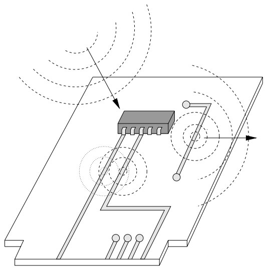
FAST METHOD FOR COMPUTATION OF ELECTROMAGNETIC COUPLING OF INTERCONNECTIONS

J. Kiwitt, A. Dietermann, M. Wójcikowski, and K. Reiß
Institut für Theoretische Elektrotechnik und Meßtechnik
Universität Karlsruhe, Kaiserstr. 12, 76128 Karlsruhe, Germany
E-mail: kiwitt@tem.etec.uni-karlsruhe.de

Intention



Electromagnetic Compatibility (EMC): PCB/system level

- interconnection line interaction:
  - internal coupling: crosstalk
  - external coupling: radiation/irradiation
- large problem space $\Longrightarrow$ only discretization of scattering objects (vs. full-discretization: FDTD, FEM etc.)
- mixed-mode analysis (vs. quasi-TEM mode: classical transmission line theory)

Used Methods

- Method of Moments (MoM) [1]
- Multi-reflection concept, derived from time-domain perspective [2], but calculated in frequency-domain
- iterative solution corresponding to physical process
- partitioning approach

1 Method of Moments

Coupled system $\longrightarrow$ linear equation system (N segments)

$$\begin{pmatrix} Z_{1,1} & \cdots & Z_{1,N} \\ \vdots & \ddots & \vdots \\ Z_{N,1} & \cdots & Z_{N,N} \end{pmatrix} \begin{pmatrix} I_1 \\ \vdots \\ I_N \end{pmatrix} = \begin{pmatrix} E_1^e \\ \vdots \\ E_N^e \end{pmatrix}$$

$$\begin{aligned} \boldsymbol{Z} &= \text{generalized impedance matrix} \\ \boldsymbol{I} &= \text{unknown current distribution vector} \\ \boldsymbol{E}^e &= \text{excitation } E\text{-field vector} \end{aligned}$$

- whole system with all couplings is described
- relatively small number of unknowns, but dense matrix
- basis and weighting functions for Z–matrix well–known

here:

- triangular current expansion
- collocation testing
- thin–wire model for interconnection lines, e. g. [3]

2 Formulation with current–to–current transfer functions

The current in segment i, induced by the current in segment j can be calculated by:

$$I_i = -\underbrace{\frac{Z_{ij}}{Z_{ii}}}_{G_{ij}} I_j$$

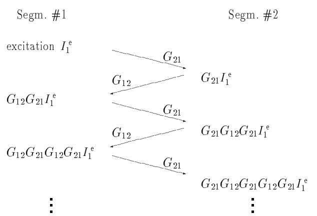
Division of each row i by element Z_{ii} leads to a formulation of the MoM with **direct current–to–current transfer functions:** G_{ij}

$$\underbrace{\begin{pmatrix} 1 & -G_{12} & \cdots & -G_{1N} \\ -G_{21} & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & -G_{N-1\,N} \\ -G_{N1} & \cdots & -G_{N\,N-1} & 1 \end{pmatrix}}_{\boldsymbol{G}} \begin{pmatrix} I_1 \\ \vdots \\ I_N \end{pmatrix} = \begin{pmatrix} I_1^e \\ \vdots \\ I_N^e \end{pmatrix}$$

3 Iterative solution

The presented iterative solution corresponds to the physical induction processes.

Iteration example: $N = 2$ segments

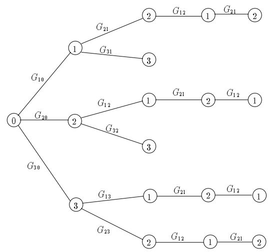


$\longrightarrow$ converging geometric series for total currents:

$$\begin{aligned} I_1 &= I_1^e \sum_{n=0}^{\infty} (G_{12}G_{21})^n & I_2 &= I_1^e\, G_{21} \sum_{n=0}^{\infty} (G_{12}G_{21})^n \\ &= I_1^e \frac{1}{1 - G_{12}G_{21}} & &= I_1^e\, G_{21} \frac{1}{1 - G_{12}G_{21}} \end{aligned}$$

$\Longrightarrow$ obvious correspondence to the closed-form solution

For $N > 2$ the occuring inductions can be calculated by following a tree like scheme:



It can be shown that the tracing of all induction processes converges towards the solution received by a matrix inversion if the spectral norm of $\boldsymbol{G}$ is smaller than 1. [2]

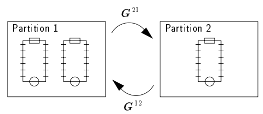
- good convergence if coupling between elements is not very strong
- problem: segments on the same wire are coupled very strongly, mostly indirectly
- solution: segments on one wire are put into one partition; internal coupling is considered by inversion of the smaller matrices

4 Partitioning and Hypermatrix Formulation

Segmentation Partitioning:

If there is a certain current in one segment of a wire, the currents in the other segments of the same wire are almost as big as the original current. They have to be put into one partition.

