TUG

Navigation

SS-2004

Chapter 7 – Celestial navigation

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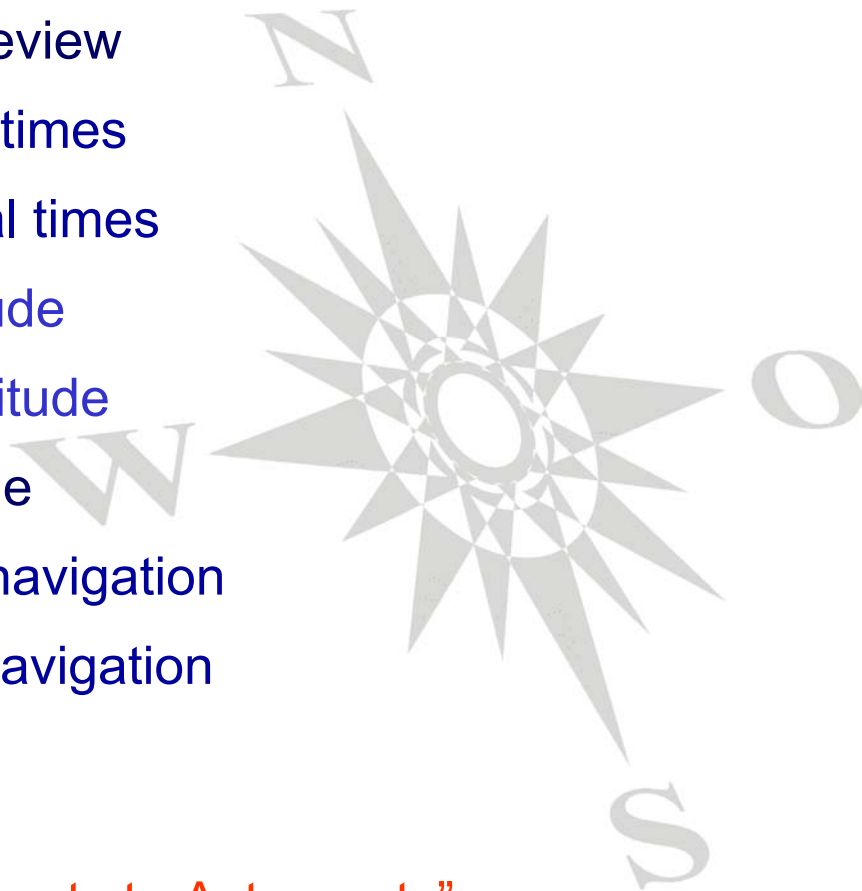
7 Celestial navigation / Initial remarks

- Definition
- Motivation
- Presentation
- Contents

7 Celestial navigation (1)

7.1 Introduction

- Historical review
  - Antique times
  - Medieval times
    - Latitude
    - Longitude
- Present time
  - Marine navigation
  - Space navigation
- Future



"From Argonauts to Astronauts"

7 Celestial navigation (2)

– Quotations

"*Marine navigators should still keep a **sextant** in good working order and know how to use it.*"

Langley R, GPS World 11(10)

"*Every textbook on navigation should include a chapter on **astronavigation**.*"

Hofmann-Wellenhof et al. (2004), Navigation – principles of positioning and guidance

7 Celestial navigation (3)

7.2 Astronomical fundamentals

- Coordinate systems (1)
  - General remarks
    - Mathematical fundamentals
      - Direction vectors (➔ unit vectors)
      - Celestial sphere
    - Mathematical calculus
  - Definitions
    - Origin
    - Orientation
      - Equatorial systems
      - Local-level system

