THEORY OF A THIN CYLINDRICAL ANTENNA IN AN ISOTROPIC MOVING PLASMA

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In the quasistatic approximation, the author computes the current distribution and input impedance of a thin cylindrical antenna in a plasma that moves at a constant nonrelativistic velocity along the antenna axis. The plasma is assumed to be nongyrotropic and homogeneous; thermal motion of charged particles is disregarded.

The theory of antennas in stationary plasmas with weak spatial dispersion has been taken up in a number of studies (see, e.g., [1-3]). At the same time it is of interest to establish a theory of a radiator in a moving plasma. In this paper we will investigate the characteristics of a thin cylindrical antenna (TCA) in a plasma that moves at constant velocity u ($v_{Te} \ll u \ll c$, v_{Te} is the mean electron thermal velocity and c is the speed of light) along the antenna axis; the plasma is assumed to be nongyrotropic and homogeneous; and thermal motion of the charged particles is not taken into account.

To determine the features of radiation in the medium in question, let us assume that the characteristic dimensions of the radiator are small as compared to the transverse electromagnetic wavelength $\lambda_\perp = (2\pi c/\omega)\sqrt{\varepsilon_0(\omega)}$, where ω is the radiation frequency, $\varepsilon_0(\omega) = 1 - \omega_0^2/\omega^2$ is the permittivity of the stationary plasma, and ω_0 is the plasma frequency. Then we can use the quasistatic approximation and thus assume that the electric field is potential so that $\mathbf{E} = -\nabla\varphi$, the potential φ satisfying the equation

$$\hat{\varepsilon}(\omega, \boldsymbol{k})\Delta\varphi = -4\pi\rho. \tag{1}$$

Here $\rho(\mathbf{r})$ is the charge distribution and $\hat{\varepsilon}(\omega, \mathbf{k})$ is the plasma permittivity operator, which has a particular numerical value for processes that have a $\exp[i(\mathbf{kr} - \omega t)]$ dependence on the coordinates and time. The potential φ can be readily obtained from (1) by convolution with respect to plane waves of the form $e^{i\mathbf{kr}}$, i.e.,

$$\varphi(\boldsymbol{r}) = 4\pi \int \frac{\rho_{\boldsymbol{k}} e^{i\boldsymbol{kr}}}{k^2 \varepsilon(\omega, \boldsymbol{k})} d\boldsymbol{k}, \tag{2}$$

where $\rho_{\mathbf{k}}$ is the Fourier component of the external charge and $\varepsilon(\omega, \mathbf{k})$ is the longitudinal permittivity of the moving plasma [4]:

$$\varepsilon(\omega, \boldsymbol{k}) = 1 - \omega_0^2/(\omega - \boldsymbol{ku})^2.$$

Before attempting to investigate antenna characteristics, it makes sense to cite some results regarding the radiation of longitudinal waves by elementary sources, in particular, by a single point charge. For such a source we have $\rho(\mathbf{r}, t) = \delta(\mathbf{r} - \mathbf{r}')e^{-i\omega t}$ ($\mathbf{r}'$ being the radius vector of the charge). To evaluate the integral in (2), we introduce the cylindrical coordinate system $(k_\perp, \theta, k_z)$ in space $\mathbf{k}$, taking the 0z axis along the velocity vector of the flow. The integral with respect to k_z is evaluated by changing to the plane of the complex variable k_z. To determine the rules for bypassing poles that lie on the real axis, we introduce weak absorption which we then let tend to zero. Ultimately the Green's function for Eq. (1) can be written in the form

$$G(\boldsymbol{R}) = G_e(\boldsymbol{R}) + G_i(\boldsymbol{R})\cdot 1(z - z') \quad (\boldsymbol{R} = \boldsymbol{r} - \boldsymbol{r}'); \tag{3}$$

$$G_e(\boldsymbol{R}) = \frac{1}{R} - \frac{ik_0}{2}[S_1(\boldsymbol{R}) - S_2(\boldsymbol{R})]; \tag{4}$$

