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Preface

The present monograph discusses a number of mathematical and conceptual methods applicable in the analysis of electromagnetic fields. The leading tone is dyadic algebra. It is applied in the form originated by J.W. Gibbs more than one hundred years ago, with new powerful identities added, making coordinate-dependent operations in electromagnetics all but obsolete. The chapters on complex vectors and dyadics are independent of the rest of the book, actually independent of electromagnetics, so they can be applied in other branches of physics as well. It is claimed that by memorizing about five basic dyadic identities (similar to the well-known bac-cab rule in vector algebra), a working knowledge of dyadic algebra is obtained. To save the memory, a collection of these basic dyadic identities, together with their most important special cases, is given as an appendix. In different chapters the dyadics are seen in action. It is shown how simply different properties can be expressed in terms of dyadics: boundary and interface conditions, medium equations, solving Green functions, generalizing circuit theory to vector field problems with dyadic impedances, finding transformations between field problems and, finally, working on multipole and image sources for different problems.

Dyadic algebra is seen especially to aid in solving electromagnetic problems involving different linear media. In recent years, the chiral medium with its wide range of potential applications has directed theoretical interest to new materials. The most general isotropic medium, the bi-isotropic medium, has made electromagnetic theory a fresh subject again, with new phenomena being looked for. The medium aspect is carried along in this text. What is normally analysed in isotropic media is done here for bi-isotropic or sometimes for bianisotropic media, if possible. Especially new is the duality transformation, which actually exists as a pair of transformations. It is seen to shed new light on the plus and minus field decomposition, which has proved useful for analysing fields in chiral media, by showing that they are nothing more than self-dual fields with respect to each of the two transformations.

In Chapter 5, Green dyadics for different kinds of media are discussed and a systematic method for their solution, without applying the Fourier transformation, is given. In Chapter 6, source equivalence and its relation to non-radiating sources is discussed, together with certain equivalent

sources: point sources (multipoles) and surface sources (Huygens' sources). Everywhere in the text the main emphasis is not on specific results but methods of analysis.

The final chapter gives a summary of the work done by this author and colleagues on the EIT, exact image theory. This is a general method for solving problems involving layered media by replacing them by image sources which are located in complex space. The EIT is presented here for the first time in book form.

The contents of this monograph reflect some of the work done and courses given by this author during the last two decades. The results should be of interest to scientists doing research work in electromagnetics, as well as to graduate students. For classroom use, there are numerous possibilities for homework problems requiring the student to fill in steps which have been omitted to keep the size of this monograph within certain limits. The EIT can also be studied independently and additional material, not found in this text, exists in print (see referece lists at section ends of Chapter 7).

The text has been typed and figures drawn by the author alone, leaving no-one else to blame. On the other hand, during graduate courses given on the material, many students have helped in checking a great number of equations. Also, the material of Chapters 1 and 2 has been given earlier as a laboratory report and a few misprints have been pointed out by some international readers. For all these I am thankful. The rest of the errors and misprints are still there to be found.

This book is dedicated to my wife Liisa. A wise man is recognized for having a wife wiser than himself. I have the pleasure to consider myself a wise man.

Helsinki I.V.L.
July 1991.

For the second edition, the main text has remained unchanged except for a small number of misprints, which have been corrected. To assist in classroom use of the book, three new appendices have been added. Appendix A contains a collection of problems on the topics of Chapters 1 – 6 and Appendix B a set of solutions for most of these problems. Appendix C gives a collection of most useful formulas in vector analysis for convenience. Appendix A (Dyadic identities) of the first edition is now relabeled as Appendix D.

Helsinki I.V.L.
March 1995.

Contents

1 Complex vectors 1

1.1 Notation 1

1.2 Complex vector identities 4

1.3 Parallel and perpendicular vectors 5

1.4 Axial representation 8

1.5 Polarization vectors 10

1.6 Complex vector bases 14

References 15

2 Dyadics 17

2.1 Notation 17

2.1.1 Dyads and polyads 17

2.1.2 Symmetric and antisymmetric dyadics 19

2.2 Dyadics as linear mappings 20

2.3 Products of dyadics 22

2.3.1 Dot-product algebra 22

2.3.2 Double-dot product 24

2.3.3 Double-cross product 25

2.4 Invariants and inverses 28

2.5 Solving dyadic equations 30

2.5.1 Linear equations 31

2.5.2 Quadratic equations 33

2.5.3 Shearers 35

2.6 The eigenvalue problem 36

2.7 Hermitian and positive definite dyadics 39

2.7.1 Hermitian dyadics 39

2.7.2 Positive definite dyadics 40

2.8 Special dyadics 41

2.8.1 Rotation dyadics 41

2.8.2 Reflection dyadics 43

2.8.3 Uniaxial dyadics 44

2.8.4 Gyrotropic dyadics 45

2.9 Two-dimensional dyadics 47
2.9.1 Eigendyadics 48
2.9.2 Base dyadics 49
2.9.3 The inverse dyadic 50
2.9.4 Dyadic square roots 50
References 51

