# Scattering of a Dipole Field by a Moving Plasma Column

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*Abstract*—**The problem of scattering of a dipole field by a plasma column moving uniformly in the axial direction is treated. In order to know the effect of the motion of the plasma on the scattering pattern, first the scattering properties of the dipole field by a stationary plasma with the aid of numerical examples are discussed. In this case two kinds of scattering patterns are illustrated: one in a cross section of the column and the other in the longitudinal section of the column. Then the scattering pattern of the dipole field by the moving plasma is investigated. From the numerical examples for the stationary plasma it is found that the scattering patterns are well-classified into three typical patterns by investigating the ratio of the backward scattering to the forward scattering with respect to the parameters which govern the scattering properties. In the case of moving plasma, it is shown that the scattering pattern in the longitudinal section will have a characteristic directional pattern, which does not appear in the case of stationary plasma, at a frequency lower than the plasma frequency.**

## I. INTRODUCTION

THE PROBLEM related to the scattering of the electromagnetic waves by a circular cylindrical medium has been studied by many workers in the field of radar, radio astronomy, diagnostics of plasma, and so forth. Especially the scattering of plane waves has been investigated in detail for the various kinds of cylindrical medium [1]–[13]. However, to date much of the activity in this area has been confined to two-dimensional problems.

In the scattering of a plane wave by a circular cylinder the polarization of the scattered wave is not generally the same with that of the incident plane wave, except for a special case in which the incident plane wave is normal to the column's axis and its electric field is polarized parallel or perpendicular to the column's axis. In the case where a plane wave is obliquely incident upon the column, the incident plane wave will have a different refractive coefficient versus a different incident angle. Therefore, the resultant field after the scattering by the column will show the polarization effect. As a special case, these problems of the plane wave scattering are called quasi-two-dimensional and two-dimensional [13].

It is now worthwhile to treat a three-dimensional problem for the interest of application, e.g., three-dimensional diagnostics of plasma, scattering of dipole field by a plasma jet, and satellite communication. In this paper we discuss the scattering of a dipole field by a circular cylindrical plasma column moving in the axial direction on the basis of the Maxwell–Minkowski theory. First, in order to know the effect of the plasma motion on the scattering we discuss the scattering properties of the dipole field by a stationary plasma with the aid of numerical examples. And it is shown that the scattering patterns are well-classified into three typical patterns by investigating the ratio of the backward scattering to the forward scattering with respect to the parameters which govern the scattering properties. Then we investigate the effect of the plasma motion on the scattering pattern. In this case, it is shown that the scattering pattern in a longitudinal section will have a characteristic directional pattern, which does not appear in the case of stationary plasma, at the frequency lower than the plasma frequency.

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## II. FORMAL SOLUTION

Consider an oscillating electric dipole situated at $(\rho_0, 0, 0)$ with respect to the cylindrical coordinate, with $\rho_0 > b$ and a circular cylindrical isotropic plasma column of radius $b$ moving at a uniform velocity, $\mathbf{v} = \mathbf{i}_z v$, in the z direction (Fig. 1). The symbol I and II refer to plasma region and free space region, respectively.

The source current due to the electric dipole can be specified by

$$\mathbf{J} = \mathbf{i}_z J\delta(\rho - \rho_0)\delta(\phi)\delta(z)/2\pi\rho. \tag{1}$$

The harmonic time dependence of the form $e^{j\omega t}$ is assumed for the source (1) and for all of the field components. In the rest frame of the plasma, the plasma medium may be characterized by a specific dielectric constant $\epsilon$, with

$$\epsilon_0\epsilon = \epsilon_0\left(1 - \frac{\omega_p{}^2}{\omega'^2}\right), \tag{2}$$

where $\omega_p$ is the plasma frequency, which is invariant under a Lorentz transformation, and $\omega'$ is the frequency measured in the rest frame of the plasma column.

In the region I, Maxwell's equations [14] are given by

$$(\nabla - j\omega\Omega) \times \mathbf{E}_1 = -j\omega\mu_0\bar{\alpha} \cdot \mathbf{H}_1 \tag{3a}$$

$$(\nabla - j\omega\Omega) \times \mathbf{H}_1 = j\omega\epsilon_0\epsilon\bar{\alpha} \cdot \mathbf{E}_1 \tag{3b}$$

$$(\nabla - j\omega\Omega) \cdot \bar{\alpha} \cdot \mathbf{E}_1 = 0 \tag{3c}$$

$$(\nabla - j\omega\Omega) \cdot \bar{\alpha} \cdot \mathbf{H}_1 = 0, \tag{3d}$$

where the subscript 1 denotes the fields measured in the region I and the following symbols are introduced.

$$\Omega = \frac{(n_p{}^2 - 1)v}{(1 - n_p{}^2\beta^2)c^2} = \frac{(n_p{}^2 - 1)\beta}{(1 - n_p{}^2\beta^2)c}\,\mathbf{i}_z$$

$$= \Omega\mathbf{i}_z, \qquad \bar{\alpha} = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$a = \frac{1 - \beta^2}{1 - n_p{}^2\beta^2}, \qquad \beta = \frac{v}{c}, \qquad n_p = \sqrt{\epsilon}, \qquad c = \frac{1}{\sqrt{\epsilon_0\mu_0}}. \tag{4}$$

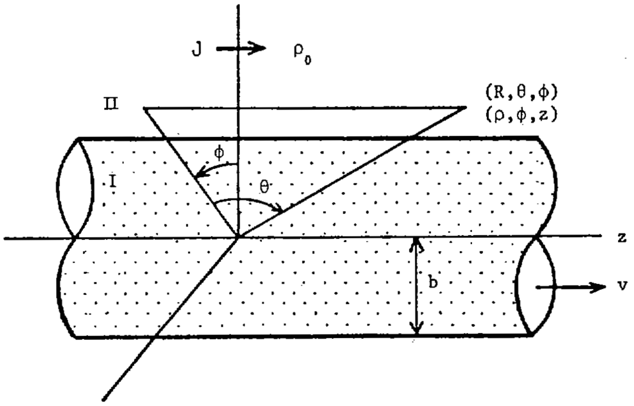


Fig. 1. Geometry of the problem.

