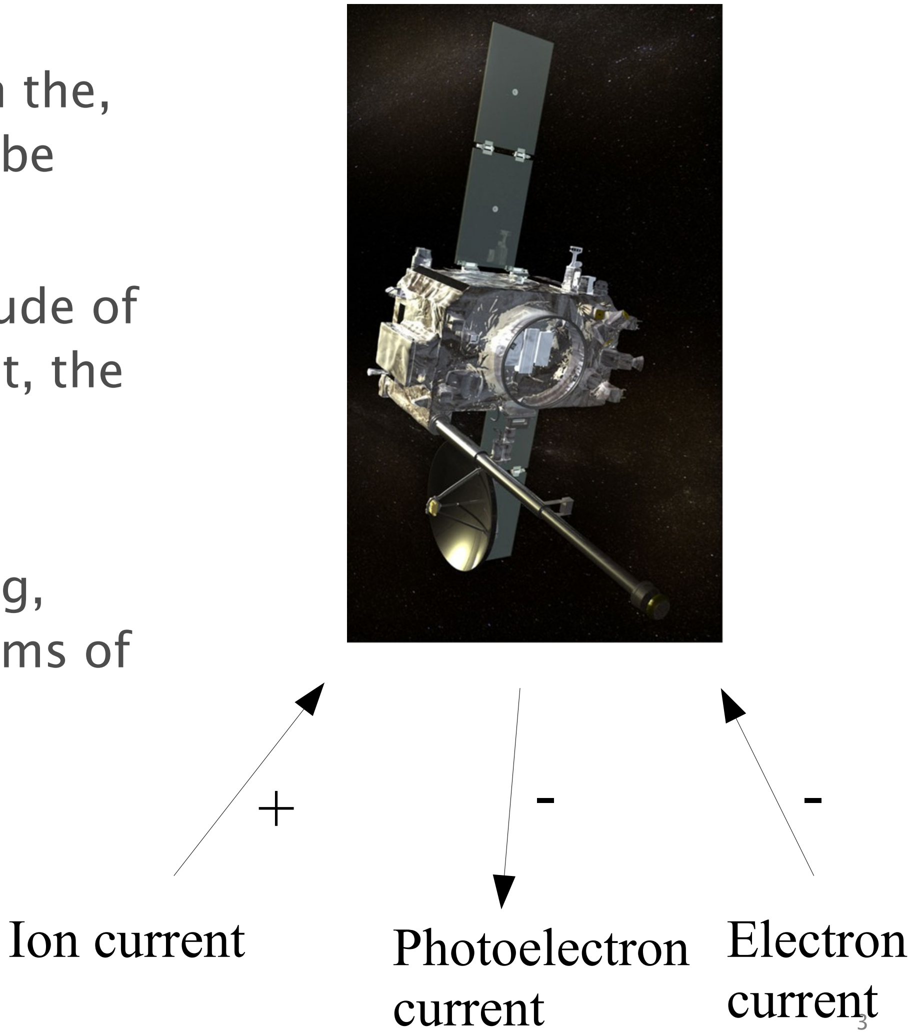
The Physics of the plasma sheath

Basics

- The space is filled with space plasma.
- The plasma interacts with an object made of conducting material (spacecraft) which is immersed in the plasma.
- A boundary zone forms around the body.
- This boundary zone is called plasma sheath.

The Physics of the plasma sheath

- 2 or 3 different currents flow to the spacecraft, depending on the environment.
- In a steady state condition the, sum of the currents must be zero.
- Depending on the magnitude of the photo-electron current, the spacecraft can be charged positively or negatively.
- Depending on the charging, there are two different forms of the sheath possible.