3 Field equations 53
3.1 The Maxwell equations 53
3.1.1 Operator equations 55
3.1.2 Medium equations 56
3.1.3 Wave equations 57
References 58
3.2 Fourier transformations 59
3.2.1 Fourier transformation in time 59
3.2.2 Fourier transformation in space 62
References 63
3.3 Electromagnetic potentials 63
3.3.1 Vector and scalar potentials 64
3.3.2 The Hertz vector 66
3.3.3 Scalar Hertz potentials 67
References 68
3.4 Boundary, interface and sheet conditions 69
3.4.1 Discontinuities in fields, sources and media 69
3.4.2 Boundary conditions 71
3.4.3 Interface conditions 74
3.4.4 Sheet conditions 77
3.4.5 Boundary and sheet impedance operators 81
References 83
3.5 Uniqueness 84
3.5.1 Electrostatic problem 85
3.5.2 Scalar electromagnetic problem 86
3.5.3 Vector electromagnetic problem 88
References 88
3.6 Conditions for medium parameters 89
3.6.1 Energy conditions 89
3.6.2 Reciprocity conditions 93
References 94

4 Field transformations 97

4.1 Reversal transformations 97
4.1.1 Polarity reversal 97
4.1.2 Time reversal 98
4.1.3 Space inversion 99
4.1.4 Transformations of power and impedance 99
References 99

4.2 Duality transformations 100
4.2.1 Simple duality 100
4.2.2 Duality transformations for isotropic media 100
4.2.3 Left-hand and right-hand transformations 101
4.2.4 Application of the duality transformations 103
4.2.5 Self-dual problems 105
4.2.6 Self-dual field decomposition 107
4.2.7 Duality transformations for bi-isotropic media 110
References 112

4.3 Affine transformations 112
4.3.1 Transformation of fields and sources 113
4.3.2 Transformation of media 115
4.3.3 Involutory affine transformations 116

4.4 Reflection transformations 117
4.4.1 Invariance of media 117
4.4.2 Electric and magnetic reflections 118
4.4.3 The mirror image principle 119
4.4.4 Images in parallel planes 121
4.4.5 Babinet's principle 122
References 124

5 Electromagnetic field solutions 125

5.1 The Green function 125
5.1.1 Green dyadics of polynomial operators 126
5.1.2 Examples of operators 128
References 129

5.2 Green functions for homogeneous media 129
5.2.1 Isotropic medium 129
5.2.2 Bi-isotropic medium 132
5.2.3 Anisotropic medium 133
References 137

5.3 Special Green functions 138
5.3.1 Two-dimensional Green function 138
5.3.2 One-dimensional Green function 140
5.3.3 Half-space Green function 141
References 141
5.4 Singularity of the Green dyadic 142
5.4.1 Constant volume current 142
5.4.2 Constant planar current sheet 144
5.4.3 Singularity for a volume source 146
5.4.4 Singularity for a surface source 147
References 148
5.5 Complex source point Green function 148
5.5.1 Complex distance function 149
5.5.2 Point source in complex space 150
5.5.3 Green function 151
References 153
5.6 Plane waves 154
5.6.1 Dispersion equations 154
5.6.2 Isotropic medium 156
5.6.3 Bi-isotropic medium 156
5.6.4 Anisotropic medium 157
References 163

6 Source equivalence 165
6.1 Non-radiating sources 165
6.1.1 Electric sources in isotropic medium 165
6.1.2 Sources in bianisotropic media 168
References 169
6.2 Equivalent electric and magnetic sources 170
6.2.1 Bianisotropic medium 170
6.2.2 Isotropic medium 171
6.2.3 Chiral medium 173
References 174
6.3 Multipole sources 174
6.3.1 Delta expansions 175
6.3.2 Multipole expansion 176
6.3.3 Dipole approximation of a multipole source 178
6.3.4 Electric and magnetic dipole approximation 181
References 181

6.4 Huygens' sources 182
6.4.1 The truncated problem 182
6.4.2 Huygens' principle 183
6.4.3 Consequences of Huygens' principle 185
6.4.4 Surface integral equations 187
6.4.5 Integral equations for bodies with impedance surface 189
6.4.6 Integral equations for material bodies 191
References 191
6.5 TE/TM decomposition of sources 192
6.5.1 Decomposition identity 193
6.5.2 Line source decomposition 194
6.5.3 Plane source decomposition 196
6.5.4 Point source decomposition 197
References 197

7 Exact image theory 199
7.1 General formulation for layered media 199
7.1.1 Fourier transformations 199
7.1.2 Image functions for reflection fields 205
7.1.3 Image functions for transmission fields 214
7.1.4 Green functions 218
References 223
7.2 Surface problems 224
7.2.1 Impedance surface 224
7.2.2 Impedance sheet 228
7.2.3 Wire grid 231
References 235
7.3 The Sommerfeld half-space problem 235
7.3.1 Reflection coefficients 236
7.3.2 Reflection image functions 238
7.3.3 Antenna above the ground 247
7.3.4 Transmission coefficients 249
7.3.5 Transmission image functions 250
7.3.6 Radiation from a loop antenna into the ground 255
7.3.7 Scattering from an object in front of an interface 257
References 259
7.4 Microstrip geometry 260
7.4.1 Reflection coefficients and image functions 260
7.4.2 Fields at the interface 267

7.4.3 The guided modes 271
7.4.4 Properties of the Green functions 272
References 274
7.5 Anisotropic half space 275
References 279

Appendix A Problems 281

Appendix B Solutions 291

Appendix C Vector formulas 307

Appendix D Dyadic identities 311

Author index 315

Subject index 317