In the region II, Maxwell's equations are

$$\nabla \times \mathbf{E}_2 = -j\omega\mu_0 \mathbf{H}_2 \tag{5a}$$
$$\nabla \times \mathbf{H}_2 = j\omega\epsilon_0 \mathbf{E}_2 + \mathbf{J} \tag{5b}$$
$$\nabla \cdot \mathbf{E}_2 = q/\epsilon_0 \tag{5c}$$
$$\nabla \cdot \mathbf{H}_2 = 0, \tag{5d}$$

where the subscript 2 denotes the fields in the region II and $\mathbf{J}$ and $q$ are the source current and charge density, respectively.

Though the primary fields of the dipole are expressed by electric type only, the fields in the region I should also have the magnetic type due to the polarization effect of the scattering on the boundary. Thus, they are written as

$$\mathbf{E}_1 = \mathbf{E}_2{}^{(e)} + \mathbf{E}_1{}^{(m)} \tag{6a}$$
$$\mathbf{H}_1 = \mathbf{H}_1{}^{(e)} + \mathbf{H}_1{}^{(m)}, \tag{6b}$$

where the superscript $(e)$ and $(m)$ denote the electric-type and magnetic-type field. In (6a) and (6b) $\mathbf{H}_1{}^{(e)}$ and $\mathbf{E}_1{}^{(m)}$ are written in terms of the electric type of the vector potential $\mathbf{A}_1{}^{(e)}$ and the magnetic type of vector potential $\mathbf{A}_1{}^{(m)}$ as follows:

$$\mathbf{H}_1{}^{(e)} = \bar{\alpha}^{-1} \cdot [(\nabla - j\omega\Omega) \times (\bar{\alpha}^{-1} \cdot \mathbf{A}_1{}^{(e)})] \tag{7a}$$
$$\mathbf{E}_1{}^{(m)} = -\bar{\alpha}^{-1} \cdot [(\nabla - j\omega\Omega) \times (\bar{\alpha}^{-1} \cdot \mathbf{A}_1{}^{(m)})]. \tag{7b}$$

On the other hand $\mathbf{E}_1{}^{(e)}$ and $\mathbf{H}_1{}^{(m)}$ are written in terms of the electric scalar potential $\phi_1{}^{(e)}$ and the magnetic scalar potential $\phi_1{}^{(m)}$ as follows:

$$\mathbf{E}_1{}^{(e)} = -j\omega\mu_0\bar{\alpha}^{-1} \cdot \mathbf{A}_1{}^{(e)} - (\nabla - j\omega\Omega)\phi_1{}^{(e)} \tag{8a}$$
$$\mathbf{H}_1{}^{(m)} = -j\omega\epsilon_0\epsilon\bar{\alpha}^{-1} \cdot \mathbf{A}_1{}^{(m)} - (\nabla - j\omega\Omega)\phi_1{}^{(m)}. \tag{8b}$$

Introducing (7a), (8a) into (3a) and then (7b), (8b) into (3b), the Gauge relation is obtained.

$$(\nabla - j\omega\Omega) \cdot \mathbf{A}_1{}^{(e)} + j\omega\epsilon_0\epsilon a^2\phi_1{}^{(e)} = 0 \tag{9a}$$
$$(\nabla - j\omega\Omega) \cdot \mathbf{A}_1{}^{(m)} + j\omega\mu_0 a^2\phi_1{}^{(m)} = 0 \tag{9b}$$

From (1) through (9) the wave equations for the vector potential are derived.

$$[(\nabla - j\omega\Omega) \cdot \bar{\alpha} \cdot (\nabla - j\omega\Omega) + a^2k^2] \begin{Bmatrix} \mathbf{A}_1{}^{(e)} \\ \mathbf{A}_1{}^{(m)} \end{Bmatrix} = 0 \tag{10}$$

where

$$k^2 = \omega^2\mu_0\epsilon_0\epsilon = \omega^2\mu_0\epsilon_0(1 - \omega_p{}^2/\omega'^2) = k_0{}^2\epsilon. \tag{11}$$

For the source current $\mathbf{J}$ given by (1), the electromagnetic fields are expressed by only the z-component of the vector potential $\mathbf{A}_1{}^{(e)}, \mathbf{A}_1{}^{(m)}$. Then the wave equations are

$$\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left\{\rho\frac{\partial}{\partial\rho}\right\} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2} + \frac{1}{a}\frac{\partial^2}{\partial z^2} - \frac{2j\omega\Omega}{a}\frac{\partial}{\partial z} - \frac{\omega^2\Omega^2}{a} + k^2a\right]\begin{Bmatrix} A_1{}^{(e)} \\ A_1{}^{(m)} \end{Bmatrix} = 0. \tag{12}$$