The Physics of the plasma sheath

Negatively charged spacecraft

- If there's no illumination, just thermal electrons and ions are present.
- A thermal electron current and a thermal ion current exists.
- If ions and electrons are in thermal equilibrium, the mean speed of the electrons will be higher: $\bar{v} = \sqrt{\frac{3kT}{m}}$
- --> more electrons hit the surface in a given time interval.
- --> the S/C will be charged negatively.
- --> the negative charge of the S/C pushes away the mobile electrons.
- --> the heavy ions are attracted.
- --> an electron depletion zone forms around the S/C
- --> the sheath thickness and S/C potential will adjust such that the currents have the same magnitude

The Physics of the plasma sheath

The electron current

- Postulation: the thermal electrons are Maxwell distributed in phase space.
- The energy $\frac{1}{2}m_e v^2 - e\phi(x)$ is conserved.
- Particle density: Zeroth moment
- Drift velocity: first moment x charge.
- V is the potential of the S/C (V<0)

$$I_e = -e\bar{n}_e A\sqrt{\frac{\kappa T_e}{2\pi m_e}}e^{\frac{eV}{\kappa T_e}}$$

$$f_e(x,v) = \frac{\bar{n}_e}{\sqrt{\frac{2\pi\kappa T_e}{m_e}}}e^{-\frac{\varepsilon}{\kappa T_e}}$$

$$\begin{aligned} n_e(x) &= \int_{-\infty}^{\infty} f_e(x,v)dv \\ &= \frac{\bar{n}_e}{\sqrt{\frac{2\pi\kappa T_e}{m_e}}}e^{\frac{e\phi(x)}{\kappa T_e}}\sqrt{\frac{2\pi\kappa T_e}{m_e}} \\ &= \bar{n}_e e^{\frac{e\phi(x)}{\kappa T_e}} \end{aligned}$$

$$\begin{aligned} j_e(x) &= -e\int_0^{\infty} v f_e(x,v)dv \\ &= -e\bar{n}_e\sqrt{\frac{\kappa T_e}{2\pi m_e}}e^{\frac{e\phi(x)}{\kappa T_e}} \end{aligned}$$

The Physics of the plasma sheath

The ion current

- All ions reach the surface.
- They fall into a potential well.
- --> accelerated.
- --> density decreases due to mass conservation. $n_i(x) \propto v_i(x)^{-1}$
- The energy is also conserved.
- Combining both conservation laws yields a formula for the particle density.
- From both expressions, an equation for the current can be found.
- Using the particle flux conservation, the final equation can be formed.

$$\frac{1}{2}m_i\bar{v}_i^2 = \frac{1}{2}m_i v_i(x)^2 + e\phi(x)$$

$$v_i(x) = \sqrt{\bar{v}_i^2 - \frac{2e\phi(x)}{m_i}}$$

$$n_i(x) = \frac{\bar{n}_i}{\sqrt{1 - \frac{2e\phi(x)}{m_i\bar{v}_i^2}}}$$

$$I_i(x) = eld\pi n_i(x) v_i(x)$$

$$I_i(x) = eld\pi\bar{n}_i\bar{v}_i$$

The Physics of the plasma sheath

The photo-electron current

- Photoelectrons are created due to photons impacting the surface (often solar origin).
- All photoelectrons reach the plasma.
- The current at the surface depends on flux and energy distribution of the photons, on the material and the geometry of the illumination.
- i_{ph} ...current density at the surface.
- A_{ϕ}...illuminated area cross section.
- Speed and density can be computed as for the ions.

$$\boxed{I_{ph} = i_{ph} A_{\phi}}$$

$$v_{ph}(x) = \sqrt{\bar{v}_{ph}^2 - \frac{2e}{m_e}(V - \phi(x))}$$

$$n_{ph}(x) = \frac{\bar{n}_{ph}}{\sqrt{1 - \frac{2e}{m_e \bar{v}_{ph}^2}(V - \phi(x))}}$$

The Physics of the plasma sheath

The potential of the S/C

- Defining the potential in a way to fulfill the boundary condition
$$\begin{aligned}\phi(0) &= V\\ \phi(\infty) &= 0\end{aligned}$$
- To calculate the potential of the spacecraft, the sum of all currents can be set to zero (steady state condition).
- Quasi-neutrality was postulated.
- An expression can be found with or without the influence of the ions.
- The influence of the ions is small.

$$V = \frac{\kappa T_e}{e} ln \left[\frac{i_{ph} A_{rel}}{e \bar{n}_e \pi} \sqrt{\frac{2\pi m_e}{\kappa T_e}} \right]$$

$$V = \frac{\kappa T_e}{e} ln \left[\left(\frac{i_{ph} A_{rel}}{e \bar{n}_e \pi} + \bar{v}_i \right) \sqrt{\frac{2\pi m_e}{\kappa T_e}} \right]$$

- A_{rel}...fraction of surface area which is illuminated.

