be compared with the previous curve for $\Omega < 1/S$ (Fig. 10) noting that the difference is because of just one oscillatory range for ρ_z in the former in contrast to the two oscillatory ranges in the latter.

The curve for $\Omega = 1/S$ is similar to the preceding and hence not given.

At β greater than or equal to C, the incident wave from below (Fig. 1) has no interaction with the slab since the normal component of the phase velocity of the wave is less than or equal to that of the slab. However, the results, given here are applicable even when $\beta > C$ if the problem is understood to be that the slab impinges on the portion of an existing incident wave above the slab. This aspect of the problem will be reported later.

References

[1] K. Landecker, "Possibility of frequency multiplication and wave amplification by means of some relativistic effect," *Phys. Rev.*, vol. 86, pp. 449–455, 1952.
[2] C. T. Tai, "Two scattering problems involving moving media," Antenna Lab., Ohio State Univ., Columbus, Rep. 1691–7, 1964.
[3] C. Yeh, "Reflection and transmission of electromagnetic waves by a moving dielectric medium," *J. Appl. Phys.*, vol. 36, pp. 3513–3517, 1965.
[4] C. Yeh and K. F. Casey, "Reflection and transmission of electromagnetic waves by a moving dielectric slab," *Phys. Rev.*, vol. 144, pp. 665–669, 1966.
[5] C. Yeh, "Brewster angle for a dielectric medium moving at relativistic speed," *J. Appl. Phys.*, vol. 38, pp. 5194–5200, 1967.
[6] ——, "Reflection and transmission of electromagnetic waves by a moving dielectric slab II. Parallel polarisation," *Phys. Rev.*, vol. 167, pp. 875–877, 1968.
[7] V. P. Pyati, "Reflection and transmission of electromagnetic waves by a moving dielectric medium," *J. Appl. Phys.*, vol. 38, pp. 652–655, 1967.
[8] S. W. Lee and Y. T. Lo, "Reflection and transmission of electromagnetic waves by a moving uniaxially anisotropic medium," *J. Appl. Phys.*, vol. 38, pp. 870–875, 1967.
[9] C. S. Tsai and B. A. Auld, "Wave interaction with moving boundaries," *J. Appl. Phys.*, vol. 38, pp. 2106–2115, 1967.
[10] T. Shiozawa, K. Hazama, and N. Kumagai, "Reflection and transmission of electromagnetic waves by a dielectric half-space moving perpendicular to the plane of incidence," *J. Appl. Phys.*, vol. 38, pp. 4459–4461, 1967.
[11] P. Daly and H. Gruenberg, "Energy relations for plane waves reflected from moving media", *J. Appl. Phys.* vol. 38, pp. 4486–4489, 1967.
[12] T. Shiozawa and N. Kumagai, "Total reflection at the interface between relatively moving media," *Proc. IEEE*, vol. 55, pp. 1243–1244, 1967.
[13] J. A. Kong and D. K. Cheng, "Wave behavior at an interface of a semi-infinite moving anisotropic medium," *J. Appl. Phys.*, vol. 39, pp. 2282–2286, 1968.
[14] D. K. Cheng and J. A. Kong, "Time-harmonic fields in source-free bi-anisotropic media," *J. Appl. Phys.*, vol. 39, pp. 5792–5796, 1968.
[15] C. Yeh, "Reflection and transmission of electromagnetic waves by a moving plasma medium," *J. Appl. Phys.*, vol. 37, pp. 3079–3082, July 1966.
[16] ——, "Reflection and transmission of electromagnetic waves by moving plasma medium, II. Parallel Polarization," *J. Appl. Phys.*, vol. 38, pp. 2871–2873, 1967.
[17] B. R. Chawla and H. Unz, "Reflection and transmission of electromagnetic waves normally incident on a plasma slab moving uniformly along a magnetostatic field," *IEEE Trans. Antennas Propagat.*, vol. AP-17, pp. 771–777, 1969.
[18] J. A. Kong and D. K. Cheng, "Reflection and transmission of electromagnetic waves by a moving uniaxially anisotropic slab," *J. Appl. Phys.*, vol. 40, pp. 2206–2212, 1969.
[19] B. R. Chawla and H. Unz, *Electromagnetic Waves in Moving Magneto-Plasmas*. Lawrence, Kansas: Kansas Univ. Press, 1971.
[20] D. Kalluri and R. C. Prasad, "Thin film reflection properties of isotropic and uniaxial plasma slabs," *Appl. Sci. Res.*, to be published.
[21] K. G. Budden, *Radio Waves in the Ionosphere*. Cambridge, England: Cambridge Univ. Press, 1961, p. 111.
[22] I. P. French, G. G. Cloutier, and M. P. Bachynski, "The absorpitivity spectrum of a uniform anisotropic plasma slab," *Can. J. Phys.*, vol. 39, pp. 1273–1290, 1961.

Experimental Investigations on the Impedance Behavior of a Cylindrical Antenna in a Collisional Magnetoplasma

BHARATHI BHAT AND BASRUR RAMA RAO

***Abstract*—An experimental study of the input impedance of a short cylindrical antenna immersed in a laboratory magnetoplasma is reported. The antenna is fixed in length and is aligned with its axis parallel to the dc magnetic field. Of particular interest is its impedance behavior in the vicinity of the resonance and cut-off conditions of the magnetoionic medium. Independent measurements are made to determine the electron density from the ion-current characteristics of a spherical Langmuir probe and the strength of the applied magnetic field with a Hall-effect probe. Measured impedances are compared with Balmain's theory. Essential features observed experimentally are a peak in the antenna input resistance near the upper hybrid resonance frequency and a peak in the antenna conductance near the cyclotron frequency. An additional small increase in the input resistance is observed near the plasma frequency when the plasma frequency is not close to either the upper hybrid resonance frequency or the cyclotron frequency. These results suggest the possibility of using a short cylindrical antenna as a diagnostic probe for measuring properties of a magnetoplasma such as the electron density and collision frequency.**

Manuscript received May 1, 1972; revised July 11, 1972. This work was supported in part by the NSF under Grant GK-24105 and in part by the NASA under Grant NGR 22-007-056.

