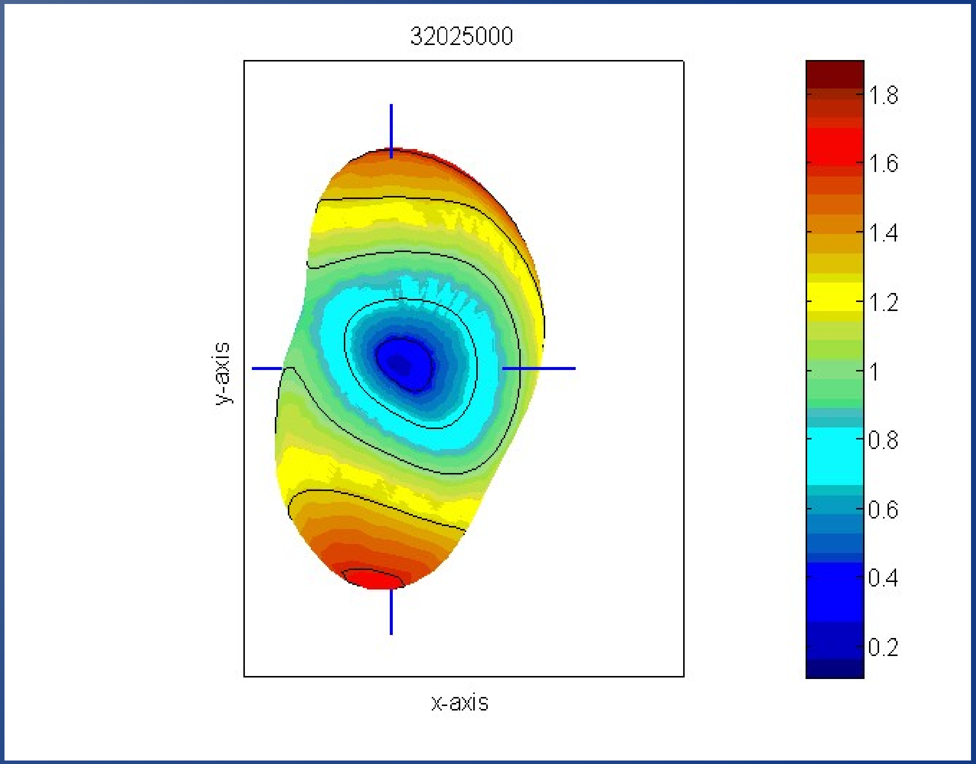
Antennas in plasma: Numerical calculation

Presentation of the PhD Thesis by Thomas Oswald

At Karl-Franzens-Universität Graz, supervised by Prof. H.O. Rucker

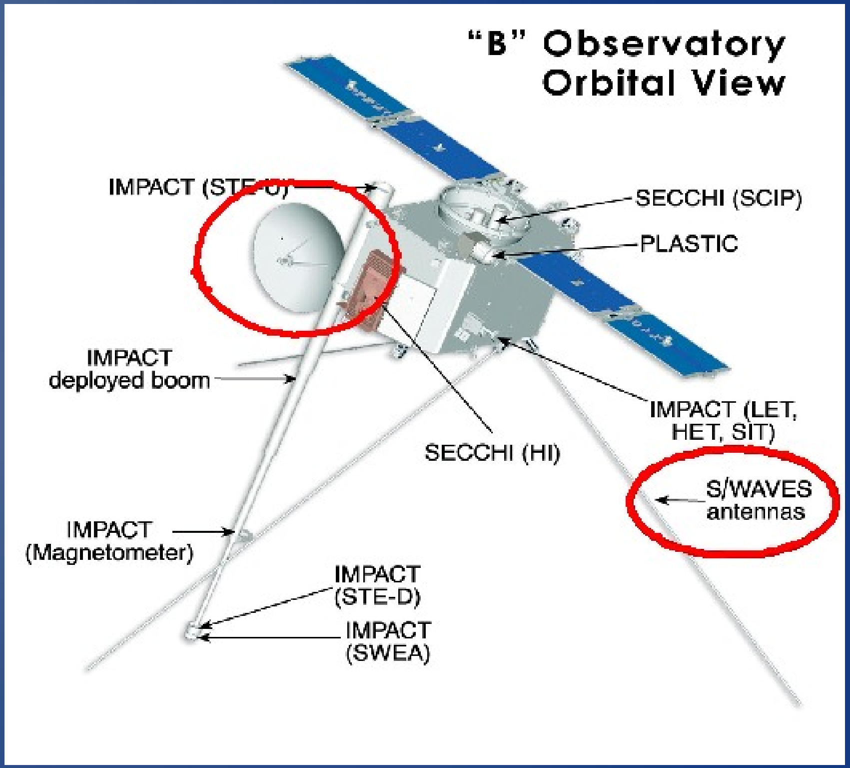
Content

- Introduction
- The method of moments (MoM)
- Theory (Plasma-Electrodynamics)
- Theory (Antenna Theory)
- Analysis of a dipole radiation
- Application on STEREO antennas
- Modeling the plasma sheath
- Conclusion + Outlook



Introduction: Radio Experiments

- Radio experiments are one of the most important experiments on spacecraft
- By receiving and analyzing radio and plasma waves created by natural phenomena, the physics of the creation process can be studied.
- Radio and plasma waves are received by antennas which are the interface between free wave propagation and guided wave propagation
- My thesis deals with the radio frequency range (kHz-MHz)

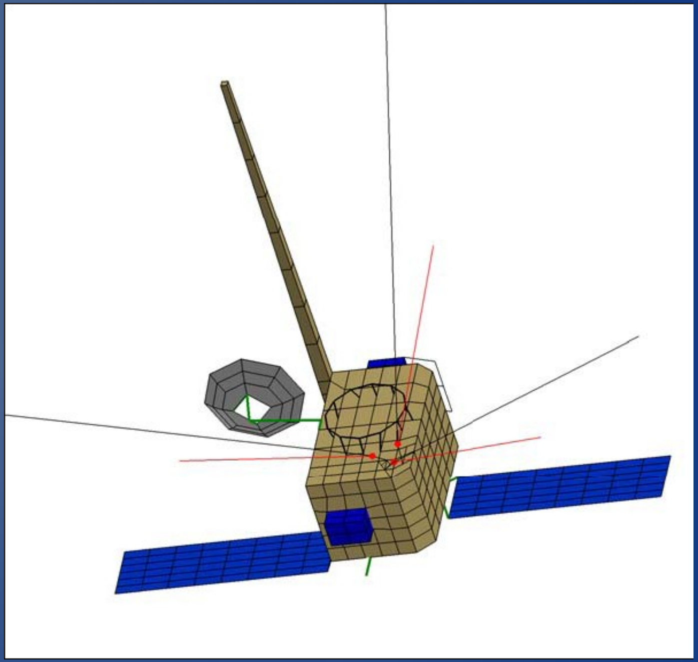


Introduction: Antennas

- Scientific antennas are monopoles which are either driven against another monopole to for a dipole, or against the spacecraft body.
- For the correct interpretation of the received data, the antenna properties must be known accurately
- Those are influenced by the spacecraft body and the surrounding plasma
- The influence is highest near the plasma resonance frequencies but also noticeable at the higher parts of the typical radio experiment frequency range.

Introduction: Properties

- The most important properties are:
  - Power patterns
  - Impedances
  - Admittances
  - Effective length vectors
- Usually a vacuum is postulated



Introduction: Calibration methods

- Numerical calculation
- Rheometry
- Anechoic chamber
- Inflight calibration

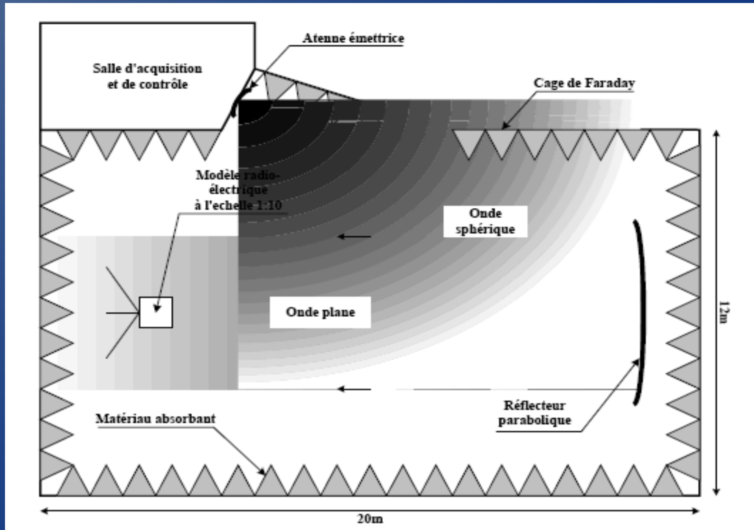
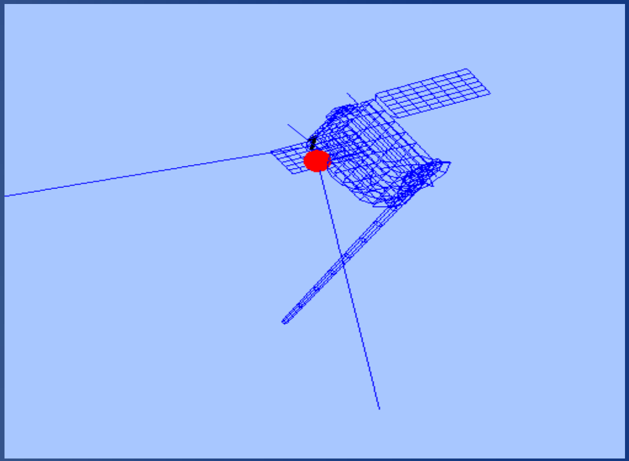
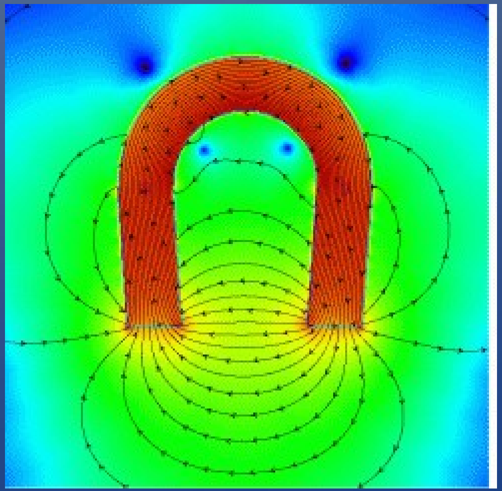


