

Electromagnetic waves in space and the STEREO/WAVES experi- ment

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Overview

- ⇒ Propagation of EM waves in vacuum and plasma
- ⇒ Antennas
- ⇒ Direction Finding
- ⇒ Antenna calibration of the STEREO/WAVES antennas

EM waves in space science

- ⇒ Transmit energy
- ⇒ Transmit information
- ⇒ Remote sensing

EM waves in space

- ➔ The basic equations are the Maxwell equations in combination with the constitutive equations

$$\begin{aligned}\nabla \cdot \mathbf{D} &= \rho \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{H} &= \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t}\end{aligned}$$

$$\begin{aligned}\mathbf{D} &= \epsilon_0 \mathbf{E} \\ \mathbf{B} &= \mu_0 \mathbf{H}\end{aligned}$$

EM waves in vacuum 1

- ⇒ In vacuum there are no source terms
- ⇒ Maxwell equations can be simplified

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \epsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t}$$

EM waves in vacuum 2

- ➡ Equations can be combined to give a wave equation

Diagram illustrating the derivation of wave equations from Maxwell's equations in vacuum:

Left side (Maxwell's equations):

$$\begin{aligned}\nabla \cdot E &= 0 \\ \nabla \cdot B &= 0 \\ \nabla \times E &= -\frac{\partial B}{\partial t} \\ \nabla \times B &= \mu_0 \epsilon_0 \frac{\partial E}{\partial t}\end{aligned}$$

Right side (Wave equations):

$$\begin{aligned}\nabla^2 E - \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} &= 0 \\ \nabla^2 B - \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2} &= 0\end{aligned}$$

Arrows indicate the following combinations:

- $\nabla \cdot E = 0$ and $\nabla \times B = \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$ combine to form the wave equation for E .
- $\nabla \cdot B = 0$ and $\nabla \times E = -\frac{\partial B}{\partial t}$ combine to form the wave equation for B .

EM waves in vacuum 3

- ⇒ And be solved by postulating a time harmonic wave $\mathbf{E} = \hat{x}E_0 e^{i(kz - \omega t)}$
- ⇒ And using a Fourier transform
- ⇒ The result is the dispersion relation

$$k^2 = \epsilon_0 \mu_0 \omega^2$$

EM waves in vacuum 4

- By substituting in the Maxwell equation the B field can be found to be

$$\mathbf{E} = \hat{\mathbf{x}} E_0 e^{i(kz - \omega t)}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\mathbf{H} = \frac{\hat{\mathbf{z}}}{\eta_0} \times \mathbf{E}$$

$$\mathbf{B} = \frac{\hat{\mathbf{z}}}{c} \times \mathbf{E}$$

$$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\Omega = 120\pi\Omega$$

The phase velocity

- ➞ When riding on a wave, the phase must be constant

$$kz - \omega t = \text{const} \rightarrow z = \frac{\omega t}{k} + \text{const}$$



$$v_{ph} = \frac{dz}{dt} = \frac{\omega}{k} = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

The principle of superposition

- ⇒ Maxwell equations are linear, so the principle of superposition applies
- ⇒ Any waveform can be synthesized by superposition of monochromatic waves

$$\mathbf{E} = \hat{\mathbf{x}} \int_0^{\infty} E_0(\omega) e^{i(kz - \omega t + \phi(\omega))} d\omega$$

Wave propagation in an isotrope plasma 1

$$P = Nqr$$

Polarization

$$\begin{aligned}\vec{F}_e &= m_e \frac{d^2 \vec{r}}{dt^2} = q\vec{E} \\ \rightarrow \frac{d^2 \vec{r}}{dt^2} &= \frac{qE_0}{m_e} e^{i(kz - \omega t)}\end{aligned}$$

Force on an
electron

$$\vec{r} = -\frac{q}{\omega^2 m_e} \vec{E}$$

Wave propagation in an isotrope plasma 2

$$\vec{r} = -\frac{q}{\omega^2 m_e} \vec{E}$$

$$P = Nqr$$

$$\vec{P} = -\frac{q^2 N}{\omega^2 m_e} \vec{E}$$

$$\vec{P} = -\epsilon_0 \frac{\omega_p^2}{\omega^2} \vec{E}$$

$$\omega_p = \sqrt{\frac{Nq^2}{m_e \epsilon_0}}$$

Wave propagation in an isotrope plasma 3

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} = \epsilon_0 \left(1 - \frac{\omega_p^2}{\omega^2}\right) \vec{E}$$

$$\vec{D} = \epsilon \vec{E}$$

$$\epsilon = \epsilon_0 \left(1 - \frac{\omega_p^2}{\omega^2}\right) \longrightarrow k^2 = \omega^2 \mu_0 \epsilon_0 \left(1 - \frac{\omega_p^2}{\omega^2}\right)$$

Dispersion relation

Wave propagation in an isotropic plasma 4

$$v_p = \frac{c}{\sqrt{1 - \frac{\omega_p^2}{\omega^2}}}$$

$$v_g = c \sqrt{1 - \frac{\omega_p^2}{\omega^2}}$$

$$k = \frac{1}{c} \sqrt{\omega^2 - \omega_p^2}$$

$$\omega < \omega_p$$

$$k = \frac{i}{c} \sqrt{\omega^2 - \omega_p^2}$$

Wave propagation in an isotropic plasma 5

$$\vec{\epsilon} = \epsilon_0 \begin{pmatrix} 1 - \frac{\omega_p^2}{\omega^2} & 0 & 0 \\ 0 & 1 - \frac{\omega_p^2}{\omega^2} & 0 \\ 0 & 0 & 1 - \frac{\omega_p^2}{\omega^2} \end{pmatrix}$$

Wave propagation in magnetized, collisionless plasma

$$k^2 = \omega^2 \mu_0 \epsilon \cdot \hat{k} = \omega^2 \mu_0 \epsilon_0 \begin{pmatrix} K' & iK'' & 0 \\ -iK'' & K' & 0 \\ 0 & 0 & K_0 \end{pmatrix} \cdot \hat{k}$$

Dispersion
relation

$$\begin{aligned} K' &= 1 - \frac{X}{1 - Y^2} = 1 - \frac{\omega_p^2}{\omega^2 - \omega_c^2} \\ K'' &= -\frac{XY}{1 - Y^2} = -\frac{\omega_p^2 \omega_c}{\omega(\omega^2 - \omega_c^2)} \\ K_0 &= 1 - \frac{\omega_p^2}{\omega^2} \end{aligned}$$

$$\begin{aligned} X &= \frac{\omega_p^2}{\omega^2} \\ Y &= \frac{\omega_c}{\omega} \end{aligned}$$

Antennas in space

- ⇒ An antenna can be used to transmit and receive EM and other plasma waves
- ⇒ Free wave propagation <--> Guided wave propagation

Some basics

- ⇒ EM fields are produced by accelerating charges
- ⇒ I case of antennas oscillating electrons
- ⇒ The radiated field can be divided into a near field and a far field
- ⇒ The nearfield part is only storing energy. No transmission

Retarding potentials 1

- ➡ It is useful to introduce the potential fields

$$\begin{aligned}\nabla \cdot \mathbf{E} &= \frac{\rho}{\epsilon_0} \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \frac{1}{\mu_0} \nabla \times \mathbf{B} &= \mathbf{j} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}\end{aligned}$$

$$\begin{aligned}\mathbf{B} &= \nabla \times \mathbf{A} \\ \mathbf{E} &= -\frac{\partial \mathbf{A}}{\partial t} - \nabla \phi\end{aligned}$$

Retarding potentials 2

⇒ The potentials are not unique

$$A + F$$

$$\phi + G$$

Does not change the physical
fields if

$$\nabla \times F = 0$$

$$\nabla G = 0$$

Lorenz gauge

- ⇒ ...so we can define the Lorenz gauge
- ⇒ Makes the system symmetrical and compatible with special relativity

$$\nabla \cdot \mathbf{A} = -\epsilon_0 \mu_0 \frac{\partial \phi}{\partial t}$$

The Helmholtz equations

- ⇒ The potential field equation can be manipulated to result in wave equations
- ⇒ These are called Helmholtz equations

$$\nabla^2 \mathbf{A} - \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu_0 \mathbf{j}$$

$$\nabla^2 \phi - \mu_0 \epsilon_0 \frac{\partial^2 \phi}{\partial t^2} = -\frac{\rho}{\epsilon_0}$$

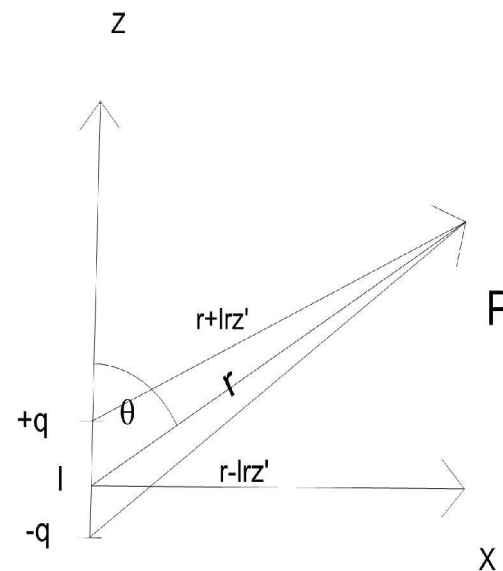
The solutions

- ➡ By postulating time harmonic behavior and Fourier transform they can be solved

$$\begin{aligned}\phi(\mathbf{r}, t) &= \int_{V'} \frac{\rho(\mathbf{r}')}{4\pi\epsilon_0|\mathbf{r} - \mathbf{r}'|} e^{-ik|\mathbf{r} - \mathbf{r}'|} dV' \\ \mathbf{A}(\mathbf{r}, t) &= \int_{V'} \frac{\mu_0 \mathbf{j}(\mathbf{r}')}{4\pi|\mathbf{r} - \mathbf{r}'|} e^{-ik|\mathbf{r} - \mathbf{r}'|} dV'\end{aligned}$$

Hertzian dipole 1

$$\begin{aligned} \frac{l}{z} &\geq z' \\ l &\ll \lambda \\ l &\rightarrow 0 \\ I = \frac{dq}{dt} &= \omega q_0 \cos \omega t = I_0 \cos \omega t \\ P = ql &= P_0 \sin \omega t \\ P_0 = ql_0 &= q_0 l \end{aligned}$$