B. Bhat is with the Division of Engineering and Applied Physics, Gordon McKay Laboratory, Harvard University, Cambridge, Mass. 02138.

B. R. Rao is with M.I.T. Lincoln Laboratory, Lexington, Mass. 02173.

I. Introduction

A KNOWLEDGE of the impedance behavior of a dipole antenna radiating in a magnetoionic medium is necessary for interpreting the data obtained from diagnostic and propagation experiments performed in the ionosphere and other planetary environments using rockets and satellites. The practical significance of this problem has resulted in a number of theoretical papers which describe the radiation and impedance characteristics of antennas immersed in magnetized plasmas (see for example, [1]–[5]). For the most part the impedance formulations of finite antennas are based on either the quasistatic approximation [2] or uniaxial approximation [3] with an assumed distribution of current on the antenna. Within the framework of these theoretical approximations, the antenna impedance behavior has been well studied. On the other hand, only a limited number of laboratory measurements on the impedance of cylindrical antennas in magnetoplasmas have been reported thus far [2], [6] and there is as yet no quantitative comparison between any experiment and theory.

The purpose of this paper is to present a comprehensive laboratory investigation of a cylindrical antenna, whose length is short compared with the free-space wavelength, immersed in a collisional magnetoplasma with dc magnetic field oriented parallel to the antenna axis. The input impedance of the antenna is studied at and in the vicinity of the various transition regions of a magnetoplasma resonance and cut-off (see Fig. 1). For a one component (electron), lossless ($Z = \nu/\omega = 0$), cold magnetoplasma, the resonance, and cut-off conditions are given by: 1) $X = 1 - Y^2$, upper hybrid resonance, $Y = 1$, cyclotron resonance; 2) $X = 1$, plasma cut-off, $X = 1 + Y$, left cut-off, $X = 1 - Y$, right cut-off. The symbols $X = \omega_p^2/\omega^2$, $Y = \omega_c/\omega$, and $Z = \nu/\omega$ are the normalized plasma parameters, where $\omega_p = (e^2 n_0/m_e\epsilon_0)^{1/2}$ is the angular plasma frequency for the electrons, $\omega_c = eB_0/m_e$ is the angular electron cyclotron frequency, ν is the electron-neutral collision frequency, ω is the angular signal frequency, B_0 is the axial static magnetic field, n_0 is the electron density, m_e and e are the electron mass and charge, respectively. The antenna operating frequency was varied (typically from 250 to 600 MHz), while the plasma, cyclotron, and collision frequencies were maintained at constant values by fixing principally the discharge current, solenoid current, and neutral gas pressure, respectively. Measurements were made at various magnetic-field strengths. The effect of collisions on antenna impedance was studied by repeating the measurements at several neutral gas pressures. Diagnostic measurements were made using a Langmuir probe to determine the electron density and electron temperature, and using a Hall-effect probe to determine the strength of the applied magnetic field so that the prevailing experimental conditions could be well defined. The impedance results are compared with Balmain's theory and the practical implications of the results are discussed.

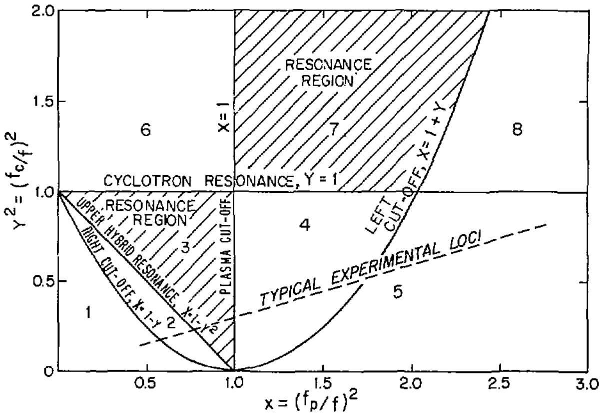

Fig. 1. CMA diagram with typical experimental loci.

II. Experimental Apparatus

A. Antenna and Plasma Chamber

Experimental investigations were accomplished using a hot cathode helium dc discharge in a glass tube having approximately a 38 cm anode-to-cathode spacing and a 6.5-cm diameter. The block diagram of the experimental apparatus is shown in Fig. 2. The antenna (a copper rod, 3.5 cm in length and 0.4 cm in diameter) protruded normally from the center of a Teflon ring press-fitted into a large brass disc which served as the anode of the discharge tube. Severe junction effects were eliminated by connecting the antenna to the air-filled coaxial line with a General Radio precision coaxial connector. A uniform axial magnetic field was provided by a large solenoid, 28 cm in length and 15 cm in diameter. A constant current regulator was used in series with a dc power supply to the plasma tube which ensured that the discharge current remained at a constant value throughout the experiment. Further details of the apparatus are given in [7].

B. Plasma Diagnostic Techniques

Electron density and electron temperature in the plasma were measured independently using the ion-characteristics of a spherical Langmuir probe; the probe consisted of a small steel ball bearing of radius (r_p) 0.078 cm, spot welded to a glass sheathed tungsten wire of radius 0.008 cm. About 0.05 cm of the supporting wire was exposed to the plasma to ensure that the glass sheath would not interfere with the collecting area of the probe surface. Also, a portion of the glass sheath close to the spherical ball was tapered to minimize the interference with particle collection. The spherical ball was located on the axis of the discharge tube at a distance of about 4.5 cm from the tip of the antenna.