The Physics of the plasma sheath

The potential distribution

- To compute the potential distribution, Poisson's equation has to be solved.
- The curve must be convex.
- Non-linear --> has to be solved numerically.
- This is the approximation for a flat surface.

$$\boxed{\frac{d^2\phi(x)}{dx^2} = -\frac{e(n_i(x) - n_{ph}(x) - n_e(x))}{\epsilon_0}}$$

$$\Downarrow$$

$$\boxed{\frac{d^2\phi(x)}{dx^2} = -\frac{e}{\epsilon_0}\left(\frac{\bar{n}_i}{\sqrt{1-\frac{2e\phi(x)}{m_i\bar{v}_i^2}}} - \frac{\bar{n}_{ph}}{\sqrt{1-\frac{2e}{m_e\bar{v}_{ph}^2}(V-\phi(x))}} - \bar{n}_e e^{\frac{e\phi(x)}{\kappa T_e}}\right)}$$

The Physics of the plasma sheath

Linearization

- If the potential energy is small in relation to the thermal and kinetic energies, a Taylor expansion can be used.
- This corresponds to the boundary area.

$$\begin{aligned} e\phi(x) &\ll m_i \bar{v}_i^2 \\ e(V-\phi(x)) &\ll m_e \bar{v}_{ph}^2 \\ e\phi(x) &\ll \kappa T_e \end{aligned}$$

$$\begin{aligned} \frac{1}{\sqrt{1-\frac{2e\phi(x)}{m_i\bar{v}_i^2}}} &\sim 1+\frac{2e\phi(x)}{m_i\bar{v}_i^2} \\ \frac{1}{\sqrt{1-\frac{2e}{m_e\bar{v}_{ph}^2}\left(V-\phi(x)\right)}} &\sim 1+\frac{2e\left(V-\phi(x)\right)}{m_e\bar{v}_{ph}^2} \\ e^{\frac{e\phi(x)}{kT_e}} &\sim 1+\frac{e\phi(x)}{\kappa T_e} \end{aligned}$$

$$\frac{d^2\phi(x)}{dx^2}+\frac{e^2}{\epsilon_0}\left(\frac{2\bar{n}_i}{m_i\bar{v}_i^2}-\frac{2\bar{n}_{ph}}{m_e\bar{v}_{ph}^2}-\frac{\bar{n}_e}{\kappa T_e}\right)\phi(x)=-\frac{e}{\epsilon_0}\left(\bar{n}_i-\left(1-\frac{2eV}{m_e\bar{v}_{ph}^2}\right)\bar{n}_{ph}-\bar{n}_e\right)$$

The Physics of the plasma sheath

Bohm criterion

- To preserve the physical content of the original equation, the curve must remain convex.
- So the term in the brackets on the left side must be smaller than zero. (negative potential)
- Neglecting the photoelectrons, this results in the Bohm criterion.

$$\frac{m_i \bar{v}_i^2}{2} > \kappa T_e$$

$$\boxed{\frac{d^2\phi(x)}{dx^2} + \frac{e^2}{\epsilon_0}\left(\frac{2\bar{n}_i}{m_i \bar{v}_i^2} - \frac{2\bar{n}_{ph}}{m_e \bar{v}_{ph}^2} - \frac{\bar{n}_e}{\kappa T_e}\right)\phi(x) = -\frac{e}{\epsilon_0}\left(\bar{n}_i - \left(1 - \frac{2eV}{m_e \bar{v}_{ph}^2}\right)\bar{n}_{ph} - \bar{n}_e\right)}$$

The Physics of the plasma sheath

Without photo-electrons

$$\frac{d^2\phi(x)}{dx^2} = -\frac{e}{\epsilon_0}\left[\bar{n}_i\left(1+\frac{2e\phi(x)}{m_i\bar{v}_i^2}\right) - \bar{n}_e\left(1+\frac{e\phi(x)}{\kappa T_e}\right)\right]$$

assuming quasi-neutrality

$$\frac{d^2\phi(x)}{dx^2} = -\frac{e^2\bar{n}_0}{\epsilon_0}\left(\frac{2}{m_i\bar{v}_i^2} - \frac{1}{\kappa T_e}\right)\phi(x)$$

