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Generalized Vector and Dyadic Analysis

Applied Mathematics in Field Theory

Second Edition

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Preface to the Second Edition

After the publication of the first edition of this book (IEEE Press, 1992), several professional friends commented that I should have used the new notations for the divergence and the curl of a vector function, namely $\nabla\!\!\!\!\!\cdot\;\mathbf{F}$ and $\nabla\!\!\!\!\!\times\;\mathbf{F}$, instead of preserving Gibbs's notations $\nabla \cdot \mathbf{F}$ and $\nabla \times \mathbf{F}$, commonly used in many books. In this edition, I have added a chapter on the history of vector analysis to point out more emphatically the contradiction and the confusion resulting from the misinterpretation of Gibbs's notations. It seems beyond doubt that the adoption of the new notations is preferable from the logical point of view.

In 1994, I had the opportunity to teach a course at the University of Michigan in which I used the method of symbolic vector and the new notations to teach vector analysis. The reaction from the students was very favorable. Similar views have been communicated to me by colleagues from other institutions. My motivation for revising the book is principally due to these encouragements.

In addition to the overhaul of the notations, the present edition considerably expands the coverage. The method of symbolic vector is now formulated not only in the curvilinear orthogonal system, but also in the general nonorthogonal curvilinear system. The reciprocal base systems, originally introduced by Gibbs, have been used very effectively in the formulation. New vector theorems and vector and dyadic identities have been derived to make the list as complete as possible. The relationship between dyadic analysis and tensor analysis has also been explained. The transformation of electromagnetic field vectors based on the special theory of relativity is explained by both the conventional method, using differential calculus, and the more sophisticated method due to Sommerfeld, with the aid of four-dimensional vector analysis.

I am most grateful to Professor Phillip Alexander of the University of Windsor and to my colleague, Mr. Richard Carnes of the University of Michigan, for technical assistance and manuscript editing. For all their help and encouragement, I want to thank Dr. John H. Bryant, Professor Fawwaz T. Ulaby, Professor John H. Kraus, Mr. Jui-Ching Cheng, Mrs. Carol Truszkowski, Ms. Patricia Wolfe, Dr. Roger DeRoo, and Dr. Jian Gong.

I owe my gratitude to Prof. Donald G. Dudley, Editor of the IEEE Press/OUP Series on Electromagnetic Wave Theory, and to Mr. Dudley Kay, IEEE Press, and his staff, particularly, Ms. Denise Phillip, for valuable suggestions in the production of this book by the IEEE Press.

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Preface to the First Edition

Mathematics is a language.
The whole is simpler than its parts.
Anyone having these desires will make these researches.
—J. Willard Gibbs

This monograph is mainly based on the author's recent work on vector analysis and dyadic analysis. The book is divided into two main topics: Chapters 1–6 cover vector analysis, while Chapter 7 is exclusively devoted to dyadic analysis. On the subject of vector analysis, a new symbolic method with the aid of a symbolic vector is the main feature of the presentation. By means of this method, the principal topics in vector analysis can be developed in a systematic way. All vector identities can be derived by an algebraic manipulation of expressions with two partial symbolic vectors without actually performing any differentiation. Integral theorems are formulated under one roof with the aid of a generalized Gauss theorem. Vector analysis on a surface is treated in a similar manner. Some basic differential functions on a surface are defined; they are different from the surface functions previously defined by Weatherburn, although the two sets are intimately related. Their relations are discussed in great detail. The advantage of adopting the surface functions advocated in this work is the simplicity of formulating the surface integral theorems based on these newly defined functions.

The scope of topics covered in this book on vector analysis is comparable to those found in the books by Wilson [21], Gans [4], and Phillips [11]. However, the topics on curvilinear orthogonal systems have been treated in great detail. One important feature of this work is the unified treatment of many theorems and formulas of similar nature, which includes the invariance principle of the differential operators for the gradient, the divergence, and the curl, and the relations between various integral theorems and transport theorems. Some quite useful topics are found in this book, which include the derivation of several identities involving the derivatives of unit vectors, and the relations between the unit vectors of various coordinate systems based on a method of gradient.

Tensor analysis is outside the scope of this book. There are many excellent books treating this subject. Since dyadic analysis is now used quite frequently in engineering sciences, a chapter on this subject, which is closely related to tensor analysis in a three-dimensional Euclidean space, may be timely.

As a whole, it is hoped that this book may be useful to instructors and students in engineering and physical sciences who wish to teach and to learn vector analysis in a systematic manner based on a new method with a clear picture of the constituent structure of this mature science not critically studied in the past few decades.

Acknowledgments for the First Edition

Without the encouragement which I received from my wife and family, and the loving innocent interference from my grandchildren, this work would never have been completed. I would like to express my gratitude to President Dr. Qian Wei-Chang for his kindness in inviting me as a Visiting Professor at The Shanghai University of Technology in the Fall of 1988 when this work was started. Most of the writing was done when I was a Visiting Professor at the Chung Cheng Institute of Technology, Taiwan, in the Spring of 1990. I am indebted to President Dr. Chen Chwan-Haw, Prof. Bor Sheau-Shong, and Prof. Kuei Ching-Ping for the invitation.

The assistance of Prof. Nenghang Fang of The Nanjing Institute of Electronic Technology, China, currently a Visiting Scholar at The University of Michigan, has been most valuable. His discussion with me about the Russian work on vector analysis was instrumental in stimulating my interest to formulate the symbolic vector method introduced in this book. Without his participation in the early stage of this work, the endeavor could not have begun. He has kindly checked all the formulas and made numerous suggestions. I am grateful to many colleagues for useful information and valuable comments. They include: Prof. J. Van Bladel of The University of Gent, Prof. Jed Z. Buchwald of The University of Toronto, Prof. W. Jack Cunningham of Yale University, Prof. Walter R. Debler and Prof. James F. Driscoll of The University of Michigan, Prof. John D. Kraus and Prof. H. C. Ko of The Ohio State University, and Prof. C. Truesdell of The Johns Hopkins University. My dear old friend Prof. David K. Cheng of Syracuse University kindly edited the manuscript and suggested the title of the book. The teachings of Prof.

Chih-Kung Jen of The Johns Hopkins Applied Physics Laboratory, formerly of Tsing Hua University, and Prof. Ronold W. P. King of Harvard University remain the guiding lights in my search for knowledge. Without the help of Ms. Bonnie Kidd, Dr. Jian-Ming Jin, and Dr. Leland Pierce, the preparation of this manuscript would not have been so professional and successful.

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