Hertzian dipole 2

$$A_z(\mathbf{r}, t) = \frac{\mu_0 I l}{4\pi r} e^{-ikr}$$

$$\phi = \frac{I l z}{2\pi\epsilon_0 i\omega} e^{-ikr} \left(\frac{1}{r^2} + \frac{1}{r^3} \right)$$

Farfield:

$$\phi = \frac{I l z}{2\pi\epsilon_0 i\omega r^2} e^{-ikr}$$

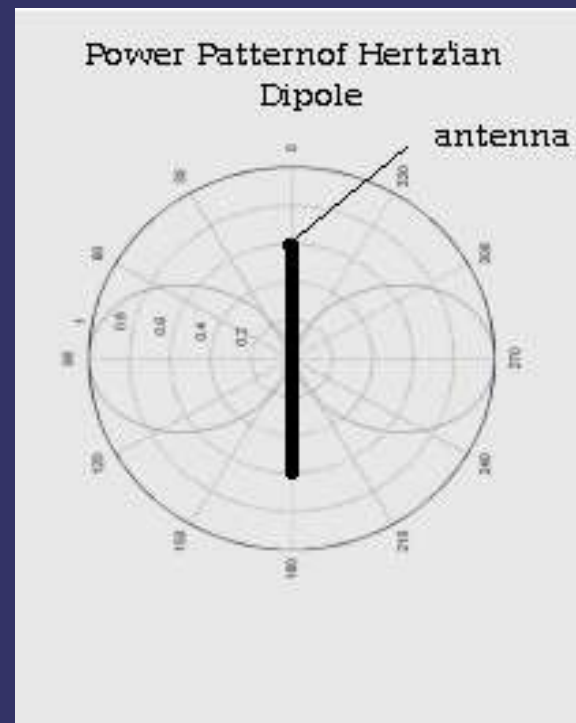
Hertzian dipole 3

The physical (far) fields and the Poynting vector

$$\mathbf{B} = \hat{\phi} \frac{i\mu_0 k I l}{4\pi r} e^{-ikr} \sin \theta$$

$$\mathbf{E} = \hat{\theta} \eta_0 \frac{ik I l}{4\pi r} e^{-ikr} \sin \theta$$

$$\mathbf{S} = \hat{\mathbf{r}} \eta_0 \left(\frac{kl I l}{4\pi r} \right)^2 \sin^2 \theta$$



Hertzian dipole 4

Other properties

$$\begin{aligned}P &= \oint_{\Sigma} \langle \mathbf{S} \rangle \cdot d\mathbf{\Sigma} \\&= \int_0^\pi r \partial\theta \int_0^{2\pi} r \cdot \langle \mathbf{S} \rangle \sin\theta \partial\phi \\&= \pi\eta_0 \left(\frac{kIl}{4\pi r} \right)^2 \int_0^\pi \sin^3\theta \partial\theta \\&= \frac{\eta_0}{12\pi} (kIl)^2\end{aligned}$$

$$R_{rad} = \frac{P}{\frac{1}{2}I^2} = \frac{\eta_0}{6\pi} (kl)^2 \approx 20(kl)^2$$

$$G(\theta, \phi) = \frac{|\langle \mathbf{S} \rangle(\theta, \phi, r \gg l)|}{\frac{P}{4\pi r^2}} = \frac{3}{2} \sin^2\theta$$

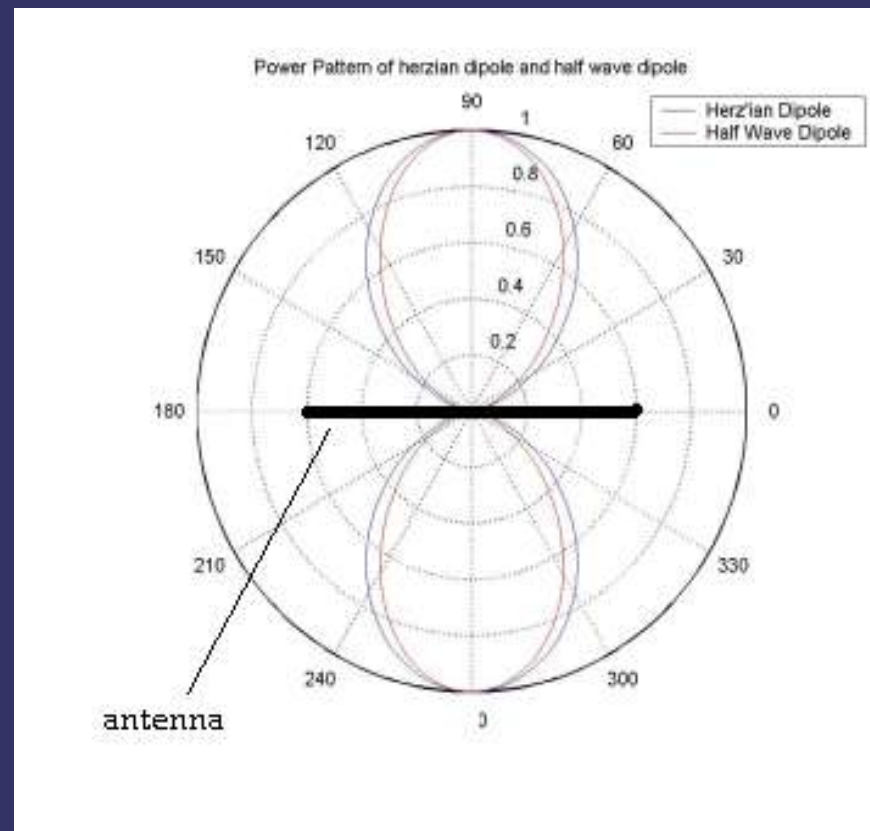
$$\eta_{rad} = \frac{P_{rad}}{P}$$

$$\Sigma_{eff}(\theta, \phi) = \frac{\lambda^2}{4\pi} G(\theta, \phi)$$

$$D(\theta, \phi) = \frac{|\langle \mathbf{S} \rangle(\theta, \phi, r \gg l)|}{\frac{P_{rad}}{4\pi r^2}} = \frac{G(\theta, \phi)}{\eta_{rad}}$$

Real antennas

- ➔ Real antennas show a slightly different behavior



Direction Finding 1

- ➔ Direction finding (DF) is the procedure to find the direction of incidence and the polarization of the incident wave
- ➔ The polarization can be represented by the normalized Stokes parameters

$$\begin{aligned}\frac{S_0}{2\eta_0} = \hat{I} &= \frac{\langle E_x^2 \rangle + \langle E_y^2 \rangle}{2\eta_0} \\ \frac{S_1}{S_0} = \hat{Q} &= \frac{\langle E_x^2 \rangle - \langle E_y^2 \rangle}{\langle E_x^2 \rangle + \langle E_y^2 \rangle} \\ \frac{S_2}{S_0} = \hat{U} &= \frac{\langle 2E_x E_y \cos \delta \rangle}{\langle E_x^2 \rangle + \langle E_y^2 \rangle} \\ \frac{S_3}{S_0} = \hat{V} &= \frac{\langle 2E_x E_y \sin \delta \rangle}{\langle E_x^2 \rangle + \langle E_y^2 \rangle}\end{aligned}$$

Direction Finding 2

➡ The basic equation:

$$V = \mathbf{h}_{eff} \cdot \mathbf{E}$$

Where \mathbf{h}_{eff} is the effective length vector

$$\mathbf{h}_{eff} = \frac{1}{I_0} \int \mathbf{J}(\mathbf{r}) d^3r$$

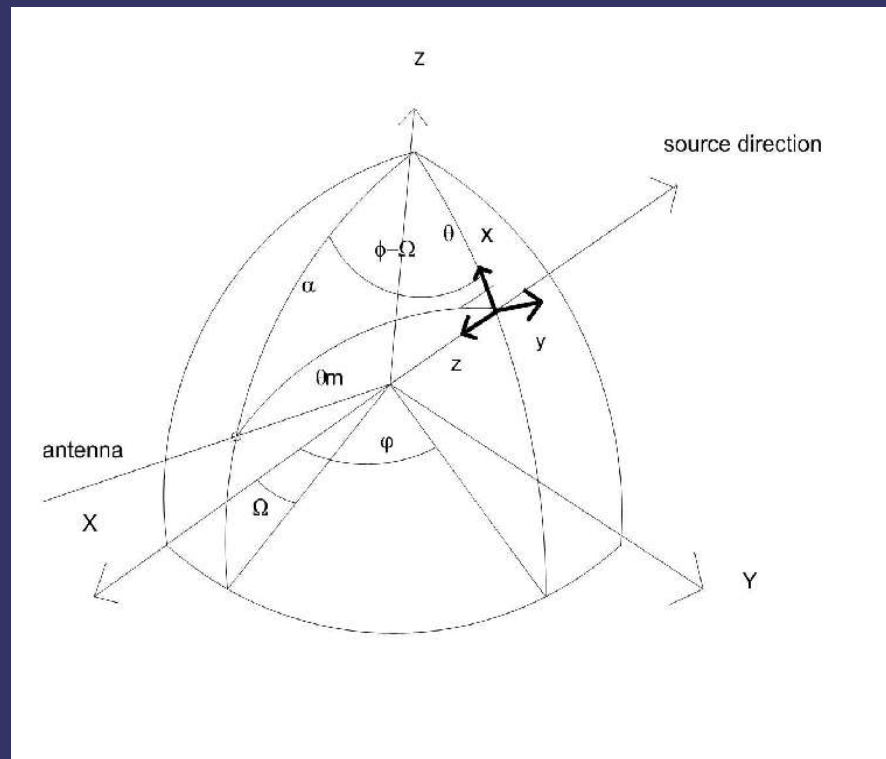
The observables

- ➔ The auto and cross correlation parameters

$$\begin{array}{c} \langle V_x V_x^* \rangle \\ \langle V_z V_z^* \rangle \\ \text{Re} \langle V_x V_z^* \rangle \\ \text{Im} \langle V_x V_z^* \rangle \end{array}$$