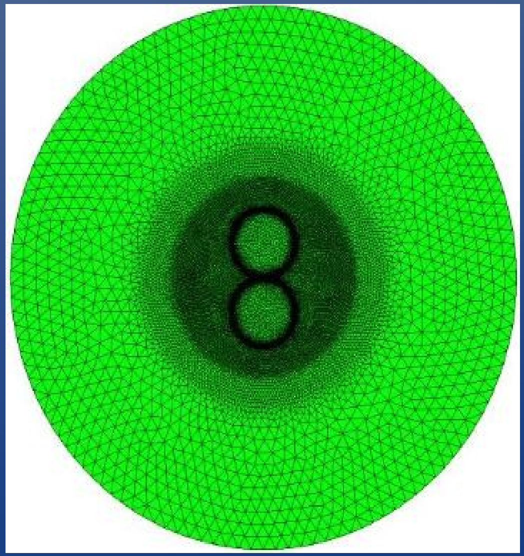
Figure 3.2 : Principe de la BCMA

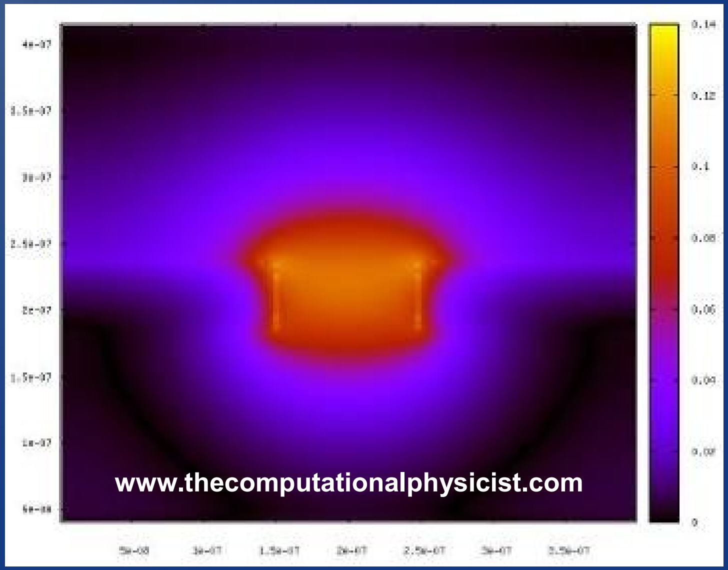
Introduction: Numerical methods



- Method of Moments(MoM)
- Finite difference, time domain (FDTD)
- Finite element (FEM)

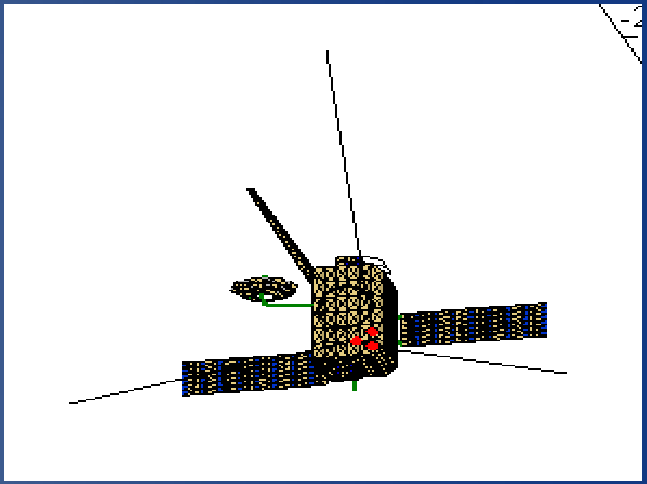


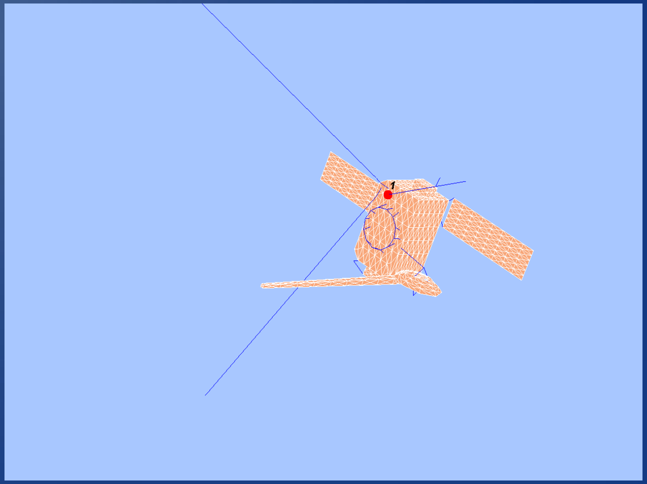




Introduction: MoM

- The MoM is a boundary value method
- The spacecraft is modeled as a grid of wires or patches
- Then the currents as a response of a 1V excitation along these wires/patches are computed
- This calculation is performed with an appropriate solver
- On basis of the current distribution, all other antenna properties (effective length vectors, impedances) can be calculated with MATLAB routines

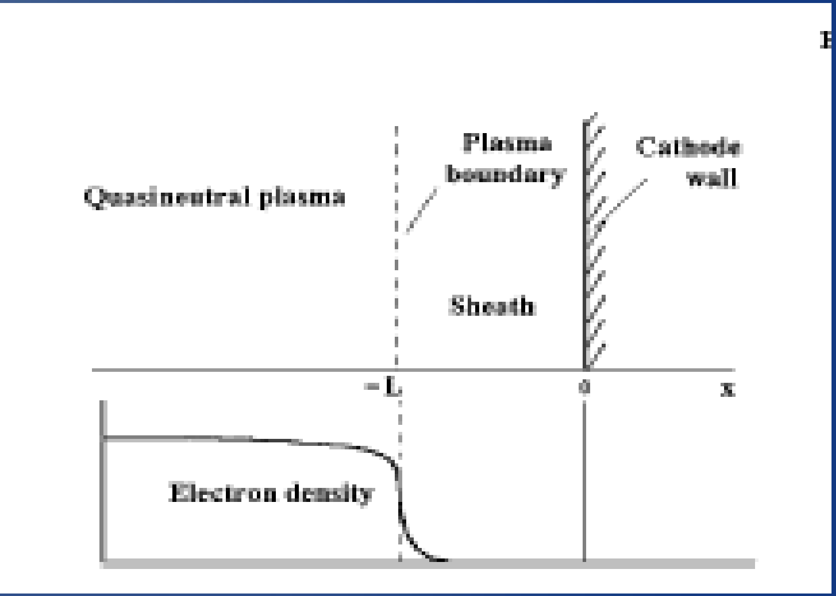




Introduction: Putting plasma into the MoM

- The effect of surrounding plasma is small but measurable at radio frequencies.
- Two Methods:

    1) The dielectric plasma mode: plasma physics in is the equivalent dielectric tensor.
    2) Modeling the plasma sheath: The plasma effect manifests itself in a capacity and resistivity which can be included easily in the calculation.



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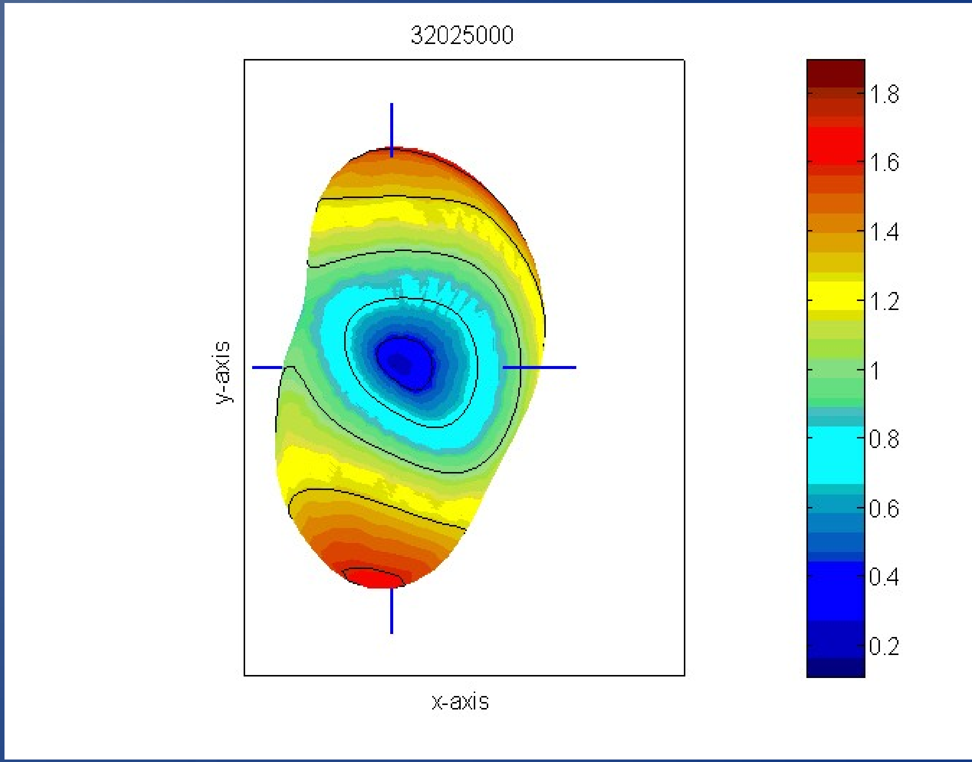


Method of Moments

- MoM is a method to solve field problems which can be described with the theory of linear spaces.
- L(j)=g
  - L...operator, functional
  - j...response function to be determined
  - g...excitation
- In the MoM this equation is converted to matrix form and solved numerically
- To solve electromagnetic scattering problems:
  - g...excitation voltage
  - j...the surface current system
- L must have certain properties which are governed by the theory of linear operators (e.g. self-adjointness,...)

Method of Moments

- A suitable inner product $\langle f, g \rangle$ has to be defined
- The inner product has to have certain properties:

$$\langle f, g \rangle = \langle g, f \rangle$$
$$\langle af + bg, h \rangle = a\langle f, h \rangle + b\langle g, h \rangle$$
$$\langle f^{¿}, f \rangle > 0 \; if \; f \neq 0$$
$$\langle f^{¿}, f \rangle = 0 \; if \; f = 0$$

- A suitable inner product for our purpose is

$$\langle f, g \rangle = \int f(x) g(x) dx$$

- The function to be determined has to be expanded:

$$\mathbf{j} = \sum_n c_n j_n$$

- J_n forms a set of independent basis functions
- C_n are coefficients to be determined
- Substituting for j in the linear equation gives

$$\sum_n c_n L j_n = g$$

Method of Moments

- The inner product with a weighting or testing function is formed:

$$\sum_n c_n \langle w_m, Lj_n \rangle = \langle w_m, g \rangle$$

- This can be written as matrix equation:

$$l_{mn} c_n = g_m$$

The solution can be found my standard methods of linear algebra.

- Then the currents can be reconstructed.
- For electromagnetic theory the equation can be written as