Equations (7) and (8) are rewritten in component form as follows:

$$\begin{Bmatrix} E_{1\rho}{}^{(e)} \\ H_{1\rho}{}^{(m)} \end{Bmatrix} = \frac{1}{j\omega\mu_0\epsilon_0\epsilon a^2}\left(\frac{\partial^2}{\partial\rho\partial z} - j\omega\Omega\frac{\partial}{\partial\rho}\right)\begin{Bmatrix} \mu_0 A_1{}^{(e)} \\ \epsilon_0\epsilon A_1{}^{(m)} \end{Bmatrix} \tag{13a}$$

$$\begin{Bmatrix} E_{1\phi}{}^{(e)} \\ H_{1\phi}{}^{(m)} \end{Bmatrix} = \frac{1}{j\omega\mu_0\epsilon_0\epsilon a^2} \cdot \frac{1}{\rho}\left(\frac{\partial^2}{\partial\phi\partial z} - j\omega\Omega\frac{\partial}{\partial\phi}\right) \cdot \begin{Bmatrix} \mu_0 A_1{}^{(e)} \\ \epsilon_0\epsilon A_1{}^{(m)} \end{Bmatrix} \tag{13b}$$

$$\begin{Bmatrix} E_{1z}{}^{(e)} \\ H_{1z}{}^{(m)} \end{Bmatrix} = \frac{1}{j\omega\mu_0\epsilon_0\epsilon a^2}\left(\frac{\partial^2}{\partial z^2} - 2j\omega\Omega\frac{\partial}{\partial z} - \omega^2\Omega^2 + k^2a^2\right)\begin{Bmatrix} \mu_0 A_1{}^{(e)} \\ \epsilon_0\epsilon A_1{}^{(m)} \end{Bmatrix} \tag{13c}$$

$$\begin{Bmatrix} H_{1\rho}{}^{(e)} \\ E_{1\rho}{}^{(m)} \end{Bmatrix} = \frac{1}{a}\frac{1}{\rho}\frac{\partial}{\partial\phi}\begin{Bmatrix} A_1{}^{(e)} \\ -A_1{}^{(m)} \end{Bmatrix} \tag{13d}$$

$$\begin{Bmatrix} H_{1\phi}{}^{(e)} \\ E_{1\phi}{}^{(m)} \end{Bmatrix} = -\frac{1}{a}\frac{\partial}{\partial\rho}\begin{Bmatrix} A_1{}^{(e)} \\ -A_1{}^{(m)} \end{Bmatrix}. \tag{13e}$$

Secondly, the electromagnetic fields and the wave equations in the region II are obtained in a similar way. They are

$$\mathbf{E}_2 = \mathbf{E}_2{}^{(e)} + \mathbf{E}_2{}^{(m)} \tag{14a}$$
$$\mathbf{H}_2 = \mathbf{H}_2{}^{(e)} + \mathbf{H}_2{}^{(m)} \tag{14b}$$

$$\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left\{\rho\frac{\partial}{\partial\rho}\right\} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2} + \frac{\partial^2}{\partial z^2} + k_0{}^2\right]\begin{Bmatrix} A_2{}^{(e)} \\ A_2{}^{(m)} \end{Bmatrix} = \begin{Bmatrix} -J\delta(\rho - \rho_0)\delta(\phi)\delta(z)/2\pi\rho \\ 0 \end{Bmatrix} \tag{15}$$

$$\begin{Bmatrix} E_{2\rho}{}^{(e)} \\ H_{2e}{}^{(m)} \end{Bmatrix} = \frac{1}{j\omega\mu_0\epsilon_0}\frac{\partial^2}{\partial\rho\partial z}\begin{Bmatrix} \mu_0 A_2{}^{(e)} \\ \epsilon_0 A_2{}^{(m)} \end{Bmatrix} \tag{16a}$$

$$\begin{Bmatrix} E_{2\phi}{}^{(e)} \\ H_{2\phi}{}^{(m)} \end{Bmatrix} = \frac{1}{j\omega\mu_0\epsilon_0}\frac{1}{\rho}\frac{\partial^2}{\partial\phi\partial z}\begin{Bmatrix} \mu_0 A_2{}^{(e)} \\ \epsilon_0 A_2{}^{(m)} \end{Bmatrix} \tag{16b}$$

$$\begin{Bmatrix} E_{2z}^{(e)} \\ H_{2z}^{(m)} \end{Bmatrix} = \frac{1}{j\omega\mu_0\epsilon_0}\left(\frac{\partial^2}{\partial z^2} + \omega^2\mu_0\epsilon_0\right)\begin{Bmatrix} \mu_0 A_2^{(e)} \\ \epsilon_0 A_2^{(m)} \end{Bmatrix} \quad (16c)$$

$$\begin{Bmatrix} H_{2\rho}^{(e)} \\ E_{2\rho}^{(m)} \end{Bmatrix} = \frac{1}{\rho}\frac{\partial}{\partial\phi}\begin{Bmatrix} A_2^{(e)} \\ -A_2^{(m)} \end{Bmatrix} \quad (16d)$$

$$\begin{Bmatrix} H_{2\phi}^{(e)} \\ E_{2\phi}^{(m)} \end{Bmatrix} = -\frac{\partial}{\partial\rho}\begin{Bmatrix} A_2^{(e)} \\ -A_2^{(m)} \end{Bmatrix}. \quad (16d)$$