- The solution, including the boundary conditions: $\boxed{\phi(x) = V e^{-\frac{x}{\lambda_{sh}}}}$
- λ_{sh} is the distance where the potential decreases by a factor of e:

$$\lambda_{sh} = \sqrt{\frac{\epsilon_0}{e^2\bar{n}_0}\left(\frac{2}{m_i\bar{v}_i^2} - \frac{1}{\kappa T_e}\right)^{-1}}$$

- Due to the Bohm criterion it is of the same order of magnitude as the Debye length.
- $\delta = \lambda_{sh}$ can be used as first approximation.

The Physics of the plasma sheath

Positively charged spacecraft

- The number of photo-electrons exceeds the number of thermal electrons.
- In a steady state condition the sum of the currents must be zero.
- -->The body builds a positive charge such that the number of photoelectrons reaching the plasma is roughly equal to the number of thermal electrons impacting on the surface.
- All other photo-electrons are not able to cross the potential difference and fall back to the surface.
- These photoelectrons form an electron sheath.
- They do not contribute to the photo-electron current.

The Physics of the plasma sheath

The currents

- All thermal electrons reach the surface --> no Boltzmann factor required.

$$I_e = -en_e dl\pi\sqrt{\frac{\kappa T_e}{2\pi m_e}}$$

- Only the photoelectrons with enough energy reach the plasma.

$$I_{ph} = A_{rel} i_{ph} l d e^{-\frac{eV}{\kappa T_{ph}}}$$

- The photoelectrons are roughly Maxwell distributed.
- The current of the back falling photo-electrons:

$$I_{ph,back} = A_{rel} i_{ph} l d(1 - e^{-\frac{eV}{\kappa T_{ph}}})$$

- In a steady state condition the photo-electrons escaping are replaced by thermal electrons.
- -->The total electron current reaching the surface :

$$I_{back} = I_{ph,back} + I_e \sim A_{rel} i_{ph} l d$$

The Physics of the plasma sheath

The spacecraft potential

- The potential of the spacecraft can be deduced by equating thermal and photo-electron current.

$$V = -\frac{\kappa T_{ph}}{e} \ln\left[\frac{e n_e \pi}{A_{rel} i_{ph}} \sqrt{\frac{\kappa T_e}{2\pi m_e}}\right]$$

The Physics of the plasma sheath

The densities

- The photo-electron density at the surface can be found by using $\mathbf{j} = \bar{\mathbf{v}} n q$.

$$n_{ph}(0) = \frac{A_{rel} i_{ph}}{\pi \bar{v}_{ph}(0) e}$$

- Since the escaping photo-electrons are replaced by thermal electrons, the total electron density must be roughly twice the outgoing photo-electron density, ignoring the charge contribution of the ions.

$$n_{e,tot}(0) = \frac{2 A_{rel} i_{ph}}{\pi \bar{v}_{ph}(0) e}$$

The Physics of the plasma sheath

The densities

- The distribution of the photo-electron density.
- The factor 2 is to include the photo-electrons which are falling back.
- The density of the thermal electrons can be derived as before.
- The heavy ions are assumed not to be influenced by the potential gradient.

$$n_{ph}(x) = 2n_{ph}(0)e^{-\frac{e(V-\phi(x))}{\kappa T_{ph}}}$$

$$n_e(x) = \frac{\bar{n}_e}{\sqrt{1 + \frac{2e\phi(x)}{m_e \bar{v}_e^2}}}$$

$$n_i(x) = \bar{n}_i$$

The Physics of the plasma sheath

The governing differential equation

- Flat surface approximation.
- Substitution of the densities.

$$\frac{d^2\phi(x)}{dx^2} = -\frac{e}{\epsilon_0}\left(\bar{n}_i - n_{ph}(x) - n_e(x)\right)$$

$$\frac{d^2\phi(x)}{dx^2} - \frac{2en_{ph}(0)}{\epsilon_0} e^{-\frac{e(V-\phi(x))}{\kappa T_{ph}}} - \frac{e\bar{n}_e}{\epsilon_0\sqrt{1+\frac{2e\phi(x)}{m_e\bar{v}_e^2}}} = -\frac{e\bar{n}_i}{\epsilon_0}$$