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$$G_i(R) = ik_0\left[K_0(k_1\rho)e^{ik_1(z-z')} - K_0(|k_2|\rho)e^{ik_2(z-z')}\right], \tag{5}$$

where $1(z - z')$ is a unit function. Here we have employed the following notation:

$$S_j \equiv S_j(\rho, z - z') = \int_0^\infty \frac{J_0(k_\perp \rho)e^{-k_\perp|z-z'|}dk_\perp}{k_\perp + i|k_j|} \quad (j = 1, 2); \tag{6}$$

$$k_0 = \omega_0/u, \quad k_1 = k + k_0, \quad k_2 = k - k_0, \quad k = \omega/u, \\ \rho^2 = (x - x')^2 + (y - y')^2, \quad R^2 = \rho^2 + (z - z')^2, \tag{7}$$

and $J_0(x)$ and $K_0(x)$ are Bessel and Macdonald functions, respectively. Furthermore, by definition, we will call $G_e(R)$ the quasistatic Green's function (or the quasistatic field), and $G_i(R)$ the plasma-wave Green's function.

It can be seen from (3)-(5) that in the half-space $z > z'$ there is a trace or "track" of the plasma oscillations that does not attenuate along the z axis. The parameter $\lambda_j \sim |k_j|^{-1}$ defines the characteristic scale of the inhomogeneities in the quasistatic field $G_e(R)$. On this scale the field drops off with increasing distance from the axis on which the charge is located. This same scale determines the oscillation wavelength on the "track." The plasma trace results from interference of two waves with wave numbers k_1 and k_2, respectively. In the limiting cases of plasma movement with very low ($u \to 0$) and very high ($u \to \infty$) velocities, we obtain from (4) and (5) the familiar expressions for a static charge field: $G = (\varepsilon_0 R)^{-1}$ and $G = R^{-1}$.

We should note that the function $S_j(\rho, z - z')$ can be represented in more tractable fashion. Differentiating (6) with respect to $|z - z'|$, we can create a differential equation for S_j that can readily be solved by the constant variation method [5], and we find [6]

$$S_j(R) = -\exp[-i|k_j(z - z')|] \int_\infty^{|k_j(z-z')|} \frac{e^{it}\,dt}{\sqrt{t^2 + k_j^2\rho^2}}, \tag{8}$$

or, integrating once by parts,

$$S_j(R) = -\frac{1}{ik_jR} + \exp[-i|k_j(z - z')|]\,T_j(|k_j|\rho, |k_j(z - z')|), \\ T_j = \int_\infty^{|k_j(z-z')|} \frac{te^{it}\,dt}{(t^2 + k_j^2\rho^2)^{3/2}}. \tag{9}$$

Then the quasistatic Green's function can be written as follows:

$$G_e(R) = (\varepsilon_0 R)^{-1} + k_0 G_e'(R)/2, \quad G_e'(R) = \exp(-ik_1|z - z'|) \\ \times T_1(k_1\rho, k_1|z - z'|) - \exp[-i|k_2(z - z')|]\,T_2(|k_2|\rho, |k_2(z - z')|). \tag{10}$$

The results enable us to solve the problem of finding the electric field of an arbitrary radiator in a moving plasma, namely,

$$\varphi(r) = \int G(r - r')\rho(r')\,dr', \tag{11}$$

where $G(\mathbf{r} - \mathbf{r}')$ is a Green's function, $\rho(\mathbf{r})$ is the volumetric charge density, and the integration is performed over the source volume.

As already noted, we regard the antenna as being thin, i.e., $a/2L \ll 1$, $k_j|a \ll 1$ (a and $2L$ are, respectively, the cylinder radius and length), and we assume that the plasma moves along the antenna axis. If we take the conductor to be ideal, as is usually done in the theory of thin antennas, we can specify the charge density in the form $\rho(\mathbf{r}) = \delta(\mathbf{r} - a)\sigma(z)/2\pi a$, where the coordinate origin is taken to be at the antenna center, the 0z coordinate axis is directed along the antenna axis, $\mathbf{r}$ in cylindrical coordinates has components (r, ϑ, z), and $\sigma(z)$ is the linear charge density. Breaking down the integral with respect to z' in (11) into two integrals with limits of integration from $-L$ to z and from z to $+L$, we can write an expression for the potential of the antenna field at an arbitrary point in space in the form

$$\varphi(r, z) = \frac{1}{2\pi\varepsilon_0}\int_0^{2\pi} d\vartheta \int_{-L}^{L} \frac{\sigma(z')\,dz'}{R} + \frac{k_0}{4\pi}\int_0^{2\pi} d\vartheta \int_{-L}^{L} G_e'(r - r')\sigma(z')\,dz' + \int_{-L}^{z} G_i(r, z - z')\sigma(z')\,dz' \tag{12}$$

for $r \geq a$ and $\varphi(r, z) = \text{const}$ for $r < a$. Here we have allowed for the fact that $\rho^2 = r^2 + a^2 - 2 a r \cos\vartheta$ in the cylindrical coordinate system, and we have used the "addition" theorem [7] for the function $K_0(|k_j|\rho)$; the expression for $G_i(r, z - z')$ differs from (5) in that ρ has been replaced by r.