Applying the Fourier transformation to the wave equation (12) we have

$$\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left\{\rho\frac{\partial}{\partial\rho}\right\} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2} - \frac{1}{a}h^2 - \frac{2\omega\Omega h}{a} - \frac{\omega^2\Omega^2}{a} + k^2 a\right]\begin{Bmatrix} \bar{A}_1^{(e)} \\ \bar{A}_1^{(m)} \end{Bmatrix} = 0 \quad (17)$$

$$[A_1^{(e)}, A_1^{(m)}] = \frac{1}{2\pi}\int_{-\infty}^{\infty} [\bar{A}_1^{(e)}, \bar{A}_1^{(m)}] e^{-jhz}\, dh \quad (18a)$$

$$[\bar{A}_1^{(e)}, \bar{A}_1^{(m)}] = \int_{-\infty}^{\infty} [A_1^{(e)}, A_1^{(m)}] e^{jhz}\, dz. \quad (18b)$$

Substituting the following relation for the isotropic moving plasma

$$[h + \omega\Omega]^2 - a^2 k^2 = a(h^2 - k_e^2) \quad (19)$$

$$k_e^2 = \omega^2\mu_0\epsilon_0(1 - \omega_p^2/\omega^2) = k_0^2(1 - \omega_p^2/\omega^2) \quad (20)$$

$$\xi_e^2 = k_e^2 - h^2 \quad (21)$$

we obtain the simpler form

$$\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left\{\rho\frac{\partial}{\partial\rho}\right\} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2} + \xi_e^2\right]\begin{Bmatrix} \bar{A}_1^{(e)} \\ \bar{A}_1^{(m)} \end{Bmatrix} = 0. \quad (22)$$

$A_1^{(e)}$ and $A_1^{(m)}$ are expanded in Fourier series by considering the periodicity in angle $\phi$ as follows:

$$[\bar{A}_1^{(e)}, \bar{A}_1^{(m)}] = \frac{1}{2\pi}\sum_{n=-\infty}^{\infty} [\bar{A}_{1n}^{(e)}, \bar{A}_{1n}^{(m)}] e^{jn\phi}. \quad (23)$$

Consequently the wave equations in the region I are given by

$$\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left\{\rho\frac{\partial}{\partial\rho}\right\} - \frac{n^2}{\rho^2} + \xi_e^2\right]\begin{Bmatrix} \bar{A}_{1n}^{(e)} \\ \bar{A}_{1n}^{(m)} \end{Bmatrix} = 0, \quad (24)$$

where $A_1^{(e)}$ and $A_1^{(m)}$ are expressed by the following integral form

$$[A_1^{(e)}, A_1^{(m)}] = \frac{1}{4\pi^2}\int_{-\infty}^{\infty} e^{-jhz}\, dh \cdot \sum_{n=-\infty}^{\infty} [\bar{A}_{1n}^{(e)}, \bar{A}_{1n}^{(m)}] e^{jn\phi}. \quad (25)$$

Similarly in the region II

$$\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left\{\rho\frac{\partial}{\partial\rho}\right\} - \frac{n^2}{\rho^2} + \xi_0^2\right]\begin{Bmatrix} \bar{A}_{2n}^{(e)} \\ \bar{A}_{2n}^{(m)} \end{Bmatrix} = \begin{Bmatrix} \dfrac{-J\delta(\rho - \rho_0)}{2\pi\rho} \\ 0 \end{Bmatrix}, \quad (26)$$

where

$$\xi_0^2 = k_0^2 - h^2, \quad (27)$$

and the corresponding formulas to (25) are

$$[A_2^{(e)}, A_2^{(m)}] = \frac{1}{4\pi^2}\int_{-\infty}^{\infty} e^{-jhz}\, dh \cdot \sum_{n=-\infty}^{\infty} [\bar{A}_{2n}^{(e)}, \bar{A}_{2n}^{(m)}] e^{jn\phi}. \quad (28)$$

The vector potentials which satisfy (25) are

$$[A_1^{(e)}, A_1^{(m)}] = \frac{-jJ}{16\pi^2}\int_{-\infty}^{\infty} e^{-jhz}\, dh \cdot \sum_{n=-\infty}^{\infty} [F_n, C_n] J_n(\xi_e\rho) e^{jn\phi}, \quad (\rho < b) \quad (29)$$

In the region II, the scattered and primary wave are superposed and thus the vector potential relating to those can be expressed in a different way.

$$[A_2^{(e)s}, A_2^{(m)s}] = \frac{-jJ}{16\pi^2}\int_{-\infty}^{\infty} e^{-jhz}\, dh \sum_{n=-\infty}^{\infty} [D_n, B_n] \cdot H_n^{(2)}(\xi_0\rho) e^{jn\phi}, \quad (\rho > b) \quad (30)$$

$$A_2^{(e)p} = \begin{cases} \dfrac{-jJ}{16\pi^2}\displaystyle\int_{-\infty}^{\infty} e^{-jhz}\, dh \sum_{n=-\infty}^{\infty} H_n^{(2)}(\xi_0\rho) \cdot J_n(\xi_0\rho_0) e^{jn\phi}, & (\rho > \rho_0) \quad (31a) \\ \dfrac{-jJ}{16\pi^2}\displaystyle\int_{-\infty}^{\infty} e^{-jhz}\, dh \sum_{n=-\infty}^{\infty} H_n^{(2)}(\xi_0\rho_0) \cdot J_n(\xi_0\rho) e^{jn\phi}, & (\rho < \rho_0), \quad (31b) \end{cases}$$

where $s$ and $p$ denote the scattered-wave and the primary-wave part, respectively. The applying the boundary conditions [15]