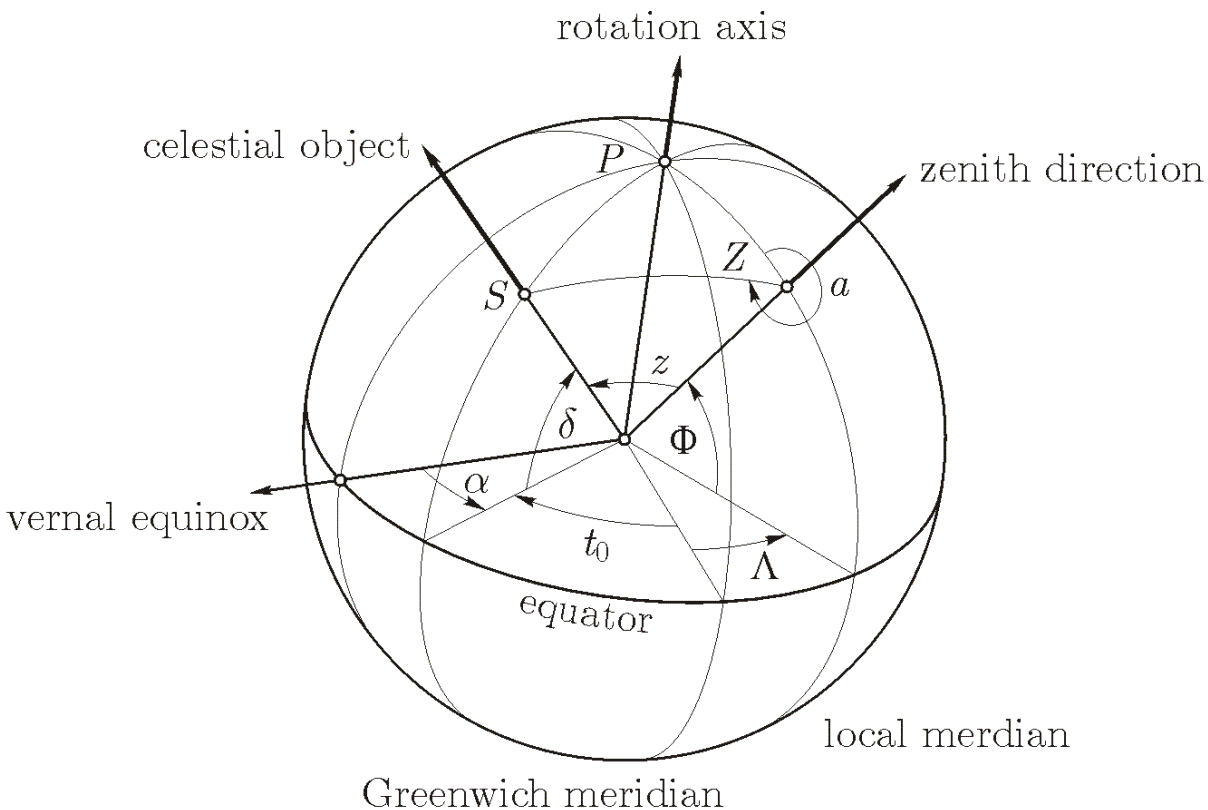
7 Celestial navigation (4)

- Coordinate systems (2)
  - Defining axes and parameters
    - Celestial (equatorial) system
      - Earth rotation axis, intersection of ecliptical and equatorial plane (direction to vernal equinox)
      - Right ascension (α), declination (δ)
    - Terrestrial (equatorial) system
      - Earth rotation axis, intersection of Greenwich meridian and equatorial plane
      - Hour angle (t_0), declination (δ)
    - Local-level system
      - Direction of the zenith, north direction
      - Azimuth (a), zenith distance (z) or elevation

      Note that latitude Φ and longitude Λ are the equatorial parameters of the zenith direction (i.e. local plumb line).

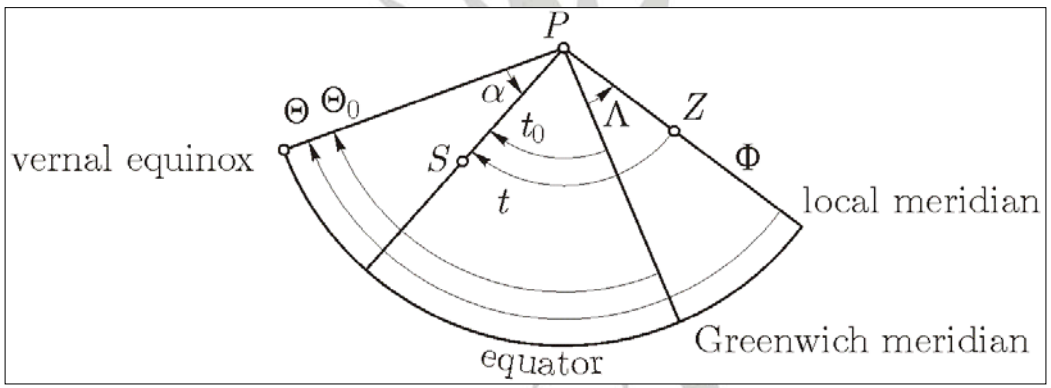
7 Celestial navigation (5)