$$\langle CC^* \rangle = \frac{1}{T} \int_0^T CC^* dt$$

The coordinate frame



Observables-parameter

$$\begin{aligned}\langle V_i V_i^* \rangle &= \hat{S}_{\eta 0} h_{eff,i}^2 [(\hat{Q} + 1)(\sin^2 \theta \cos^2 \alpha_i - \\ &\quad - \frac{1}{2} \sin(2\alpha_i) \sin(2\theta) \cos^2(\varphi - \Omega_i) + \\ &\quad + \sin^2 \alpha_i \cos^2 \theta \cos^2(\varphi - \Omega_i)) + \\ &\quad + (1 - \hat{Q}) \sin^2 \alpha_i \sin^2(\varphi - \Omega_i) + \\ &\quad + \hat{U}(-\sin \theta \sin 2\alpha_i \sin(\varphi - \Omega_i) + \sin^2 \alpha_i \cos \theta \sin(2\varphi - 2\Omega_i))]\end{aligned}$$

$$\begin{aligned}Re \langle V_i V_j^* \rangle &= \hat{S}_{\eta 0} h_{eff,i} h_{eff,j} [(\hat{Q} + 1)(\sin^2 \theta \cos \alpha_i \cos \alpha_j - \\ &\quad - \frac{1}{2} \sin(2\theta)(\sin \alpha_j \cos \alpha_i \cos(\varphi - \Omega_j) + \sin \alpha_i \cos \alpha_j \cos(\varphi - \Omega_i)) + \\ &\quad + \sin \alpha_i \sin \alpha_j \cos^2 \theta \cos(\varphi - \Omega_i) \cos(\varphi - \Omega_j)) + \\ &\quad + (1 - \hat{Q}) \sin \alpha_i \sin \alpha_j \sin(\varphi - \Omega_i) \sin(\varphi - \Omega_j) - \\ &\quad - \hat{U}(\sin \theta(\sin \alpha_i \cos \alpha_j \sin(\varphi - \Omega_i) + \\ &\quad + \sin \alpha_j \cos \alpha_i \sin(\varphi - \Omega_j)) - \\ &\quad - \cos \theta \sin \alpha_i \sin \alpha_j (\sin(\varphi - \Omega_j) \cos(\varphi - \Omega_i) + \\ &\quad + \sin(\varphi - \Omega_i) \cos(\varphi - \Omega_j)))]\end{aligned}$$

$$\begin{aligned}Im \langle V_i V_j^* \rangle &= -\hat{S}_{\eta 0} h_{eff,i} h_{eff,j} \hat{V} [(\sin \theta(\sin \alpha_i \cos \alpha_j \sin(\varphi - \Omega_i) - \\ &\quad - \sin \alpha_j \cos \alpha_i \sin(\varphi - \Omega_j)) + \\ &\quad + \cos \theta \sin \alpha_i \sin \alpha_j (\sin(\varphi - \Omega_j) \cos(\varphi - \Omega_i) + \\ &\quad + \sin(\varphi - \Omega_i) \cos(\varphi - \Omega_j)))]\end{aligned}$$

Analytical solution for the direction

$$\begin{aligned} \tan \varphi = & [Im \langle V_X V_Z^* \rangle h_{eff,Y} \sin \alpha_Y \tan \Omega_Y \cos \Omega_Y - \\ & - Im \langle V_Y V_Z^* \rangle h_{eff,X} \sin \alpha_X \tan \Omega_X \cos \Omega_X] \times \\ & \times [Im \langle V_X V_Z^* \rangle h_{eff,Y} \sin \alpha_Y \cos \Omega_Y - \\ & - Im \langle V_Y V_Z^* \rangle h_{eff,X} \sin \alpha_X \cos \Omega_X]^{-1} \end{aligned}$$

Attention: Phi is not unique

$$\begin{aligned} \tan \theta = & [\langle V_Z V_Z^* \rangle h_{eff,X} h_{eff,Y} \sin \alpha_X \sin \alpha_Y \\ & \times (\cos(\varphi - \Omega_Y) \sin(\varphi - \Omega_X) - \cos(\varphi - \Omega_X) \sin(\varphi - \Omega_Y))] \\ & \times [Re \langle V_X V_Z^* \rangle \sin \alpha_Y \sin(\varphi - \Omega_Y) h_{eff,Y} h_{eff,Z} \\ & - Re \langle V_Y V_Z^* \rangle \sin \alpha_X \sin(\varphi - \Omega_X) h_{eff,X} h_{eff,Z} \\ & + \langle V_Z V_Z^* \rangle h_{eff,X} h_{eff,Y} \\ & \times (\cos \alpha_Y \sin \alpha_X \sin(\varphi - \Omega_X) - \cos \alpha_X \sin \alpha_Y \sin(\varphi - \Omega_Y))]^{-1} \end{aligned}$$

The matrix equation for the Stokes parameters

$$\mathbf{M}\mathbf{x} = \mathbf{b}$$

$$\mathbf{M} = \begin{bmatrix} (A_X^0 + B_X^0) & (A_X^0 - B_X^0) & 2A_X B_X & 0 \\ (A_Z^0 + B_Z^0) & (A_Z^0 - B_Z^0) & 2A_Z B_Z & 0 \\ (A_X A_Z + B_X B_Z) & (A_X A_Z - B_X B_Z) & (A_X B_Z + A_Z B_X) & 0 \\ 0 & 0 & 0 & -(-A_X B_Z + A_Z B_X) \end{bmatrix}$$

$$\mathbf{x} = \begin{bmatrix} \hat{S} \\ \hat{SQ} \\ \hat{SU} \\ \hat{SV} \end{bmatrix}$$

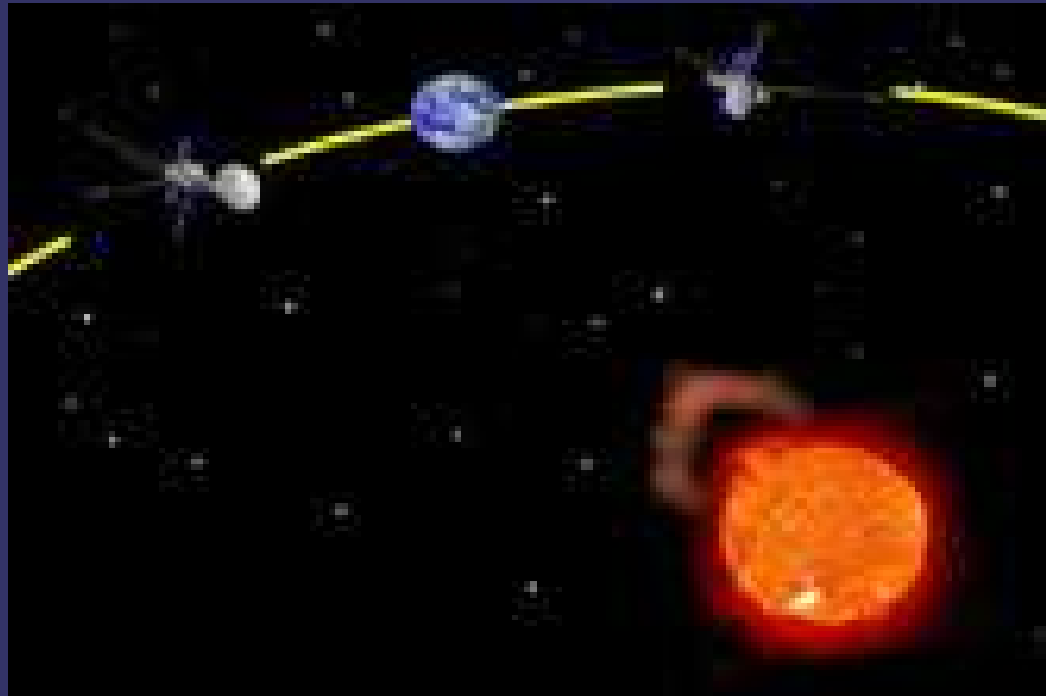
$$\begin{aligned} A_i &= \cos \alpha_i \sin \theta - \sin \alpha_i \cos \theta \cos(\varphi - \Omega_i) \\ B_i &= -\sin \alpha_i \sin(\varphi - \Omega_i) \end{aligned}$$

$$\mathbf{b} = \begin{bmatrix} \frac{\langle V_X V_X^* \rangle}{\eta_0 h_{eff,X}^2} \\ \frac{\langle V_Z V_Z^* \rangle}{\eta_0 h_{eff,Z}^2} \\ \frac{Re\langle V_X V_Z^* \rangle}{\eta_0 h_{eff,X} h_{eff,Z}} \\ \frac{Im\langle V_X V_Z^* \rangle}{\eta_0 h_{eff,X} h_{eff,Z}} \end{bmatrix}$$

The STEREO mission

- ➔ Two spacecraft, one ahead and one behind earth, slowly drifting apart at a rate of 22 degrees by year
- ➔ To extend our knowledge about the physics of the solar system
- ➔ Research on space weather, CMEs and sun-earth-connection (SEC)
- ➔ For the first time stereoscopic methods are used which include remote and in-situ measurements of the same events

The STEREO mission



SWAVES

- ➔ Measures electric fields
- ➔ Frequency 40kHz-16MHz
- ➔ Measures electron density and temperature with quasi thermal noise analysis
- ➔ 3 orthogonal monopole-stacer-antennas, directed away from the sun, 6m length
- ➔ “Direction Finding” (DF) mode provides all auto- and cross correlation parameters
- ➔ 2 spacecraft render it possible to pinpoint the source of the EM radiation via triangulation
- ➔ The equipment on the 2 s/c will track those radio sources from less than $2 R_s$ to 1AU and beyond