$$E_{1\phi} = E_{2\phi}^{p} + E_{2\phi}^{s} \quad (32a)$$

$$E_{1z} = E_{2z}^{p} + E_{2z}^{s} \quad (32b)$$

$$H_{1\phi} = H_{2\phi}^{p} + H_{2\phi}^{s} \quad (32c)$$

$$H_{1z} = H_{2z}^{s}. \quad (32d)$$

TABLE I
ELEMENTS OF MATRIX OF COEFFICIENTS

| $a_m$ | $b_m$ | $c_m$ | $d_m$ | $f_m$ |
|---|---|---|---|---|
| $\frac{1}{j\omega\mu_0}\frac{nh}{b}H_n^{(2)}(\xi_0 b)$ | $-\frac{n(h+\omega\Omega)}{j\omega\mu_0 a^2 b}J_n(\xi_e b)$ | $-\xi_0 H_n^{(2)\prime}(\xi_0 b)$ | $\frac{1}{a}\xi_e J_n'(\xi_e b)$ | $H_n^{(2)}(\xi_0\rho_0)\xi_0 J_n'(\xi_0 b)$ |
| 0 | 0 | $\xi_0^2 H_n^{(2)}(\xi_0 b)$ | $-\frac{1}{\epsilon a}\xi_e^2 J_n(\xi_e b)$ | $-\xi_0^2 H_n^{(2)}(\xi_0\rho_0)J_n(\xi_0 b)$ |
| $\xi_0 H_n^{(2)\prime}(\xi_0 b)$ | $-\frac{1}{a}\xi_e J_n'(\xi_e b)$ | $\frac{1}{j\omega\epsilon_0}\frac{nh}{b}H_n^{(2)}(\xi_0 b)$ | $-\frac{1}{j\omega\epsilon_0\epsilon}\frac{n(h+\omega\Omega)}{a^2 b}J_n(\xi_e b)$ | $-\frac{nh}{j\omega\epsilon_0 b}H_n^{(2)}(\xi_0\rho_0)J_n(\xi_0 b)$ |
| $\xi_0^2 H_n^{(2)}(\xi_0 b)$ | $\frac{1}{a}\xi_e^2 J_n(\xi_e b)$ | 0 | 0 | 0 |

the following set of equations in the coefficients $B_n - F_n$ is obtained.

$$\begin{bmatrix} a_1 & b_1 & c_1 & d_1 \\ 0 & 0 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ a_4 & b_4 & 0 & 0 \end{bmatrix} \begin{bmatrix} B_n \\ C_n \\ D_n \\ F_n \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ 0 \end{bmatrix}. \tag{33}$$

Each element is tabulated in Table I. The primes denote differentiation with respect to the argument. The electromagnetic fields in both the regions can be evaluated completely with the aid of (33).

## III. FAR FIELD

The contribution of the vector potential to the radiation field can be obtained by the usual saddle-point method. When the medium moves at a relativistic velocity, the pole and the saddle point in (29)–(31) often become close to each other. Consequently, in that case the application of the saddle-point method is difficult (e.g., if the medium surrounding the source is a dielectric, this difficulty appears at a relativistically large velocity). If a moving medium is an isotropic plasma whose dielectric constant is given by the pole (2), which gives the propagation constant of the surface waves, it always exists in the region $|h/k_0| > 1$ according to the relation (19) in the same manner as in the case of the stationary plasma. Accordingly, the pole and the saddle point will not become close to each other and so the saddle-point method is applicable to the present case in much the same way as for the stationary plasma. And the far fields are obtained by applying the saddle-point method to (30) and (31a) and then applying the results to (16). The subscript 2 is deleted hereafter. The far field of the electric type consists of two parts: the electromagnetic field of the primary wave and that of the scattered wave. They are

$$E_\theta{}^{(e)} = E_\theta{}^{(e)p} + E_\theta{}^{(e)s}$$

$$= -\frac{J}{8\pi^2}\sqrt{\frac{\mu_0}{\epsilon_0}}\frac{\omega_p}{c}(\Phi)^{(e)}(\theta,\phi)\frac{e^{-jk_0 R}}{R} \tag{34a}$$

$$H_\phi{}^{(e)} = \sqrt{\frac{\epsilon_0}{\mu_0}}E_\theta{}^{(e)}. \tag{34b}$$

On the other hand, the fields of the magnetic type have only the scattered wave

$$E_\phi{}^{(m)} = E_\phi{}^{(m)s} = -\frac{J}{8\pi^2}\sqrt{\frac{\mu_0}{\epsilon_0}}\frac{\omega_p}{c}(\Phi)^{(m)}(\theta,\phi)\frac{e^{-jk_0 R}}{R} \tag{35a}$$

$$H_\theta{}^{(m)} = -\sqrt{\frac{\epsilon_0}{\mu_0}}E_\phi{}^{(m)}, \tag{35b}$$

where $\Phi^{(e)}$ and $\Phi^{(m)}$ are

$$\Phi^{(e)}(\theta,\phi) = \frac{\omega}{\omega_p}\cos\theta\cdot\sum_{n=0}^{\infty}(2\cos n\phi - \delta_{n,0})\cdot e^{-j(n+1)\pi/2}\cdot[J_n(k_0\rho_0\cos\theta) + D_{ns}] \tag{36a}$$

$$\Phi^{(m)}(\theta,\phi) = -\frac{\omega}{\omega_p}\cos\theta\cdot\sum_{n=0}^{\infty}2j\sin n\phi\cdot e^{-j(n+1)\pi/2}\cdot B_{ns}, \tag{36b}$$

where $\delta_{n,0}$ is the Kronecker delta. Then the Poynting vector for the far field is given as

$$\langle\mathbf{S}\rangle = \tfrac{1}{2}\,\mathrm{Re}\,[\mathbf{E}\times\mathbf{H}^*] = \frac{1}{2}\sqrt{\frac{\epsilon_0}{\mu_0}}[|E_\theta{}^{(e)}|^2 + |E_\phi{}^{(m)}|^2]\,\mathbf{i}_R, \tag{37}$$

