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FINITE ELEMENT METHOD FOR ELECTROMAGNETICS

Antennas, Microwave Circuits, and Scattering Applications

IEEE Series on
Electromagnetic Wave Theory

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Preface

The finite element method (FEM) and its hybrid versions (finite element-boundary integral, finite element-absorbing boundary condition, finite element-mode matching, etc.) is one of the most successful frequency domain computational methods for electromagnetic simulations. It combines geometrical adaptability and material generality for modeling arbitrary geometries and materials of any composition. The latter is particularly important in electromagnetics since nearly most applications dealing with antennas, microwave circuits, scatterers, motor and generator modeling, etc. require the simulation of nonmetallic/composite materials. Also, the hybridization of the finite element method with integral equation techniques leads to fully rigorous approaches which combine the best aspects of volume and surface formulation techniques.

Because of its unique features, the finite element method is becoming the workhorse for electromagnetic modeling and simulations. Many research and development codes are now available from universities and industry, and these have demonstrated the utility and capability of the method. Also, a number of commercial finite element analysis packages are currently available. Typically, these packages do not yet incorporate the more rigorous hybrid versions of the FEM. However, they are rapidly evolving to more sophisticated and capable packages which incorporate new technologies in geometrical modeling, simulation engines, and solvers.

With the increasing importance of electromagnetics simulation packages using the FEM, this book should serve as a valuable text for students, practicing engineers, and researchers in electromagnetics. The original goal of writing the book was to serve as a text for beginning graduate students interested in the application of the finite element method and its hybrid versions to electromagnetics. However, the authors also recognized a need to report (in a coherent manner) the many recent advances in applying the method(s) to traditional and new problems in electromagnetics. The result is a book that can serve both beginning students and more advanced practitioners. The first half of the book has already been used in the classroom as part of a course on numerical electromagnetics at the University of Michigan. The second half of the book covers primarily work on three-dimensional (3D) developments and applications which have primarily appeared in the literature over the past 5 years.

The book assumes that the reader is a first-year graduate student who has likely taken one advanced course in electromagnetics beyond the standard undergraduate courses. For practicing engineers, it is assumed that the reader is familiar with concepts of electromagnetic radiation and has an understanding of Maxwell's equations and their implications. No previous experience in numerical methods is necessary, but such experience will, of course, help the reader in understanding the procedure of casting analytical equations into discrete systems for numerical solution.

For classroom use, it is expected that the first four chapters will be thoroughly covered with the exception of Chapter 2, which describes a variety of basis/expansion functions. At the introductory stage, only the initial sections of Chapter 2 need be covered. The reader may then return to Chapter 2 as needed. Chapters 3 and 4 [one-dimensional (1D) and two-dimensional (2D) formulations and applications] are written in a step-by-step process with the assumption that this is the first exposure of the reader to numerical methods and the finite element method in general. Chapters 5 through 7 introduce the finite element method and its hybrid versions for 3D simulations with applications to microwave circuits, scattering, and conformal antennas. These chapters are written at a more advanced level and cover the latest applications and successes of the method in electromagnetics. Chapter 5 (closed-domain 3D applications) is a straightfoward extension of the two-dimensional development in Chapter 4 and can be part of a quarter or semester course which includes Chapters 1–5. FEM implementations with absorbing boundary conditions and the finite element-boundary integral method for 3D applications are described in Chapters 6 and 7, respectively. These are realistic practical simulations and should be of particular interest to practicing engineers and researchers in the field. Their 2D counterparts are described in Chapter 4 at a significant level of detail along with explicit formulas for developing computer codes.

Chapter 8 describes some recent developments on the implementation of the boundary integral methods for mesh truncation in conjunction with fast integral methods. Fast integral methods have shown dramatic reductions in CPU and memory. They are currently the subject of research and will impact the utility and development of the finite element-boundary integral method.

Finally, Chapter 9 presents an overview of storage techniques for sparse systems, iterative solvers, preconditioning, parallelization, and a variety of details pertinent to the development of finite element codes. These items were not mixed with the earlier chapters which discuss the mathematics and applications for the FEM. Thus, the reader can refer to Chapter 9 at different stages, and as needed, when developing finite element codes.

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Acknowledgments

Interest in the finite element method (FEM) at the Radiation Laboratory of the University of Michigan began in 1987 by the first author and his graduate students. The motivation was to model large domains without restrictions in geometry and material composition. At that time, two graduate students, Timothy J. Peters and Kasra Barkeshli, had completed successful implementations of boundary integral solutions using k-space methods. This $O(N \log N)$ approach paved the way for a fully $O(N)$ finite element-boundary integral algorithm which combined the rigor of the boundary integral for mesh truncation and the generality of the FEM for volume/domain modeling. The first of these hybrid implementations was developed by Dr. Jian-Ming Jin, a graduate assistant of John Volakis, resulting in a highly successful finite element-boundary integral computer program. Versions of this code are still in use by government, industrial, and academic researchers in the United States. Another graduate student of Professor Volakis, Dr. Jeffery D. Collins, furthered this work to a body-of-revolution with an integral mesh enclosure. His later students—among them the co-authors, Dr. Chatterjee and Dr. Kempel, and Dr. Daniel C. Ross, Dr. Jian Gong, and Dr. Tayfun Özdemir—made significant contributions toward the understanding of 3D problems in antennas, scattering, and microwave circuits.

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