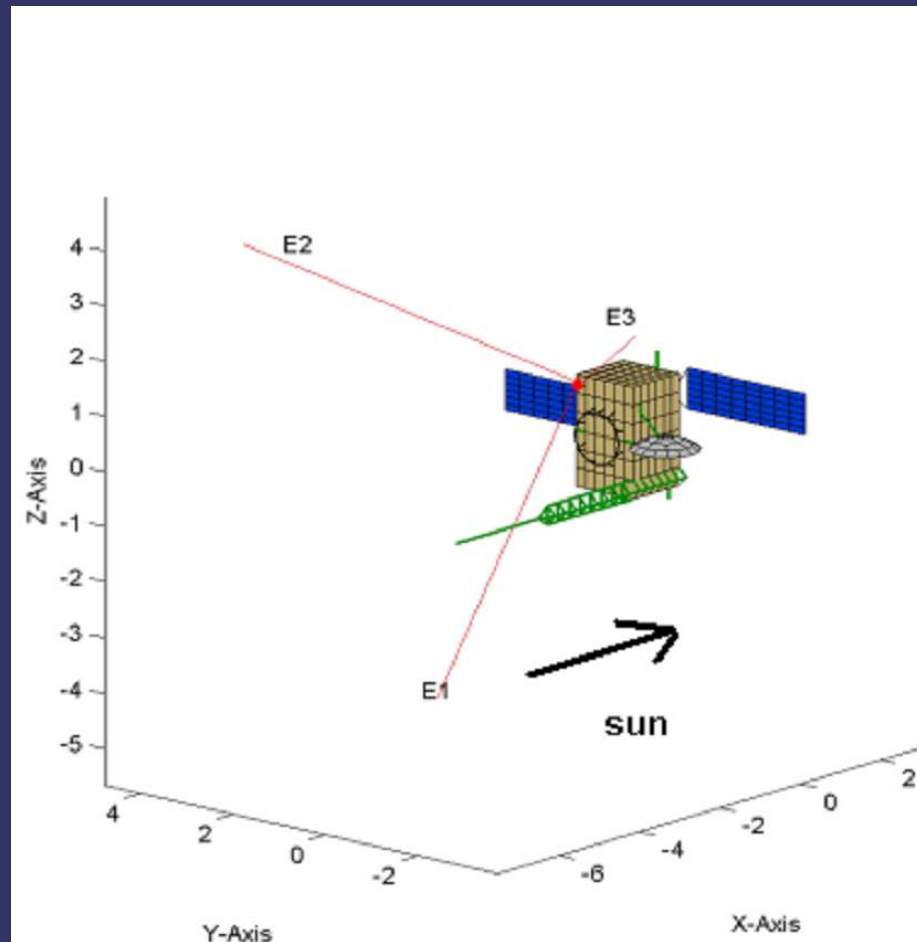
Why is antenna calibration necessary ?

- ➔ To perform “Direction Finding”(DF), antenna properties must be known to a high degree of accuracy
- ➔ The receiving properties can be quantified by the effective length vector
- ➔ The effective length vector represents the antenna as it behaves electrically
- ➔ It is influenced by the geometry of the spacecraft
- ➔ Depends, in general, upon frequency and direction of incidence and is a complex vector, but at low frequency it can be treated as a constant real vector
- ➔ In this quasistatic range, DF is possible

Methods to determine the effective length vector

- I. Numerical electromagnetic code
- II. Rheometry
- III. EMC chamber
- IV. In-flight Calibration

The numerical method



The numerical method

- ⇒ The spacecraft is modelled as a grid of wires
- ⇒ Then the currents along these wires are computed
- ⇒ On base of the current distribution, all other antenna properties (effective length vectors, impedances) can be calculated

Computation of the current distribution

- The equation governing the current distribution is the electric field integral equation (EFIE)
- Simplifications:
 - x Thin currents along the center of the wires
 - x No transverse currents

$$\vec{E}_i = \frac{i\eta}{4\pi k} \int \vec{J}_s(r') G(r, r') dS$$

The Method of Moments

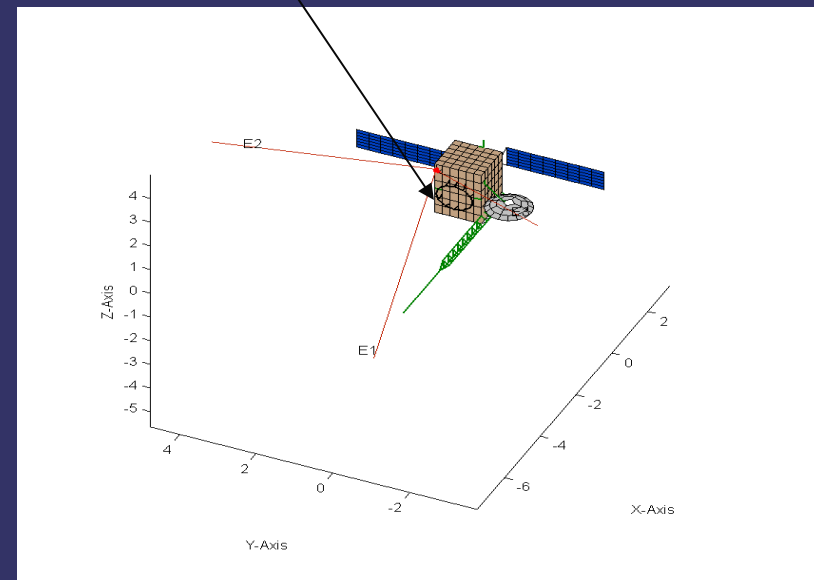
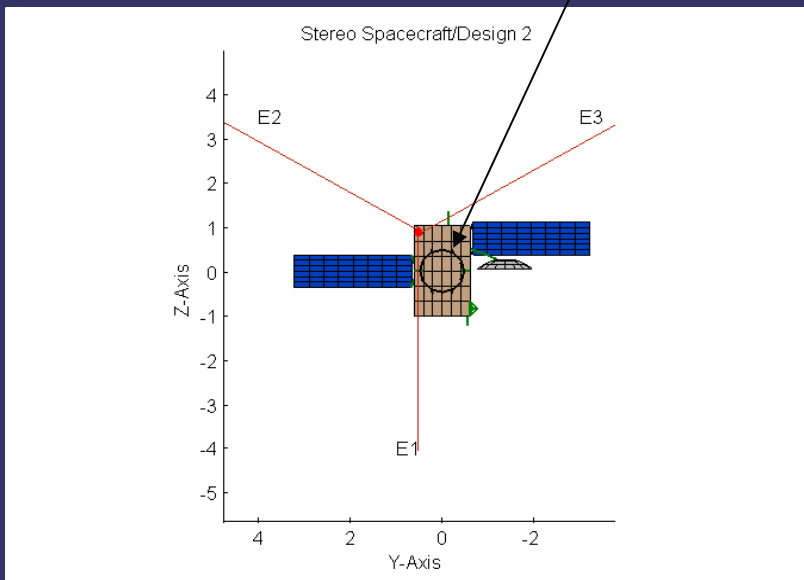
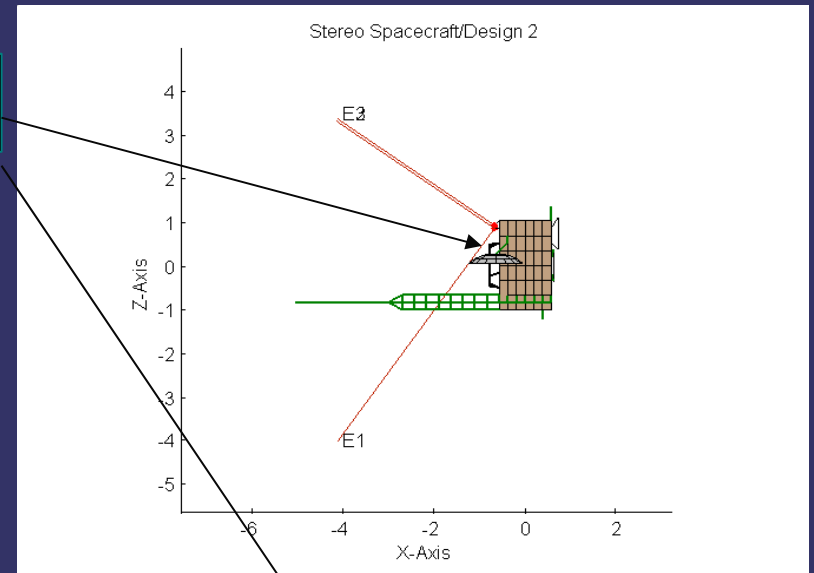
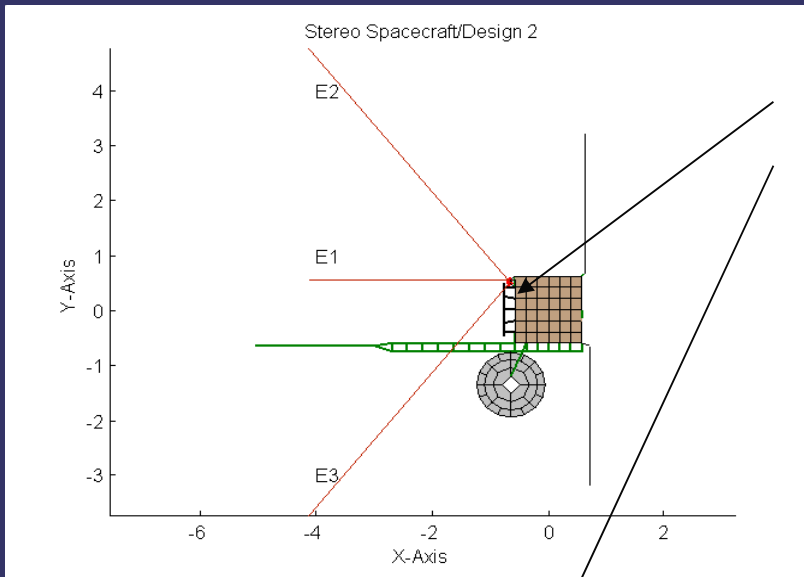
- The Method of Moments (MoM) can be used to solve integral equations
- A modified version of the antenna scatterers analysis program (ASAP) is used to calculate the currents

The Matlab toolbox

- ➡ The effective length vectors and impedances are calculated by the Matlab toolbox created in the space research institute
- ➡ Calculations were performed for open feeds and capacitances of 90pF