where $\mathbf{i}_R$ is the unit vector of the radial direction. The radial part of $\langle S\rangle$ and the scattering pattern $F(\theta,\phi)$ are described by using (36a) and (36b) as

$$\langle S_R\rangle = \frac{1}{2}\sqrt{\frac{\mu_0}{\epsilon_0}}\left(\frac{J}{8\pi^2 R}\frac{\omega_p}{c}\right)^2\cdot[|(\Phi)^{(e)}(\theta,\phi)|^2 + |(\Phi)^{(m)}(\theta,\phi)|^2] \tag{38a}$$

$$= \frac{1}{2}\sqrt{\frac{\mu_0}{\epsilon_0}}\left(\frac{J}{8\pi^2 R}\frac{\omega_p}{c}\right)^2 F(\theta,\phi). \tag{38b}$$

For the sake of convenience we introduce the parameter $M, N, L$, and $\lambda$ as follows:

$$L = \frac{\omega_p}{c}\rho_0 \tag{39a}$$

$$M = \frac{\omega_p}{c} b = \text{normalized radius} \tag{39b}$$

$$N = \frac{\omega}{\omega_p} = \text{normalized frequency} \tag{39c}$$

$$\lambda = \sqrt{|\cos^2\theta - N^{-2}|} \tag{39d}$$

$$\rho = R\cos\theta \tag{40a}$$

$$z = R\sin\theta\left(-\frac{\pi}{2} < \theta < \frac{\pi}{2}\right). \tag{40b}$$

For $\cos^2\theta - N^{-2} > 0$, $B_{ns}$, and $D_{ns}$ are written by using (39) and (40) as

$$\begin{aligned} D_{ns} = {} & H_n^{(2)}(LN\cos\theta)\Bigg[\{J_n(MN\lambda)\}^2 \\ & \cdot \Bigg[n^2\frac{\lambda^2}{\cos^2\theta}\left(\sin\theta - \frac{\sin\theta + c\Omega}{a}\frac{\cos^2\theta}{\lambda^2}\right)^2 \\ & \cdot H_n^{(2)}(MN\cos\theta)J_n(MN\cos\theta) - \frac{\lambda^2}{\cos^2\theta} \\ & \cdot \{nH_n^{(2)}(MN\cos\theta) - MN\cos\theta H_{n+1}^{(2)}(MN\cos\theta)\} \\ & \cdot \{nJ_n(MN\cos\theta) - MN\cos\theta J_{n+1}(MN\cos\theta)\}\Bigg] \\ & + J_n(MN\lambda)\{nJ_n(MN\lambda) - MN\lambda J_{n+1}(MN\lambda)\} \\ & \cdot [\epsilon\{nH_n^{(2)}(MN\cos\theta) - MN\cos\theta H_{n+1}^{(2)} \\ & \cdot (MN\cos\theta)\}\, J_n(MN\cos\theta) + H_n^{(2)}(MN\cos\theta) \\ & \cdot \{nJ_n(MN\cos\theta) - MN\cos\theta J_{n+1}(MN\cos\theta)\}] \\ & - \epsilon\frac{\cos^2\theta}{\lambda^2}\{nJ_n(MN\lambda) - MN\lambda J_{n+1}(MN\lambda)\}^2 \\ & \cdot H_n^{(2)}(MN\cos\theta)J_n(MN\cos\theta)\Bigg] \Big/ D_{e1}, \end{aligned} \tag{41a}$$

$$\begin{aligned} B_{ns} = {} & \frac{2n}{\pi}\frac{\lambda^2}{\cos^2\theta}\left\{\sin\theta - (\sin\theta + c\,\Omega)\frac{\cos^2\theta}{a\lambda^2}\right\} \\ & \cdot H_n^{(2)}(LN\cos\theta)\{J_n(MN\lambda)\}^2/D_{e1}, \end{aligned} \tag{41b}$$

$$\begin{aligned} D_{e1} = {} & \frac{\lambda^2}{\cos^2\theta}\{[nH_n^{(2)}(MN\cos\theta) - MN\cos\theta H_{n+1}^{(2)} \\ & \cdot (MN\cos\theta)]\, J_n MN\lambda)\}^2 \\ & + \epsilon\frac{\cos^2\theta}{\lambda^2}\{[nJ_n(MN\lambda) - MN\lambda J_{n+1}(MN\lambda)] \\ & \cdot H_n^{(2)}(MN\cos\theta)\}^2 - \frac{n^2\lambda^2}{\cos^2\theta} \\ & \cdot \left(\sin\theta - (\sin\theta + c\Omega)\frac{\cos^2\theta}{a\lambda^2}\right)^2 \\ & \cdot \{J_n(MN\lambda)H_n^{(2)}(MN\cos\theta)\}^2 \\ & - (\epsilon + 1)\{nH_n^{(2)}(MN\cos\theta) - MN\cos\theta H_{n+1}^{(2)} \\ & \cdot (MN\cos\theta)\} \times \{nJ_n(MN\lambda) - MN\lambda J_{n+1}(MN\lambda)\} \\ & \cdot J_n(MN\lambda)H_n^{(2)}(MN\cos\theta), \end{aligned} \tag{41c}$$

where $H_n^{(2)}$ is $n$th order of the second kind of Hankel function. For $\cos^2\theta - N^{-2} < 0$, the similar representations are obtained by replacing the real $\lambda$ with $j\lambda$, and replacing $J_n(MN\lambda)$ with $j^n I_n(MN\lambda)$, where $I_n$ is $n$th order of the first kind of the modified Bessel function.