The electron-current characteristic ($\ln I_e$ versus voltage) of the probe was obtained by the voltage sweep technique similar to the one employed by Scott and Rao [8] and the electron temperature was determined from the slope of the Boltzmann line in the presence of magnetic field (as suggested by Bickerton and von Engel [9]). With the electron temperature known, the electron num-

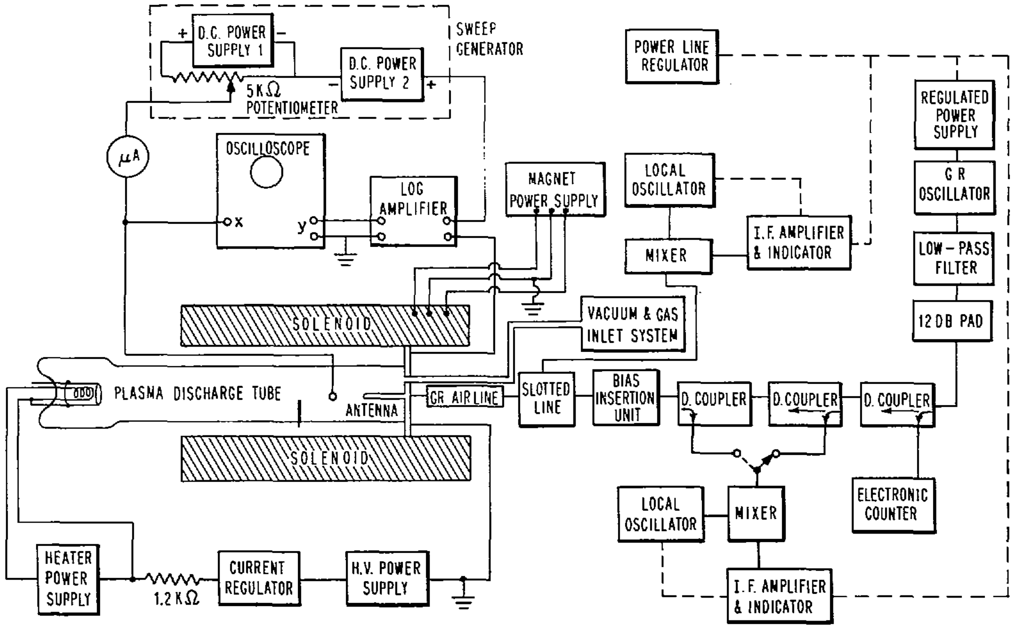

Fig. 2. Block diagram of experimental apparatus.

ber density was calculated from the ion-current characteristic using the asymptotic theory of Lam [10], [11]. This theory is applicable [12], [13] for spherical probes in the range $1 < r_p/\lambda_D < 100$ and $r_{Li}/r_p \gg 1$, where λ_D is the Debye wavelength for electrons and r_{Li} is the ion Larmor radius. The highest value of λ_D encountered in this experiment was 0.056 cm so that the ratio r_p/λ_D was always greater than 1, and for the strongest magnetic field used (100 G), the ratio $r_{Li}/r_p \simeq 160$ (assuming that $T_e = 50T_i$, where T_i is the ion temperature).

III. Theoretical Predictions of Antenna Impedance

Within the framework of the quasi-static approximation and for an assumed triangular distribution of current on the antenna, the input impedance Z_A of a short antenna in a plasma with axial magnetic field is [2]

$$Z_A = R_A + jX_A = \frac{1}{j2\pi\epsilon_0 hK_\perp}\left[\ln\frac{h}{\rho} - 1 + \ln\left(\frac{K_\perp}{K_{||}}\right)^{1/2}\right] \tag{1}$$

where R_A and X_A are the input resistance and reactance of the antenna, h and ρ are the height and radius of the antenna, ϵ_0 is the permittivity of free space and $K_\perp = 1 - XU/(U^2 - Y^2)$, $K_{||} = 1 - X/U$, $U = 1 - jZ$, and $Z = \nu/\omega$.

Balmain uses the triangular current distribution on the basis that the antenna is short compared with the free space wavelength. Furthermore, it is stated that under lossless conditions the quasi-static differential equation is hyperbolic in the resonant regions 3, 7 and 8 of the CMA diagram (Fig. 1) where $K_\perp/K_{||}$ is negative and that, consequently, the logarithmic term in (1) contributes a real part to the antenna impedance. However, under lossless conditions an antenna which is electrically short in free space does not remain so in the resonant regions of a magnetoplasma. Thus the assumption of a triangular distribution of current and the validity of (1) are questionable under lossless conditions. Realistic plasmas, however, always involve some collisions. The inclusion of the slightest contribution from collisions suppresses the infinities in the refractive index (and, therefore, in the propagation constant) of the pertinent characteristic waves. Under these conditions and provided the effect of collisions is sufficiently large, the antenna can be regarded as electrically short and (1) can be a good representation of antenna impedance.

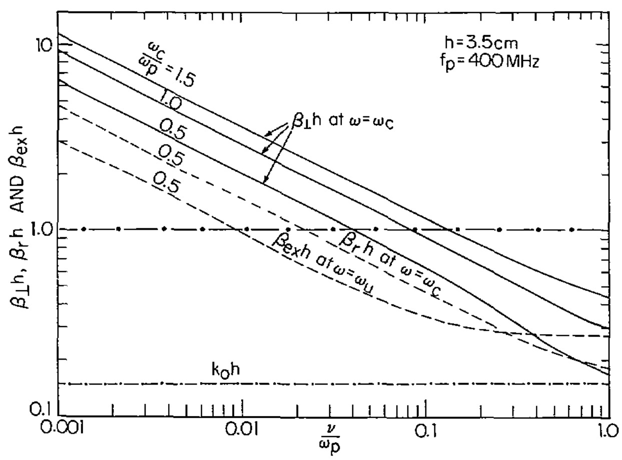

Fig. 3. $\beta_\perp h$, $\beta_r h$, and $\beta_{ex} h$ as functions of ν/ω_p.