The model

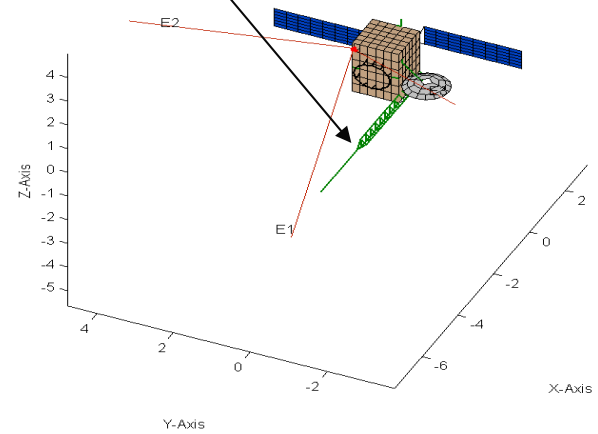
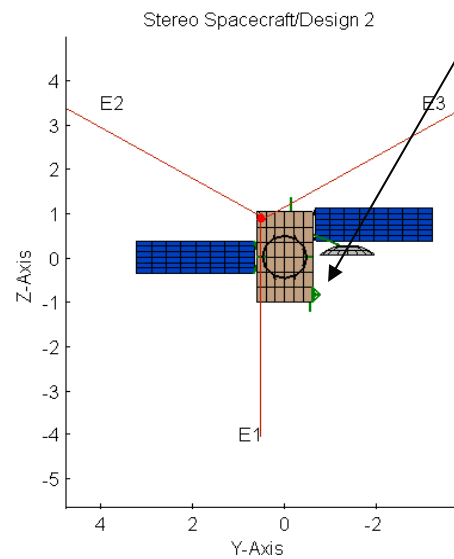
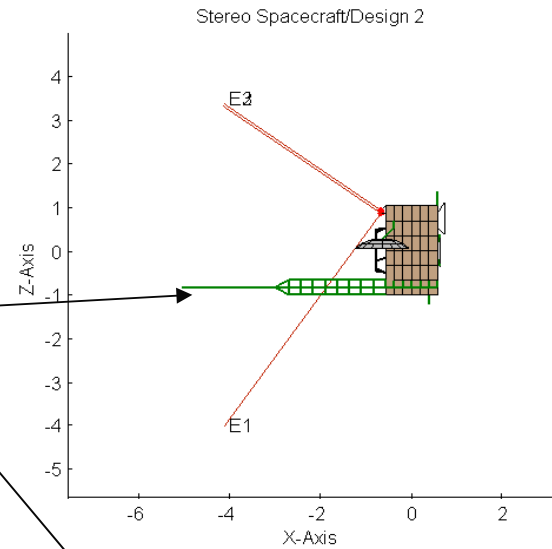
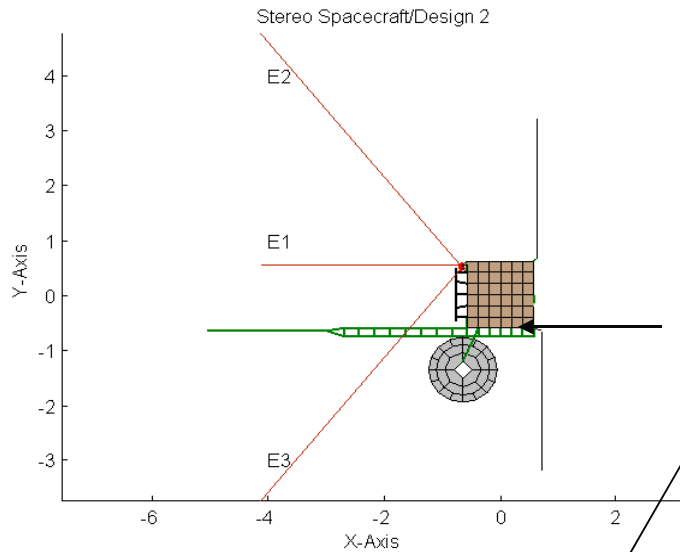
RING



The model

RING

BOOM

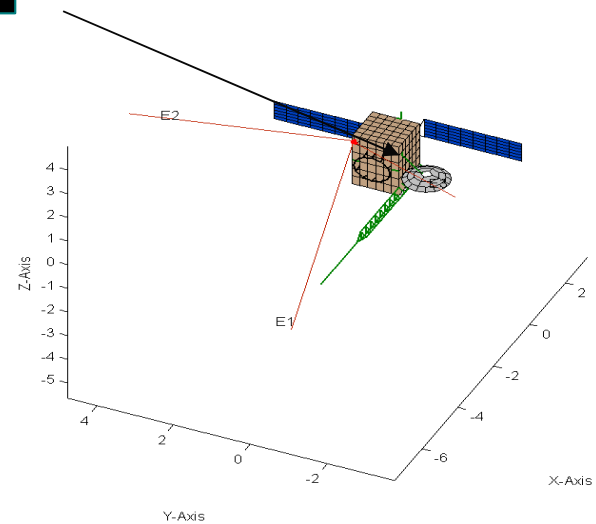
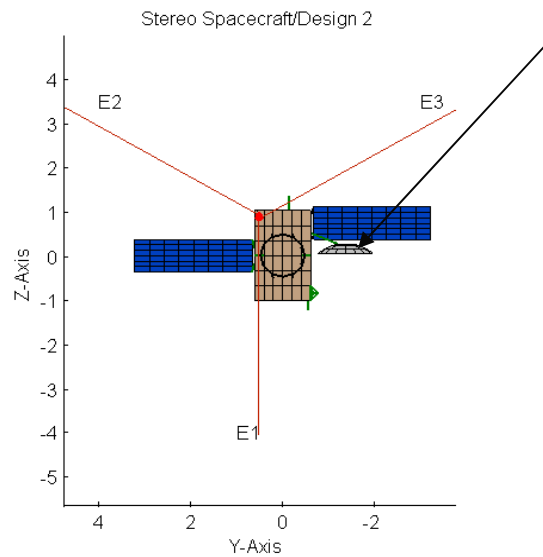
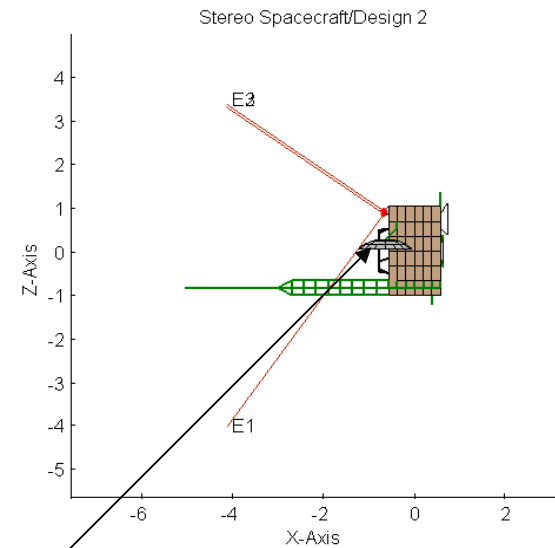
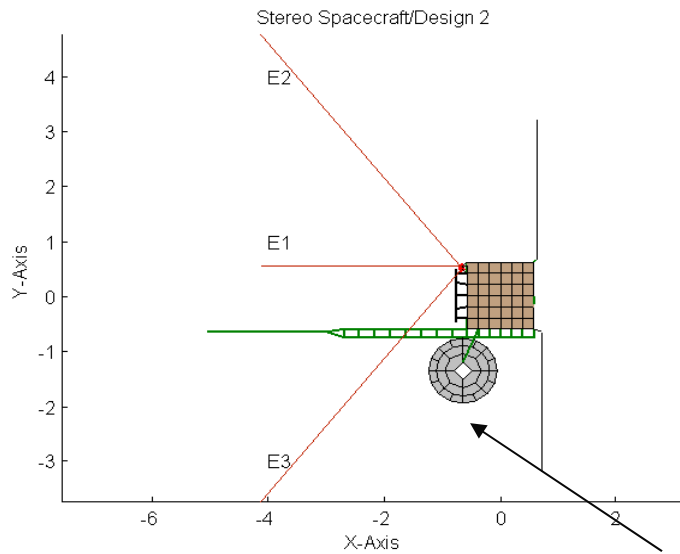


The model

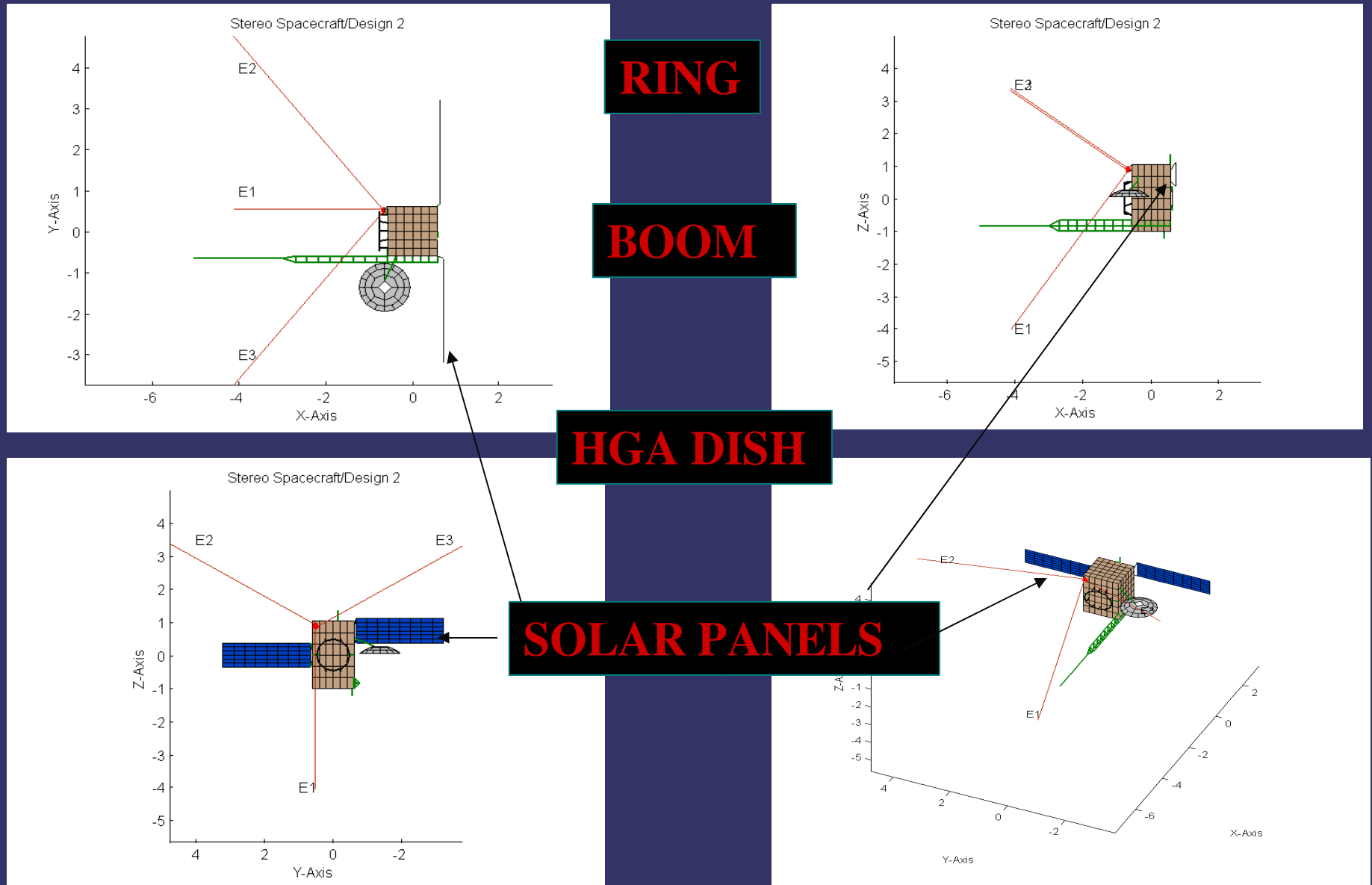
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HGA DISH



