

# **ELECTROMAGNETIC SIMULATION USING THE FDTD METHOD**

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# **ELECTROMAGNETIC SIMULATION USING THE FDTD METHOD**

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IEEE Microwave Theory and Techniques Society, *Sponsor*



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To  
Sully and Jane

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# Guide to the Book

## PURPOSE

This book has one purpose only: to enable the reader or student to learn how to do three-dimensional electromagnetic simulation using the finite-difference time-domain (FDTD) method. It does not attempt to explain the theory of FDTD simulation in great detail. It is not a survey of all possible approaches to the FDTD method nor is it a “cookbook” of applications. It is aimed at those who would like to learn to do FDTD simulation in a reasonable amount of time.

## FORMAT

This book is tutorial in nature. Every chapter attempts to address an additional level of complexity. The text increases in complexity in two major ways:

Dimension of Simulation	Type of Material
One-dimensional	Free space
Two-dimensional	Dielectric material
Three-dimensional	Lossy dielectric material
	Frequency-dependent material

The first section of Chapter 1 is one-dimensional simulation in free space. From there it progresses to more complicated media. In Chapter 2, the simulation of frequency-dependent media is addressed. Chapter 3 introduces two-dimensional simulation, including the simulation of plane waves and how to implement the perfectly matched layer. Chapter 4 introduces three-dimensional simulation. This is the approach taken throughout the book.

## SPECIFIC CHOICES DEALING WITH SOME TOPICS

There are many ways to handle individual topics having to do with FDTD simulation. This book does not attempt to address all of them. In most cases, one single approach is taken and used

throughout the book for the sake of clarity. My philosophy is that when first learning the FDTD method, it is better to learn one specific approach and learn it well rather than to be confused by switching among different approaches. In most cases, the approach being taught is this author's preference. That does not make it the only approach or even the best; it is just the approach that this author has found to be effective. In particular, the following are some of the choices that have been made.

1. **The use of normalized (Gaussian) units.** Maxwell's equations have been normalized by substituting

$$\tilde{E} = \sqrt{\frac{\epsilon_0}{\mu_0}} E.$$

This is a system called *Gaussian units*, which is frequently used by physicists. The reason for using it here is simplicity in the formulations. The  $E$  field and the  $H$  fields have the same order of magnitude. This has an advantage in formulating the perfectly matched layer (PML), which is a crucial part of FDTD simulation.

2. **The PML for boundary conditions.** The absorbing boundary conditions (ABCs) are an important topic in FDTD simulation. The ABCs prevent spurious reflections from the edge of the problem space. There are numerous approaches to this, but this book will use the perfectly matched layer (PML) for two- and three-dimensional simulation exclusively. (A simpler boundary condition will be used in one dimension just for convenience.) The reason is its effectiveness and versatility in working with different media.
3. **Maxwell's equations with flux density.** There is some leeway in forming the time-domain Maxwell's equations from which the FDTD formulation is developed. The following is used in Chapter 1:

$$\frac{\partial \mathbf{E}}{\partial t} = \frac{1}{\epsilon_0} \nabla \times \mathbf{H} - \frac{\sigma}{\epsilon_0} \mathbf{E} \quad (1)$$

$$\frac{\partial \mathbf{H}}{\partial t} = \frac{1}{\mu_0} \nabla \times \mathbf{E}. \quad (2)$$

This is a straightforward formulation and among the most commonly used. However, by Chapter 2, the following formulation using the flux density is adopted:

$$\frac{\partial \mathbf{D}}{\partial t} = \nabla \times \mathbf{H} \quad (3)$$

$$\mathbf{D} = \epsilon \mathbf{E} \quad (4)$$

$$\frac{\partial \mathbf{H}}{\partial t} = -\frac{1}{\mu_0} \nabla \times \mathbf{E}. \quad (5)$$

In this formulation, it is assumed that the materials being simulated are nonmagnetic; that is,  $\mathbf{H} = \frac{1}{\mu_0} \mathbf{B}$ . However, we will be dealing with a broad range of dielectric properties, so Eq. (4) could be a complicated convolution. There is a reason for using this formulation: Eqs. (3) and (5) remain the same regardless of the material; any complicated mathematics stemming from the material is in Eq. (4). We will see that the solution of Eq. (4) can be looked upon as a digital filtering problem. In fact, the use of signal processing techniques in FDTD simulation will be a recurring theme in this book.

## Z TRANSFORMS

As mentioned above, the solution of Eq. (4) for most complicated material can be viewed as a digital filtering problem. That being the case, the most direct approach to solving the problem is

to take Eq. (4) into the Z domain. Z transforms are a regular part of electrical engineering education, but not that of physicists, mathematicians, and others. In teaching a graduate class on FDTD simulation, I begin the semester by teaching two topics in parallel: FDTD simulation and Z transforms. When we have progressed to the simulation of complicated dispersive materials, the students are ready to apply the Z transform theory. This had two distinct advantages over and above the simulation applications: (1) Electrical engineering students have had another application of Z transforms to strengthen their understanding of signal processing and filter theory; and (2) physics students and others now know and can use Z transforms, something that had not previously been part of their formal education. Based on my positive experience, I would encourage anyone using this book when teaching an FDTD course to consider this approach. However, I have left the option open to simulate dispersive methods with other techniques. The sections on Z transforms are optional and may be skipped. An appendix of Z transform theory is provided.

## PROGRAMMING EXERCISES

The philosophy behind this book is that the student will learn by doing. Therefore, the majority of exercises involve programming. Each chapter has one or more FDTD programs written in C. If there is more than one program per chapter, typically only the first will be a complete program listing. The subsequent programs will not be complete, but will only show changes as compared to the first program. For instance, section 1 of Chapter 1 describes one-dimensional FDTD simulation in free space. The program `fd1d_1.1.c` at the end of the chapter is a simulation of a pulse in free space. Section 3 describes how a simple absorbing boundary condition is implemented. The program `fd1d_1.2.c` is not a complete program, but shows the changes necessary to `fd1d_1.1.c` to implement the boundary condition. Furthermore, important lines of code are highlighted in bold-face.

## PROGRAMMING LANGUAGE

The programs at the end of each chapter are written in the C programming language. The reasons for this are the almost universal availability of C compilers on UNIX workstations, and the fact that so many engineers and scientists know C. However, the reader should keep one fact in mind: most researchers who do large scientific programming use FORTRAN because FORTRAN was written for scientific programming. The structured style of C may have aesthetic appeal, but it typically runs slower than FORTRAN. This is particularly true of supercomputers.

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