Now we will obtain an equation which relates the charge induced on the antenna to the external emf. For this we should set $r = a$ in (12). As $a \to 0$ we replace the Macdonald function by the approximation $K_0(x) \approx \ln(2/\gamma x)$, $|x| \ll 1$ ($\gamma \approx 1.78$ is the Euler–Mascheroni constant) and we neglect $k_j^2 a^2$ in the denominator of the integrand in (9). To evaluate the static term

$$I = \frac{1}{2\pi\varepsilon_0} \int_0^{2\pi} d\vartheta \int_{-L}^{L} \frac{\sigma(z')\,dz'}{R}, \quad R^2 = (z - z')^2 + 4a^2 \sin^2 \frac{\vartheta}{2}$$

we set $\sigma(z') = \sigma(z) + [\sigma(z') - \sigma(z)]$. We can neglect the term with a^2 in R in the member containing the difference $\sigma(z') - \sigma(z)$. Allowing for the fact that $L \gg a$, and considering points that are not too close to the ends of the antenna, we have

$$I = \sigma(z) \ln \frac{4L^2}{a^2} + \sigma(z) \ln\left(1 - \frac{z^2}{L^2}\right) + \int_{-L}^{L} \frac{\sigma(z') - \sigma(z)}{|z - z'|}\,dz'$$

(where we have used the familiar relation [6] $\int_0^{\pi} \ln \sin\vartheta\, d\vartheta = -\pi \ln 2$). In this expression we can neglect the second term on the right side, since it is important only near the ends of the antenna $|z| \sim L$; this region of z values is inconsequential for computing the antenna characteristics. We can also clearly discard the third term, since it does not contain large parameters. Finally, using the boundary condition $\varphi(a, z) = \Phi(z)$ $(-L < z < L)$, where $\Phi(z)$ is the specified potential distribution on the conductor surface, we can write the equation of a TCA in a moving plasma as follows:

$$B\sigma(z)/\varepsilon_0(\omega) + \Phi_i[\sigma, z] + \Phi_e'[\sigma, z] = \Phi(z), \tag{13}$$

where

$$\Phi_i[\sigma, z] = ik_0 \int_{-L}^{z} M(z - z')\sigma(z')\,dz', \quad M(z) = A_1 e^{ik_1 z} - A_2 e^{ik_2 z}; \tag{14}$$

$$\Phi_e'[\sigma, z] = \frac{k_0}{4\pi} \int_0^{2\pi} d\vartheta \int_{-L}^{L} G_e'(r - r') \Big|_{r = a \to 0} \sigma(z')\,dz'; \tag{15}$$

$$A_j = \ln(2/\gamma|k_j|a), \quad B = \ln(4L^2/a^2). \tag{16}$$

Equation (13) contains three large parameters (16). The parameter B is static, while the parameters A_j are of the same order as in [8]. This becomes clear if we consider that Eq. (13) was obtained without allowing for the "intrinsic" spatial dispersion of the plasma and the problem does not contain parameters not determined by the antenna geometry. Depending on the relationships among the large parameters, we will investigate (13) in the limiting cases of "short" and "long" antennas (see below), and also in the general case.

1. "Short" antenna: $B \ll A_j$, or, equivalently, $|k_j| L \ll 1$ ($u \to \infty$). This inequality means that we are dealing with an antenna in a vacuum. In the zero approximation we can assume that σ is independent of z; we have $\Phi_i[\sigma, z] \approx 0$. We use (8) for the functions $S_j(R)$. Then the denominator of the static term [first term on the left side of (13)] will not contain $\varepsilon_0(\omega)$. Allowing for the familiar relations [6]

$$\int_0^{\infty} \frac{\sin ax\,dx}{\sqrt{\beta^2 + x^2}} = \frac{\pi}{2}[I_0(a\beta) - L_0(a\beta)] \qquad (a > 0,\ \mathrm{Re}\,\beta > 0),$$