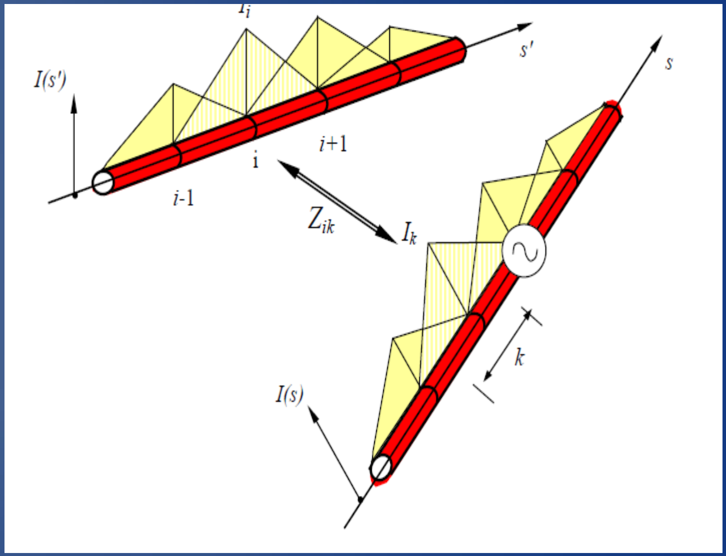
$$-\mathbf{V} = \mathbf{ZC}$$

- When the antenna is excitec by a unit voltage, V or g is a vector with zeros and a single entry which is 1 at the feed.
- When the excitation is by an incoming wave, the vector is

$$V_i = \mathbf{E}_i(\mathbf{r}_i) \cdot \mathbf{s}_i$$

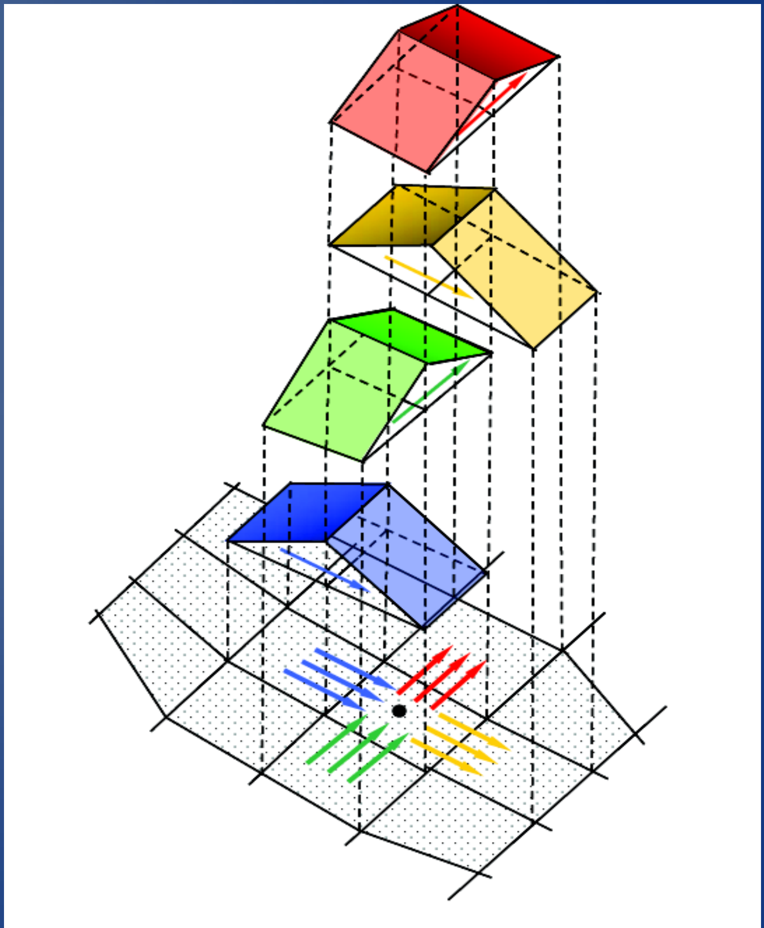
MoM: Basis and Testing functions

- In infinite set of combinations of basis and testing functions lead to success
- They differ in accuracy and convergence.
- For basis functions, constant functions (MEC), linear function (Concept II) and sinusoidal functions (ASAP, NEC, MEC) are common.
- For patches, spacial functions have to be used.



MoM: Basis and Testing functions

- For testing functions it is common to use the same functions as basis functions (Galerkin method).
- An other method is to use Dirac delta functions (collocation method).
- The collocation method facilitates the integration to get the matrix elements.
- This way the function is forced to fit at a single point.



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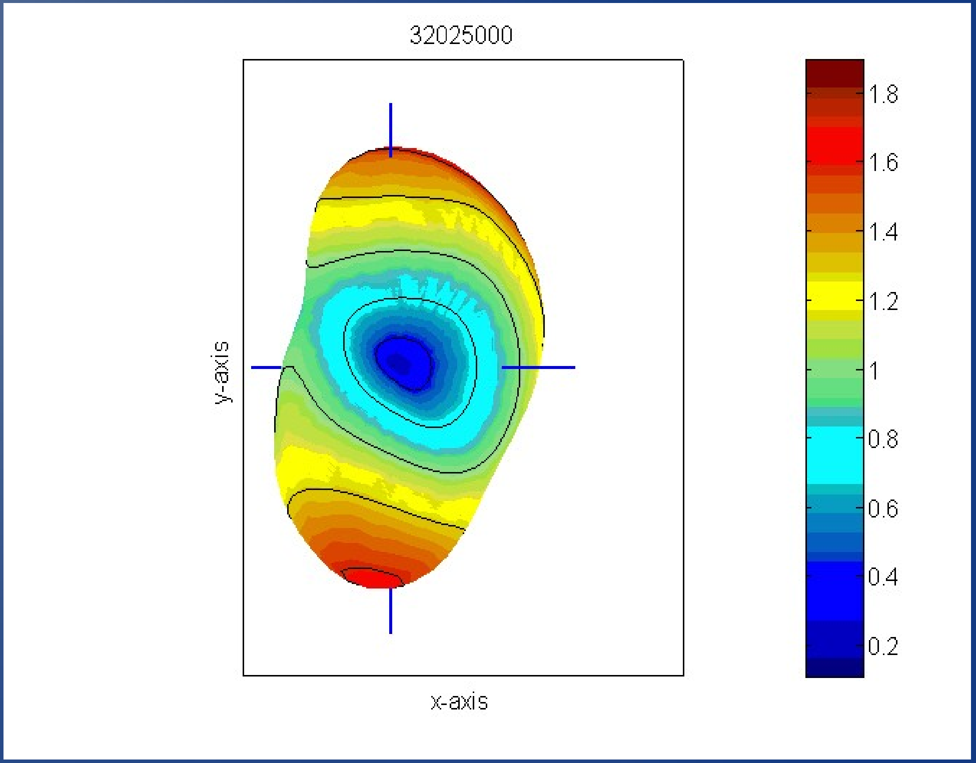

Theory: Electrodynamics

- Starting point: Maxwell

$$
\begin{aligned}
\nabla \cdot \mathbf{D}(\mathbf{r},t) &= \rho(\mathbf{r},t) \\
\nabla \cdot \mathbf{B}(\mathbf{r},t) &= 0 \\
\nabla \times \mathbf{E}(\mathbf{r},t) &= -\frac{\partial \mathbf{B}(\mathbf{r},t)}{\partial t} \\
\nabla \times \mathbf{H}(\mathbf{r},t) &= \mathbf{j}(\mathbf{r},t) + \frac{\partial \mathbf{D}(\mathbf{r},t)}{\partial t}
\end{aligned}
$$

- And Constitutive equations

$$\mathbf{D}(\mathbf{r},t) = \int_{-\infty,t} \int_V \hat{\epsilon}_{ij}(t,t',\mathbf{r},\mathbf{r}') E_j(\mathbf{r}',t') d^3\mathbf{r}' dt'$$

- A similar equation would exist for the magnetic field but is not necessary, because all the physics can be included in the equivalent dielectric tensor.

- If the material is linear and space and time are homogeneous, the equations can be Fourier transformed. When external sources are not present:

$$
\begin{aligned}
\mathbf{k} \cdot \mathbf{E}(\mathbf{k},\omega) &= 0 \\
\mathbf{k} \cdot \mathbf{B}(\mathbf{k},\omega) &= 0 \\
\mathbf{k} \times \mathbf{E}(\mathbf{k},\omega) &= \omega \mathbf{B}(\mathbf{k},\omega) \\
\mathbf{k} \times \mathbf{B}(\mathbf{k},\omega) &= -\mu_0 \epsilon_0 \omega \mathbf{E}(\mathbf{k},\omega)
\end{aligned}
$$

- and

$$\mathbf{D} = \epsilon_0 \epsilon_{\mathbf{r}}(\mathbf{k},\omega) \mathbf{E}(\mathbf{k},\omega) = \epsilon(\mathbf{k},\omega) \mathbf{E}(\mathbf{k},\omega)$$

- where

$$\epsilon(\mathbf{k},\omega) = \int_{0,\infty} \int_V \hat{\epsilon}_{ij}(t-t',\mathbf{r}-\mathbf{r}') e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} d^3(\mathbf{r}-\mathbf{r}') d(t-t')$$

Theory: Electrodynamics

- Often potential fields are used to facilitate finding solutions:

$$\begin{aligned}\mathbf{B}(\mathbf{r},t) &= \nabla \times \mathbf{A}(\mathbf{r},t)\\ \mathbf{E}(\mathbf{r},t) &= -\frac{\partial \mathbf{A}(\mathbf{r},t)}{\partial t} - \nabla\phi(\mathbf{r},t)\end{aligned}$$

- They are not unique, so gauge conditions are used:

$$\begin{aligned}\nabla\cdot\mathbf{A} + \varepsilon_0\mu_0\frac{\partial\phi}{\partial t} &= 0\\ \nabla\cdot\mathbf{A} &= 0\\ \phi &= 0\end{aligned}$$

- Lorenz gauge, Coloumb gauge, temporal (Weyl) gauge

- In plasma it is often better to use the fields directly, since there is no gain in solving for A instead of E.
- Maxwell's equation can be manipulated to form a wave equation: $\mathbf{k}\times\mathbf{k}\times\mathbf{E} + \omega^2\mu\epsilon\mathbf{E} = 0$
- Or: $\mathbf{n}\times\mathbf{n}\times\mathbf{E} + \varepsilon_r\mathbf{E} = 0$ using the diffraction index $\mathbf{n} = \frac{\mathbf{k}}{\omega\sqrt{\mu_0\epsilon_0}} = \frac{\mathbf{k}c}{\omega},$
- Since

$$\mathbf{n}\times\mathbf{n}\times\mathbf{E} = \mathbf{nnE} - \mathbf{n}^2\mathbf{E}$$

$$(\mathbf{nn} - \mathbf{n}^2\mathbf{I} + \varepsilon_r)\mathbf{E} = 0$$

Theory: Electrodynamics

$$(\mathbf{nn} - \mathbf{n}^2\mathbf{I} + \epsilon_r)\mathbf{E} = 0$$

$$\mathbf{T} \cdot \mathbf{E} = 0$$

$$\mathbf{T} = \begin{pmatrix} n_1^2 - \mathbf{n}^2 + \epsilon_{r11} & n_1 n_2 + \epsilon_{r12} & n_1 n_3 + \epsilon_{r13} \\ n_1 n_2 + \epsilon_{r21} & n_2^2 - \mathbf{n}^2 + \epsilon_{r22} & n_2 n_3 + \epsilon_{r23} \\ n_1 n_3 + \epsilon_{r31} & n_2 n_3 + \epsilon_{r32} & n_3^2 - \mathbf{n}^2 + \epsilon_{r33} \end{pmatrix}$$

Theory: The equivalent dielectric tensor

- Includes all plasma physics in the dielectric model.
- Describes the linear response of a medium to electromagnetic disturbances.
- Is a tensor in general.
- Can be complex. The imaginary part can be interpreted as a damping effect.

$$\epsilon_0 \nabla \cdot \mathbf{E}(\mathbf{r}, t) = \rho_e(\mathbf{r}, t)$$

$$\otimes$$

$$\frac{\partial \rho_e}{\partial t} + \nabla \cdot \mathbf{j} = 0$$

$$\downarrow$$

$$\epsilon_0 \nabla \cdot \mathbf{E}(\mathbf{r}, \omega) - \frac{\nabla \cdot \mathbf{j}(\mathbf{r}, \omega)}{i\omega} = 0$$

$$\otimes$$

Ohm's law

$$\downarrow$$

$$\nabla \cdot \left(\epsilon_0 - \frac{\sigma}{i\omega} \right) \mathbf{E}(\mathbf{r}, \omega) = 0$$

Theory: Cold isotropic plasma

- The cold isotropic dielectric function is a scalar function of the frequency.
- It is real valued.
- Can easily be derived by using the Polarization:

$$\mathbf{P} = nq\mathbf{r}$$

- And solving Newton's equation.

$$F_e = m_e \frac{d^2\mathbf{r}}{dt^2} = q\mathbf{E}$$

For r. One gets:

$$\mathbf{P} = -\frac{q^2 n}{\omega^2 m_e}\mathbf{E}$$

- Using

$$\omega_p = \sqrt{\frac{nq^2}{m_e \epsilon_0}}$$

- One gets

$$\mathbf{P} = -\epsilon_0 \frac{\omega_p^2}{\omega^2}\mathbf{E}$$

- Hence

$$\epsilon = \epsilon_0 \left(1 - \frac{\omega_p^2}{\omega^2}\right)$$

- The isotropic dielectric tensor can be inserted to all MoM solvers, so this result can be used immediately

Theory: Cold magnetized plasma

- The dielectric tensor of magnetized cold plasma is well known.
- It can be derived by using Newton's equation with the magnetic field.

$$F = m\frac{d\mathbf{v}}{dt} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B}_0)$$

- The subscript is to show that it is an external static field.