In the experiments described in this paper the plasma parameters and the magnetic field were adjusted to ensure that, in the frequency range of measurement, the antenna would be electrically short not only in comparison with the free-space wavelength but also with the shortest wavelength of any one of the characteristic waves in the medium or the traveling-wave component of the current on the antenna. For the case $\omega_c/\omega_p = 0.5$, Fig. 3 shows

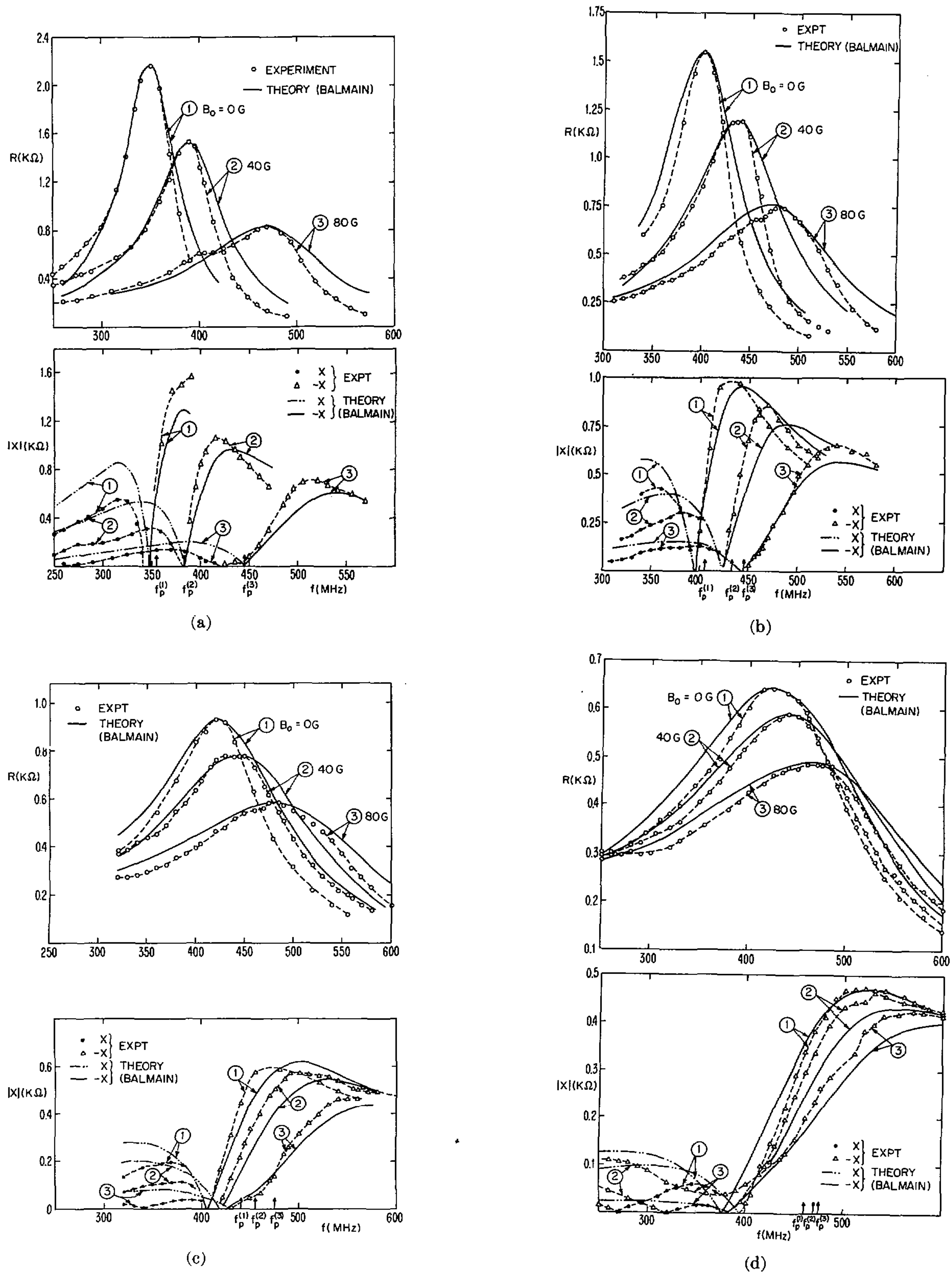

Fig. 4. Comparison of theoretical (Balmain) and experimental results for impedance of monopole antenna in magnetoplasma. (Gas: helium; $\rho_0 = 0.213$ cm, $h = 3.5$ cm.) (a) Neutral gas pressure = 76 μm of Hg and discharge current = 120 mA. (b) Neutral gas pressure = 110 μm of Hg and discharge current = 120 mA. (c) Neutral gas pressure = 215 μm and discharge current = 80 mA. (d) Neutral gas pressure = 325 μm and discharge current = 60 mA.

$\beta_{ex}h$ and $\beta_r h$ as functions of ν/ω_p, computed at the upper hybrid resonance ($X = 1 - Y^2$ or $\omega = \omega_u$) and the cyclotron resonance ($Y = 1$ or $\omega = \omega_c$), respectively. β_{ex} and β_r are the real parts of the propagation constants $k_{ex} = k_0(K_r K_l / K_\perp)^{1/2}$ (extraordinary wave) and $k_r = k_0(K_r)^{1/2}$ (right circularly polarized wave), respectively. Fig. 3 also shows $\beta_\perp h$ plotted as a function of ν/ω_p at the cyclotron resonance for $\omega_c/\omega_p = 0.5$, 1.0, and 1.5, where $\beta_\perp$ is the real part of $k_\perp = k_0(K_\perp)^{1/2}$, the propagation constant of the current wave on the antenna [14], [15]. Thus at $Y = 1$ the larger the ratio of ω_c/ω_p, the larger should be the value of ν/ω_p to satisfy the condition $\beta_\perp h < 1$; a similar argument applies to $\beta_{ex}h$ and $\beta_r h$ at the resonance conditions. For the experimental results reported in Figs. 4(a)–(d), ω_c/ω_p varies between 0 and 0.6, ν/ω_p varies between 0.1 and 1.1. Thus for these experimental conditions the antenna used in this experiment is electrically short even in the resonant regions and Balmain's quasi-static impedance expression (1) is apparently applicable.

