Navigation

SS-2004

Chapter 3 – Mathematical fundamentals

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3 Mathematical fundamentals (1)

3.1 Reference systems and frames

3.1.1 Introduction

– **General remarks**
  - Triple of numbers (coordinates) locate a 3D point.
  - Coord. or reference systems yield a consistent representation.
  - Coord. are in principle time-dependent (esp. in navigation).
  - Time is one-dimensional, space is three-dimensional; *"a mass point may be at different time epochs at the same location but it cannot be at one epoch at different places".*
  - Reference systems require a choice of **origin** (e.g., barycenter, geocenter, topocenter) and a **orientation** defined by orthogonal axes (in case of $\mathbf{x}_3$-axis, e.g., earth rotation axis, local zenith).
  - Distinguish between reference system (concept) and reference frame (realization).

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– **Hierarchy of reference systems**
  - Inertial, quasi-inertial, and noninertial systems
  - Terrestrial (earth-fixed, i.e., fixed to the rotating earth) and celestial (space-fixed) systems
  - Equatorial systems ($\mathbf{x}_3$-axis corresponds to the earth rotation axis)
  - Local-level (horizon) systems ($\mathbf{x}_3$-axis corresponds to the local zenith, i.e., is tangent to the slightly curved plumb line)
  - Body (vehicle, i.e., land vehicles, vessel, aircraft) systems
  - Sensor-related and image reference systems
  - Model-related and mapping reference systems
  - Time systems (based, e.g., on earth rotation as solar and sidereal time, or on periodic processes as atomic time)

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3.1.2 Definitions of systems and frames

– **Inertial system/frame**
  - Non-accelerated, i.e., at rest or subject to a uniform translational motion (constant velocity along a straight line)
  - Laws of Newtonian mechanics may be applied:
    (1) a body at rest or in uniform translation preserves its status if no forces are applied;
    (2) $F=ma$ is the linear relation between force F applied to a body of mass m and acceleration a experienced by that body.
  - Approximate realization by a **quasi-inertial system**, e.g., a geocentric system with celestial (space-fixed) orientation as in the case of a **celestial equatorial system** (origin: geocenter, $\mathbf{x}_1$-axis points to the vernal equinox, $\mathbf{x}_3$-axis: mean direction of the earth rotation axis, $\mathbf{x}_2$-axis completes the system to a right-handed Cartesian system); definition of the vernal equinox by very distant extragalactic radio sources (quasars) or bodies of the solar system (planets, moon, satellites).

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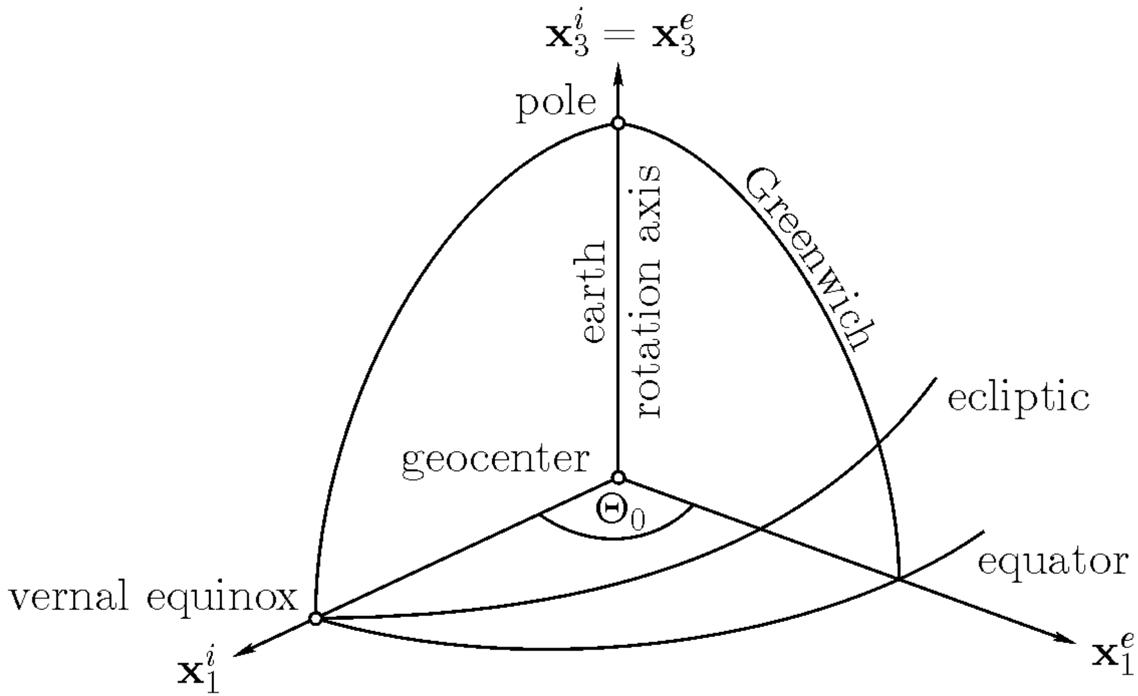

– **Terrestrial equatorial system/frame**

- Origin: geocenter**;** $\mathbf{x}_1$-axis points towards the Greenwich meridian, $\mathbf{x}_3$-axis: mean direction of the earth rotation axis, $\mathbf{x}_2$-axis completes the system to a right-handed Cartesian system.
- Often denoted as earth-centered-earth-fixed (ECEF)
- Examples: ITRF released by IERS, WGS-84