- Coordinate systems (3)
  - Graphical representation



7 Celestial navigation (6)

- Coordinate systems (4)
  - Transformations (1)
    - Celestial and terrestrial (equatorial) system
      - Greenwich meridian



$$\mathbf{r}^{e} = \mathbf{R}_3(\Theta_0)\,\mathbf{r}^{i} \quad \text{or} \quad \Theta_0 = \alpha + t_0$$

      - Local meridian

$$\Theta = \Theta_0 + \Lambda \quad \text{and} \quad t = t_0 + \Lambda$$

- Coordinate systems (5)
  - Transformations (2)
    - Terrestrial (equatorial) and local-level system
      - Vector calculus

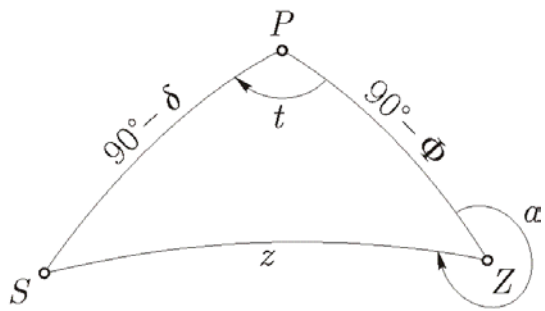
$$\mathbf{r}^l = \mathbf{R}^l_e \mathbf{r}^e \quad , \quad \mathbf{R}^l_e = \mathbf{R}^l_e(\Phi, \Lambda) = \begin{bmatrix} \mathbf{n}^T \\ \mathbf{e}^T \\ \mathbf{u}^T \end{bmatrix}$$

$$\sin z \cos a = \mathbf{n} \cdot \mathbf{r}^e = -\sin\Phi \cos\delta \cos t + \cos\Phi \sin\delta$$

$$\sin z \sin a = \mathbf{e} \cdot \mathbf{r}^e = -\cos\delta \sin t$$

$$\cos z = \mathbf{u} \cdot \mathbf{r}^e = \cos\Phi \cos\delta \cos t + \sin\Phi \sin\delta$$

      - Spherical trigonometry
        (nautical or navigational triangle PZS)



7 Celestial navigation (8)

- Coordinate systems (6)
  - Main tasks of celestial navigation

$$\mathbf{r}^l = \mathbf{R}_e^l(\Phi, \Lambda)\,\mathbf{r}^e$$

    - Rotation matrix (➔ position)
    - Local-level direction vector (➔ constellation)
    - Equatorial direction vector (➔ identification)
  - Differential relations
    - Application
      - Linearization
      - Graphical interpretation (“line of position”)
    - Examples

$$dz = -\cos a\, d\Phi - \sin a \cos\Phi\, dt$$

$$da = \sin a \cot z\, d\Phi + (\sin\Phi - \cos a \cot z \cos\Phi)\, dt$$

7 Celestial navigation (9)

- Time systems
  - General remarks
  - Definitions
    - Solar times
    - Sidereal times
  - Transformations
    - Local and Greenwich times
    - Universal Time (UT) und Greenwich Sidereal Time (GST)

$$\Theta_o = 1.002\,737\,9093\ \mathrm{UT} + \vartheta_o$$

$$\vartheta_o = 24110.54841^s + 8\,640184.812866^s\,T + 0.093104^s\,T^2 + \ldots$$