Numerical computations of (1) [for finite ν/ω] show that $|Z_A|$ has a pronounced maximum in the vicinity of upper hybrid resonance $X = 1 - Y^2$, and that the antenna conductance has a pronounced maximum in the vicinity of cyclotron resonance $Y = 1$. The antenna reactance changes sign near $X = 1 - Y^2$ as well as near $Y = 1$. Galejs [3] used a variational method with a two-term sinusoidal type trial function for the antenna current and obtained a similar expression in the quasi-static limit as that of Balmain. Seshadri [16] considered the impedance behavior of an infinitely long antenna in a collisionless ($Z = 0$) cold magnetoplasma. The axial component of the current near the driving point as derived by Seshadri is

$$I_{z\to 0} = C \ln\left(\frac{1}{z}\right)\left[\frac{(1 - X)(1 - X - Y^2)}{1 - Y^2}\right]^{1/2} \qquad (2)$$

where C is a constant whose value depends on the antenna radius. From this expression it is seen that the antenna impedance reaches infinity at the upper hybrid resonance and at plasma cut-off, and becomes zero at the cyclotron resonance.

IV. Experimental Results

A. *Impedance Behavior in Vicinity of Upper Hybrid Resonance and Plasma Cut-Off*

Figs. 4(a)–(d) depict the influence of the magnetic field on the antenna input impedance at neutral gas pressures of 76, 110, 215, and 325 μm of Hg, respectively, for the case $f_c < 0.6 f_p$. Both the theoretical (Balmain) and experimental results are plotted as functions of the operating frequency over the range 250–600 MHz. For a specific combination of neutral gas pressure and discharge current, impedance measurements were made first with no magnetic field and subsequently with magnetic fields of 40 and 80 G.

The cyclotron frequency f_c is calculated from the known strength of the applied magnetic field which was accurately determined using the Hall-effect probe. The plasma frequency f_p and the effective collision frequency ν_{eff} are determined using a curve fitting technique [7] similar to the one described by Scott and Rao [8]. The theoretical curve is obtained by matching $R_{\max}^{\text{theory}}$ with $R_{\max}^{\text{expt}}$ (when there are two maxima, $R_{\max}^{\text{expt}}$ corresponds to the value at the higher frequency) using the plasma frequency and the collision frequency as the varying parameters. The arrow marks, labelled $f_p^{(1)}$, $f_p^{(2)}$, and $f_p^{(3)}$, show the plasma frequencies at magnetic field strengths of 0, 40, and 80 G, respectively. The various experimental parameters and the corresponding measured values of f_p, f_c, and ν_{eff} are presented in Table I.

The influence of a magnetic field on the antenna input impedance for the case $f_c < f_p$ may be summarized as follows (see Figs. 4(a)–(d)).

1) The resistance peak decreases in magnitude and the curve becomes broader as the magnetic field increases. For sufficiently high magnetic fields two distinct resistance maxima are observed. These are attributed to the effects of plasma cut-off and upper hybrid resonance conditions. For extremely low collisions ($\nu/\omega_p \sim 0.001$) sharp resistance peaks should occur at the plasma frequency and the upper hybrid resonance frequency according to the predictions of Seshadri [16]. Since the plasma under investigation is collision-dominated ($\nu/\omega_p > 0.1$), these peaks are broadened and their positions also shifted from their corresponding locations for no collisions. The magnitude of this shift and the damping effect are larger for higher neutral gas pressures because of the correspondingly higher electron-neutral collision frequencies.

2) There is a general shift of the resistance peak to a higher frequency when the magnetic field is applied. This is due to the slightly higher plasma frequency brought about by the constricting effect of the magnetic field and the subsequent increase in the electron number density. Measurements made with a spherical Langmuir probe also showed an increase in electron density with the application of magnetic field. A comparison of the plasma frequencies obtained from the Langmuir probe measurements with those deduced from the antenna impedance measurements is shown in Table II. Maximum relative error in the plasma frequency between these two measurements is approximately 20 percent. It should be noted that the spherical probe measures the local electron density at an axial distance of 4.5 cm from the tip of the antenna. Furthermore, Lam's theory, which was used to interpret the probe data, neglects the effects of collisions and assumes a mono-energetic distribution of ions. A comparison of resistance curves corresponding to 0, 40, and 80 G at various neutral gas pressures shows that the shift in resistance maxima is greater for lower pressures than for higher pressures. It was further observed that the effect of magnetic field on the positive column is more pronounced at lower pressures than at higher pressures; with an increase in the magnetic field the glow on the axis of the tube became increasingly brighter.