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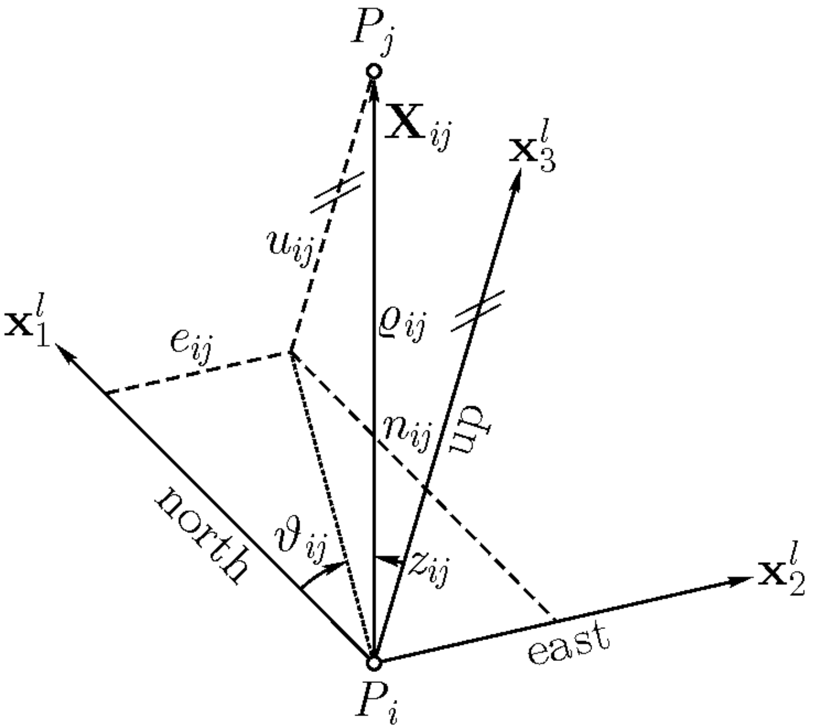
– Local-level (horizon) system/frame

- Origin: e.g., topocenter; $\mathbf{x}_1$-axis points to north, $\mathbf{x}_2$-axis points to east, $\mathbf{x}_3$-axis points “up” (left-handed) or “down” (right-handed).
- Serves as a direct reference with respect to geodetic observations:

$$\mathbf{X}_{ij} = \begin{bmatrix} n_{ij} \\ e_{ij} \\ u_{ij} \end{bmatrix}$$

$$= \varrho_{ij} \begin{bmatrix} \sin z_{ij} \, \cos \vartheta_{ij} \\ \sin z_{ij} \, \sin \vartheta_{ij} \\ \cos z_{ij} \end{bmatrix}$$



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– Body system/frame

- Origin: specific point within an object (car, ship, airplane, etc.), e.g., center of mass; axes of a right-handed Cartesian frame coincide with principal rotation axes of the object.
- Used to determine the relative orientation (attitude) of the object or a platform with respect to a local-level frame; the attitude parameters are often called roll (*r*), pitch (*p*), and yaw (*y*).

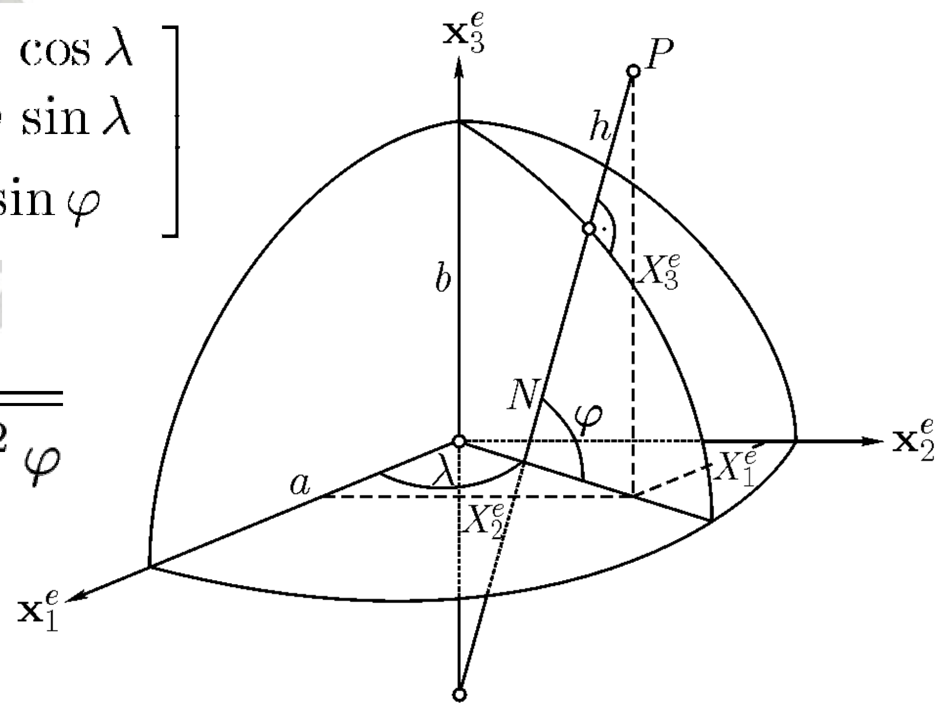
3.1.3 Transformations

– **Transformation between different types of coordinates:**

Cartesian coordinates X_1, X_2, X_3 $\Leftrightarrow$ ellipsoidal coordinates φ, λ, h

$$\begin{bmatrix} X_1^e \\ X_2^e \\ X_3^e \end{bmatrix} = \begin{bmatrix} (N+h)\cos\varphi\,\cos\lambda \\ (N+h)\cos\varphi\,\sin\lambda \\ \left(\frac{b^2}{a^2}N+h\right)\sin\varphi \end{bmatrix}$$

$$N = \frac{a^2}{\sqrt{a^2\cos^2\varphi + b^2\sin^2\varphi}}$$



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– Transformation between different types of frames

- **General remarks:** $\mathbf{X}^q = \mathbf{R}_p^q \mathbf{X}^p$

  The orthogonal matrix $\mathbf{R}_p^q$ rotates the *p*-frame into the *q*-frame.