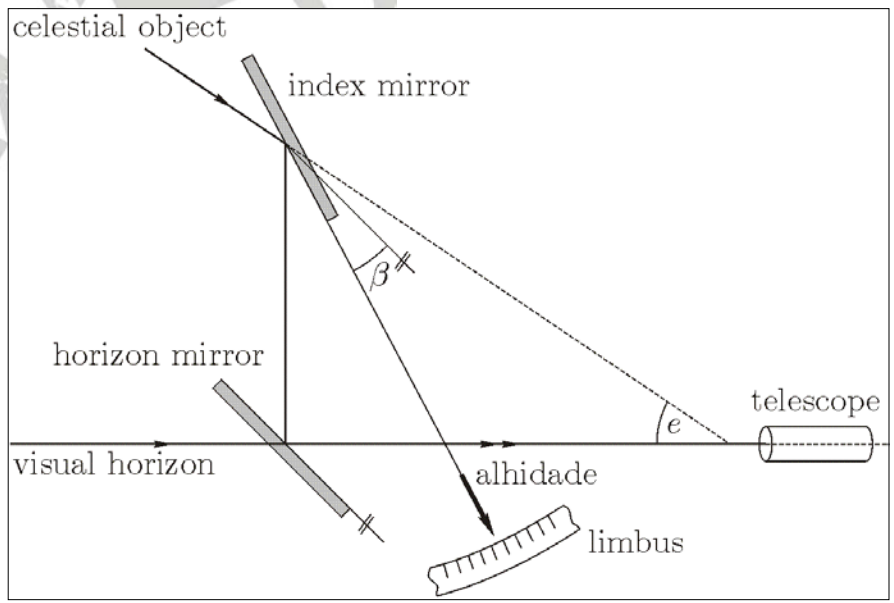
The parameter T is the time interval (expressed in Julian centuries) between the standard epoch J2000.0 and the day of observation at 0^h UT.

  - Calendar computations
    - Civil date versus Julian date
    - Auxiliary dates

7 Celestial navigation (10)

7.3 Astrofix by measured altitudes

- General remarks
- Measurement of altitudes
  - Sextant
    - Basic principle
    - http://www.mat.uc.pt/~helios/Mestre/Novemb00/H61iflan.htm
  - Systematic errors
  - Practical hints
    - Observation period
    - Optimal configuration
    - Identification
    - Accuracy
      - Observations
      - Results



7 Celestial navigation (11)

- Mathematical model
  - Definition

$$\mathbf{r}_1 \cdot \mathbf{u} = \cos z_1$$

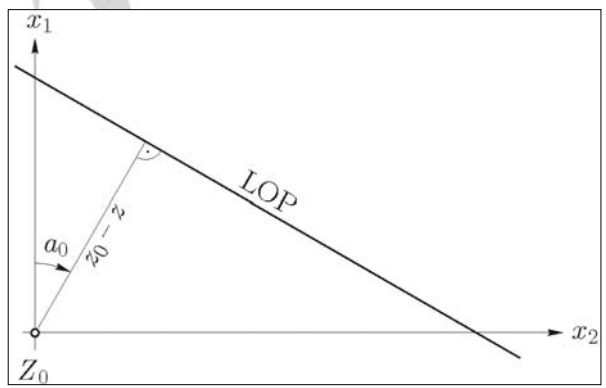
$$\mathbf{r}_2 \cdot \mathbf{u} = \cos z_2$$

$$\mathbf{u} \cdot \mathbf{u} = 1$$

  - Solution
    - Rigorous solution

$$\mathbf{u} = \frac{1}{\mathbf{g} \cdot \mathbf{g}} \left[\mathbf{g} \times \mathbf{h} \pm \sqrt{\mathbf{g} \cdot \mathbf{g} - \mathbf{h} \cdot \mathbf{h}}\, \mathbf{g} \right]; \quad \mathbf{g} = \mathbf{r}_1 \times \mathbf{r}_2 \; ; \quad \mathbf{h} = \cos z_2\, \mathbf{r}_1 - \cos z_1\, \mathbf{r}_2$$

    - Linearization by redundant observations
    - Solution using LOPs
      - Analytical solution
      - Graphical solution



  - Numerical example

7.4 Star tracker

- General remarks
- Star sensor

  www.ball.com/aerospace

  www.llnl.gov/sensor_technology
- Mathematical model
  - Position

    $$\mathbf{r}^b = \mathbf{r}^l = \mathbf{R}_e^l(\Phi,\Lambda)\,\mathbf{r}^e$$
  - Attitude

    $$\mathbf{R}_e^l(\Phi,\Lambda)\,\mathbf{r}^e = \mathbf{r}^l = \mathbf{R}_b^l(\alpha_1,\alpha_2,\alpha_3)\,\mathbf{r}^b$$
- Reduction of star coordinates
  - Apparent positions
  - Mean positions

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