IX. PLASMAS

9.3: THE IMPEDANCE OF A SHORT DIPOLE ANTENNA IN A MAGNETOPLASMA

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SUMMARY

The impedance of a short dipole antenna in a lossy magnetoplasma has been calculated theoretically for any orientation of the dipole with respect to the D. C. magnetic field. Since it is assumed that the antenna is very short, quasi-static field equations are used in the analysis. In this paper the theoretical impedance calculations are discussed and compared with laboratory measurements for the case in which the magnetic field and dipole axis are parallel.

The quasi-static field expressions and differential equation are derived from Maxwell's equations under the assumption that k_o^2 is very small (where $k_o = \omega\sqrt{\mu_o \epsilon_o}$). The electric field is given by the gradient of a scalar potential:

$$\bar{E} = -\nabla \Psi \tag{1}$$

Since the potential Ψ results from an assumed current distribution $\bar{J}$, the differential equation for Ψ is

$$\nabla \cdot \bar{\bar{K}} \, \nabla \Psi = \frac{\nabla \cdot \bar{J}}{j\omega \epsilon_o} \tag{2}$$

in which $\bar{\bar{K}}$ is the relative permittivity tensor, given by

$$\bar{\bar{K}} = \begin{bmatrix} K' & jK'' & 0 \\ -jK'' & K' & 0 \\ 0 & 0 & K_o \end{bmatrix}, \qquad \begin{aligned} K' &= 1 - \frac{X(1-jZ)}{(1-jZ)^2 - Y^2} \\ K'' &= \frac{XY}{(1-jZ)^2 - Y^2} \\ K_o &= 1 - \frac{X}{1-jZ} \end{aligned}$$

X, Y, and Z are the relative plasma, cyclotron and collision frequencies respectively. The quasi-static approximation can be applied not only to the field equations but also to the Poynting theorem which becomes

$$\int_V \bar{E} \cdot \bar{J}^* dv = j\omega \left[\int_V \bar{E} \cdot \bar{D}^* dv + \int_S \Psi \, \bar{D}^* \cdot \hat{n} \, ds \right] \tag{3}$$

Some information about the quasi-static fields can be obtained by studying the form of equation (2) which can be written as

$$K'(\Psi_{xx} + \Psi_{yy}) + K_o \Psi_{zz} = \frac{\nabla \cdot \bar{J}}{j\omega\epsilon_o} \qquad (4)$$

For the case of a lossless plasma (Z = O), equation (4) is elliptic when K'/K_o is positive and hyperbolic when K'/K_o is negative. When the equation is elliptic, the potential is a smooth function of the spatial coordinates; when the equation is hyperbolic, discontinuities in the divergence of the source current cause infinite electric field discontinuities which extend outward along the conical characteristic surfaces of the differential equation (the axes of the cones are parallel to the z axis). The elliptic and hyperbolic regions are shown graphically in Figure 1.