- The result:

$$\bar{\epsilon} = \epsilon_0 \begin{pmatrix} 1-\frac{\omega_p^2}{\omega^2-\Omega^2} & -\imath\frac{\omega_p^2\Omega}{\omega(\omega^2-\Omega^2)} & 0 \\ \imath\frac{\omega_p^2\Omega}{\omega(\omega^2-\Omega^2)} & 1-\frac{\omega_p^2}{\omega^2-\Omega^2} & 0 \\ 0 & 0 & 1-\frac{\omega_p^2}{\omega^2} \end{pmatrix}$$

- Or when factorizing for lh and rh polarization:

$$\begin{aligned}
\varepsilon_{11} &= \varepsilon_0\left[1-\sum_s\left(\frac{\omega_{p,s}^2}{\omega(\omega+\Omega_s)}+\frac{\omega_{p,s}^2}{\omega(\omega-\Omega_s)}\right)\right] \\
\varepsilon_{12} &= \varepsilon_0\left[-\imath\sum_s\left(\frac{\omega_{p,s}^2}{\omega(\omega+\Omega_s)}-\frac{\omega_{p,s}^2}{\omega(\omega-\Omega_s)}\right)\right] \\
\varepsilon_{21} &= \varepsilon_0\left[\imath\sum_s\left(\frac{\omega_{p,s}^2}{\omega(\omega+\Omega_s)}-\frac{\omega_{p,s}^2}{\omega(\omega-\Omega_s)}\right)\right] \\
\varepsilon_{22} &= \varepsilon_0\left[1-\sum_s\left(\frac{\omega_{p,s}^2}{\omega(\omega+\Omega_s)}+\frac{\omega_{p,s}^2}{\omega(\omega-\Omega_s)}\right)\right] \\
\varepsilon_{33} &= \varepsilon_0\left[1-\sum_s\frac{\omega_{p,s}^2}{\omega^2}\right]
\end{aligned}$$

Theory: Kinetic isotropic plasma...electrostatic approximation 1

- Kinetic theory finds many wave-modes.
- Above characteristic frequencies and without magnetic field, 2 types are important:
  - Langmuir waves
  - EM waves
- Basis: Vlasov equation

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla_r - \frac{e}{m_e}\mathbf{E}\cdot\nabla_v\right) f_e(\mathbf{r},\mathbf{v},t) = 0$$

- And perturbation theory.

- E-field || k
- After Fourier transform one gets:

$$f_1 = \frac{e}{m_e \imath (kv_{||} - \omega)} E_1 \frac{\partial f_0}{\partial v_{||}}$$

one can use scalar potential and Poisson eq. as second function:

$$\begin{aligned} f_1 &= -\frac{e}{m_e(kv_{||} - \omega)} k\phi \frac{\partial f_0}{\partial v_{||}} \\ -k^2\phi &= \frac{e}{\epsilon_0}\int_{-\infty}^{\infty} f_1 dv_{||} \end{aligned}$$

- Results in:

$$1 - \frac{e^2}{km_e\epsilon_0}\int_{-\infty}^{\infty}\frac{1}{kv_{||} - \omega}\frac{\partial f_0}{\partial v_{||}} dv_{||} = 0$$

Theory: Kinetic isotropic plasma...electrostatic approximation 2

- Can be identified as dispersion relation. Hence:

$$\epsilon_{||} = 1 - \frac{\omega_{pe}}{kn_e}\int_{-\infty}^{\infty} \frac{1}{kv_{||} - \omega}\frac{\partial f_0}{\partial v_{||}} dv_{||}$$

- The integral can not be evaluated since it contains a pole.
- Landau found a solution by making the frequency complex:

$$(\omega \rightarrow \omega + \delta\imath)$$

- Integrated along a contour around.
- This leads to the famous collision-less Landau-Damping.
- ...which means the Langmuir waves are so heavily damped that we will never receive such waves from a distant source in the RF-range.
- But these waves are important for quasi-static thermal noise analysis.

Theory: Kinetic isotropic plasma...EM waves

- Using the same procedure but retaining the vector form, one gets:

$$f_1 = \frac{e}{m_e\imath(\mathbf{k}\cdot\mathbf{v}-\omega)}\mathbf{E}_1\frac{\partial f_0}{\partial \mathbf{v}}$$

- Substituting this into the equation for current density:

$$\mathbf{j} = n_e e\overline{v} = e\int_{-\infty}^{\infty}\mathbf{v} f_1 d^3\mathbf{v}$$

$$\mathbf{j} = \frac{e^2}{m_e\imath}\int_{-\infty}^{\infty}\frac{1}{(\mathbf{k}\cdot\mathbf{v}-\omega)}\mathbf{v}\frac{\partial f_0}{\partial \mathbf{v}}\cdot\mathbf{E}_1 d^3\mathbf{v}$$

- The electric field is independent of v, so can be pulled out of the integral:

$$\overline{\sigma} = \frac{e^2}{m_e\imath}\int_{-\infty}^{\infty}\frac{1}{(\mathbf{k}\cdot\mathbf{v}-\omega)}\mathbf{v}\frac{\partial f_0}{\partial \mathbf{v}}d^3\mathbf{v}$$

- Finally:

$$\overline{\epsilon} = \mathbf{I} + \frac{\omega_{pe}^2}{n_e\omega}\int_{-\infty}^{\infty}\frac{1}{(kv_z-\omega)}\mathbf{v}\frac{\partial f_0}{\partial \mathbf{v}}d^3\mathbf{v}$$

- The integration over the longitudinal part is difficult. But at high frequencies: $\omega \gg k\overline{v}$
- The the integrand can be approximated:

$$\overline{\epsilon} \approx \mathbf{I} - \frac{\omega_{pe}^2}{n_e\omega^2}\int_{-\infty}^{\infty}\mathbf{v}\frac{\partial f_0}{\partial \mathbf{v}}d^3\mathbf{v} = \mathbf{I}\left(1-\frac{\omega_{pe}^2}{\omega^2}\right)$$

- At T=10^5K and 100kHz: $\frac{\omega}{kv_{th}} \approx 10^{14}$
- So for this subject no kinetic treatment is necessary !

Theory: Other dielectric tensors

- There is a complicated dielectric tensor for magnetized kinetic plasmas. This tensor is not relevant due to the result of the last slide.
- At high temperatures the kinetic model would be necessary. Then also the relativistic effects and the radiation would have to be taken into account.
- Quantum mechanical model necessary ? The criterion is

$$\hbar\omega \ll mc^2$$

- For electrons:

$$\omega \ll 7.77 \cdot 10^{20} rad s^{-1}$$

- Which is always true in the scope of this subject.

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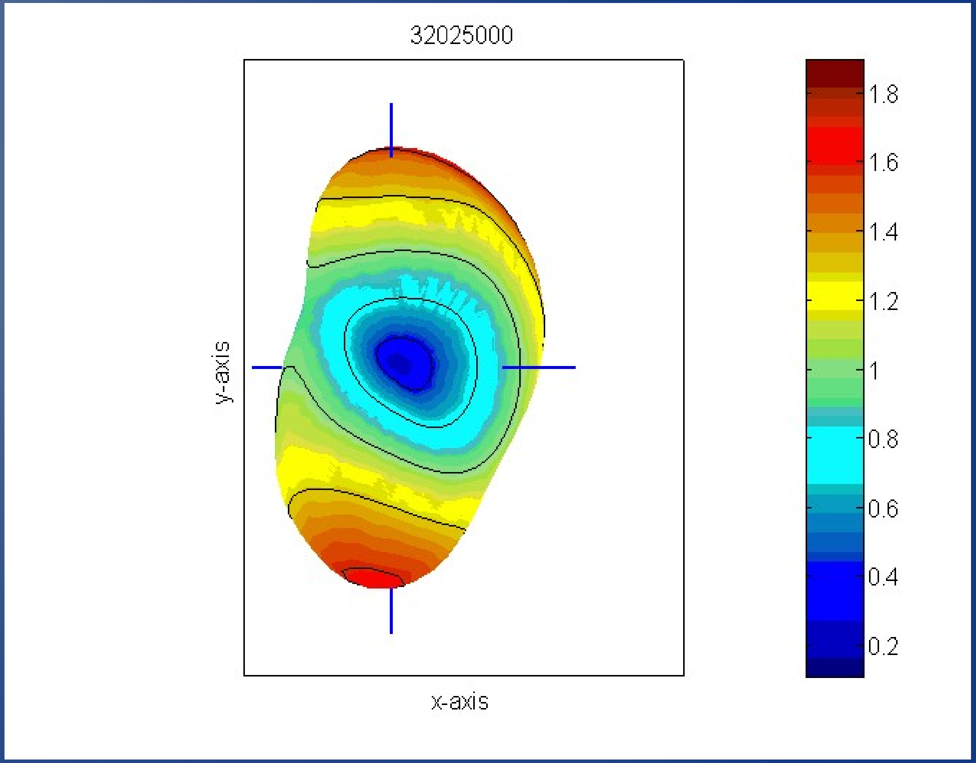

Antenna Theory: Green's function

- In this context Green's function is a dyadic function.
- It maps an infinitesimal source to the responding field.
- The wave equation for the electric field :

$$\nabla \times \nabla \times \mathbf{E}(\mathbf{r}, \omega) - k_0^2 \epsilon_r \cdot \mathbf{E}(\mathbf{r}, \omega) - \mu_0 \imath \omega \mathbf{j}_{ant}(\mathbf{r}, \omega) \quad = \quad 0$$

- It can be solved when the Green's function is found:

$$\mathbf{E}(\mathbf{r}, \omega) = \int_{V'} \mathbf{G}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{j}_{ant}(\mathbf{r}', \omega) dV'$$

- This equation is independent of the medium. The whole plasma physics is insider Green's function.
- Combining the two equations gives:

$$\nabla \times \nabla \times \mathbf{G}(\mathbf{r}, \mathbf{r}') - k_0^2 \epsilon_r \cdot \mathbf{G}(\mathbf{r}, \mathbf{r}') \quad = \quad \mu_0 \imath \omega \mathbf{I} \delta(\mathbf{r} - \mathbf{r}')$$

- Fourier transforming and manipulating gives:

$$\left(k^2 \mathbf{I} - \mathbf{k}\mathbf{k} - k_0^2 \epsilon_r\right) \cdot \Gamma(\mathbf{k}) = \mu_0 \imath \omega \mathbf{I} \delta(\mathbf{k})$$

- And finally

$$\Gamma(\mathbf{k}) = \left(k^2 \mathbf{I} - \mathbf{k}\mathbf{k} - k_0^2 \epsilon_r\right)^{-1} \frac{\mu_0 \imath \omega}{(2\pi)^{\frac{3}{2}}}$$

- To use, a FT to real space has to be performed.