The Physics of the plasma sheath

The linearized equation

- Taylor expansion.
- Assuming quasi-neutrality.

$$\frac{1}{\sqrt{1+\frac{2e\phi(x)}{m_e \bar{v}_e^2}}} \sim 1 - \frac{2e\phi(x)}{m_e \bar{v}_e^2}$$

$$e^{\frac{e\phi(x)}{kT_{e\,ph}}} \sim 1 + \frac{e\phi(x)}{\kappa T_{ph}}$$

$$\frac{d^2\phi(x)}{dx^2} - \frac{2e^2}{\epsilon_0}\left(\frac{n_{ph}(0)}{\kappa T_{ph}} e^{-\frac{eV}{\kappa T_{ph}}} + \frac{\bar{n}_0}{m_e \bar{v}_e^2}\right)\phi(x) = \frac{2en_{ph}(0)}{\epsilon_0} e^{-\frac{eV}{\kappa T_{ph}}}$$

- The solution is the sum of an exponential function and a constant.
- Does not fulfill the boundary conditions.
- One can define the shielding length.

$$\lambda_{sh} = \left[\frac{2e^2}{\epsilon_0}\left(\frac{n_{ph}(0)}{\kappa T_{ph}} e^{-\frac{eV}{\kappa T_{ph}}} + \frac{\bar{n}_0}{m_e \bar{v}_e^2}\right)\right]^{-\frac{1}{2}}$$

The Physics of the plasma sheath

To do

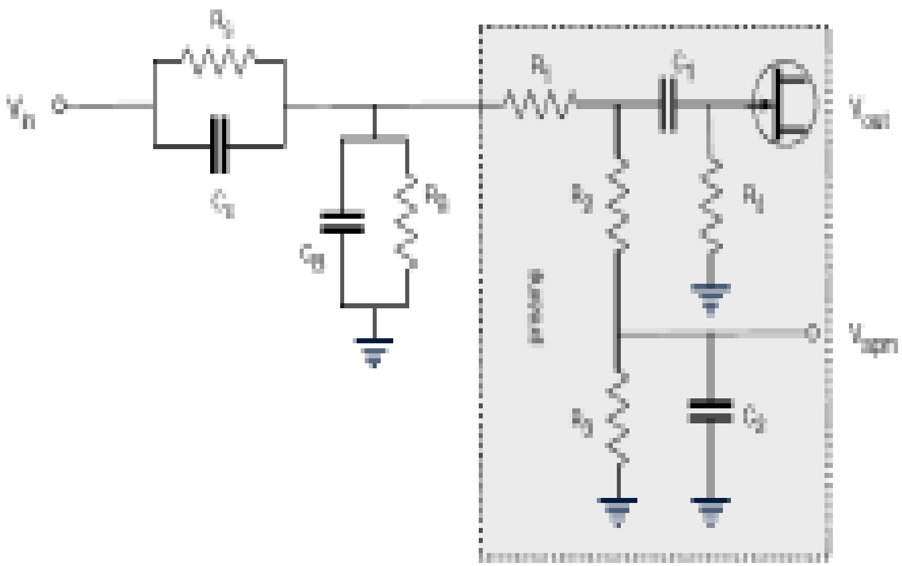
- For a cylindrical antenna the solution of the Poisson's equation in cylindrical polar coordinates is interesting.

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\phi(r)}{dr}\right) =: -\frac{e}{\epsilon_0}\left(\bar{n}_i - n_{ph}(x) - n_e(x)\right)$$

- The equation can be brought in the form of a Sturm-Liouville problem.
- The solution is a modified Bessel's equation.
- Details are not yet worked out.

The Physics of the plasma sheath

- Spacecraft antennas are usually coupled to the surrounding space plasma.
- The electromagnetic coupling can be modeled by a system of a resistance and a capacitance.
- In rarefied plasma the coupling can not take place without photoelectrons.
- The sheath thickness must not be larger than the antennas.