## IV. NUMERICAL RESULTS

Let us discuss the scattering patterns of the dipole field by a moving cylindrical plasma column with the aid of numerical examples. The scattering pattern on two different cross sectional planes are illustrated. Pattern (i) is the one in a cross of the column, or $\theta = 0$ plane, and pattern (ii) is the one in a longitudinal section including the dipole and column's axis, or $\phi = 0$ plane. The scattering pattern has a strong dependence of the different choice of the parameters $M$, $N$, and $L$; and further on, whether the plasma is in motion or not. In order to know the effect of motion clearly we first show some numerical results of the scattering patterns for the stationary plasma ($\beta = 0$). Subsequently, as is noticed, the effect of motion appears clearly in the pattern (ii).

### A. Case for the Stationary Plasma

Let us discuss the scattering properties of the stationary plasma column with the aid of numerical examples. Certainly, when one deals with a nondispersive scattering medium such as a dielectric cylinder, the product of $M$ and $N$ often becomes the important parameter governing the feature of a scattering pattern and hence is a good parameter for the classification of the patterns [5]. In the present case the plasma cylinder is a dispersive medium, and therefore, the shape of the pattern is modified by the combination of each parameter $M, N$, and $L$. The scattering properties are rather complicated, as is seen in Fig. 2. However, we can see the effect of the parameter $M, N$, and $L$ on the scattering pattern more clearly by investigating the ratio of the backward scattering to the forward scattering $F(0, 0)/F(0, \pi)$. A new parameter $L/M$ gives some periodicity in the ratio $F(0, 0)/F(0, \pi)$, as is seen in Fig. 3. Expectantly the ratio approaches 1 with increasing $L/M$, suggesting the decrease of plasma's effect on the radiation from the electric dipole. Furthermore, it is found that the pattern (i) exhibits an eight-figured pattern for $F(0, 0)/F(0, \pi) < 1$, dipole-like char-

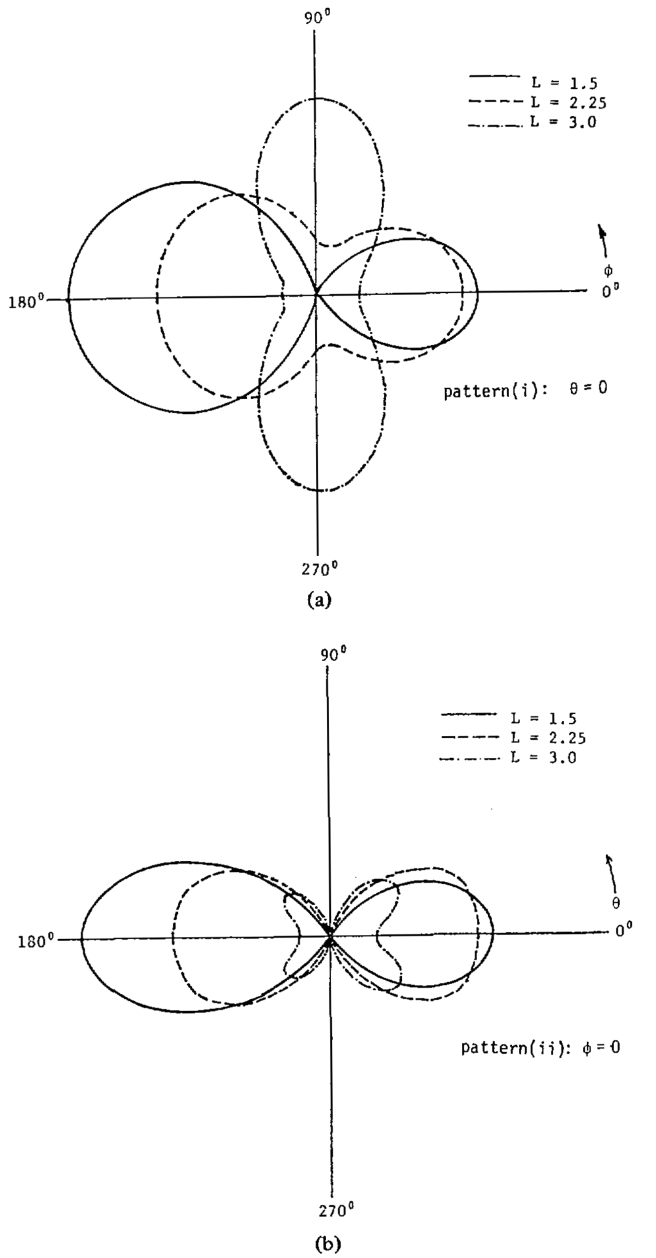


Fig. 2. Scattering pattern ($M = 1, N = 1.05, \beta = 0$).

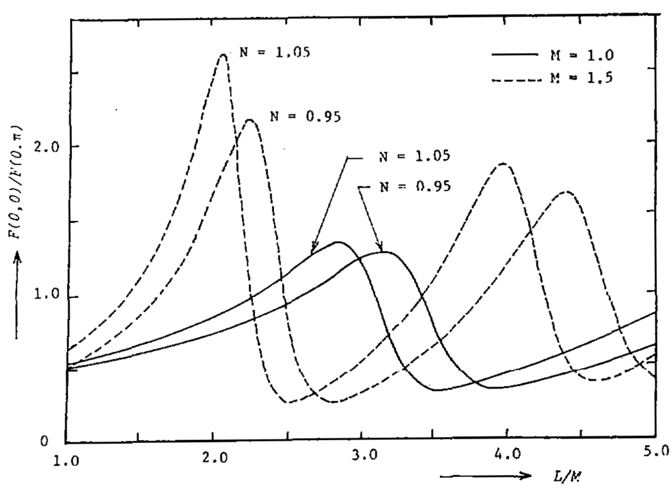


Fig. 3. Ratio $F(0, 0)/(F(0, \pi)$ versus $L/M$.