$$\int_0^{\infty} \frac{\cos ax\,dx}{\sqrt{\beta^2 + x^2}} = K_0(a\beta) \qquad (a > 0,\ \mathrm{Re}\,\beta > 0),$$

where $I_0(x)$, $K_0(x)$, $L_0(x)$ are Bessel, Macdonald, and Struve functions, respectively, we obtain as $|k_j| L \to 0$ that $\Phi_e'[\sigma, z] \to 0$. Ultimately, Eq. (13) reduces to the algebraic equation

$$B\sigma(z) = \Phi(z). \tag{17}$$

The potential is defined to within an arbitrary constant. The constant is determined from the condition that the conductor be electrically neutral (for an emf that is periodic in time). We note that this condition is equivalent to the current at the antenna ends being zero, and conversely. If the external emf is specified as a delta-function $P(z) = P_0\delta(z)$ (P_0 = const), then for a linear current density

$$I(z) = i\omega \int_{-L}^{z} \sigma(z')\,dz'$$

we obtain, as was to be expected, a "triangular" distribution:

$$I(z) = I_0\left(1 - \frac{|z|}{L}\right), \quad I_0 = I(0) = \frac{i\omega P_0 L}{2B}. \tag{18}$$

Here the components of the input impedance

$$Z = P_0/I_0 = R + iX$$

are determined from the formulas

$$R = 0, \quad X = -2B/\omega L. \tag{19}$$

By solving the initial equation in the next approximation, we can obtain the radiation resistance of a "short" antenna; the expression is the same as Eq. (24) below for the radiation resistance of a "long" antenna and, as $|k_j|L \to 0$, it assumes the form

$$R \approx \frac{k_0 L^2}{2\omega}\left(k_2^2 A_2 - k_1^2 A_1\right). \tag{20}$$

2. "Long" antenna: $|k_j|L \gg 1$ $(u \to 0)$, i.e., the antenna is in an isotropic plasma with permittivity $\varepsilon_0(\omega)$. In this case $T_j \to 0$, and hence $\Phi'_e[\sigma, z] \to 0$. As a result, Eq. (13) can be written as follows:

$$\sigma(z) = \mu\{\Phi(z) - \Phi_i[\sigma, z]\}, \tag{21}$$

where we have introduced the small parameter $\mu = \varepsilon_0/B$. Since $|\mu| \ll 1$, we will utilize the perturbation method and seek the solution of (21) in the form of a power series in μ:

$$\sigma = \sigma_0 + \mu\sigma_1 + \mu^2\sigma_2 + \ldots$$

Substituting this series in (21) and equating coefficients for equal powers of μ, we arrive at the following system of algebraic equations:

$$\begin{aligned} \sigma_0 &= 0, \\ \sigma_1 &= \Phi(z) - \Phi_i[\sigma_0, z], \\ \sigma_2 &= -\Phi_i[\sigma_1, z], \\ \sigma_3 &= -\Phi_i[\sigma_2, z]; \\ &\ldots\ldots\ldots \end{aligned} \tag{22}$$

The functional $\Phi_i[\sigma_0, z]$ vanishes in view of its linearity. Consequently the series begins with $\sigma_1(z)$. For a linear current density in first approximation we again obtain the obvious "triangular" distribution (18), but with amplitude $I_1(0) = i\omega P_0\varepsilon_0 L/2B$ proportional to the permittivity of a stationary plasma. Here the input resistance and reactance of the antenna are determined by the formulas

$$R = 0, \quad X = -2B/\omega\varepsilon_0 L. \tag{23}$$

To compute the real part of the impedance, we must solve the second-approximation equation of system (29). In this case, obviously, it suffices to compute $\mathrm{Re}\, I_2(0)$, since $\mathrm{Im}\, I_2(0)$ yields only a correction to the reactance. Computations lead to the result

$$R = -\frac{8k_0}{\omega L^2}\left(\frac{A_1}{k_1^2}\sin^4\frac{k_1 L}{2} - \frac{A_2}{k_2^2}\sin^4\frac{k_2 L}{2}\right). \tag{24}$$

It can be seen from (24) that, depending on the geometrical dimensions of the antenna, the radiation frequency, and the plasma parameters, the radiation resistance may change sign. Radiation in the region of anomalous Doppler frequencies may predominate over radiation in the region of normal frequencies if the following inequality holds:

$$(\omega + \omega_0)^2 \sin^4\frac{(\omega-\omega_0)L}{2u} \ln\frac{2u}{\gamma|\omega-\omega_0|a} < (\omega-\omega_0)^2 \sin^4\frac{(\omega+\omega_0)L}{2u} \ln\frac{2u}{\gamma(\omega+\omega_0)a}. \tag{25}$$