The Physics of the plasma sheath

Negatively charged spacecraft

- The sheath resistance is the gradient of the V-I curve.

$$R_s = \frac{\partial V}{\partial I}$$

- Explicit expression for negatively charged S/C after [Gurnett 2000]

$$R_s = \frac{\kappa T_e}{e(I_{ph} + I_i)}$$

- The resistance can be neglected at high frequencies.
- For the capacitance the general equation for cylindrically shaped capacitors can be used.

$$C_s = l_a \frac{2\pi\epsilon_0}{\ln\left(\frac{\delta}{r_a}\right)}$$

The Physics of the plasma sheath

Positively charged spacecraft

- The expressions for the positively charged spacecraft.
- $\overline{\epsilon_r}$ is the relative impedance tensor which depends on the model used.
- Changes continuously along the sheath.
- As a first approximation the mean value could be used.

$$R_s = \frac{\partial V}{\partial I}$$

$$R_s = -\frac{\kappa T_{ph}}{e I_{ph}}$$

$$C_s = l_a \frac{2\pi\epsilon_0\bar{\epsilon}_r}{\ln\left(\frac{\delta}{r_a}\right)}$$

The Physics of the plasma sheath

Application

- STEREO operates in solar wind conditions at 1AU.
- The photo-electron production rate is higher than the thermal electron impact rate. --> positive charge.
- i_{ph}...$10^{-4}Am^{-2}$
- A_{rel}...0.5
- l=6m
- d=1in (0.0254m) on average
- Mean energy of photo-electrons=1.5eV
- Mean energy of thermal electrons=10eV

$$A_{rel} i_{ph} l d \sim 7.6 \cdot 10^{-6} A$$

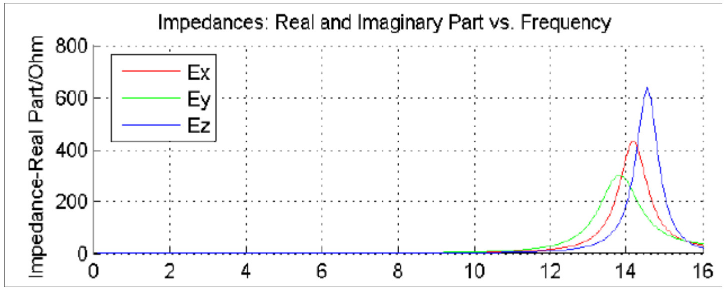
$$I_e = -e n_e d\pi \sqrt{\frac{\kappa T_e}{2\pi m_e}} \sim -2 \cdot 10^{-7} A$$

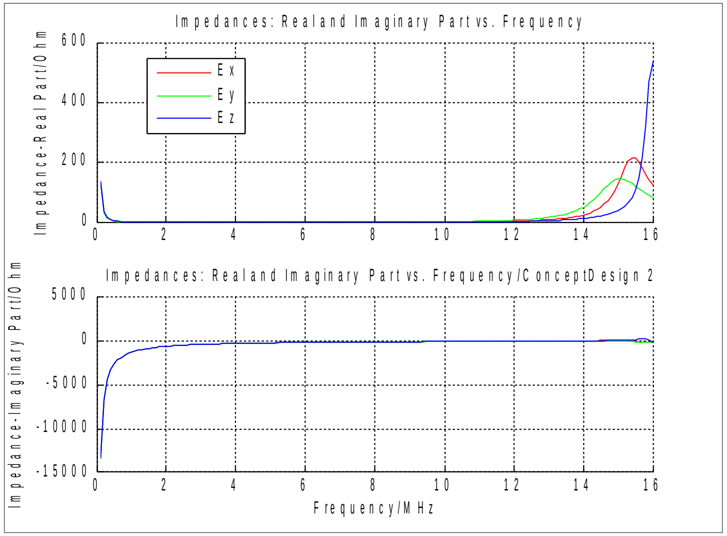
- -->V=5.5V
- $\overline{n}_e=10^6m^{-3}$
- -->$n_{ph}(0)=2x10^8m^{-3}$
- -->2.5% of the photo-electrons reach the plasma.
- -->λ_{sh} =85cm, using the photo-electron Debye length at the surface.

The Physics of the plasma sheath

Application

- Using the appropriate equations, one finds:
  - $R_s = 0.2 M\Omega$
  - $C_s = 87 pF$
- Via these parameters the sheath can be included into the numerical antenna calibration (wire-grid).
- No calculations for the effective length vectors where done so far.
- Computation of the impedances show that the inclusion of the sheath has an effect.