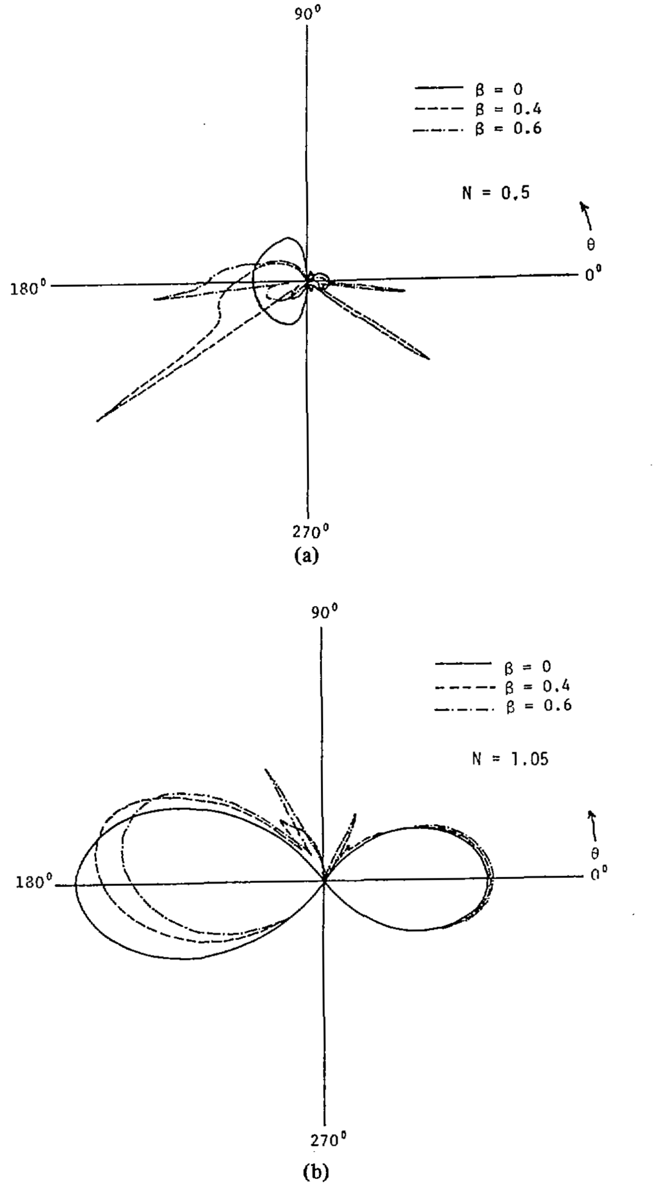


Fig. 4. Scattering pattern (ii) ($M = 1, N = 1.5$).

acter for $F(0, 0)/F(0, \pi) \cong 1$, and $\infty$-figured pattern for $F(0, 0)/F(0, \pi) > 1$. From the figure of the ratio $F(0, 0)/F(0, \pi)$ versus $L/M$ we can predict the shape of the scattering pattern for the various sets of the parameter $M, N$, and $L$.

### *B. Case for the Moving Plasma*

We discuss the scattering pattern of a dipole field by a moving plasma column with the aid of numerical calculations. Generally speaking the lobe of the scattering pattern shifts in the direction of the plasma motion. Pattern (ii) has a certain advantage over pattern (i) in grasping the physical feature of the pattern. Therefore, we concentrate our discussion on pattern (ii) particularly. As seen in Fig. 4 the patterns are well classified into groups having similar angle dependence in the scattering pattern according to the parameter $N$. It comes from the fact that (17) for the plasma in motion is written as (24), which does not include the term related to the plasma's motion; and consequently, the plasma medium in motion exhibits the entirely different physical nature whether the frequency of the incident electromagnetic waves is lower than the plasma frequency ($N < 1$) or higher ($N > 1$).

As seen in Fig. 4(a), which is a typical numerical example for $N < 1$, the directional patterns having a strong dependence on the value of $\beta$ appear almost symmetrical with respect to the column's axis. This symmetrical feature comes from the fact that each $n$th term of $\Phi^{(e)}$ in (36a) takes an equal, absolute value for the same value of $\theta$ whether $\phi = 0$, $\phi = \pi$. While, $\Phi^{(m)}$ gives 0 for the pattern(ii) because of nonpolarization in this plane. The directional pattern shifts strikingly with increasing $\beta$ from the direction $\theta = -\pi/2$ to the direction $\theta = \pi/2$. These tendencies are to be contrasted with Fig. 4(b), which is a typical numerical example for $N > 1$. In regard to the convergence of the series (36), sufficiently accurate results are obtained by taking terms up to tenth order even for the largest value of the product $M$ and $N$.

## V. CONCLUSION

The foregoing has addressed the problem of scattering of a dipole field by a moving plasma column. Numerical calculations of the effect of a moving plasma on the scattering pattern were presented, and it was found that the effect of the moving plasma appears more remarkably in the scattering patterns in the longitudinal section including both the plasma column's axis and the dipole rather than in the cross section of the plasma column. Furthermore, we have discussed the feature of the sharp directional lobe of scattering pattern which appears under certain conditions.

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