The above method of determining the input impedance can be conveniently checked by computing the radiation response of the antenna and then comparing the resultant expression with the radiated energy W determined in accordance with the above formulas, i.e.,

$$W = \frac{1}{2} P_0 \operatorname{Re} I(0), \quad I(0) = \mu^2 I_3(0). \tag{26}$$

Computing the radiation response from the formula

$$W = -\frac{1}{2} \operatorname{Re} \int_{-L}^{L} E_i[\sigma, z] I^*(z)\, dz, \quad E_i[\sigma, z] = -\frac{d\Phi_i[\sigma, z]}{dz},$$

where as $\sigma(z)$ and $I(z)$ we should take the solution of the equation in first approximation, (the asterisk denotes the complex conjugate), we obtain, after extremely laborious computations,

$$W = -\frac{\omega k_0 \varepsilon_0^2 P_0^2}{B^2}\left(\frac{A_1}{k_1^2}\sin^4\frac{k_1 L}{2} - \frac{A_2}{k_2^2}\sin^4\frac{k_2 L}{2}\right). \tag{27}$$

If we now compute W using (26), it turns out that the resultant expression is the same as (27).

3. In the general case, for an arbitrary relationship among the large parameters of the problem A_j and B, we introduce the small parameter

$$\chi = 1/\max\{A_1, B/\varepsilon_0\}, \quad |\chi| \ll 1, \tag{28}$$

and again employ the perturbation method. We eventually arrive at a system of Volterra integral equations:

$$\begin{aligned}
&\alpha\sigma_0(z) + ik_0\beta \int_{-L}^{z} K(z - z')\sigma_0(z')\, dz' = 0,\\
&\alpha\sigma_1(z) + ik_0\beta \int_{-L}^{z} K(z - z')\sigma_1(z')\, dz' = \Phi(z) - \Phi_e'[\sigma_0, z],\\
&\alpha\sigma_2(z) + ik_0\beta \int_{-L}^{z} K(z - z')\sigma_2(z')\, dz' = -\Phi_e'[\sigma_1, z],\\
&\alpha\sigma_3(z) + ik_0\beta \int_{-L}^{z} K(z - z')\sigma_3(z')\, dz' = -\Phi_e'[\sigma_2, z],\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots,
\end{aligned} \tag{29}$$

where we have allowed for the fact that the functional $\Phi_e'[\sigma, z]$ does not contain large parameters, and we have introduced the following notation:

$$\begin{aligned}
K(z) = e^{ik_1 z} - \delta e^{ik_2 z}, \quad \delta = A_2/A_1,\\
\alpha = B/\varepsilon_0 A_1, \quad \beta = 1,
\end{aligned} \tag{30}$$

if $A_1 > B$, and

$$\alpha = 1, \quad \beta = \varepsilon_0 A_1/B,$$

if $A_1 < B$. The zero-approximation equation of system (29) has a trivial solution. Then the first-approximation equation (and we will limit ourselves to this) is a Volterra integral equation of the second kind. It can be solved operationally; for this we define the Laplace transformation as follows:

$$F(p) = \int_0^\infty f(z - L) e^{-pz}\, dz, \tag{31}$$

where $f(z)$ is a function that is defined for all $z \geq -L$. The transform $F(p)$ of $f(z)$ differs from the ordinary Laplace transform only in that the argument of the original is shifted by an amount $-L$. In restoring the original with a specified transform on the basis of the usual formulas of operational calculus, therefore, we must shift the argument by an amount $+L$. If the correspondence formulas for the transform contain derivatives $f^{(k)}(0)$, they must be replaced by $f^{(k)}(-L)$. Otherwise, the formulas of operational calculus remain valid for a transform defined in accordance with rule (31).