Antenna theory: Cold isotropic Green's function

- For the isotropic cold plasma the solution turns out to be:

$$\mathbf{G}(\mathbf{r},\mathbf{r}') = -\left(\nabla\nabla + k_0^2\epsilon_r \mathbf{I}\right) g(\mathbf{r}-\mathbf{r}')$$

- Where g is the Green's function in vacuum:

$$g(\mathbf{r}-\mathbf{r}') = -\frac{\epsilon_r^{-1}}{4\pi\imath\omega\epsilon_0}\frac{e^{-\imath k|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}$$

- So the electric field equation is:

$$\mathbf{E}(\mathbf{r},\omega) = \int_{V'} \left(\nabla\nabla + k^2\mathbf{I}\right)\frac{\epsilon_r^{-1}}{4\pi\imath\omega\epsilon_0}\frac{e^{-\imath k|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\cdot \mathbf{j}_{ant}(\mathbf{r}',\omega)dV'$$

- And the electric field integral equation, EFIE would be

$$\hat{\mathbf{e}}_{\mathrm{n}} \times \mathbf{E}_{\mathrm{i}}(\mathbf{r},\omega) = \hat{\mathbf{e}}_{\mathrm{n}} \times \int_{S'} \left(\nabla\nabla + k^2\mathbf{I}\right) g(\mathbf{r}-\mathbf{r}')\cdot \mathbf{j}_{ant,s}(\mathbf{r}',\omega)dA'$$

- This equation is the boundary condition of an electric field on a conducting surface. It says, that the tangential field of the incident field is equal to the tangential component of the scattered field.
- This equation is used in NEC to compute the mutual impedances between the segments to create the matrix.
- Certain assumptions are made for facilitation: thin wires, where only axial currents are used.

Antenna theory: cold magnetized plasma

- Using a special form of the radiation tensor:

$$\lambda = \begin{pmatrix} 1+n^2\cos^2\theta - \frac{\omega_p^2}{\omega^2-\Omega^2} & \imath\frac{\omega_p^2\Omega}{\omega(\omega^2-\Omega^2)} & -n^2\sin\theta\cos\theta \\ -\imath\frac{\omega_p^2\Omega}{\omega(\omega^2-\Omega^2)} & 1+n^2-\frac{\omega_p^2}{\omega^2-\Omega^2} & 0 \\ -n^2\sin\theta\cos\theta & 0 & 1+n^2\sin^2\theta-\frac{\omega_p^2}{\omega^2} \end{pmatrix}$$

- Green's function can be written as

$$\begin{aligned} \mathbf{G}(\mathbf{r},\mathbf{r}') &= \frac{\mu_0 \imath\omega}{(2\pi)^3}\int_{-\infty}^{\infty}\lambda^{-1}e^{\imath(k\cdot(\mathbf{r}-\mathbf{r}'))}d\mathbf{k} \\ &= \frac{\mu_0 \imath\omega}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{adj(\lambda)}{\det(\lambda)}e^{\imath(k\cdot(\mathbf{r}-\mathbf{r}'))}d\mathbf{k} \end{aligned}$$

- To my knowledge the integral can not be solved analytically.
- It would have to be solved together with:

$$\mathbf{E}(\mathbf{r},\omega) = \int_{V'}\mathbf{G}(\mathbf{r},\mathbf{r}')\cdot\mathbf{j}_{ant}(\mathbf{r}',\omega)dV'$$

- Numerically as the inner integral in a system of 2 nested integrals.

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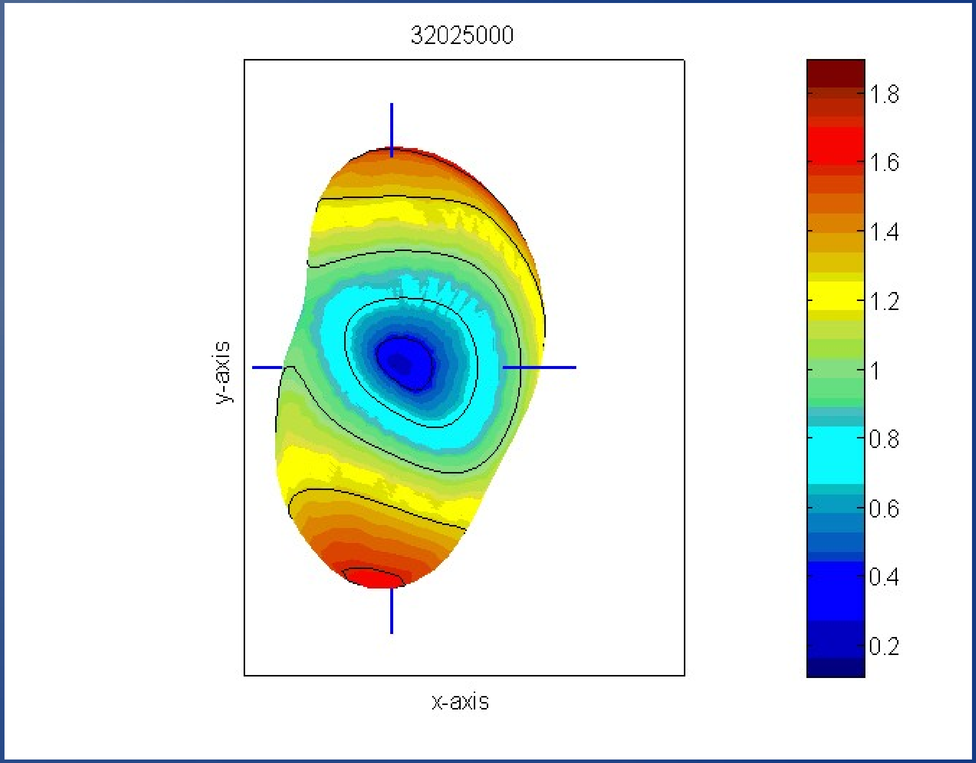
- Modeling the plasma sheath
- Conclusion + Outlook



Dipole Radiation

- A 1D MoM solver was written and calculations where performed with a 6m dipole.
- The MoM code uses pulse functions.
- A second solver, using sinusoidal functions, was written and produces the same results.
- The feed is across a segment.
- The isotropic cold plasma model was implemented.

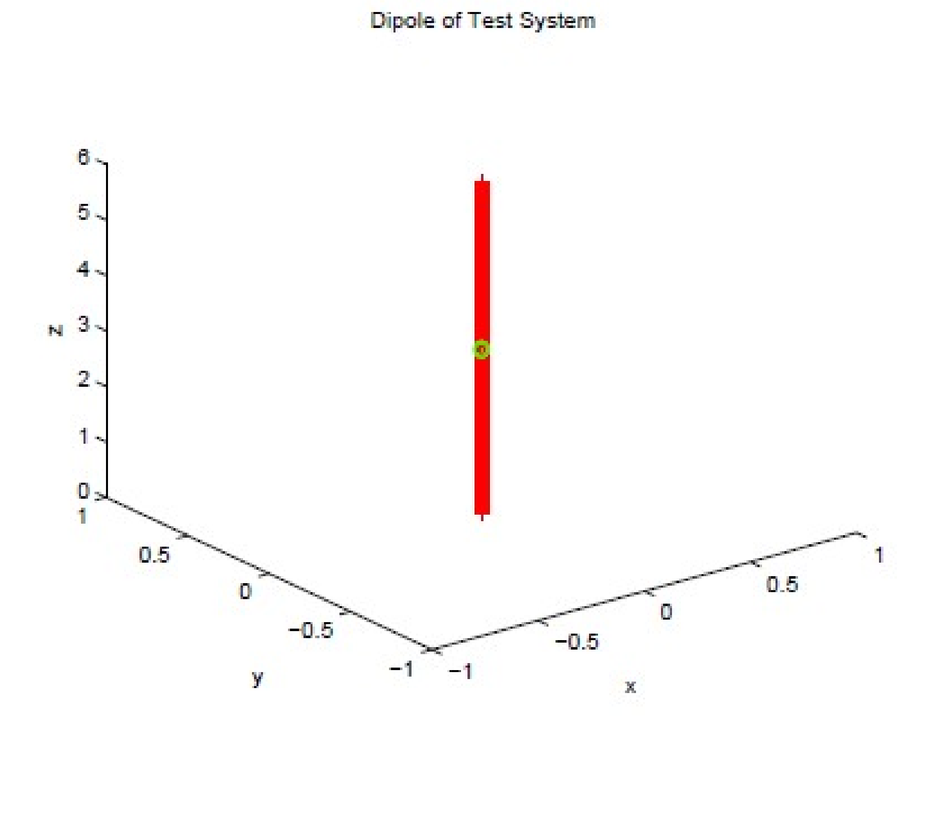


Fig. 5.1: 6 meter dipole used for calculation

Dipole Radiation

- After many test-calculations in vacuum, field calculations where performed, using currents computed with vacuum conditions.
- In this simple model, the main difference is an elongation of wavelength.
- This corresponds to a smaller effective length vector.
- A lower field intensity is expected because energy is stored in the oscillation of the plasma particles: ordered kinetic energy.