  The clockwise rotations about the coordinate axes are given by:

$$\mathbf{R}_1(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$\mathbf{R}_2(\alpha) = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

$$\mathbf{R}_3(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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- **Celestial and terrestrial equatorial frame**

  $\mathbf{R}_e^i = \mathbf{R}_3(-\Theta_0)$ rotates the *e*-frame into the *i*-frame.

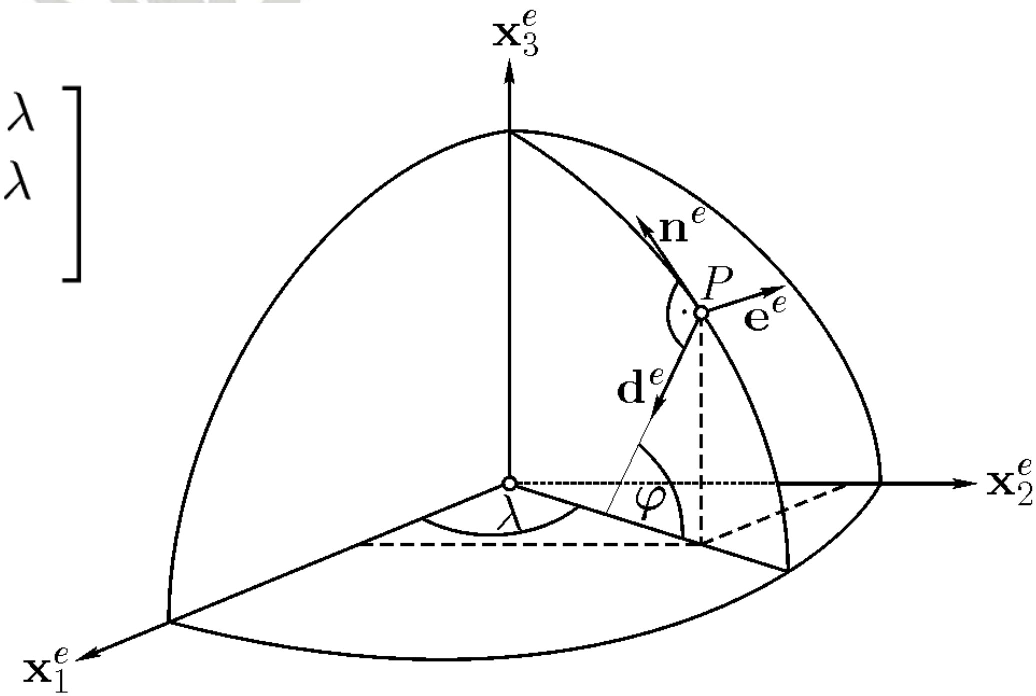
- **Terrestrial equatorial and local-level frame**

  The axes of the local-level frame in the global frame are given by:

$$\mathbf{d}^e = \begin{bmatrix} -\cos\varphi\,\cos\lambda \\ -\cos\varphi\,\sin\lambda \\ -\sin\varphi \end{bmatrix}$$

$$\mathbf{n}^e = -\frac{\partial \mathbf{d}^e}{\partial \varphi}$$

$$\mathbf{e}^e = -\frac{1}{\cos\varphi}\frac{\partial \mathbf{d}^e}{\partial \lambda}$$



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The transformation from the local-level frame to the global frame is described by the following rotation matrix:

$$\mathbf{R}_l^e = \begin{bmatrix} -\sin\varphi\cos\lambda & -\sin\lambda & -\cos\varphi\cos\lambda \\ -\sin\varphi\sin\lambda & \cos\lambda & -\cos\varphi\sin\lambda \\ \cos\varphi & 0 & -\sin\varphi \end{bmatrix}$$

Only difference vectors are transformed (origins not identical). If the local-level frame refers to the plumb line at *P* the ellipsoidal coordinates are replaced by astronomical coordinates Φ, Λ.

- **Local-level and body frame**
  Transformation from the body frame to the local-level frame is usually performed by three sequential rotations about the $\mathbf{x}^b$-axes. In principle, the composed rotation matrix may look like:

$$\mathbf{R}_b^l = \mathbf{R}_3(\alpha_3)\,\mathbf{R}_2(\alpha_2)\,\mathbf{R}_1(\alpha_1)$$

3.2 Principles of position determination

3.2.1 Introduction

– **General remarks**
  - Position (set of coordinates) vs. location (topological relation)
  - Absolute positioning (position fixing) vs. relative positioning (dead reckoning)

– **Use of principal tasks**
  - First principal task: $\mathbf{X}_2 = \mathbf{X}_1 + \mathbf{X}_{12}$
  - Second principal task: $\mathbf{X}_{12} = \mathbf{X}_2 - \mathbf{X}_1$

3.2.2 Dead Reckoning

– **Definition:**

Repeated application of the first principal task!

– **Possible realizations**

- **Rho-theta-technique:**
  baseline is calculated from terrestrial measurements (range “rho” and oriented direction “theta”, plus zenith angle in 3D)

$$\mathbf{X}_{12} = \varrho_{12} \begin{bmatrix} \cos\vartheta_{12} \\ \sin\vartheta_{12} \end{bmatrix}$$

- **Inertial navigation:**
  baseline is determined by a double integration of accelerations measured along coordinate axes (controlled by gyroscopes)

$$\dot{\mathbf{X}}(t) - \dot{\mathbf{X}}_1 = \int_{t_1}^{t} \ddot{\mathbf{X}}(\tau)\,d\tau\,, \quad \mathbf{X}_{12} = \int_{t_1}^{t_2} \dot{\mathbf{X}}(t)\,dt$$

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3.2.3 Position fixing

- **General remarks**
  - Position determination uses measurements to (or from) fixed reference points
  - Measurements used: directions (angles), ranges, pseudoranges, range rates (radial velocities), zenith (elevation) angles
  - Measurements give rise to lines of position (LOP, 2D) or surfaces of position (SOP, 3D)