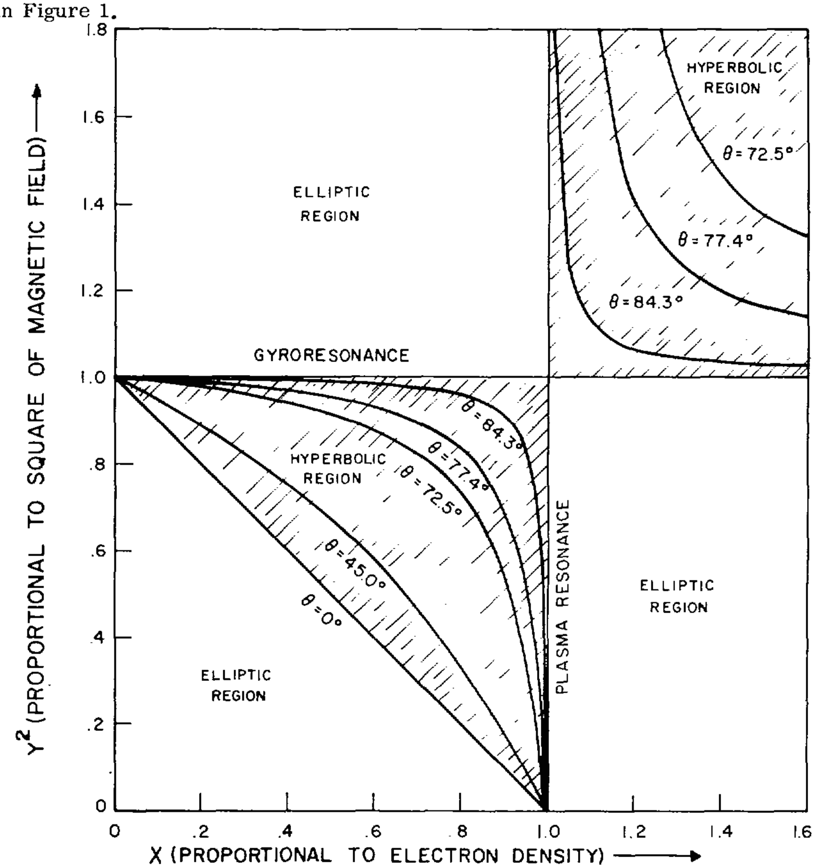

Fig. 1 The elliptic and hyperbolic regions. Note: θ is the angle (with respect to the z axis) of the characteristic cone when the differential equation is hyperbolic.

The electric field calculations for a given current distribution are carried out using Fourier transforms. The current distribution is assumed to be filamentary in cross-section and triangular along the dipole axis (the current is maximum at the center of the dipole and vanishes at both ends). Under hyperbolic conditions the calculated fields exhibit the conical discontinuities anticipated by the foregoing discussion of the differential equation. The quasi-static field formula for an infinitesimal dipole is identical to the first near field term in the expansion of Mittra and Deschamps. [1]

The input impedance computation can be carried out by assuming the current $\bar{J}$ to be spread over the cylindrical surface of the dipole. If the input current is I, the input impedance is given by

$$Z_{in} = -\frac{1}{I^2}\int_S \bar{E} \cdot \bar{J}\, ds \tag{5}$$

where S is the dipole surface. If the radius of the dipole is much smaller than its half-length, the input impedance is found to be

$$Z_{in} = \frac{a}{j\omega\pi\epsilon_0 K' L\sqrt{F}}\left[\ln\frac{L}{\rho} - 1 - \ln\frac{a+\sqrt{F}}{2F}\right] \tag{6}$$

where L = dipole half-length

ρ = dipole radius

$F = \sin^2\theta + a^2\cos^2\theta$

θ = angle between the dipole and the z axis

$a = \sqrt{\frac{K'}{K}}$ (choose the square root having a positive real part).

Examination of the above formula reveals that Zin can have a positive real part under lossless hyperbolic conditions, a result which indicates power flow into the plasma. The same indication of power flow can be obtained by carrying out the surface integration in the quasi-static form of Poynting's theorem (equation 3). It should be noted that an input impedance with a real part has also been found by Kaiser[2] in the quasi-static analysis of a biconical antenna.

A typical set of impedance calculations is shown in Figure 2 for the case in which the antenna axis is parallel to the D. C. magnetic field. The calculations of Figure 2 are for a monopole over a ground plane and are half the values for the corresponding dipole. The impedance loci are for varying electron density (X) and the parameter is the D. C. magnetic field (Y). Each locus has a "kink" near plasma resonance (X = 1); the kink arises as a result of the shift from an elliptic to a hyperbolic region as shown in Figure 1. Figure 1 also shows that the line $X - Y^2 = 1$ is an elliptic-hyperbolic boundary for $X < 1$, $Y^2 < 1$; however, the impedance loci exhibit no unusual behavior at $X = 1-Y^2$.