$$E_{ok} = \sum_s \frac{1}{2} n_s m_s v_s^2$$

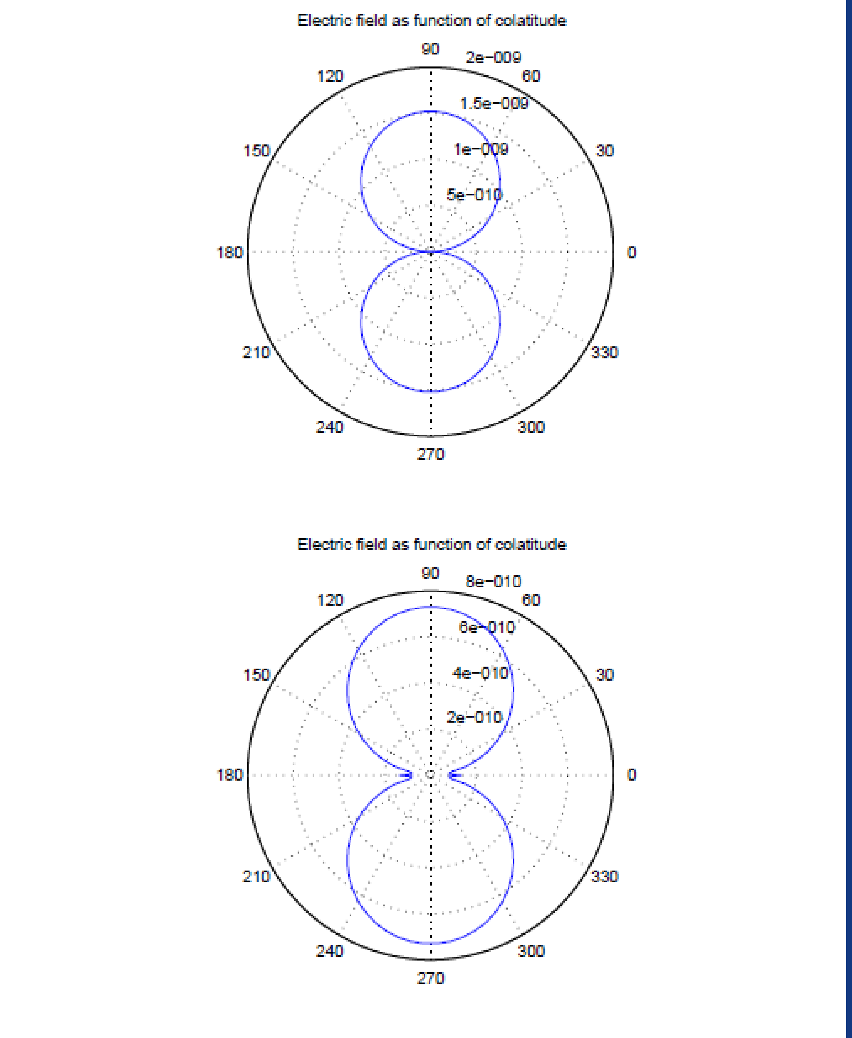
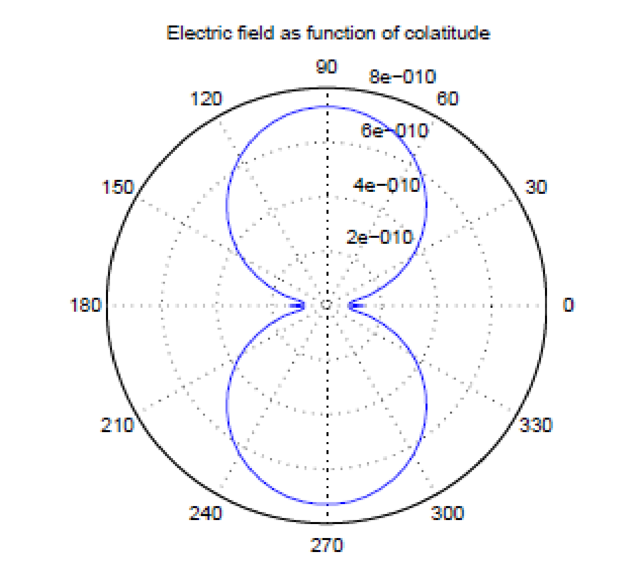
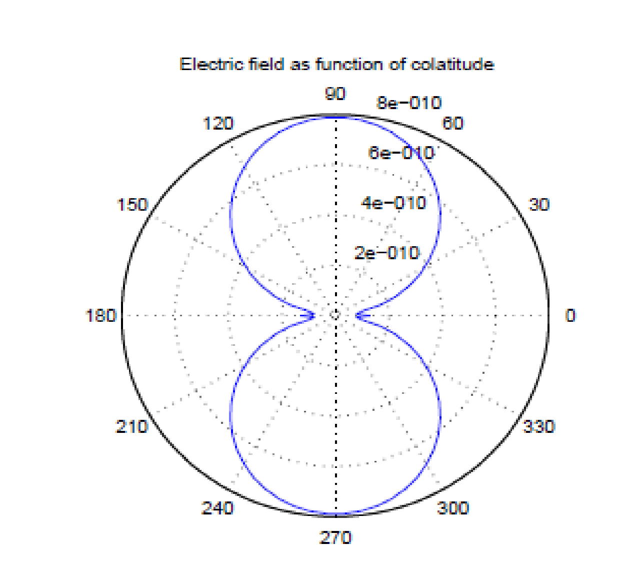


Fig. 5.9: Electric field strength [$V m^{-1}$] at 10000m distance in vacuum (top) and isotropic cold plasma (bottom) with $\frac{\omega_{pe}^2}{\omega^2} = 0.9$ at 300kHz

Dipole Radiation

- Including the effect of the ions is of magnitude $\sqrt{2000}$ due to the mass relation of the particles.





Dipole Radiation

- Investigating the currents and Impedances, it can be seen that the net effect is a reduction of the lengths of the effective length vectors.
- This is the unrealistic situation where the relation of plasma frequency to frequency is constant over the whole frequency range.

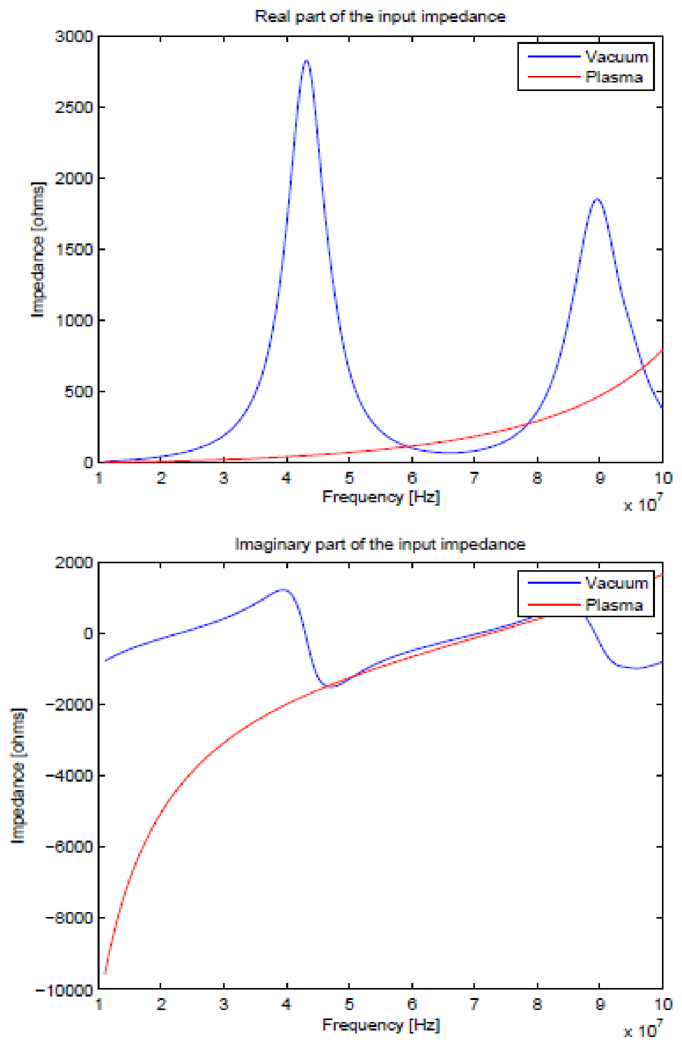


Fig. 5.18: Impedance curves of the dipole with and without plasma. Fixed $\frac{\omega_{pe}^2}{\omega^2} = 0.9$

Dipole Radiation

- Here a realistic case.
- ω_{pe} = 10MHz
- This result suggests that a vacuum approximation is valid for many situations.

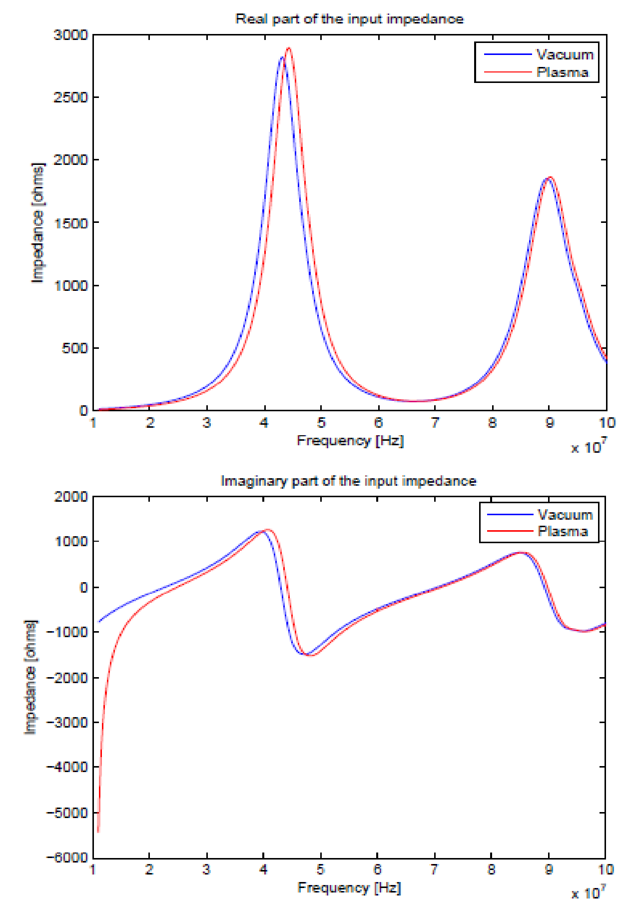


Fig. 5.19: Impedance curves of the dipole with and without plasma. $\omega_{pe} = 10MHz$

Dipole Radiation

- But near resonances the situation can be different.

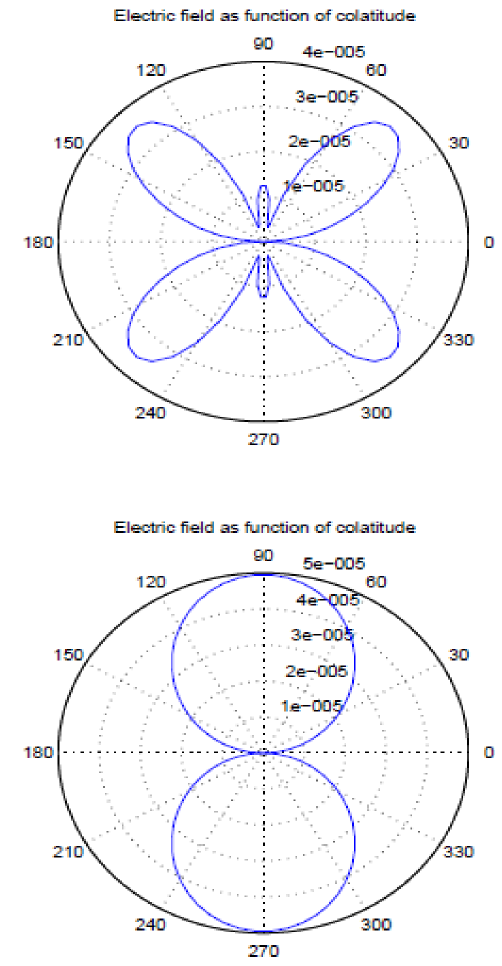


Fig. 5.20: Electric field strength $[Vm^{-1}]$ at 10000m distance in vacuum (top) and in plasma at $\frac{3}{2}\lambda$ resonance condition (bottom)