- **Typical resection methods**
  - Theta-theta-fixing / multiple plotting
  - Rho-rho-fixing / multiple ranging
  - Pseudorange position fixing / hyperbolic positioning
  - Generic position fixing (e.g., rho-theta-fixing)

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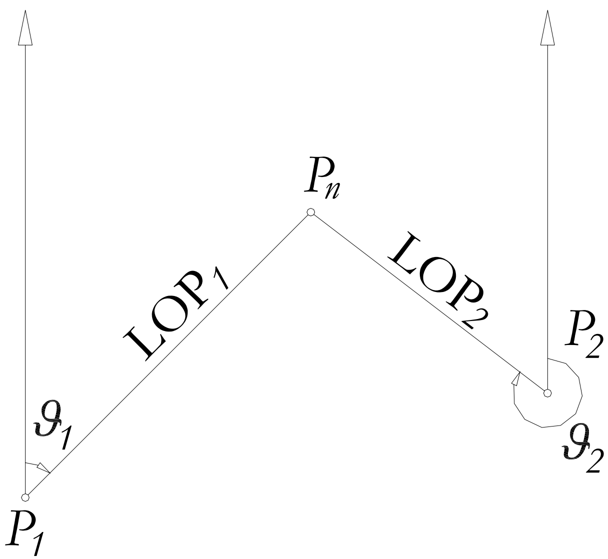
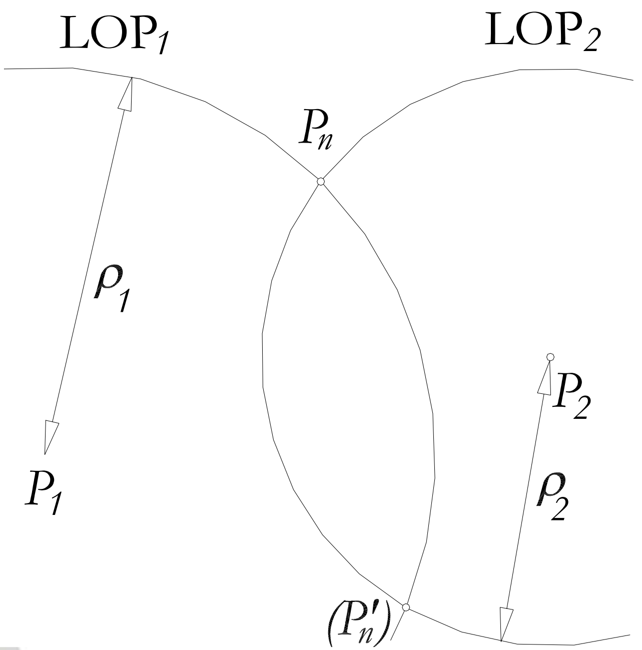
– **Theta-theta-fixing**



– **Rho-rho-fixing**



Critical configuration:

unknown point is close to or is far away from the reference baseline.

3D: third measurement is required!

– Pseudorange position fixing

- Pseudorange:

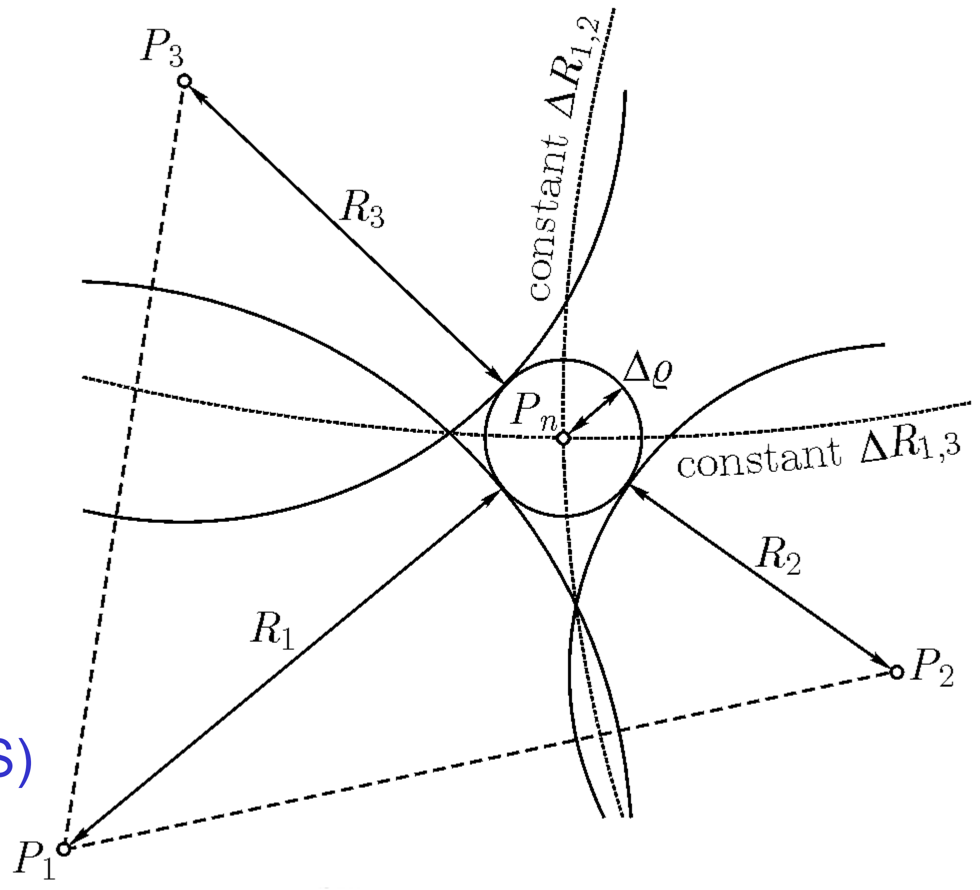
  $$R_i = \varrho_i + \Delta\varrho$$

- Range differences:

  $$R_i - R_1 = \varrho_i - \varrho_1$$

- Critical configuration: line (2D), cone (3D)