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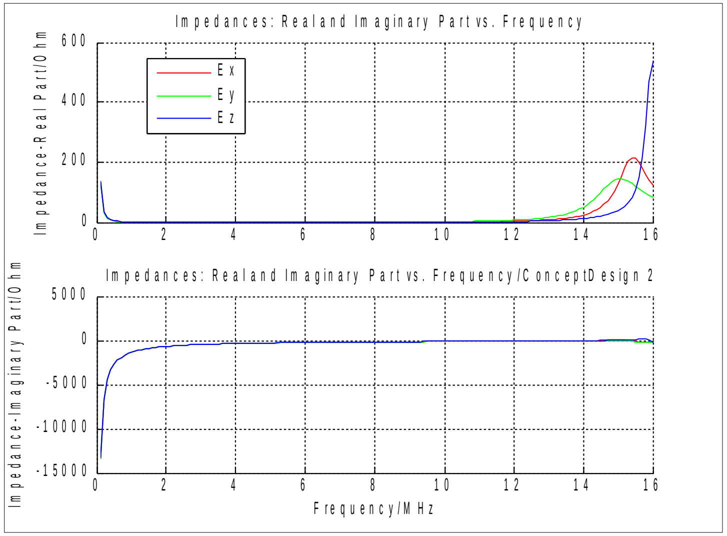
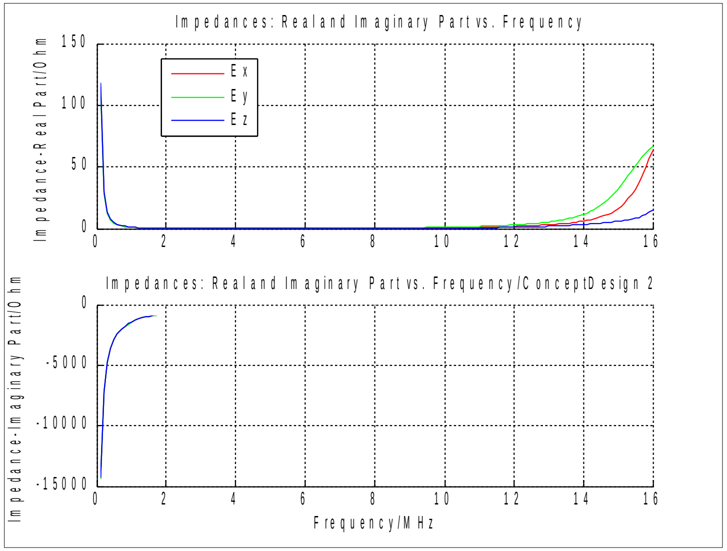
Application

- Stuart Bale estimates
  - $R_s = 0.75 M\Omega$
  - $C_s = 40 pF$
- The figures show the impedances using Bale's values in relation to the results using our theoretical model.

