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Donald G. Dudley
University of Arizona, Tucson

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Dedicated to Professor Robert S. Elliott

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Preface

This book is written for the serious student of electromagnetic theory. It is a principal product of my experience over the past 25 years interacting with graduate students in electromagnetics and applied mathematics at the University of Arizona.

A large volume of literature has appeared since the latter days of World War II, written by researchers expanding the basic principles of electromagnetic theory and applying the electromagnetic model to many important practical problems. In spite of widespread and continuing interest in electromagnetics, the underlying mathematical principles used freely throughout graduate electromagnetic texts have not been systematically presented in the texts as preambles. This is in contrast to the situation regarding undergraduate electromagnetic texts, most of which contain preliminary treatments of fundamental applied mathematical principles, such as vector analysis, complex arithmetic, and phasors. It is my belief that there should be a graduate electromagnetic theory text with linear spaces, Green's functions, and spectral expansions as mathematical cornerstones. Such a text should allow the reader access to the mathematics and the electromagnetic applications without the necessity for consulting a wide range of mathematical books written at a variety of levels. This book is an effort to bring the power of the mathematics to bear on electromagnetic problems in a single text.

Since the mastery of the foundations for electromagnetics provided in this book can involve a considerable investment of time, I should like to indicate some of the potential rewards. When the student first begins a study of electromagnetic theory at the graduate level, he/she is confronted with a large array of series expansions and transforms with which to reduce the differential equations and boundary conditions in a wide variety of canonical problems in Cartesian, cylindrical, and spherical coordinates. Often, it seems to the student that experience is the only way to determine specifically which expansions or transforms to use in a given problem. In addition, convergence properties seem quite mysterious. These issues can be approached on a firm mathematical base through the foundations provided in this book. Indeed, the reader will find that different differential operators with their associated boundary conditions lead to specific expansions and transforms that are "natural" in a concrete mathematical sense for the problem being considered. My experience with graduate students has been that mastery of the foundations allows them to appreciate why certain expansions and transforms are used in the study of canonical problems. Then, what is potentially more important, the foundations allow them to begin the more difficult task of formulating and solving problems on their own.

I first became interested in Green's functions and spectral representations during my graduate studies at UCLA in the 1960s. I was particularly influenced by the treatment of the spectral representations of the delta function by Bernard Friedman [1], whose book at that time formed the cornerstone of the Applied Mathematics Program at UCLA in the College of Engineering. Subsequently, examples of spectral representations began appearing in texts on electromagnetic theory, such as [2]–[4], and, more recently, [5]. However, no text specifically devoted to Green's functions and spectral expansions and their application to electromagnetic problems has been forthcoming.

The material in this book forms a two-semester sequence for graduate students at the University of Arizona. The first three chapters contain the mathematical foundations, and are covered in a course offered every year to electrical engineering and applied mathematics graduate students with a wide range of interests. Indeed, the first three chapters in this book could be studied by applied mathematicians, physicists, and engineers with no particular interest in the electromagnetic applications. The fourth and fifth chapters are concerned with the electromagnetics, and are covered in a course on advanced electromagnetic theory, offered biennially. In this book, I have presumed that the reader has a working knowledge of complex variables. In addition, in the last two chapters, I have assumed that the reader has studied an introductory treatment of electromagnetics at the graduate level, as can be found, for example, in the texts by Harrington

[6], Ishimaru [7], or Balanis [8]. I have therefore felt no necessity to include a chapter on Maxwell's equations or a chapter on analytic function theory, presupposing reader familiarity.

Chapter 1 is an introduction to modern linear analysis. It begins with the notion of a linear space. Structure is added by the introduction of the inner product and the norm. With the addition of suitable convergence criteria, the space becomes a Hilbert space. Included in the discussion of Hilbert space are the concepts of best approximation and projection. The chapter concludes with a discussion of operators in Hilbert space. Emphasis is placed on the matrix representation of operations, a concept that leads naturally to the Method of Moments, one of the most popular techniques for the numerical solution to integral equations occurring in electromagnetic boundary value problems.

Chapter 2 covers Green's functions for linear, ordinary, differential operators of second order. The chapter begins with a discussion of the delta function. The Sturm–Liouville operator is introduced and discussed for three cases, which we title SLP1, SLP2, and SLP3. A clear distinction is made between self-adjoint and nonself-adjoint operators. In addition, the concepts of limit point and limit circle cases are introduced and explored through examples applicable to electromagnetic problems.