- Accuracy measure: DOP factor (e.g., GPS)



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– Summary of LOPs and SOPs

Observation	LOP	SOP
Direction	Straight line	Plane
Angle	Straight line	–
Zenith angle	–	Cone
Range	Circle	Spherical shell
Pseudorange	Circle with biased radius	Spherical shell with biased radius
Range difference	Hyperbolic line	Hyperbolic shell
Range rate	Straight line	Cone

3.3 Principles of velocity determination

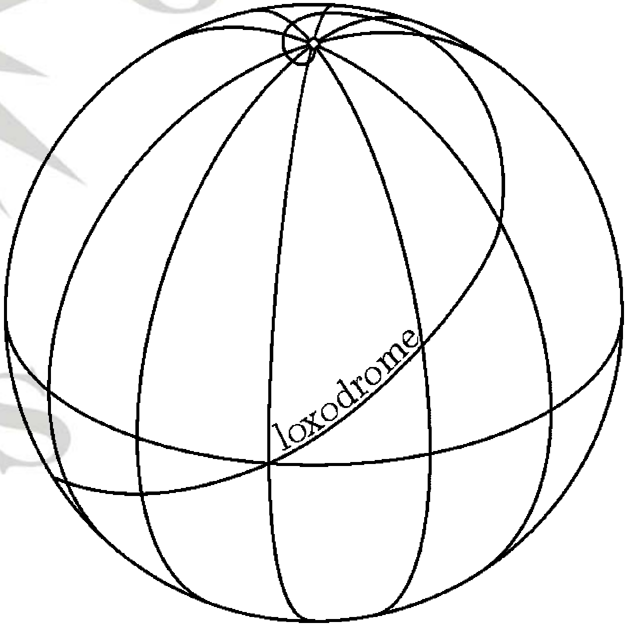
3.3.1 Velocity vector

– **Definition:**

A velocity vector is given by its norm (length), sometimes denoted as speed, and its unit direction vector defined by the course angle.

– **Analytic representation:**

$$\mathbf{v} = v\,\mathbf{v}_0 = v \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix}$$



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3.3.2 Course angle

- **General remarks:**
  - A course angle strongly depends on the kind of the course line.
  - Course lines are treated as spatial curves on curved surfaces.
  - Loxodrome (rhumb lines) vs. orthodrome (geodesic line)
  - Loxodrome … constant azimuth
  - Orthodrome … shortest distance
  - Analytic representation on sphere or ellipsoid

- **Spherical approximation of the loxodrome course:**

$$\tan\alpha = \frac{R\cos\varphi\, d\lambda}{R\, d\varphi} \quad \rightarrow \quad d\lambda = \tan\alpha\,\frac{d\varphi}{\cos\varphi}$$

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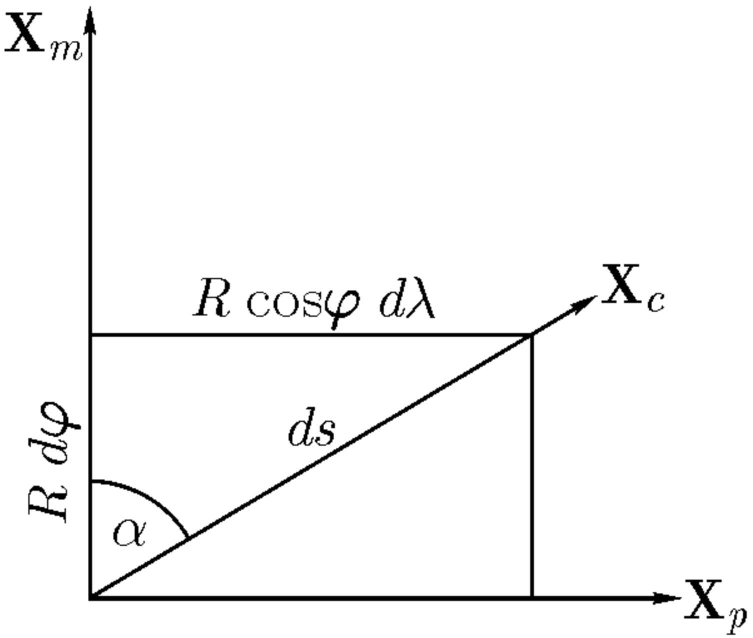
Integration between *A* and *B*:

$$\lambda_B - \lambda_A = \tan\alpha \int_A^B \frac{d\varphi}{\cos\varphi}$$



Mercator integral:

$$\int \frac{d\varphi}{\cos\varphi} = \ln\tan(\varphi/2 + \pi/4)$$

Final result:

$$\alpha = \arctan\left[\frac{\lambda_B - \lambda_A}{\ln\frac{\tan(\varphi_B/2+\pi/4)}{\tan(\varphi_A/2+\pi/4)}}\right]$$

3.4 Principles of attitude determination

3.4.1 Definitions

- Attitude parameters appear as rotation angles about, e.g., the local level frame, i.e., roll, pitch and yaw rotate the local-level frame to the body frame.
- Exemplary order of rotations: rotation by yaw about $\mathbf{x}_3$, by pitch about $\mathbf{x}_2$, and by roll about $\mathbf{x}_1$.
- The rotation matrix is given by: $\mathbf{R}_l^b = \mathbf{R}_1(r)\,\mathbf{R}_2(p)\,\mathbf{R}_3(y)$

$$\mathbf{R}_l^b = \begin{bmatrix} \cos p \cos y & \cos p \sin y & -\sin p \\ \begin{matrix}\sin r \sin p \cos y \\ -\cos r \sin y\end{matrix} & \begin{matrix}\sin r \sin p \sin y \\ +\cos r \cos y\end{matrix} & \sin r \cos p \\ \begin{matrix}\cos r \sin p \cos y \\ +\sin r \sin y\end{matrix} & \begin{matrix}\cos r \sin p \sin y \\ -\sin r \cos y\end{matrix} & \cos r \cos p \end{bmatrix}$$

