Appendix B

Vector and Dyadic Identities

Vector Identities

1. $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \mathbf{b} \cdot (\mathbf{c} \times \mathbf{a}) = \mathbf{c} \cdot (\mathbf{a} \times \mathbf{b})$
2. $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\,\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\,\mathbf{c}$
3. $\nabla (ab) = a\nabla b + b\nabla a$
4. $\nabla (a\mathbf{b}) = a\nabla \mathbf{b} + (\nabla a)\mathbf{b}$
5. $\nabla \cdot (a\mathbf{b}) = a\nabla \cdot \mathbf{b} + \mathbf{b} \cdot \nabla a$
6. $\nabla \times (a\mathbf{b}) = a\nabla \times \mathbf{b} - \mathbf{b} \times \nabla a$
7. $\nabla (\mathbf{a} \cdot \mathbf{b}) = (\nabla \mathbf{a}) \cdot \mathbf{b} + \mathbf{a} \cdot \nabla \mathbf{b} = \mathbf{a} \times \nabla \times \mathbf{b} + \mathbf{b} \times \nabla \times \mathbf{a} + \mathbf{a} \cdot \nabla \mathbf{b} + \mathbf{b} \cdot \nabla \mathbf{a}$
8. $\nabla \cdot (\mathbf{a} \times \mathbf{b}) = \mathbf{b} \cdot \nabla \times \mathbf{a} - \mathbf{a} \cdot \nabla \times \mathbf{b}$
9. $\nabla \cdot (\mathbf{ab}) = (\nabla \cdot \mathbf{a})\mathbf{b} + \mathbf{a} \cdot \nabla \mathbf{b}$
10. $\mathbf{a} \times \nabla \times \mathbf{b} = (\nabla \mathbf{b}) \cdot \mathbf{a} - \mathbf{a} \cdot \nabla \mathbf{b}$
11. $\nabla \times (\mathbf{a} \times \mathbf{b}) = \nabla (\mathbf{ba} - \mathbf{ab}) = \mathbf{a}\nabla \mathbf{b} - \mathbf{b}\nabla \mathbf{a} - \mathbf{a} \cdot \nabla \mathbf{b} - \mathbf{b} \cdot \nabla \mathbf{a}$
12. $\nabla \times \nabla \times \mathbf{a} = \nabla\nabla \cdot \mathbf{a} - \nabla \cdot \nabla \mathbf{a}$
13. $\nabla \cdot \nabla a = 0$
14. $\nabla \times \nabla \mathbf{a} = 0$

Dyadic Identities

15. $\mathbf{a} \cdot (\mathbf{b} \times \bar{\bar{c}}) = -\mathbf{b} \cdot (\mathbf{a} \times \bar{\bar{c}}) = (\mathbf{a} \times \mathbf{b}) \cdot \bar{\bar{c}}$
16. $\mathbf{a} \times (\mathbf{b} \times \bar{\bar{c}}) = \mathbf{b}(\mathbf{a} \cdot \bar{\bar{c}}) - (\mathbf{a} \cdot \mathbf{b})\bar{\bar{c}}$
17. $\nabla(a\bar{\bar{b}}) = a\nabla\bar{\bar{b}} + (\nabla a) \cdot \bar{\bar{b}}$
18. $\nabla(a\bar{\bar{b}}) = a\nabla\bar{\bar{b}} + (\nabla a) \times \bar{\bar{b}}$
19. $\nabla(\mathbf{a} \times \bar{\bar{b}}) = (\nabla \mathbf{a}) \cdot \bar{\bar{b}} - \mathbf{a} \cdot \nabla\bar{\bar{b}}$
20. $\nabla\nabla\bar{\bar{a}} = \nabla\nabla\bar{\bar{a}} - \nabla\nabla\bar{\bar{a}}$
21. $\nabla\nabla\bar{\bar{a}} = 0$
22. $\nabla\nabla\bar{\bar{a}} = 0$
23. $\mathbf{a} \cdot \bar{\bar{b}} = [\bar{\bar{b}}]^{\mathrm{T}} \cdot \mathbf{a}$
24. $\mathbf{a} \times \bar{\bar{b}} = -\left\{[\bar{\bar{b}}]^{\mathrm{T}} \times \mathbf{a}\right\}^{\mathrm{T}}$
25. $[\bar{\bar{c}}]^{\mathrm{T}} \cdot (\mathbf{a} \times \bar{\bar{b}}) = -[\mathbf{a} \times \bar{\bar{c}}]^{\mathrm{T}} \cdot \bar{\bar{b}}$