From the solution of the first-approximation equation for a linear current density we obtain

$$I(z) = \frac{i\omega\chi P_0}{\alpha\varphi(2L)}\{\varphi(L)\varphi_1(L + z) - \varphi(2L)\varphi_1(z) + f_0[\varphi_1(2L)\cdot 1(z) + \varphi_1(L)]z - f_0^2 \delta |z| - f_0 L\varphi(L)\}. \tag{32}$$

Here

$$\varphi(\xi) = \tilde{\varphi}(\xi) - f_0\xi, \quad \tilde{\varphi}(\xi) = \delta_1(e^{p_1\xi} - 1) - \delta_2(e^{p_2\xi} - 1),$$

$$\delta_j = \gamma_j/p_j, \; \gamma_j = (p_j - ik_1)(p_j - ik_2)/p_j(p_1 - p_2), \quad f_0 = k_1k_2/p_1p_2,$$

$$p_{1,2} = \frac{i}{2\alpha}\{[\beta(\delta - 1) \pm \varepsilon]k_0 + 2\alpha k\}$$

for $\varepsilon^2 > 0$ and

$$p_{1,2} = \frac{1}{2\alpha}\{\pm\varepsilon k_0 + i[2\alpha k + \beta(\delta - 1)k_0]\}$$

for $\varepsilon^2 < 0$, where

$$\varepsilon^2 = \beta^2(1 - \delta)^2 + 4\alpha[\alpha - \beta(1 + \delta)].$$

In the limiting cases $|k_j|L \ll 1$ and $|k_j|L \gg 1$, we obtain distribution (18) from (32) (where we should take into account that $\chi = B^{-1}$ for a "short" antenna). For the radiation resistance and reactance we have the following expressions (to be specific, we will assume that $\varepsilon^2 > 0$):

$$R = \frac{\alpha}{\omega\chi}\frac{(\varphi_1^2 - \varphi_2^2)\psi_2 - 2\varphi_1\varphi_2\psi_1}{|\varphi|^4}, \quad X = \frac{\alpha}{\omega\chi}\frac{(\varphi_2^2 - \varphi_1^2)\psi_1 - 2\varphi_1\varphi_2\psi_2}{|\varphi|^4}, \tag{33}$$

where

$$\varphi \equiv \varphi(L) = \varphi_1 + i\varphi_2, \quad \psi \equiv \varphi(2L) = \psi_1 + i\psi_2,$$

$$\varphi_1 = |\delta_1|\sin|p_1|L - |\delta_2|\sin|p_2|L - f_0L,$$

$$\varphi_2 = 2\left(|\delta_1|\sin^2\frac{|p_1|L}{2} - |\delta_2|\sin^2\frac{|p_2|L}{2}\right).$$

In the limiting cases in question, we obtain formulas (19) and (23), respectively, from (33).

To analyze solution (32), it suffices to find the imaginary part of $I(z)$. The appropriate computations lead to the following result:

$$\begin{aligned}\operatorname{Im} I(z) = \frac{\omega\chi P_0}{\alpha|\psi|^2}\Big\{&(\varphi_1\psi_1 + \varphi_2\psi_2)[|\delta_1|\sin|p_1|(L + z) - |\delta_2|\sin|p_2|(L + z)]\\ &+ 2(\varphi_1\psi_2 - \varphi_2\psi_1)\left[|\delta_1|\sin^2\frac{|p_1|(L + z)}{2} - |\delta_2|\sin^2\frac{|p_2|(L + z)}{2}\right] - |\psi|^2(|\delta_1|\sin|p_1|z - |\delta_2|\sin|p_2|z)\cdot 1(z)\\ &- f_0L(\varphi_1\psi_1 + \varphi_2\psi_2)\left[1 - \left(\frac{|\psi|^2}{\varphi_1\psi_1 + \varphi_2\psi_2}\cdot 1(z) - 1\right)\frac{z}{L}\right]\Big\}.\end{aligned} \tag{34}$$

Expression (34) implies that the current distribution along the antenna differs from a "triangular" distribution in the general case. The function $\operatorname{Im} I(z)$ depends linearly on z and oscillates about the 0z axis. As estimates show, the oscillation amplitude is small if

$$\frac{|\omega \pm \omega_0|}{u}L \sim 1. \tag{35}$$

In conclusion, we should note that as $\omega \to \omega_0$ Eq. (13) becomes invalid. If we take a dipole as an elementary source, however, the Green's function (2), as calculations show, does not have singularities as $\omega \to \omega_0$. This can also be seen from (4)-(6). For the case $\omega \to \omega_0$ we can develop a perturbation method, and this will be the subject of another paper. Moreover, we can develop an analogous theory of TCA in moving plasma allowing for the intrinsic spatial dispersion of the plasma. In the latter case the antenna equation is a Fredholm integral equation of the first kind if the mean electron thermal velocity is equal to the plasma flow velocity.

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