3.4.2 Rigorous solution strategy

- Attitude determination may start with two (difference) vectors in both frames via $\mathbf{x}^b = \mathbf{R}_l^b\,\mathbf{x}^l$

- Three conditions must be fulfilled:

$$\|\mathbf{u}^l\| = \|\mathbf{u}^b\| \quad \text{and} \quad \|\mathbf{v}^l\| = \|\mathbf{v}^b\|$$

$$\mathbf{u}^l \cdot \mathbf{v}^l = \mathbf{u}^b \cdot \mathbf{v}^b$$

- Two more corresponding vectors are computed:

$$\mathbf{X}_3^l = \mathbf{X}_1^l \times \mathbf{X}_2^l \text{ and } \mathbf{X}_3^b = \mathbf{X}_1^b \times \mathbf{X}_2^b$$

- Rotation matrix and, consequently, attitude parameters are gained from the following matrix equation:

$$\mathbf{B} = \mathbf{R}_l^b\,\mathbf{L} \quad \rightarrow \quad \mathbf{R}_l^b = \mathbf{B}\,\mathbf{L}^{-1}$$

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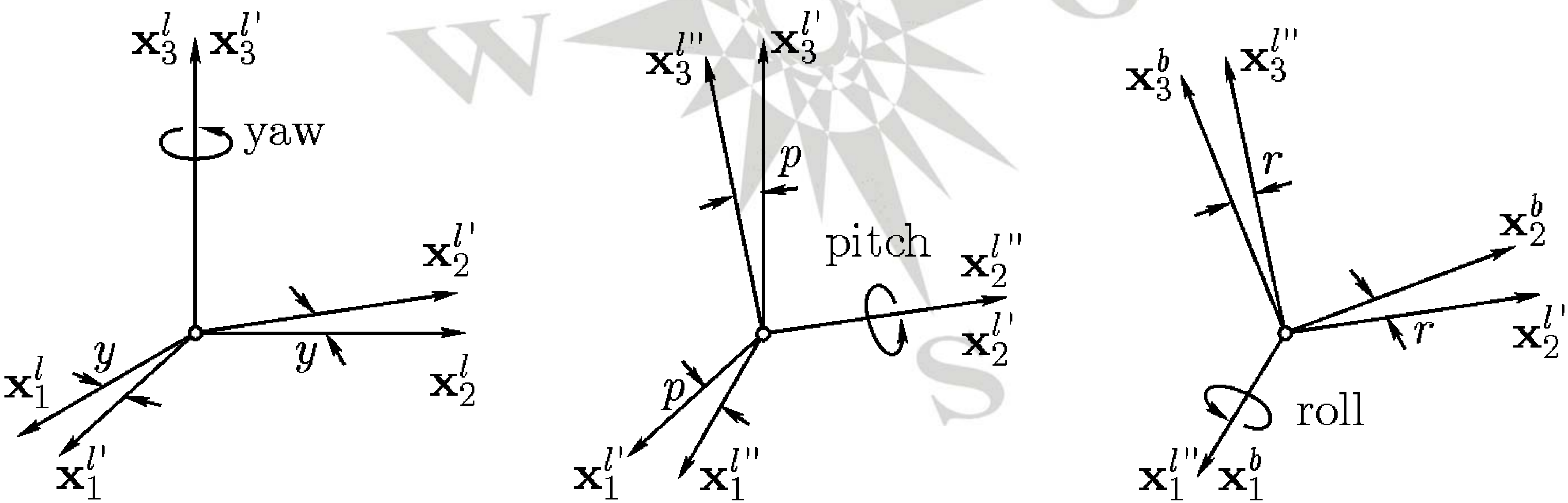
- Results:

$$\tan r = \frac{R_{23}}{R_{33}}$$

$$\tan p = -\frac{R_{13}}{\sqrt{R_{11}^2 + R_{12}^2}} \qquad \tan y = \frac{R_{12}}{R_{11}}$$



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3.5 Accuracy measures

3.5.1 Specifications

– **One-dimensional case** $$P_1(-\alpha < z < \alpha) = \frac{1}{\sqrt{2\pi}} \int_{-\alpha}^{+\alpha} e^{-z^2/2} dz$$

α	Probability	Notation
0.67	50.0 %	Linear Error Probable (LEP)
1.00	68.3 %	1σ level, rms (1D)
1.96	95.0 %	95% confidence level
2.00	95.4 %	2σ level
3.00	99.7 %	3σ level

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– **Two-dimensional case**

$x = z_1^2 + z_2^2$ follows χ^2-distribution: $P_2(0 < x < \alpha) = 1 - e^{-\alpha/2}$

$\sqrt{\alpha}$	Probability	Notation
1.00	39.4 %	1σ or standard ellipse, rms (2D)
1.18	50.0 %	Circular Error Probable (CEP)
$\sqrt{2}$	63.2 %	distance rms (drms)
2.00	86.5 %	2σ ellipse
2.45	95.0 %	95% confidence level
$2\sqrt{2}$	98.2 %	2drms
3.00	98.9 %	3σ ellipse

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– **Three-dimensional case**

$x = z_1^2 + z_2^2 + z_3^2$: $$P_3(0 < x < \alpha) = \frac{1}{\sqrt{2\pi}} \int_0^\alpha \sqrt{x}\, e^{-x^2/2} dx$$