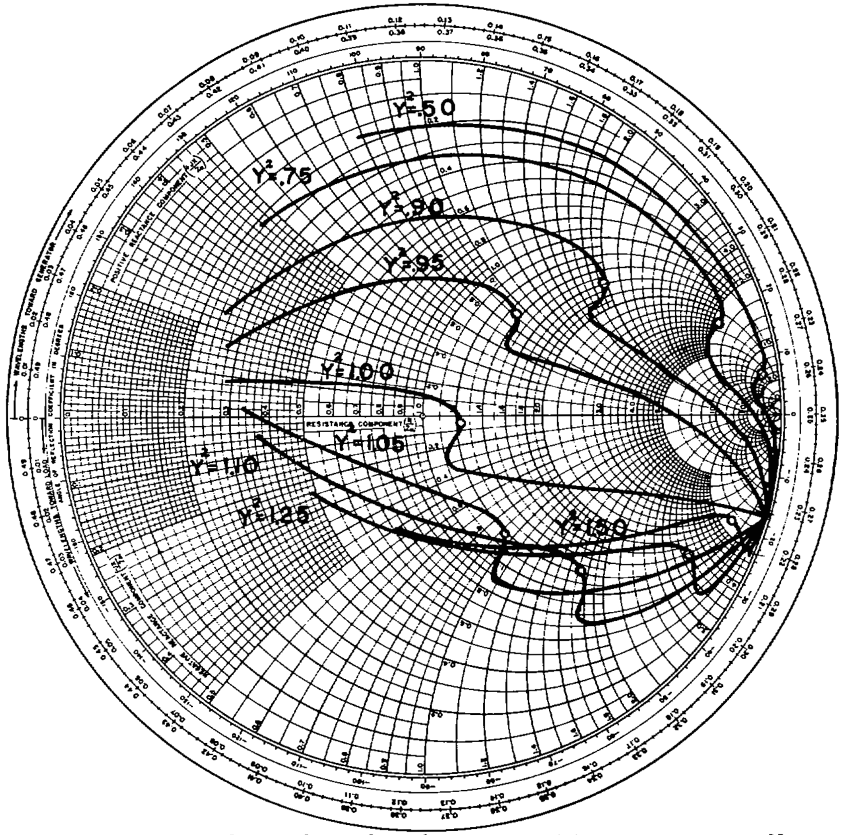

Fig. 2 Theoretical impedance loci for neon at 4.3 mm. pressure. Note: $6 \geqslant X \geqslant 0$ and the small circles indicate $X = 1$. The frequency is 1.6 Gc and the monopole dimensions are L $\boxminus$ 8.0 mm. and $L/\rho = 12.6$.

The quasi-static theory for anisotropic media may be reduced to isotropic form if the space coordinates are suitably scaled. The dimensional scaling between the magnetoplasma coordinates (unprimed) and the free space coordinates (primed) is given by

$$
\begin{aligned}
x' &= \sqrt{K' K_0}\; x \\
y' &= \sqrt{K' K_0}\; y \\
z' &= K'\; z
\end{aligned}
\tag{7}
$$

This is the most useful member of a family of such scalings. Scaling applied to equation (4) converts it to Poisson's equation. Under lossless, elliptic conditions for $X < 1$ the scaling has geometrical significance; the cylindrical dipole is shown to have a free space equivalent with a different length and with an elliptical cross-section. The work of Y. T. Lo[3] on equivalent antenna cross-sections gives the radius of the corresponding cylindrical antenna. The dimensions of this antenna are

$$L' = L\sqrt{K'}\ \sqrt{K_0 \sin^2\theta + K' \cos^2\theta} \tag{8}$$

$$\rho' = \frac{\rho}{2}\left[\frac{K'\sqrt{K_0}}{\sqrt{K_0 \sin^2\theta + K'\cos^2\theta}} + \sqrt{K'K_0}\right] \tag{9}$$

where L, ρ are the length and radius of the magnetoplasma dipole and L', ρ' are the length and radius of the free space dipole. The impedance of a short, thin cylindrical dipole in free space (as given by Schelkunoff[4], for instance), is

$$Z_{in} = \frac{1}{j\omega\pi\epsilon_0 L'}\left[\ln\frac{L'}{\rho'} - 1\right] \tag{10}$$

Substitution of (8) and (9) in (10) gives the impedance expression (6).

Since the impedance depends on the assumed current distribution used in the electric field calculation, two additional current distributions were considered. In one case the current was considered to be spread over the cylindrical dipole surface; in the other case the distribution had zero slope at the ends and center of the dipole. In each case the impedance formula obtained was essentially identical to the formula derived assuming a filamentary, triangular current.

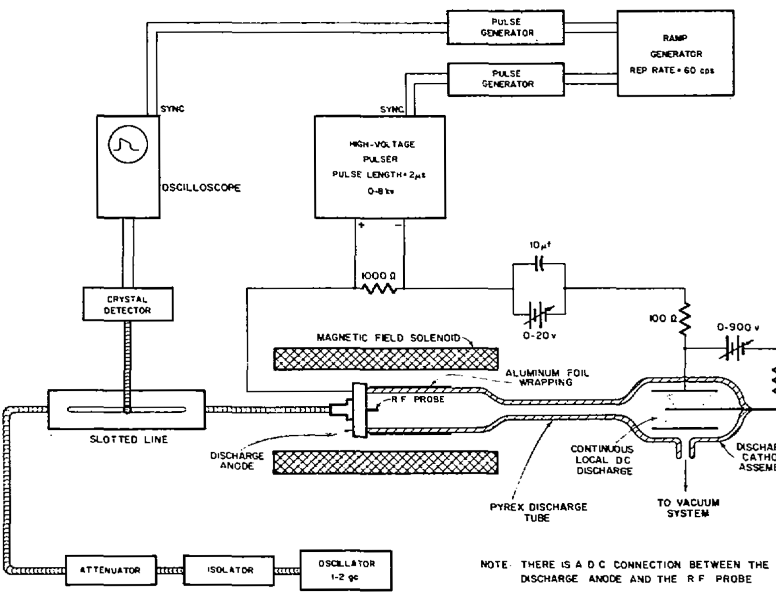

Fig. 3 The experimental apparatus.