The model

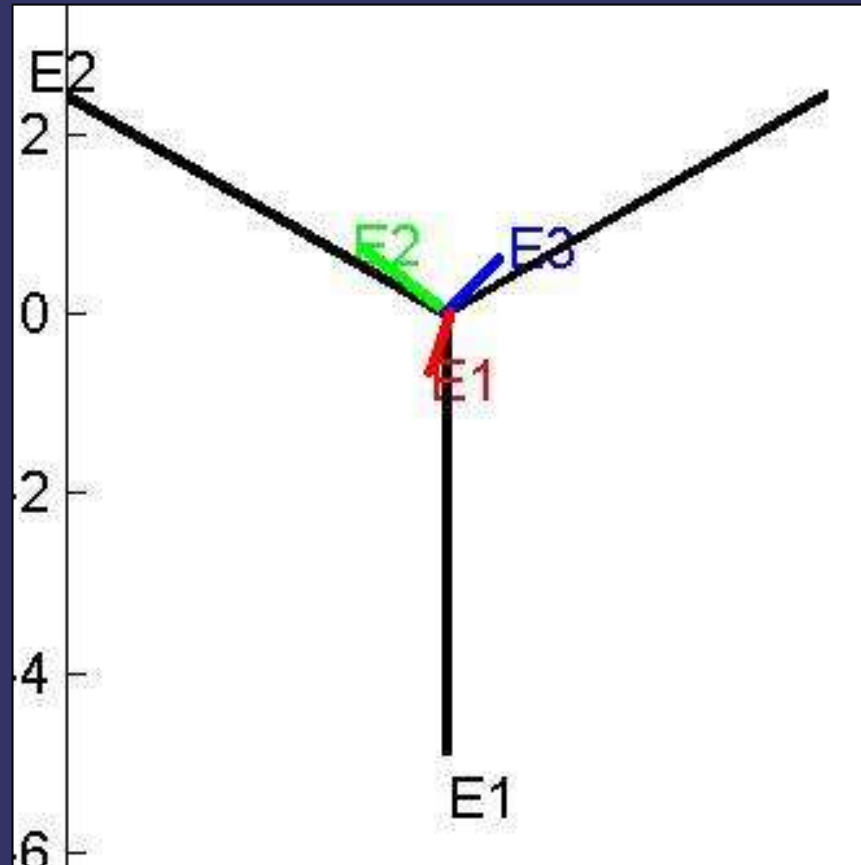
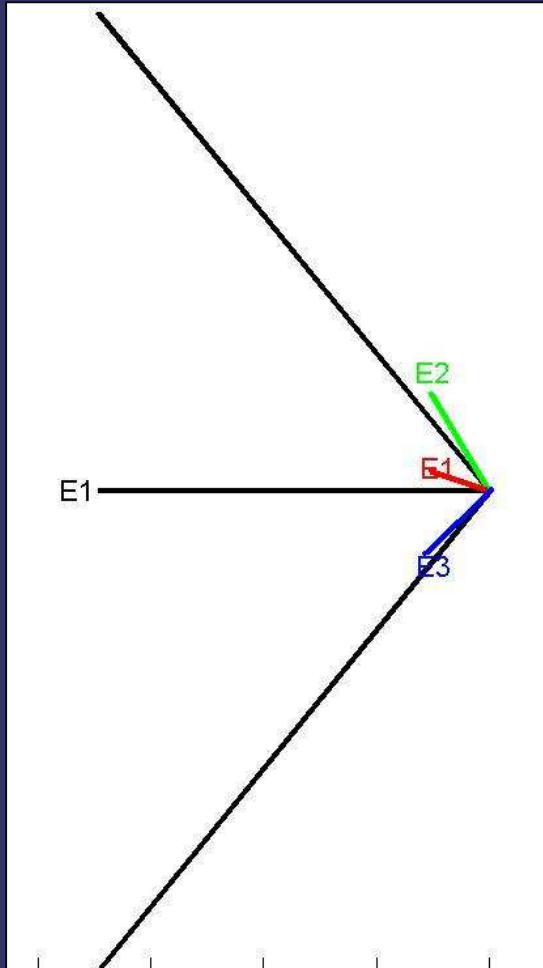


The results

Table 1. Effective length vectors at 500kHz

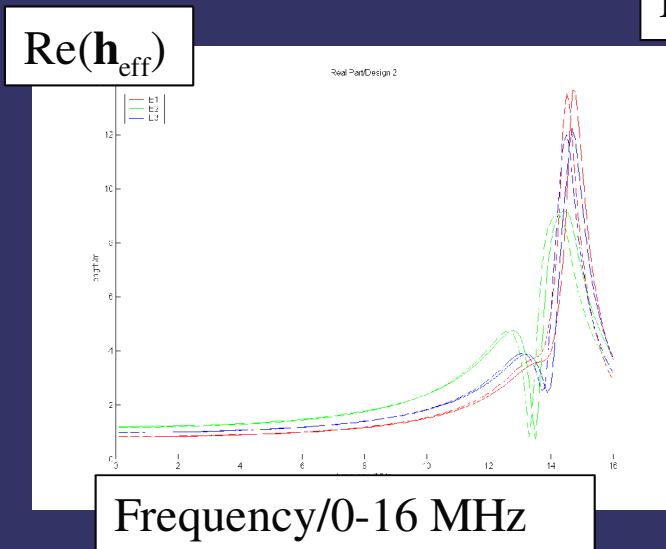
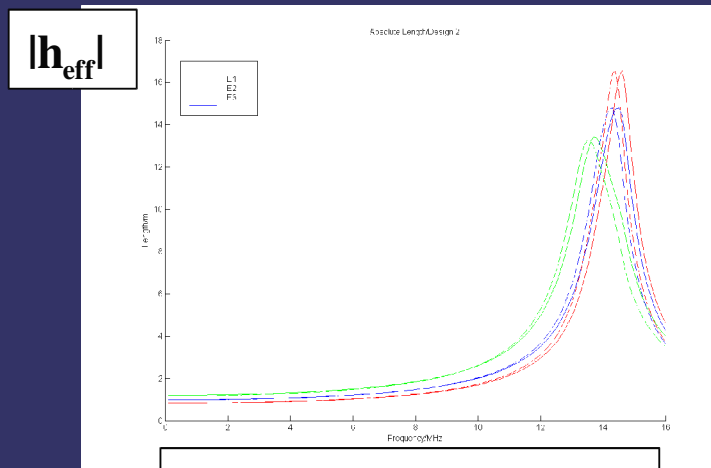
		Spacecraft A	Spacecraft B	Physical antennas
E1	Length/m	0.83	0.84	6.00
	$\zeta/^{\circ}$	128.3	127.2	125.26
	$\xi/^{\circ}$	15.0	13.3	0.0
E2	Length/m	1.21	1.18	6.00
	$\zeta/^{\circ}$	117.1	117.4	125.26
	$\xi/^{\circ}$	125.8	125.2	120.0
E3	Length/m	0.99	0.98	6.00
	$\zeta/^{\circ}$	123.5	123.3	125.26
	$\xi/^{\circ}$	-137.2	-135.4	-120.0