$\sqrt{\alpha}$	Probability	Notation
1.00	19.9 %	1σ or standard ellipsoid, rms (3D)
1.53	50.0 %	Spherical Error Probable (SEP)
$\sqrt{3}$	61.0 %	Mean Radial Sph. Error (MRSE)
2.00	73.8 %	2σ ellipsoid
2.80	95.0 %	95% confidence level
3.00	97.1 %	3σ ellipsoid

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3.5.2 Accuracy equivalences

Based on assumptions on error distribution, different accuracy measures may be compared:
e.g., CEP=2m less accurate than rms(3D)=3m!

rms (1D)	CEP	rms (2D)	R95 (2D)	2drms	rms (3D)	SEP	$\Downarrow$ * $\Rightarrow$
1	0.44	0.53	0.91	1.1	1.1	0.88	rms (1D)
	1	1.2	2.1	2.4	2.5	2.0	CEP
		1	1.7	2	2.1	1.7	rms (2D)
			1	1.2	1.2	0.96	R95 (2D)
				1	1.1	0.85	2drms
					1	0.79	rms (3D)
						1	SEP

3.6 Least squares estimation

3.6.1 Least squares adjustment by parameters

- Basic observation equation (*n* unknowns, *l* observations):

$$\mathbf{z} = \mathbf{H}\,\mathbf{x} + \mathbf{v} \qquad \mathbf{v} \sim N(\mathbf{0}, \mathbf{R})$$

- Minimization problem:

$$\mathbf{v}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{v} = (\mathbf{z} - \mathbf{H}\,\hat{\mathbf{x}})^{\mathrm{T}}\mathbf{R}^{-1}(\mathbf{z} - \mathbf{H}\,\hat{\mathbf{x}}) \overset{!}{=} \min$$

- BLUE for the state vector and corresponding covariance matrix:

$$\hat{\mathbf{x}} = \left(\mathbf{H}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{H}\right)^{-1}\mathbf{H}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{z}$$

$$\mathbf{P} = \left(\mathbf{H}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{H}\right)^{-1}$$

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3.6.2 Recursive least squares adjustment

- Partitioning of the observation model ($l = l_0 + l_1$):

$$\begin{bmatrix} \mathbf{z}_0 \\ \mathbf{z}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{H}_0 \\ \mathbf{H}_1 \end{bmatrix} \mathbf{x} + \begin{bmatrix} \mathbf{v}_0 \\ \mathbf{v}_1 \end{bmatrix} \qquad \mathbf{R} = \begin{bmatrix} \mathbf{R}_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_1 \end{bmatrix}$$

- Solution regarding l_0 only:

$$\mathbf{P}_0 = \left(\mathbf{H}_0^{\mathrm{T}} \mathbf{R}_0^{-1} \mathbf{H}_0\right)^{-1}$$

$$\hat{\mathbf{x}}_0 = \mathbf{P}_0 \mathbf{H}_0^{\mathrm{T}} \mathbf{R}_0^{-1} \mathbf{z}_0$$

- Solution regarding l_1 in addition:

$$\mathbf{K}_1 = \mathbf{P}_0 \mathbf{H}_1^{\mathrm{T}} \left(\mathbf{H}_1 \mathbf{P}_0 \mathbf{H}_1^{\mathrm{T}} + \mathbf{R}_1\right)^{-1}$$

$$\hat{\mathbf{x}}_1 = \hat{\mathbf{x}}_0 + \mathbf{K}_1 \left(\mathbf{z}_1 - \mathbf{H}_1 \hat{\mathbf{x}}_0\right)$$

$$\mathbf{P}_1 = \mathbf{P}_0 - \mathbf{K}_1 \mathbf{H}_1 \mathbf{P}_0$$

3.6.3 Discrete Kalman filtering

– **General remarks**

- A nonstationary random process is considered, i.e., the state vector and its stochastic behavior become a function of time.
- Filtering in the sense of updating involves a dynamic model of the motion to achieve a more realistic processing of the signals.
- Estimation is possible during data gaps and resistance against sensor errors is increased.
- Kalman filtering yields a state vector of minimal variance in the sense of optimal filtering.
- The Kalman filter method is a favorite for integrating navigation sensors (sensor fusion) as in multi-sensor systems.

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– **Linear observation and dynamic model**
  (nonlinear models are treated by extended Kalman filtering)

- Observation equation $\mathbf{z}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{v}_k$
- Measurement noise $\mathbf{v}_k \sim N(\mathbf{0}, \mathbf{R}_k)$
- Dynamic model $\mathbf{x}_{k+1} = \mathbf{\Phi}_k \mathbf{x}_k + \mathbf{w}_k$
- System noise $\mathbf{w}_k \sim N(\mathbf{0}, \mathbf{Q}_k)$

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– Recursive Kalman filter algorithm

- Gain computation *(Kalman weight)*

$$\mathbf{K}_k = \tilde{\mathbf{P}}_k \mathbf{H}_k^{\mathrm{T}} \left(\mathbf{H}_k \tilde{\mathbf{P}}_k \mathbf{H}_k^{\mathrm{T}} + \mathbf{R}_k \right)^{-1}$$

- Measurement update (*correction step*)

$$\hat{\mathbf{x}}_k = \tilde{\mathbf{x}}_k + \mathbf{K}_k (\mathbf{z}_k - \mathbf{H}_k \tilde{\mathbf{x}}_k)$$

$$\mathbf{P}_k = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \tilde{\mathbf{P}}_k$$

- Time update (*prediction step*)

$$\tilde{\mathbf{x}}_{k+1} = \mathbf{\Phi}_k \hat{\mathbf{x}}_k$$

$$\tilde{\mathbf{P}}_{k+1} = \mathbf{\Phi}_k \mathbf{P}_k \mathbf{\Phi}_k^{\mathrm{T}} + \mathbf{Q}_k$$