The dipole impedance is also affected by the excitation of longitudinal plasma oscillations. For the case of an isotropic plasma, this effect is estimated to be important in the ionosphere but negligible in the laboratory experiment to be described.

Laboratory measurements of short monopole impedance were carried out in pulsed, decaying discharges in neon and helium for the case of the antenna axis parallel to the D. C. magnetic field (see Figure 3). The ion sheath around the antenna was collapsed by the use of positive bias with respect to the discharge cathode. The RF input level was kept low to minimize heating of the plasma electrons. A set of representative impedance loci is shown in Figure 4 for the same conditions as were used in the calculations of Figure 2. Each locus represents the impedance variation during the decay or afterglow period in the plasma following the discharge pulse. The differences between theoretical and experiment-

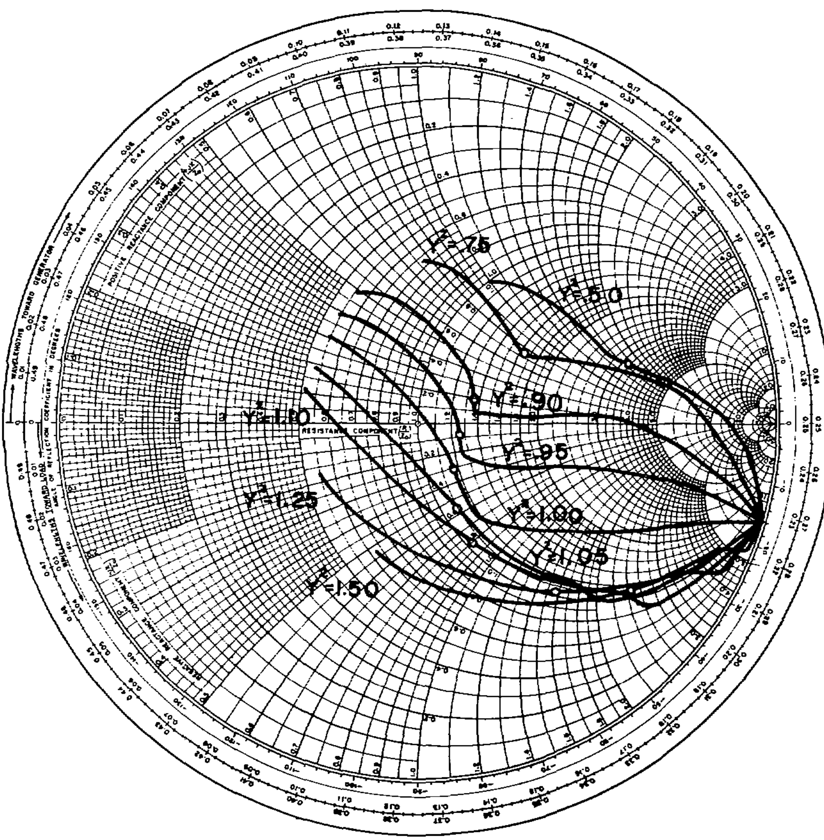

Fig. 4 Experimental impedance loci for the conditions indicated in Figure 2.

al loci are believed to result largely from electron density non-uniformity caused by plasma diffusion to the antenna surface.

In Figure 4 the plasma resonance points are marked on the impedance loci. These points were obtained by using the monopole as a "resonance probe"[5] when the D. C. magnetic field is zero. Zero magnetic field is required since in this geometry a magnetic field destroys the plasma resonance effect on which the resonance probe technique depends.

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3. Y. T. Lo, "A Note on the Cylindrical Antenna of Noncircular Cross Section", Journal of Applied Physics, Vol. 24, No. 10, pp. 1338-1339, October 1953.

4. S. A. Schelkunoff and H. T. Friis, "Antennas; Theory and Practice", Chapter 10, John Wiley & Sons., Inc., 1952.

5. K. Takayama, H. Ikegami and S. Miyazaki, "Plasma Resonance in a Radio Frequency Probe", Physical Review Letters, Vol. 5, No. 6, pp. 238-240, September 15, 1960.