Quasi-static effective length vectors

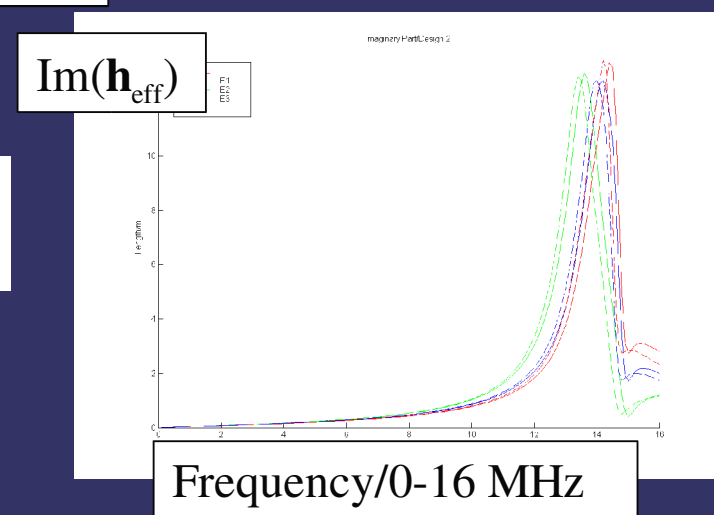


S/C A at 500 kHz

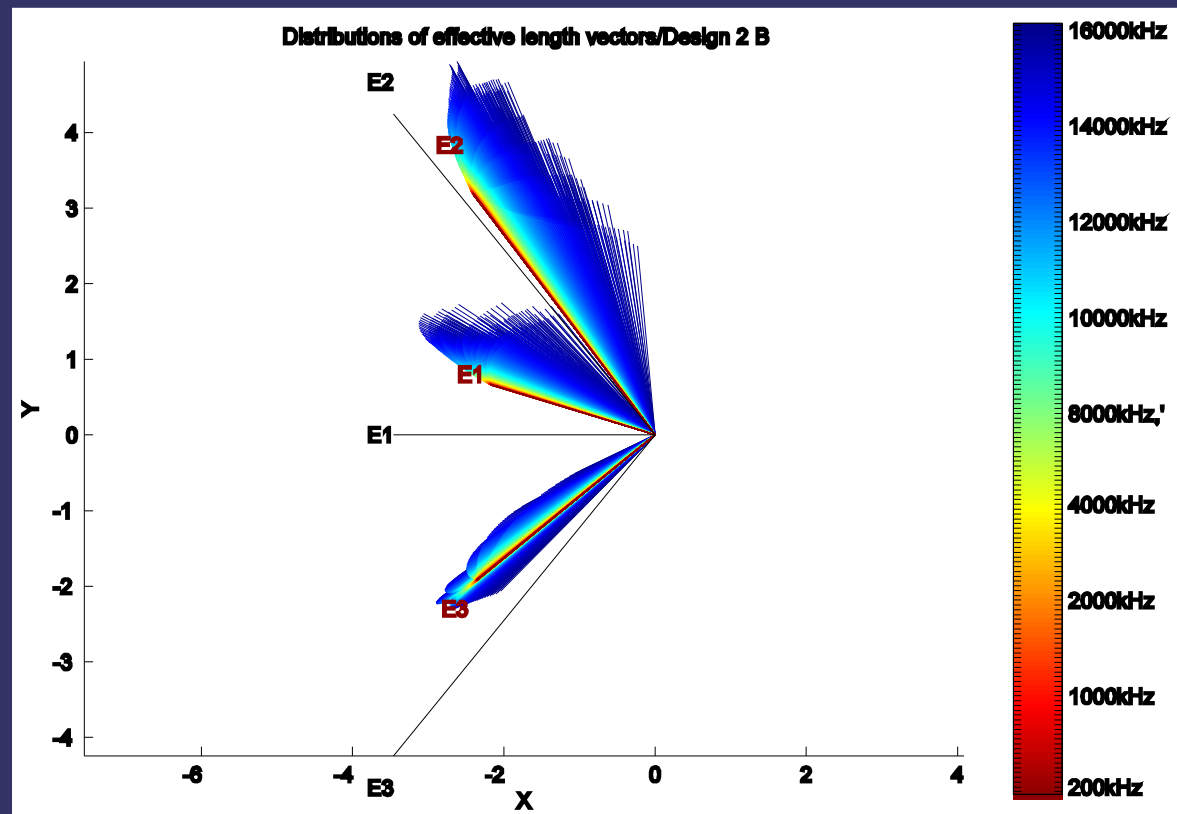
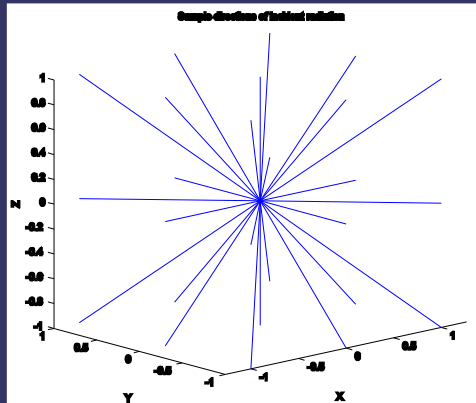
The effective length vector as function of frequency



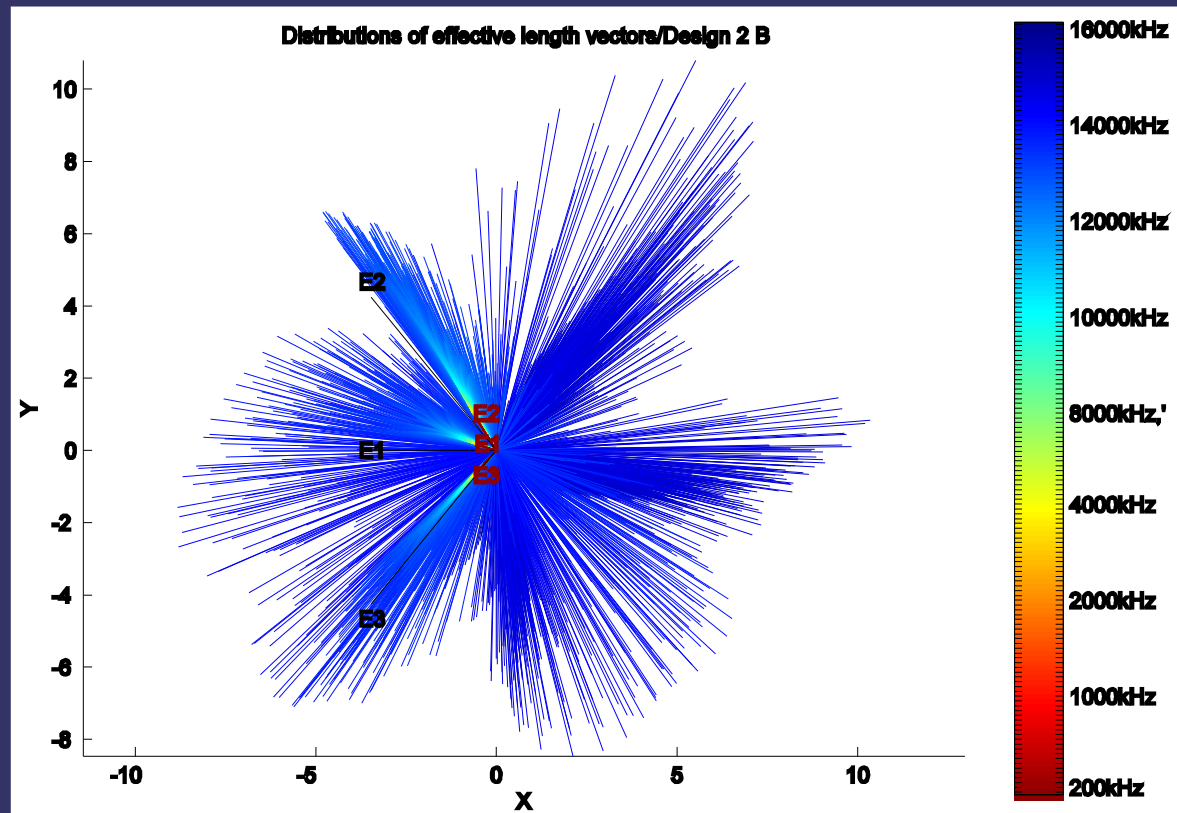
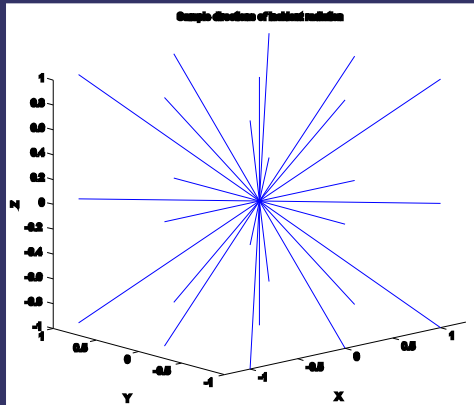
1. Resonance
at 14 MHz