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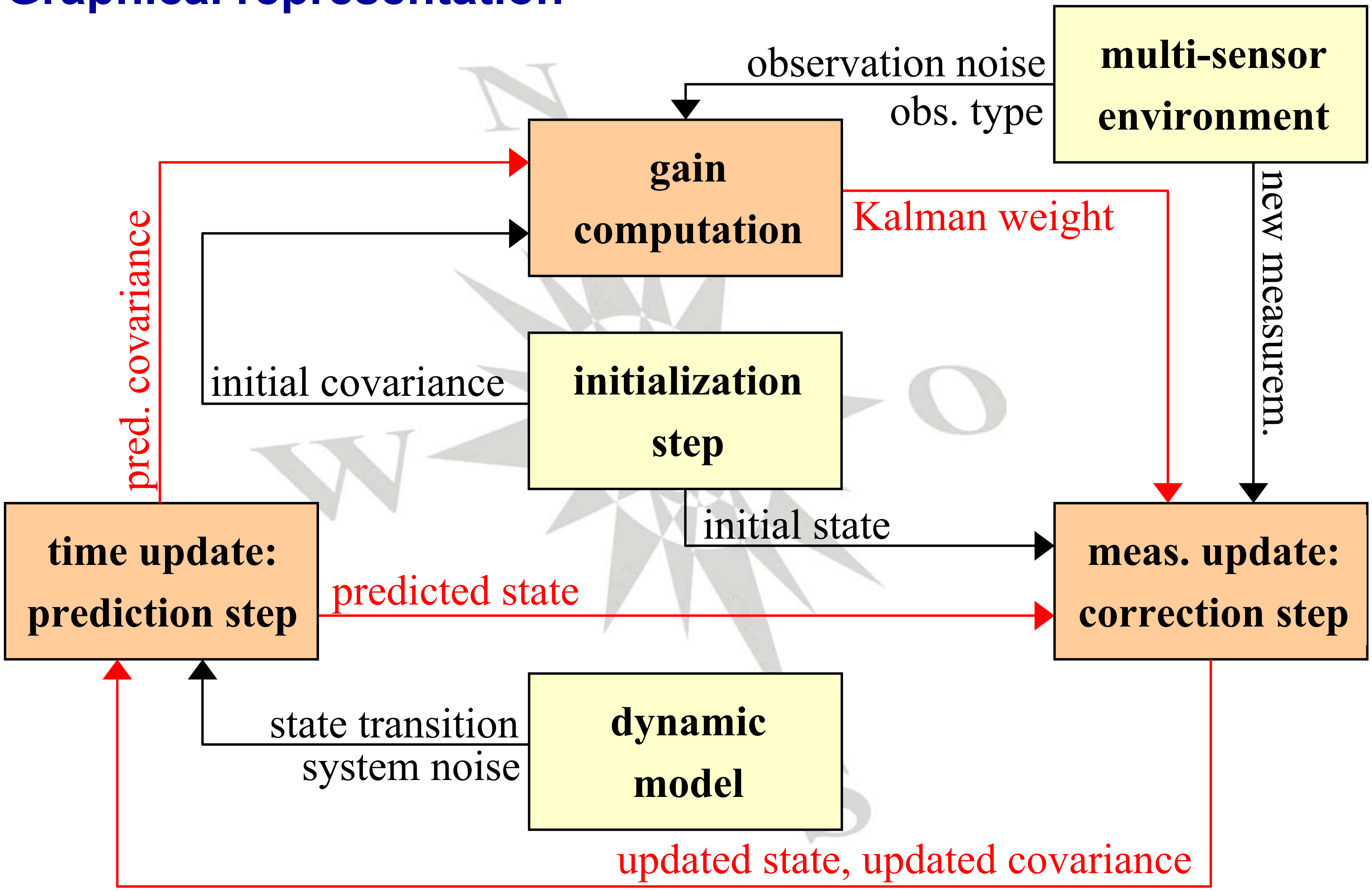
– Graphical representation



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– Example: direct coordinate observations via GPS

- Observation equation

$$\begin{bmatrix} x_{GPS} \\ y_{GPS} \end{bmatrix}_k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}_k$$

- Gain computation

$$\mathbf{H}_k = \mathbf{I} \rightarrow \mathbf{K}_k = \tilde{\mathbf{P}}_k \left(\tilde{\mathbf{P}}_k + \mathbf{R}_k \right)^{-1}$$

- Case 1

$$\mathbf{R}_k \sim \mathbf{0} \Rightarrow \mathbf{K}_k \sim \mathbf{I}$$

$$\hat{\mathbf{x}}_k \sim \tilde{\mathbf{x}}_k + \mathbf{I}(\mathbf{z}_k - \tilde{\mathbf{x}}_k)$$

- Case 2

$$\mathbf{R}_k \sim \infty \Rightarrow \mathbf{K}_k \sim \mathbf{0}$$

$$\hat{\mathbf{x}}_k \sim \tilde{\mathbf{x}}_k + \mathbf{0}(\mathbf{z}_k - \tilde{\mathbf{x}}_k)$$

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3.7 Principles of routing and guidance

3.7.1 Graph theory

- A graph is defined as a set of nodes and a set of edges but also contains complex structures like paths, cycles, etc.
- Graph theory serves as the mathematical basis for modeling the vector-type structure of navigational networks (→ digital maps).
- Graph theory describes graph properties and algorithms which are useful for routing and guidance within navigation systems.

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3.7.2 Combinatorial optimization

- The task is the linear combination of elements to find an optimum with respect to an objective function.
- Within navigation combinatorial optimization arises together with routing (route planning) problems.
- Combinatorial optimization contains graph-theoretical optimization and linear programming.
- Graph-theoretical optimization directly uses the topological behavior of a graph for the purpose of path finding.
- Linear programming appears with respect to complex routing (tour planning).
  In a linear program, the unknown parameters have to optimize the objective function and, in addition, have to fulfill a set of inequalities and/or equalities.

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