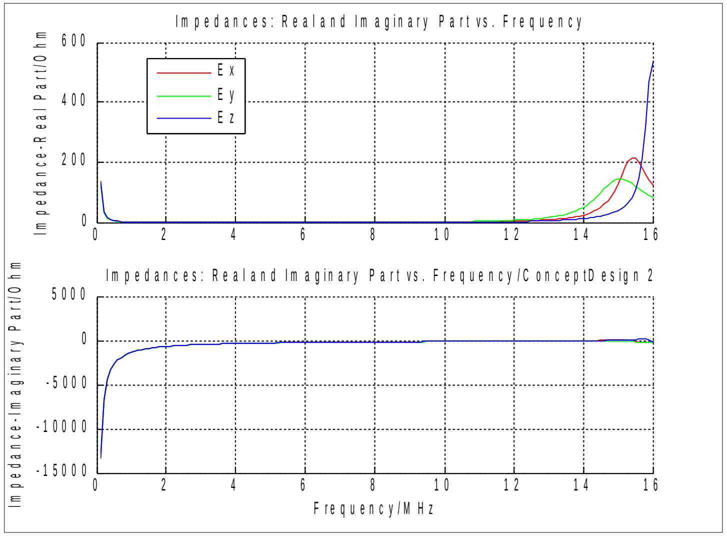


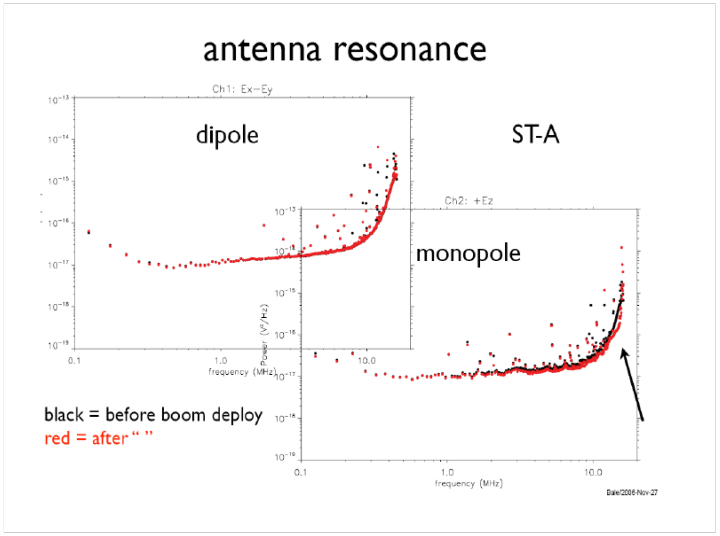


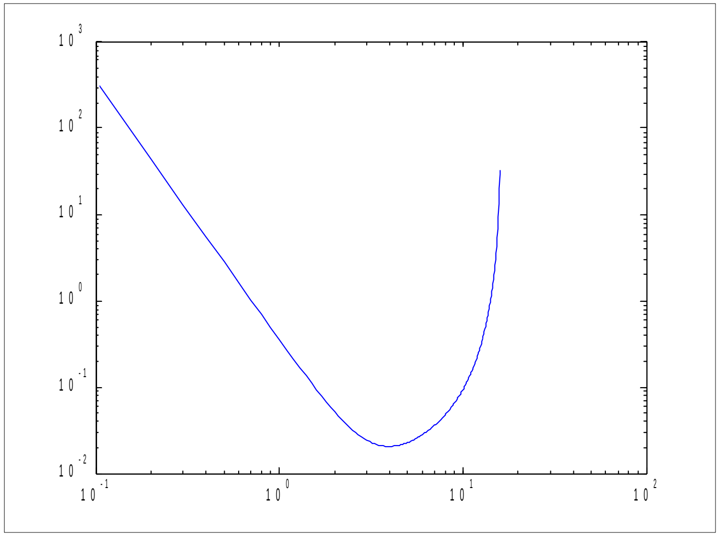
Application

The Physics of the plasma sheath

- Comparing the location of the second resonance of antenna E_z with measured data shows that the inclusion of the sheath capacitance has a corrective effect.
- Further and more accurate data is needed to verify the model.







The Physics of the plasma sheath

Final Remarks

- The surface of a body immersed in space plasma interacts in a complicated way with the plasma.
- The body is charged negatively or positively and a sheath is formed.
- Models approximating the physics of the sheaths were derived.
- An exact solution of the potential/density is not possible by analytical means.
- Solutions for the linearized models describing the sheath of a flat surface, were presented. They are not valid across the whole sheath and therefor do not fulfill the boundary conditions.
- Solutions for the cylindrical surface can be found but were not presented.
- Representing the sheath by a combination of resistivity and capacitance, the coupling of the antenna to the space plasma can be included into the numerical antenna calibration.

The Physics of the plasma sheath

Final Remarks

- As an example, the STEREO mission was used.
- The effect on the computed impedances was shown and compared to Stuart Bales estimation.
- Further calculations regarding the effect on the effective length vectors have to be performed.
- Further, more accurate data is needed for verification.
- The other antennas have resonance peaks within the frequency of the receiver and are therefor more suited.
- Many estimations have to be used in the model.
- A uniform sheath around the spacecraft has been assumed. This is not realistic, because the photoelectric effect takes only place at the side directed towards the sun.
- The velocity of the spacecraft has been neglected.
- The model for the density of the photoelectrons has to be rethought.