3) An examination of the reactance curves also shows that the application of the magnetic field causes a shift in the frequency at which the reactance reverses sign

TABLE I

				f = 250 - 600 MHz		
Neutral Gas Pressure and Discharge Current	B_0(Gauss)	f_p(MHz)	$\nu_{eff}(10^6\ sec^{-1})$	$X = (f_p/f)^2$	$Y^2 = (f_c/f)^2$	$Z = \nu_{eff}/\omega$
76μ , 120 mA	0	355	431	2.01 - 0.35	0	0.27 - 0.114
	40	384	543	2.36 - 0.41	0.2 - 0.035	0.35 - 0.144
	80	443	812	3.13 - 0.55	0.8 - 0.14	0.52 - 0.22
110μ , 120 mA	0	405	606	2.62 - 0.46	0	0.39 - 0.16
	40	434	725	3.01 - 0.52	0.2 - 0.035	0.46 - 0.19
	80	447	900	3.20 - 0.56	0.8 - 0.14	0.57 - 0.24
160μ , 80 mA	0	385	775	2.37 - 0.41	0	0.49 - 0.21
	40	404	875	2.61 - 0.45	0.2 - 0.035	0.56 - 0.23
	80	413	975	2.72 - 0.47	0.8 - 0.14	0.62 - 0.26
215μ , 80 mA	0	440	1037	3.09 - 0.54	0	0.66 - 0.4
	40	456	1150	3.33 - 0.58	0.2 - 0.035	0.73 - 0.31
	80	475	1300	3.61 - 0.63	0.8 - 0.14	0.83 - 0.35
325μ , 60 mA	0	460	1500	3.39 - 0.59	0	0.95 - 0.4
	40	471	1600	3.55 - 0.62	0.2 - 0.035	1.02 - 0.42
	80	475	1600	3.61 - 0.63	0.8 - 0.14	1.02 - 0.42

TABLE II

COMPARISON OF PLASMA FREQUENCY DETERMINED BY SPHERICAL LANGMUIR PROBE AND FROM ANTENNA IMPEDANCE MEASUREMENT

Neutral Gas Pressure (microns of Hg)	Discharge Current (mA)	Magnetic Field (Gauss)	Plasma Frequency: Antenna (MHz)	Plasma Frequency: Langmuir Probe (MHz)
76	120	0	355	294
		40	384	366
		80	443	375
110	120	0	405	379
		40	434	443
		80	447	482
160	80	0	385	350
		40	404	389
		80	413	442
215	80	0	440	429
		40	456	436
		80	475	462
325	60	0	460	375
		40	471	383
		80	475	405

near $X = 1$ (as in the case of an isotropic plasma) and the inductive reactance is substantially reduced. At lower pressures (Figs. 4(a) and (b)) the reactance changes from inductive to capacitive very close to the plasma frequency, whereas at higher pressures (Figs. 4(c) and (d)) the reactance reverses sign at a frequency much below the plasma frequency.

4) In general there is good qualitative agreement between experiment and theory (Balmain). However, it is interesting to note that the theoretical curves have a single smooth resistance maximum, whereas the experimental results for moderately high magnetic fields (80 G) show two broad maxima. Measurements of the impedance of a short dipole antenna in the ionosphere made by Stone, Weber, and Alexander [17] also show two resistance maxima in the vicinity of $X = 1$ and $X = 1 - Y^2$.

It should be pointed out that a second resistance maximum attributed to the plasma cut-off may not always be observed. For example, in Figs. 4(a) and (b) the resistance curves for 40 G show a single smooth maximum. This is probably due to the merging of the two maxima into a single broad peak since the cyclotron frequency is small. A similar effect is also observed at higher neutral gas pressures where the collisional damping causes the two resistance maxima to merge into a single broad maximum (e.g., Fig. 4(d)). It may be noted that for moderately high magnetic fields, the antenna reactance does not reverse sign in the vicinity of the plasma frequency but remains capacitive throughout. A similar effect may be noticed by increasing only the collisional losses while keeping the magnetic field fixed.

B. Impedance Behavior in Vicinity of Cyclotron Resonance

Fig. 5 shows the measured impedances and admittances in the vicinity of cyclotron resonance. The arrow labelled f_c on the x axis locates the cyclotron frequency as measured by the Hall-effect probe, that labelled f_p locates the plasma frequency as measured by the Langmuir probe. In this case the experimental loci passing close to the point defined by $X = 1$ and $Y = 1$ were deliberately chosen in order to investigate the impedance behavior when the plasma frequency and the cyclotron frequency are close to one another. The influence of the cyclotron damping on the plasma cut-off is apparent from the resistance curves; the resistance increase expected near $X = 1$ is completely obliterated, and only the dominant resistance peak due to the upper hybrid resonance is observed. As predicted by the theory, the antenna conductance has a dominant peak in the vicinity of cyclotron resonance $Y = 1$. Furthermore, the antenna susceptance remains negative (or reactance positive) over a narrow