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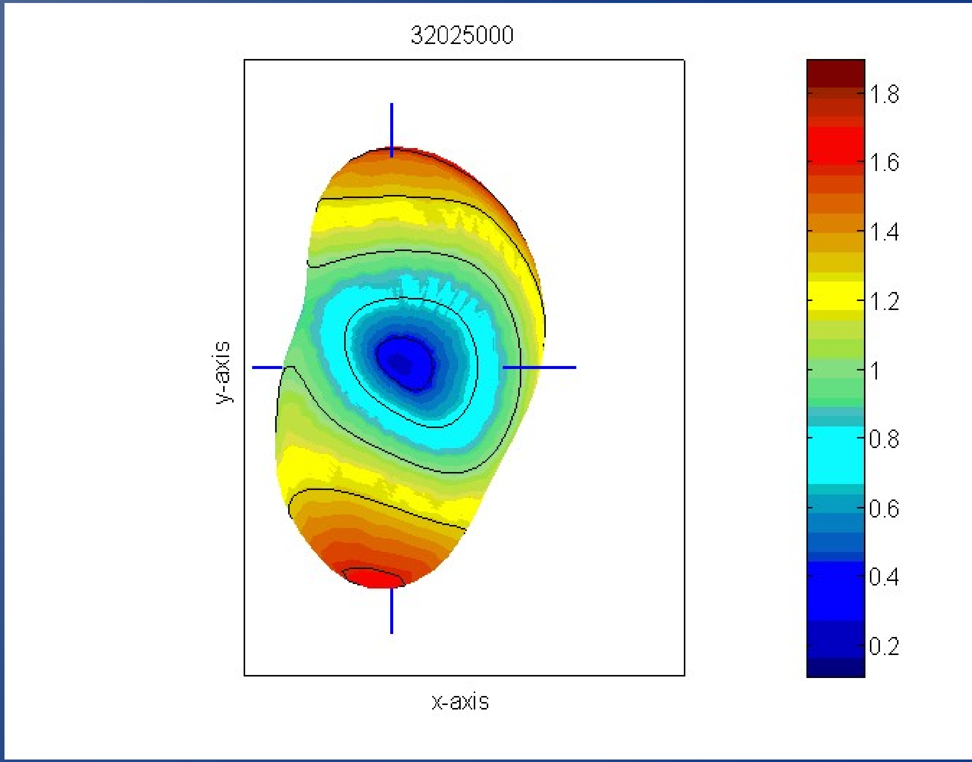
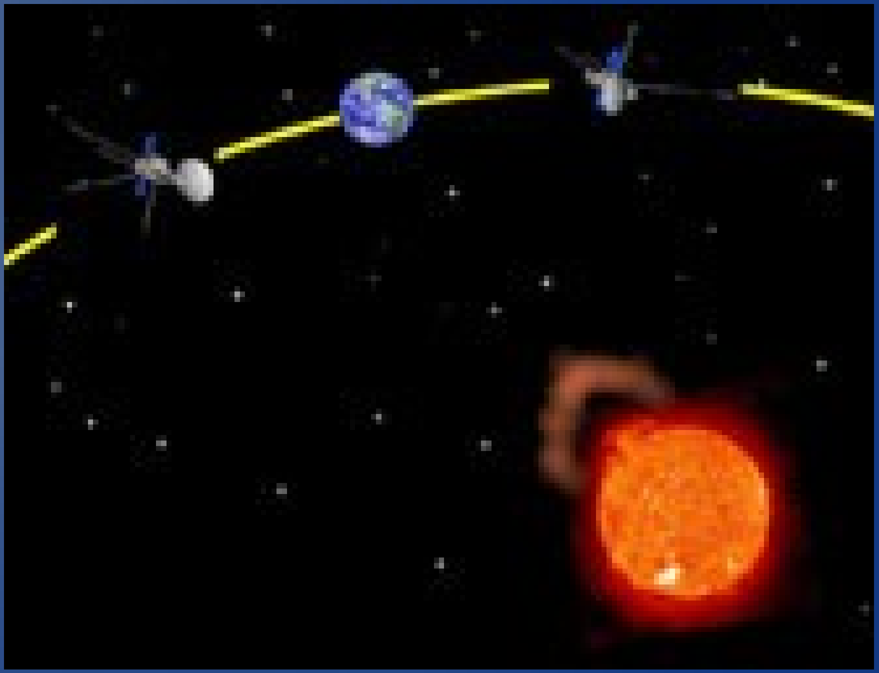


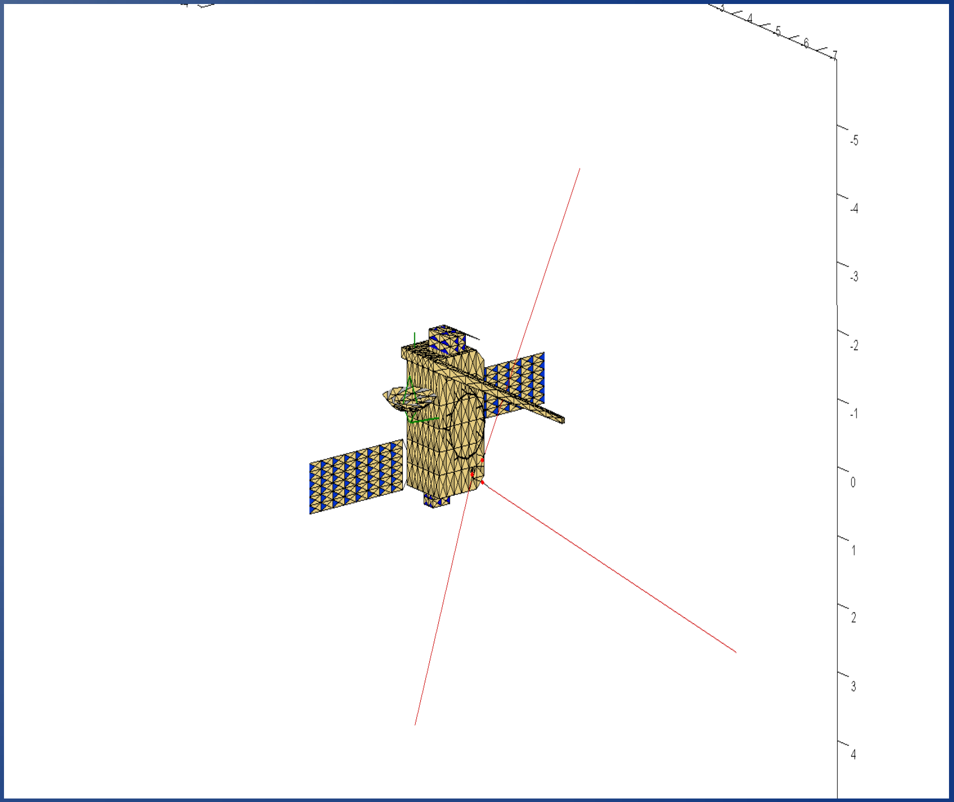
STEREO Antennas

- Calculations of the S-WAVES antennas where performed.
- The Stereo mission comprises 2 spacecraft.
- Both are nearly at earth orbit around the sun, one ahead of earth, one behind.
- They drift apart to get a stereographic view with different angles.
- Solar radiation is received with the antennas.



STEREO Antennas

- The calculations where performed with Concept II.
- In Concept the real and imaginary part of the permittivity can be altered.
- By setting the real part, a simple cold isotropic plasma can be simulated.



STEREO Antennas: Impedances

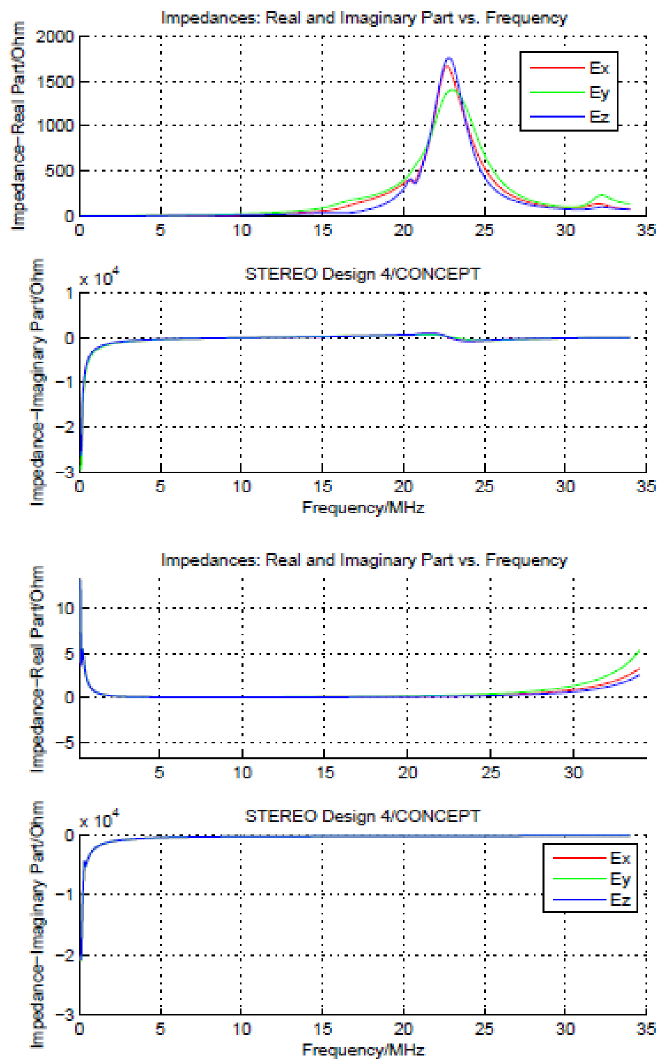


Fig. 6.1: The impedance of the S/WAVES antennas in vacuum (top) in relation to cold plasma with $\epsilon_r = 0.1$ (bottom)

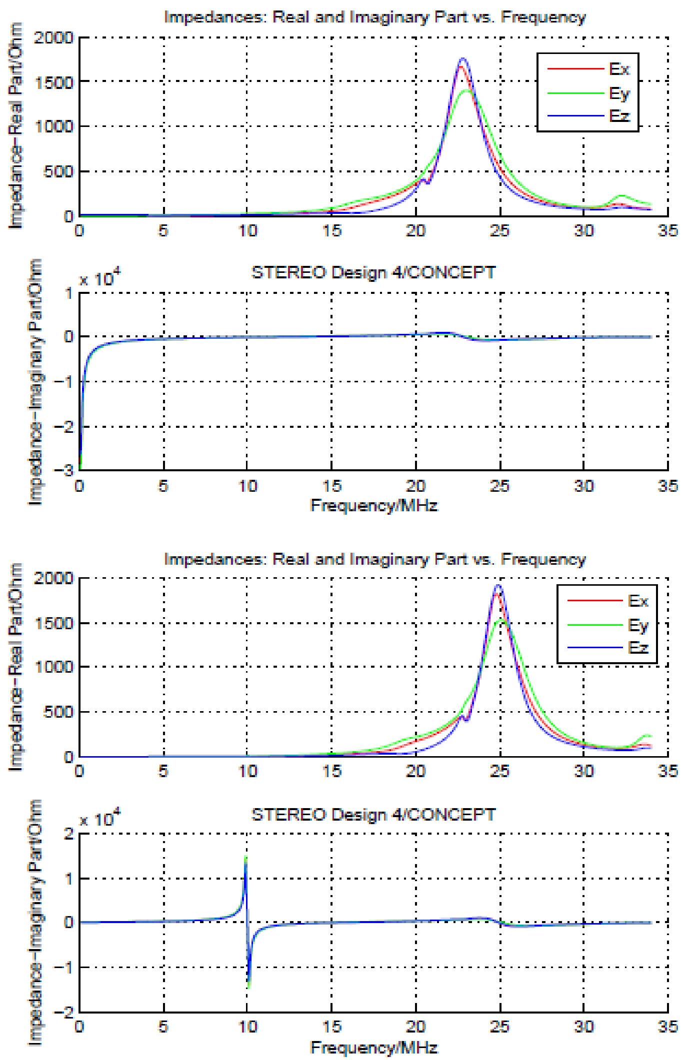


Fig. 6.2: The impedance of the S/WAVES antennas in vacuum (top) in relation to cold plasma with $f_{pe} = 10MHz$ (bottom)

STEREO Antennas: Effective length vectors

Tab. 6.1: Effective length vectors in vacuum: $f = 300kHz$

	$length/m$	$\zeta/^\circ$	$\xi/^\circ$
E_x	1.35	119.9	-135.3
E_y	1.64	114.4	127.3
E_z	1.09	124.7	15.5

Tab. 6.2: Effective length vectors in cold plasma: f=300kHz, $f_{pe} = 100kHz$

	$length/m$	$\zeta/^\circ$	$\xi/^\circ$
E_x	1.26	119.6	-135.0
E_y	1.53	114.1	127.2
E_z	1.02	124.3	15.2