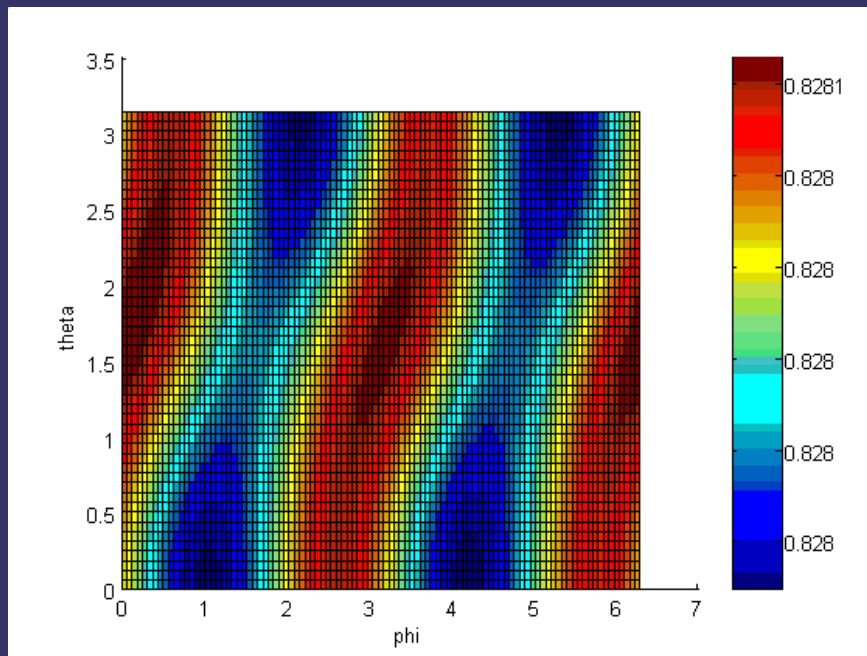
Variation of the length with direction



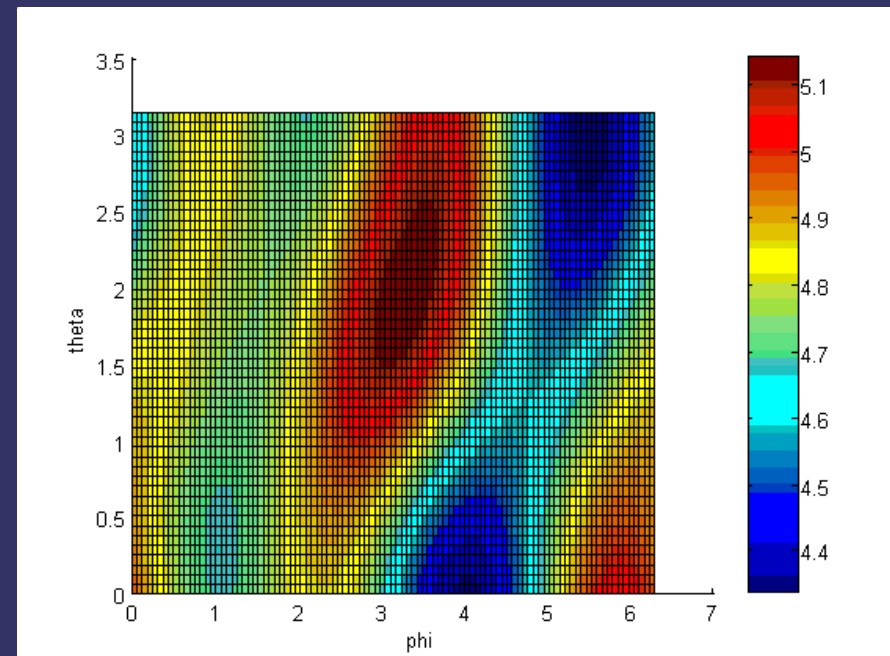
Variation of the length with direction



Variation of the length with frequency



500kHz



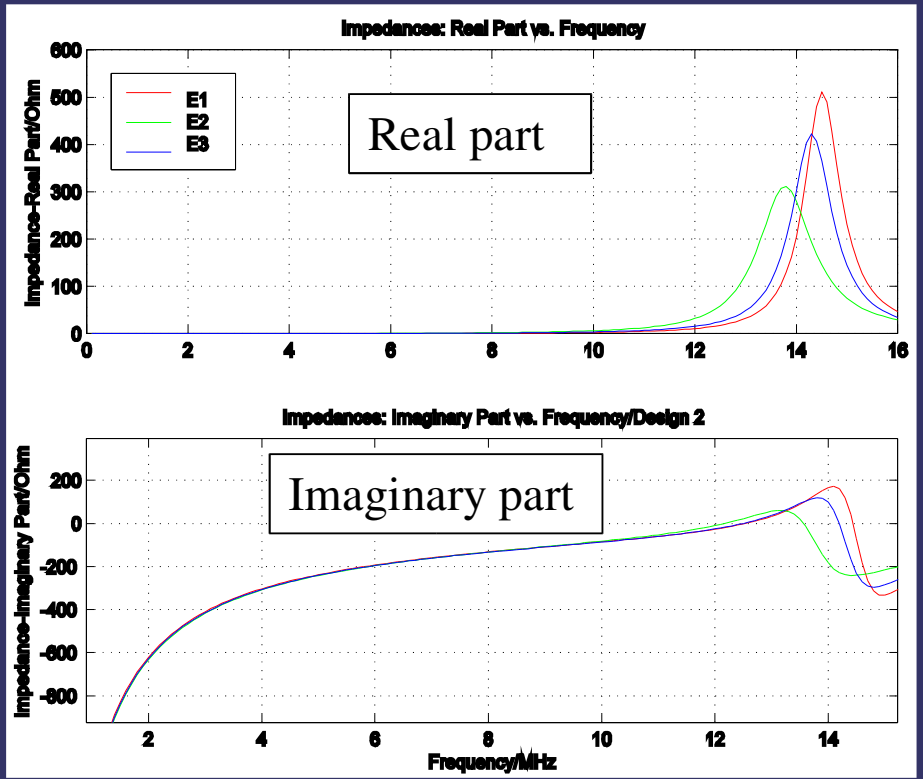
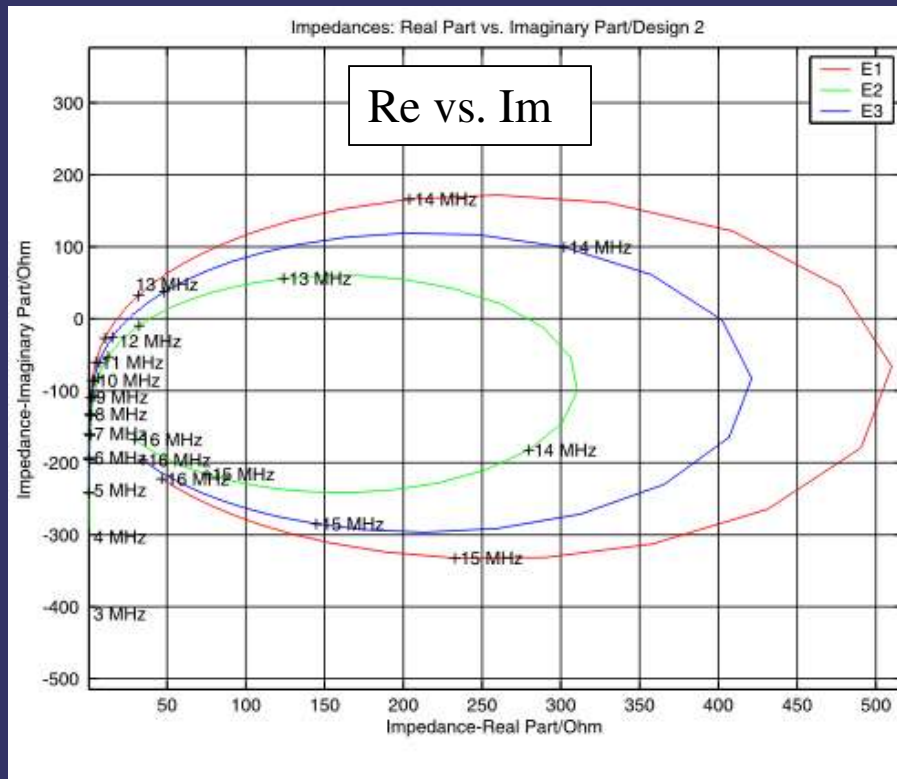
13.5MHz

Variation due to the HGA orientation

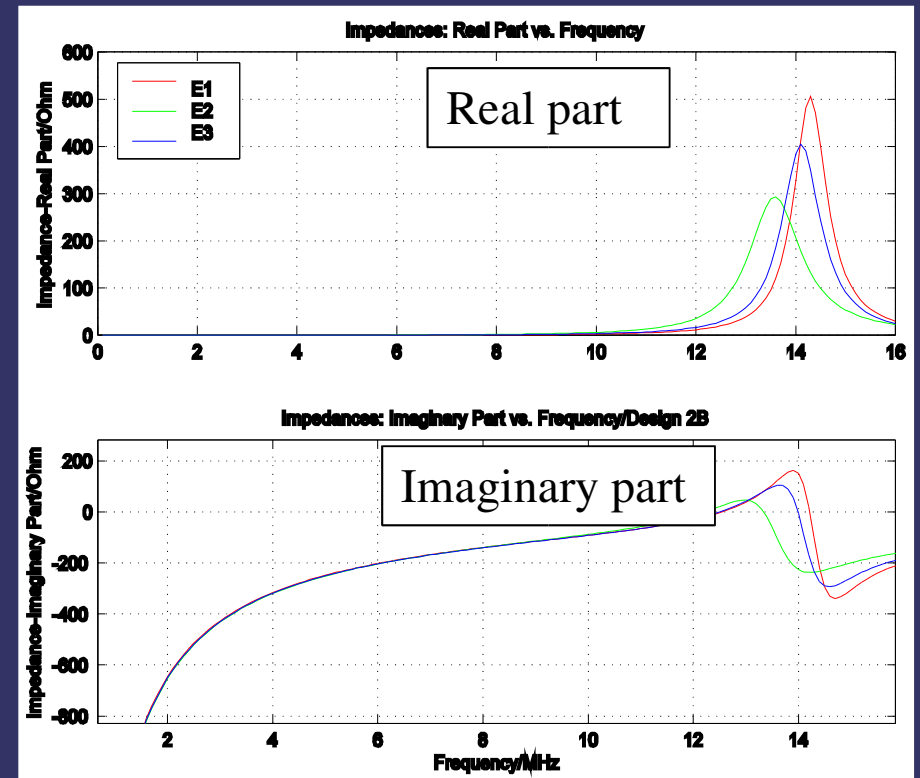
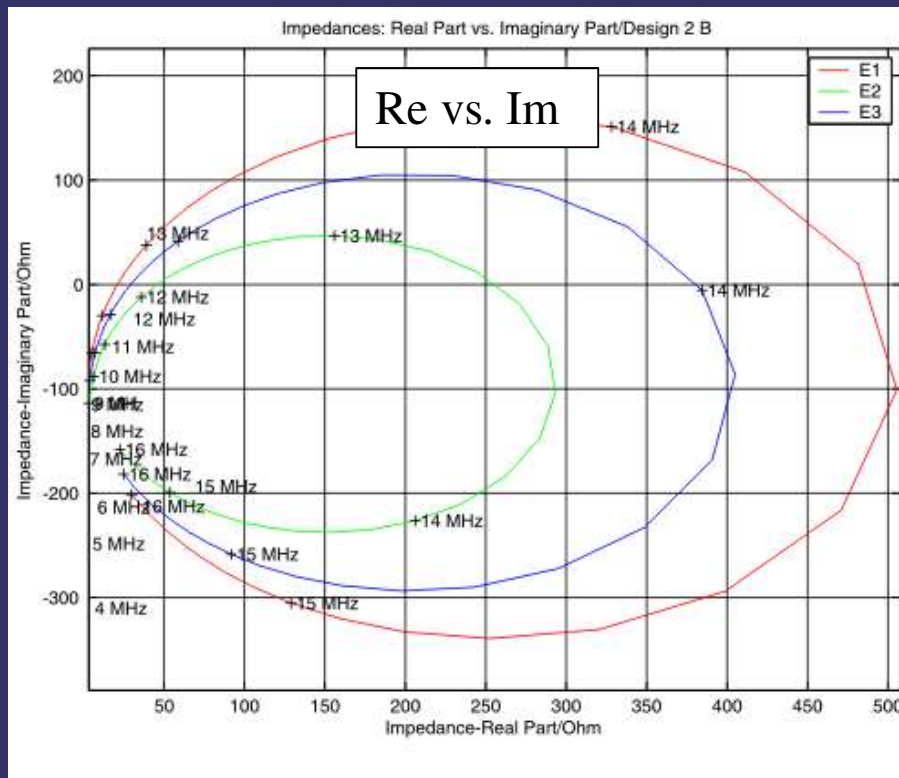
Table 2. Variation of E1 due to HGA angle variation

HGA angle	h_{eff}	ζ	ξ
-90	0.83	128.5	14.5
-80	0.83	128.5	14.5
-70	0.83	128.5	14.4
-60	0.83	128.4	14.5
-50	0.83	128.4	14.5
-40	0.83	128.4	14.5
-30	0.83	128.4	14.2
-20	0.83	128.3	14.7
-10	0.83	128.3	14.9
0	0.83	128.2	15.0
10	0.83	128.1	15.0
20	0.83	128.0	15.1
30	0.83	127.8	15.1
40	0.83	127.7	15.2
50	0.83	127.7	15.2
60	0.83	127.6	15.3
70	0.83	127.6	15.3
80	0.83	127.7	15.4
90	0.83	127.7	15.4

The impedances: Spacecraft A



The impedances: Spacecraft B



Thank You for Your attention !