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# METHODS IN ELECTROMAGNETIC WAVE PROPAGATION

## SECOND EDITION

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UNIVERSITY OF DUNDEE

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# PREFACE TO THE SECOND EDITION

There has been much activity in the areas covered by this book since the first edition. Numerous papers have appeared on all the topics and the constraints of space have meant that not all of them can be quoted. To the many authors whose research I have had to omit I offer my apologies. It is hoped that a sufficient selection has been included for the interested reader to become acquainted with the progress that has been made and to follow up developments.

Once again I am most grateful to my wife and Mrs Ross for their forbearance, unfailing assistance, and encouragement.

D.S.J.

# PREFACE TO THE FIRST EDITION

Modern methods of tackling problems associated with electromagnetic waves involve a judicious mixture of analysis and computation. The analysis occurs in the mathematical formulation and in establishing that it has the requisite properties. Conversion to a form suitable for the computer entails numerical analysis, whose justification may also rest on a considerable body of analysis. Therefore, the aim of these two volumes is to develop a suitable framework of theory and numerical analysis with applications to various aspects of the propagation of electromagnetic waves. An attempt has been made to couch the explanation in as comprehensible a language as possible and to assume a starting point as early as commensurate with the size of the text. To assist with the understanding numerous exercises have been inserted at convenient points and some of these are open-ended so that any instructor has plenty of freedom in determining the mode of treatment. Complementary material will be found in D. S. Jones, *Acoustic and electromagnetic waves*, Oxford University Press (1986). The first five chapters are devoted to the provision of a theoretical background and the topic of guided waves. The first chapter sets out the fundamentals of numerical analysis which are essential in handling a problem numerically. Propagation in waveguides can be approached from three different points of view. One possibility is a direct numerical attack based on difference equations; this is the subject matter of Chapter 2. Another angle is to consider the problem as one of finding the eigenvalues of an operator. This avenue is explored in Chapter 3, which also treats the cavity resonator from a theoretical standpoint. The third route employs variational methods and these are considered in Chapter 4. Chapter 5 returns to numerical techniques with particular emphasis on variational methods, integral equations and finite elements.

Chapters 6 to 9 deal with radiating waves whether produced directly from a transmitter or indirectly by scattering from an irradiated obstacle. Antennas are discussed in Chapter 6 with separate sections for wires, solids, and dielectrics. The analysis in Chapter 6 is concerned with the frequency domain. The changes necessary in the time domain are examined in Chapter 7, including singularity expansion method. The well-known geometric theory of diffraction receives an extensive review in Chapter 8. Finally, Chapter 9 investigates inverse scattering, embracing holography and adaptive arrays as well as other applications.

Again my thanks are due to my wife and Mrs Ross for their constant help and encouragement.

Dundee
December 1986

D.S.J.

*This book is dedicated with deep affection to*
*the Streather family and, in particular*
*Bessie, Kittie, Nell, Flo, Alice, Peg, and Frank*

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