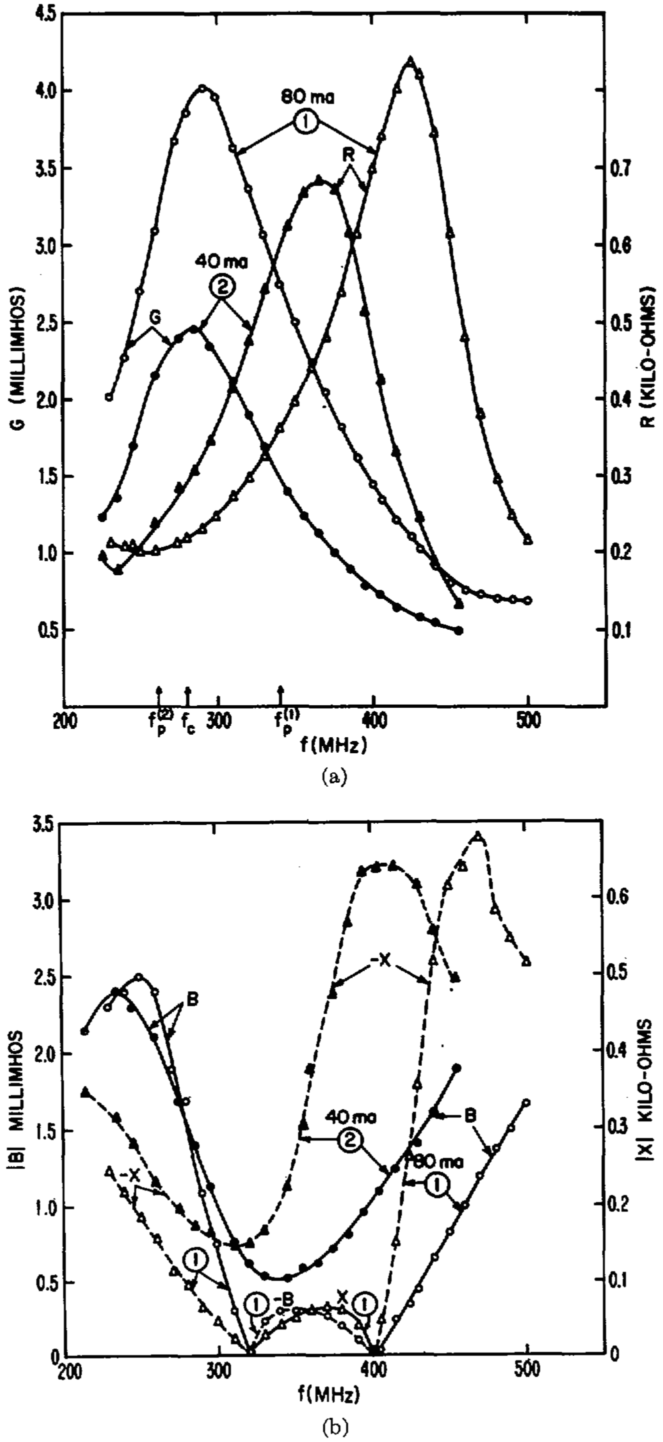

Fig. 5. Measured impedance and admittance of monopole antenna in magnetoplasma. (Gas: helium; $\rho_0 = 0.213$ cm, $h = 3.5$ cm; neutral gas pressure = 110 μm of Hg; $B_0 = 100$ G.) ①—Discharge current = 80 mA; $f_p^{(1)} = 340$ MHz. ②—Discharge current = 40 mA; $f_p^{(2)} = 262$ MHz.

range determined by the plasma frequency, the cyclotron frequency, and the collision frequency. It is evident from the susceptance curve labelled (2) that when the plasma frequency and the cyclotron frequency are sufficiently close to one another, the antenna susceptance remains positive throughout.

V. Discussion

Apart from radiation and collisional losses, other mechanisms such as the electroacoustic effects and ion sheath effects can contribute to the measured antenna impedance in a collisional magnetoplasma. In the case of an isotropic collisional plasma, Scott and Rao [8] have shown that the electroacoustic mode makes a negligible contribution to the input impedance of an electrically short antenna when collisions are a dominant loss mechanism. In particular, they have shown that for collision frequency $\nu_e = 0.16\omega$ the maximum ratio of the electroacoustic component to the electromagnetic component of resistance was 0.185 for electron temperatures as high as 10 eV. In this experiment the values of ν_e/ω and the electron temperature are almost of the same order of magnitude as those used by Scott and Rao. Moreover, experimentally determined electron temperatures plotted as a function of PR (see Fig. 6) where P is the neutral gas pressure in mm of Hg and R is the radius of the discharge tube in cm, show that the application of magnetic field reduces the electron temperature as measured by the spherical probe. The effect of magnetic field is smaller at higher gas pressures. Measurements of electron temperature, reported by Bickerton and von Engel [9], also show a decrease of electron temperature with magnetic field. Theoretical plots of electron temperature following Langmuir's theory of free ion fall [9, fig. 5] and CPR curve [18, fig. 8.16] valid for zero magnetic field, and modified Schottky theory [9, fig. 5] valid when the mean free path of ions is small compared with the radius are also presented in Fig. 6. It can be seen that in the absence of magnetic field, the measured electron temperature as a function of pressure approximates Langmuir's theory, whereas for sufficiently high magnetic fields (120 G), the electron temperature follows the modified Schottky theory.

A contribution to the measured antenna impedance by a possible ion sheath formation surrounding the antenna was examined by applying positive dc bias voltages to the antenna with respect to ground. No noticeable change was observed in the antenna impedance over the operating ranges of interest. This was to be expected as the collisional losses were sufficiently high and the Debye wavelengths were much smaller than the radius of the antenna.

VI. Concluding Summary

The following general trends in antenna impedance behavior have been observed experimentally: 1) a dominant resistance peak occurs near $X = 1 - Y^2$; 2) a dominant conductance peak occurs near $Y = 1$; 3) a small resistance increase occurs near $X = 1$; 4) the reactance is negative for frequencies below f_c, positive between f_c and f_p, and negative above f_p (for $f_c < f_p$); and 5) increasing collisions causes a considerable shift of the resistance maxima and reactance zeros from their corresponding locations for no collisions.

It is interesting to note that exceptions to the foregoing general trends do arise under certain conditions. For

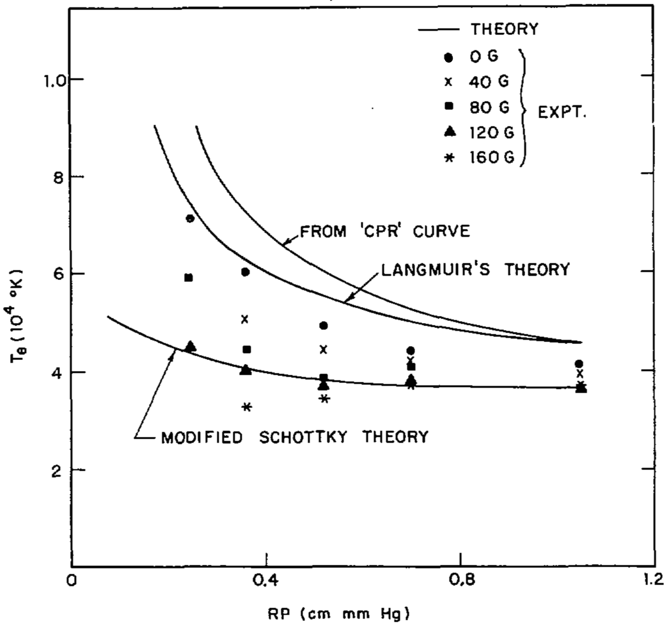

Fig. 6. Comparison of theoretical and experimental electron temperatures. (Gas: helium; discharge tube radius R = 6.5 cm.)