- The segmentation space is divided into N_p partitions.
- Each partition i holds n_i segments.



Mathematical Formulation:

- currents I_j $\longrightarrow$ current *vectors* $\boldsymbol{I}^j$
- transfer functions G_{ij} $\longrightarrow$
  transfer *matrices* $\boldsymbol{G}^{ij} = (\boldsymbol{Z}^{ii})^{-1}\,\boldsymbol{Z}^{ij}$
  $$\boldsymbol{I}^i = \boldsymbol{G}^{ij}\ \boldsymbol{I}^j$$
- matrix equation $\boldsymbol{G}\,\boldsymbol{I} = \boldsymbol{I}^e \longrightarrow$ *hypermatrix* equ. [4]

$$\underbrace{\begin{pmatrix} \mathbf{1} & -\boldsymbol{G}^{1,2} & \cdots & -\boldsymbol{G}^{1,N_p} \\ -\boldsymbol{G}^{2,1} & \mathbf{1} & & \vdots \\ \vdots & & \ddots & \vdots \\ -\boldsymbol{G}^{N_p,1} & \cdots & \cdots & \mathbf{1} \end{pmatrix}}_{\mathbf{G} = \mathbf{1} - \widetilde{\mathbf{G}}} \underbrace{\begin{pmatrix} \boldsymbol{I}_1 \\ \vdots \\ \boldsymbol{I}_{N_p} \end{pmatrix}}_{\mathbf{I}} = \underbrace{\begin{pmatrix} \boldsymbol{I}^e{}_1 \\ \vdots \\ \boldsymbol{I}^e{}_{N_p} \end{pmatrix}}_{\mathbf{I}^e}$$

5 Solution with Multi–Inductions

The solution of the hypermatrix equation according to the multi-inductions scheme is received by a Neumann series [5] for hypermatrices:

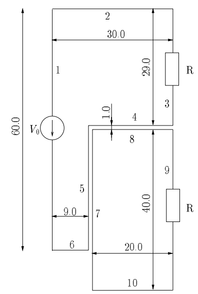
$$\begin{aligned} \mathbf{I} &= \mathbf{G}^{-1}\,\mathbf{I}^e \\ &= (\mathbf{1} - \widetilde{\mathbf{G}})^{-1}\,\mathbf{I}^e \\ &= \sum_{r=0}^{\infty} \widetilde{\mathbf{G}}^r\,\mathbf{I}^e, \quad \|\widetilde{\mathbf{G}}\| < 1 \end{aligned}$$

where

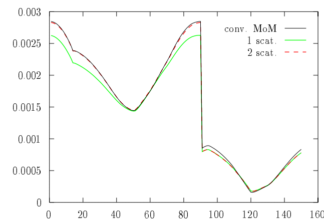
$$\mathbf{1} = \text{identity hypermatrix}$$

- fast convergence for values of $\|\widetilde{\mathbf{G}}\|$ not very close to one that means a certain part of the signal energy has to vanish at every reflecton
- normally only a few orders r must be taken into account (3-10)
- in cases where almost the whole energy is reflected many many times within the system, the convergence becomes bad

5.1 Example



Geometry of a testing example; wire radius 0.1 mm, $V_0 = 1$ V, $R = 1$ kΩ



Current distribution on the wires in the order of their number, calculated with one and two induction processes in comparison with a direct solution (1 GHz)

In this example only 2 induction processes had to be considered.

5.2 Implementation

- object–oriented implementation style for matrix/hypermatrix handling derived from [6]
- programming language: C++ on DEC AlphaStation 200–4/166

6 Complexity

- described algorithm is most efficient for a great amount of small partitions
- for a system of N_p partitions with n segments in each partition the number of necessary multiplications m for r reflections can be estimated as follows:

Solution by Gauss algorithm or LU-decomposition:

$$m_{LU} \approx \frac{(N_p n)^3}{3}$$

Solution by proposed multi-induction algorithm:

$$m_{MI} \approx N_p \frac{n^3}{3} + r\,(N_p n)^2$$

Considered for a fixed number of segments in one partition, direct matrix inversion increases with the 3. power of the number of partitions. The number of multiplications needed for the multi-induction algorithm consists of two terms. Although the first term $\sim N_p$ and the second $\sim N_p^2$ the first one normally is much bigger. The gain in comparison to LU-decomposition is obvious. For high numbers of partitions the second becomes important. It increases with N_p^2. This is similar to other iterative methods like conjugate gradients. But they are not as effective for the dense matrices received by the MoM, as they are for sparse matrices received by FEM or FD. For systems with hundreds of unknows you normally need much more iterations than just the few needed in our approach, which correspond to the physical inductions.

7 Conclusion

- fast method for interaction of high-speed interconnection lines (crosstalk, radiation)
- iterative solution by transfer functions
- partitioning
- avoids inversion of large matrices
- reveals potential of time–domain perspective

References

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[2] J. Kiwitt, A. Dietermann, K. Reiß: *A New Strategiy for the Calculation of Radiation and EMI by Successive Cross-Coupling Computations*. EMC Zurich'97.

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