Differential Forms in Electromagnetics

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Differential forms can be fun. Snapshot at the time of the 1978 URSI General Assembly in Helsinki Finland, showing Professor Georges A. Deschamps and the author disguised in fashionable sideburns.

This treatise is dedicated to the memory of Professor Georges A. Deschamps (1911–1998), the great proponent of differential forms to electromagnetics. He introduced this author to differential forms at the University of Illinois, Champaign-Urbana, where the latter was staying on a postdoctoral fellowship in 1972–1973. Actually, many of the dyadic operational rules presented here for the first time were born during that period. A later article by Deschamps [18] has guided this author in choosing the present notation.

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Preface

The present text attempts to serve as an introduction to the differential form formalism applicable to electromagnetic field theory. A glance at Figure 1.2 on page 18, presenting the Maxwell equations and the medium equation in terms of differential forms, gives the impression that there cannot exist a simpler way to express these equations, and so differential forms should serve as a natural language for electromagnetism. However, looking at the literature shows that books and articles are almost exclusively written in Gibbsian vectors. Differential forms have been adopted to some extent by the physicists, an outstanding example of which is the classical book on gravitation by Misner, Thorne and Wheeler [58].

The reason why differential forms have not been used very much may be that, to be powerful, they require a toolbox of operational rules which so far does not appear to be well equipped. To understand the power of operational rules, one can try to imagine working with Gibbsian vectors without the bac cab rule $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \mathbf{b}(\mathbf{a} \cdot \mathbf{c}) - \mathbf{c}(\mathbf{a} \cdot \mathbf{b})$ which circumvents the need of expanding all vectors in terms of basis vectors. Differential-form formalism is based on an algebra of two vector spaces with a number of multivector spaces built upon each of them. This may be confusing at first until one realizes that different electromagnetic quantities are represented by different (dual) multivectors and the properties of the former follow from those of the latter. However, multivectors require operational rules to make their analysis effective. Also, there arises a problem of notation because there are not enough fonts for each multivector species. This has been solved here by introducing marking symbols (multihooks and multiloops), easy to use in handwriting like the overbar or arrow for marking Gibbsian vectors. It was not typographically possible to add these symbols to equations in the book. Instead, examples of their use have been given in figures showing some typical equations. The coordinate-free algebra of dyadics, which has been used in conjunction with Gibbsian vectors (actually, dyadics were introduced by J.W. Gibbs himself in the 1880s, [26–28]), has so far been missing from the differential-form formalism. In this book one of the main features is the introduction of an operational dyadic toolbox. The need is seen when considering problems involving general linear media which are defined by a set of medium dyadics. Also, some quantities which are represented by Gibbsian vectors become dyadics in differential-form representation. A collection of rules for multivectors and dyadics is given as an appendix at the end of the book. An advantage of differential forms when compared to Gibbsian vectors often brought forward lies in the geometrical content of different (dual) multivectors, best illustrated in the aforementioned book on gravitation. However, in the present book, the analytical aspect is emphasized because geometrical interpretations do not help very much in problem solving. Also, dyadics cannot be represented geometrically at all. For complex vectors associated with time-harmonic fields the geometry becomes complex.

It is assumed that the reader has a working knowledge on Gibbsian vectors and, perhaps, basic Gibbsian dyadics as given in [40]. Special attention has been made to introduce the differential-form formalism with a notation differing from that of Gibbsian notation as little as possible to make a step to differential forms manageable. This means balancing between notations used by mathematicians and electrical engineers in favor of the latter. Repetition of basics has not been avoided. In particular, dyadics will be introduced twice, in Chapters 1 and 2. The level of applications to electromagnetics has been left somewhat abstract because otherwise it would need a book of double or triple this size to cover all the aspects usually presented in books with Gibbsian vectors and dyadics. It is hoped such a book will be written by someone. Many details have been left as problems, with hints and solutions to some of them given as an appendix.

The text is an outgrowth of lecture material presented in two postgraduate courses at the Helsinki University of Technology. This author is indebted to two collaborators of the courses, Dr. Pertti Lounesto (a world-renown expert in Clifford algebras who sadly died during the preparation of this book) from Helsinki Institute of Technology, and Professor Bernard Jancewicz, from University of Wroclaw. Also thanks are due to the active students of the courses, especially Henrik Wallén. An early version of the present text has been read by professors Frank Olyslager (University of Ghent) and Kurt Suchy (University of Düsseldorf) and their comments have helped this author move forward.

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Koivuniemi, Finland
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