example, if $f_c < 0.2f_p$, the upper hybrid frequency and plasma frequency are so close to one another that their resistance maxima merge to yield only a single maximum. If $f_c \sim f_p$, the two opposing effects on resistance are superimposed, thus somewhat obliterating the plasma cut-off but not affecting the conductance peak near $Y = 1$ which is still observed. Also the reactance may not necessarily change sign. If f_c is sufficiently close to f_p and the magnetic-field strength or the collision frequency is large, the reactance remains capacitive throughout the entire frequency range. The resistance maximum in the vicinity of $X = 1 - Y^2$ and the conductance maximum in the vicinity of $Y = 1$, which are always observed, offer a method of determining the plasma frequency, cyclotron frequency, and effective collision frequency in a magnetoplasma.

Acknowledgment

The authors are grateful to Prof. R. W. P. King for his encouragement and guidance, Dr. D. H. Preis for numerous useful discussions and the careful reading of the manuscript, and Dr. L. D. Scott for his invaluable assistance in the improvement of measuring techniques.

References

[1] S. R. Seshadri, "Radiation resistance of elementary electric-current sources in a magneto-ionic medium," *Proc. Inst. Elec. Eng.*, vol. 112, pp. 1856–1869, 1965.

[2] K. G. Balmain, "The impedance of a short dipole antenna in a magnetoplasma," *IEEE Trans. Antennas Propagat.*, vol. AP-12, pp. 605–617, Sept. 1964.

[3] J. Galejs, "Impedance of a finite insulated cylindrical antenna in a cold plasma with a longitudinal magnetic field," *IEEE Trans. Antennas Propagat.*, vol. AP-14, pp. 727–736, Nov. 1966.

[4] W. S. Ament, J. C. Katzin, M. Katzin, and B. Y. C. Koo, "Impedance of a cylindrical dipole having a sinusoidal current distribution in a homogeneous anisotropic ionosphere," *J. Res. Nat. Bur. Stand.* vol. 68D, pp. 379–405, Apr. 1964.

[5] J. P. Lafon and H. Weil, "Impedance of cylindrical and helical antennas in lossy magnetoplasma including conditions of refractive index resonance," *Radio Sci.*, vol. 6, pp. 99–111, Jan. 1971.

[6] H. Ito and Y. Mushiake, "Impedance of an antenna in anisotropic plasma," *Electron. Commun. Jap.* vol. 51B, no. 2, pp. 64–71, 1968.

[7] B. Bhat, "Cylindrical antennas in magnetoplasma," Ph.D. dissertation, Harvard Univ. Cambridge, Mass., 1971.

[8] L. D. Scott and B. R. Rao, "A short cylindrical antenna as a diagnostic probe for measuring collision frequencies in a collision-dominated nonMaxwellian plasma," *IEEE Trans. Antennas Propagat.*, vol. AP-17, pp. 777–786, Nov. 1969.

[9] R. J. Bickerton and A. von Engel, "The positive column in a longitudinal magnetic field," *Proc. Phys. Soc. London (Gen.)*, vol. 69, pt. 4, pp. 468–481, Apr. 1956.

[10] S. H. Lam, "Unified theory for the Langmuir probe in a collisionless plasma," *Phys. Fluids*, vol. 8, pp. 73–87, Jan. 1965.

[11] ——, "Plasma diagnostics with moderately large Langmuir probes," *Phys. of Fluids*, vol. 8, pp. 1002–1004, 1965.

[12] F. F. Chen, C. Etievant, and D. Mosher, "Measurement of low plasma densities in a magnetic field," *Phys. Fluids*, vol. 11, pp. 811–821, Apr. 1968.

[13] F. F. Chen, "Numerical computation for ion probe characteristics in a collisionless plasma," *Plasma Phys. (Accelerators-Thermonuclear Res.)*, vol. 7, pp. 47–68, Jan.–Feb. 1965.

[14] S. W. Lee and Y. T. Lo, "Current distribution and input admittance of an infinite cylindrical antenna in anisotropic plasma," *IEEE Trans. Antennas Propagat.*, vol. AP-15, pp. 244–252, Mar. 1967.

[15] Y. Mushiake, "Electromagnetic waves along an infinitely long and thin conducting wire in a magneto-ionic medium," *J. Res. Nat. Bur. Stand., vol. 69D*, pp. 503–510, Apr. 1965.

[16] S. R. Seshadri, "Input admittance of a cylindrical antenna in a magnetoionic medium," *J. Appl. Phys.*, vol. 39, pp. 2407–2412, Apr. 1968.

[17] R. G. Stone, R. R. Weber, and J. K. Alexander, "Measurement of antenna impedance in the ionosphere, I. Observing frequency below the electron gyro frequency," *Planet. Space Sci.*, vol. 14, pp. 631–639, July 1966; and R. G. Stone, J. K. Alexander and R. R. Weber, "Measurement of antenna impedance in the ionosphere, II. Observing frequency greater than the electron gyro frequency," *Planet. Space Sci.*, vol. 14, pp. 1007–1016, Oct. 1966.

[18] J. D. Cobine, *Gaseous Conductors.* New York: Dover, 1958.