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# COMPUTATIONAL METHODS FOR ELECTROMAGNETICS

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Andrew F. Peterson

*School of Electrical and Computer Engineering  
Georgia Institute of Technology*

Scott L. Ray

*Modeling and Information Sciences Laboratory  
Dow AgroSciences*

Raj Mittra

*Department of Electrical and Computer Engineering  
Pennsylvania State University*

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# Preface

A decade ago, when the task of developing this book was initiated, there were few available texts on computational techniques for electromagnetics. Although a large number have appeared since then, none attempt to treat both integral and differential equation formulations in a unified manner. The present text is intended to fill that gap and is designed for graduate-level classroom use or self-study. Its primary focus is open-region formulations, and while resonant cavity and antenna applications are touched on in places, the majority of the material is presented in the context of electromagnetic scattering. We have attempted to provide enough detail to enable a reader to implement the concepts in software. In addition to a few subroutines in Appendix C, a collection of related computer programs is available through the Internet. Earlier drafts of the material were tested in graduate courses taught at the University of Illinois and the Georgia Institute of Technology as well as in a number of continuing education courses. The authors sincerely appreciate the comments of former students, colleagues, and the dozen or more reviewers who offered critiques during the book's development.

*Andrew F. Peterson  
Scott L. Ray  
Raj Mittra*

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