Chapter 3 introduces the spectral representation of the delta function. The theory is applied by example to various operators and boundary conditions. Included are important representations associated with the limit point and limit circle cases introduced in the previous chapter. A wide variety of spectral representations are presented in a form suitable for use in solving electromagnetic boundary value problems in multiple dimensions. These representations are augmented by further examples in the Problems.

Chapter 4 contains a discussion of fundamental electromagnetic sources represented by delta functions. The sources are analyzed using spectral representations and Green's functions in Cartesian, cylindrical, and spherical conditions. A variety of useful alternative representations emerge. Included are sheet sources, line sources, ring sources, shell sources, and point sources.

In Chapter 5, the ideas developed in the previous chapters are applied to a sample of electromagnetic boundary value problems. No attempt is made to produce an exhaustive collection. Rather, the purpose of the chapter is to demonstrate the power of the structure developed in the first three chapters. Static problems included involve the rectangular box and rectangular cylinder. Dynamic problems include propagation in a parallel plate waveguide, scattering by an iris obstacle in a parallel plate waveguide, aperture diffraction, and scattering by a conducting cylinder. Emphasis has been placed on the power of alternative representations by including useful alternatives in the examples on the parallel plate waveguide and scattering from a conducting circular cylinder.

My graduate students over the past 25 years have had a major influence on this book. All have contributed through classroom and individual discussions. Many too numerous to mention have made suggestions and corrections in early drafts. Specifically, I should like to acknowledge some special help. In the early 1980s, K. A. Nabulsi and Amal Nabulsi painstakingly typed a portion of my handwritten class notes. These typed notes were produced before the advent of modern computational word processors, and formed the basis for my subsequent writing of Chapters 1–3 of this book. Dr. Nabulsi, now a Professor in Saudi Arabia, sent me a gifted student, Muntasir Sheikh, for doctoral training. Mr. Sheikh has critically read the entire book manuscript and offered suggestions and corrections. In addition, Charles Trantanella, Michael Pasik, and Jacob Adopley have carefully read portions of the manuscript.

In the mid-1970s, I had the good fortune to be a part of the creation of the now greatly successful Program in Applied Mathematics at the University of Arizona. W. A. Johnson was my first student to graduate through the program. Because of him, I became acquainted with three professors in the Department of Mathematics, C. L. DeVito, W. M. Greenlee, and W. G. Faris. These four mathematicians have had a lasting influence on the way I have come to consider many of the mathematical issues involved in electromagnetic theory.

Among my colleagues, there are several who have had a marked influence on this book. R. E. Kleinman, University of Delaware, has consistently encouraged me to pursue my mathematical interests applied to electromagnetic theory. L. B. Felsen, Polytechnic University, has influenced me in many ways, scientifically and personally. In addition, his comments concerning modern research applications led me to some important additions in Chapter 5. K. J. Langenberg, University of Kassel, has read in detail the first three chapters and offered important advice and criticism. R. W. Ziolkowski, University of Arizona, has taught a course using the material contained in Chapters 1–3 and offered many suggestions and corrections. I. Stakgold, University of Delaware, made me aware of the recent mathematical literature on limit point and limit circle problems.

Many reviewers, anonymous and known, have made comments that have led me to make changes and additions. I would particularly like to mention Ehud Heyman, Tel Aviv University, whose comments concerning

alternative representations led me to strengthen this material in Chapter 5. I would also like to thank Dudley Kay and the staff at IEEE Press whose competence and diligence have been instrumental in the production phase of this book project.

With Chalmers M. Butler, Clemson University, a distinguished educator and cherished friend, I have had the good fortune to have a 20-year running discussion concerning methods of teaching electromagnetics to graduate students. Part of the fun has been that we have not always agreed. However, one issue upon which there has been no disagreement is the importance of presenting electromagnetics to students in a structurally organized manner, stressing the common links between wide ranges of problems. I have drawn strength, satisfaction, and pleasure from our association.

My family has always seemed to understand my many interests, and this book has been a major one for more years than I should like to recall. It is with love and affection that I acknowledge my wife, Marjorie A. Dudley; my children, Donald L. Dudley and Susan D. Benson; and the memory of my former wife, Marjorie M. Dudley. Love truly does "make the world go 'round."

Finally, it is with gratitude that I dedicate this book to my teacher, mentor, and friend, Robert S. Elliott, University of California at Los Angeles, a consummate scholar without whom none of this would have occurred.

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