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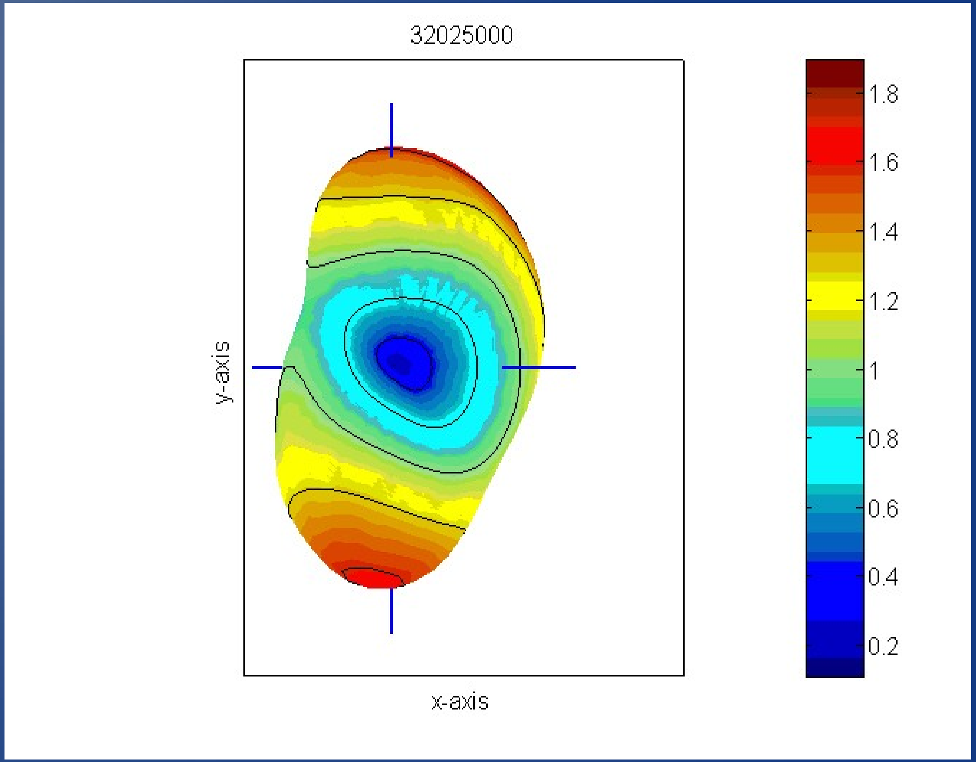
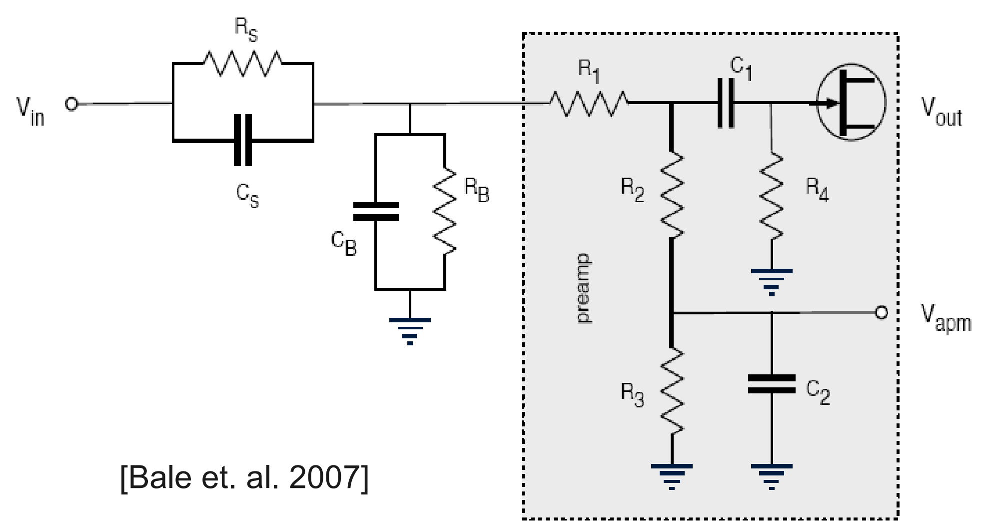


Plasma sheath

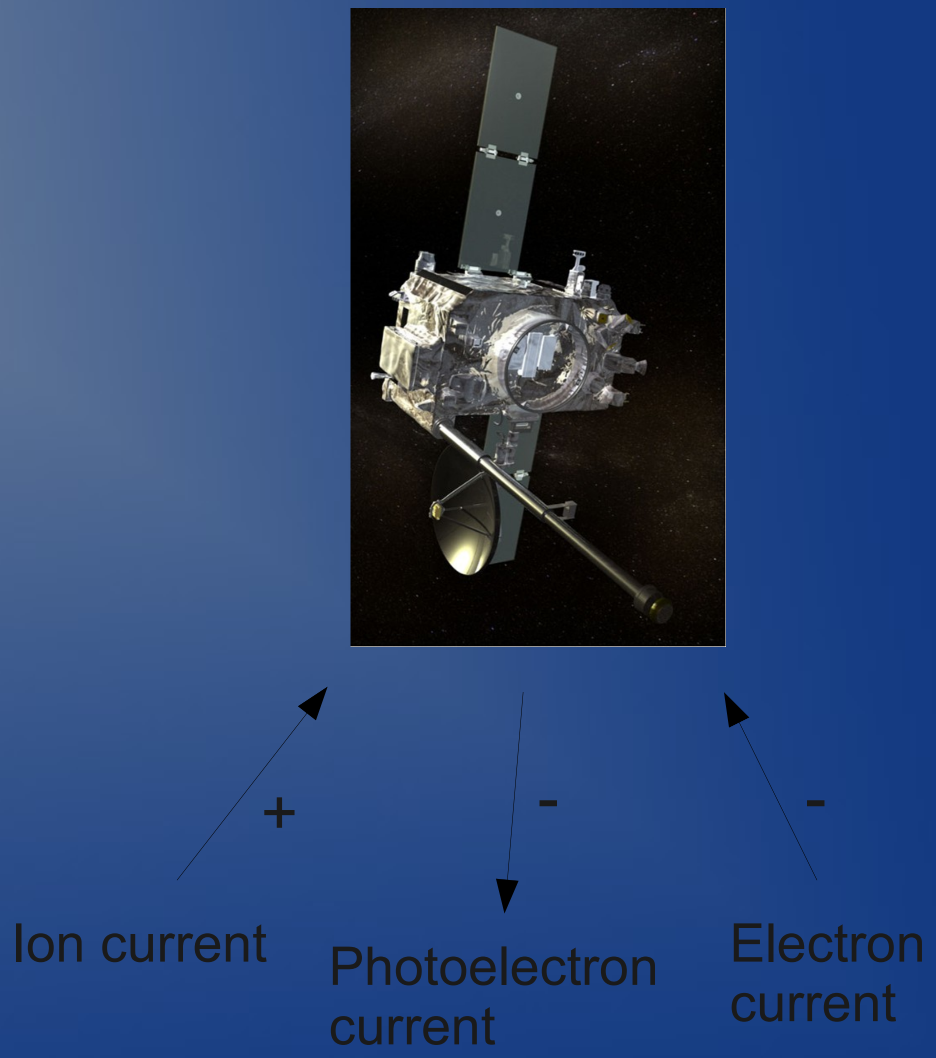
- Conductive surfaces interact with surrounding plasma.
- Around the immersed object a plasma sheath forms.
- This sheath can vary, depending on the charge of the object (positive or negative).
- The sheath can be modeled as capacity which is easily included in the antenna calculation.



[Bale et. al. 2007]

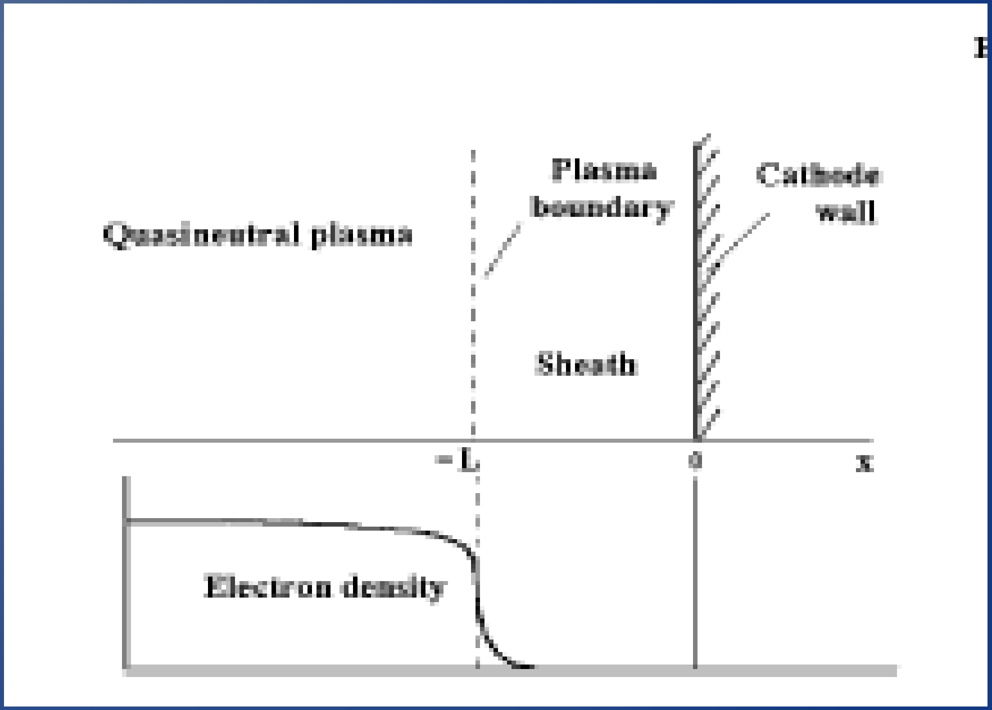
Plasma sheath

- 2 or three currents flow between S/C and plasma.
- In equilibrium the sum of the currents is zero.
- Depending on the photoelectron current the S/C is charged positively or negatively.
- Different physical effects.



Plasma sheath, negative charge

- No solar radiation → S/C negatively charged.
- If electrons and ions are in thermal equilibrium, mean electron speed is higher.
- → more electrons hit the surface.
- → negative charge builds up.
- → thermal electrons are pushed away by charge.
- → Ions are attracted.
- → electrons depletion sheath forms.
- → the thickness of the sheath regulates itself in a way that the sum of the currents are equal.



Plasma sheath, negative charge

- Thermal electrons are postulated to be Maxwell distributed.

$$f_e(x,v) = \frac{\bar{n}_e}{\sqrt{\frac{2\pi\kappa T_e}{m_e}}} e^{-\frac{\varepsilon}{\kappa T_e}}$$

- Conservation of energy:

$$\frac{1}{2} m_e v^2 - e\phi(x)$$

- Particle density: zeroth moment.

$$\begin{aligned} n_e(x) &= \int_{-\infty}^{\infty} f_e(x,v)dv \\ &= \frac{\bar{n}_e}{\sqrt{\frac{2\pi\kappa T_e}{m_e}}} e^{\frac{e\phi(x)}{\kappa T_e}} \sqrt{\frac{2\pi\kappa T_e}{m_e}} \\ &= \bar{n}_e e^{\frac{e\phi(x)}{\kappa T_e}} \end{aligned}$$

- Current: first moment x charge

$$j_e(x) = -e\int_0^{\infty} v f_e(x,v)dv$$

$$= -e\bar{n}_e \sqrt{\frac{\kappa T_e}{2\pi m_e}} e^{\frac{e\phi(x)}{\kappa T_e}}$$

$$\boxed{I_e = -e\bar{n}_e A \sqrt{\frac{\kappa T_e}{2\pi m_e}} e^{\frac{eV}{\kappa T_e}}}$$

Plasma sheath, negative charge

- All ions reach the surface.
- They fall into a potential well.
- $\rightarrow$ acceleration
- $\rightarrow$ density decreases due to mass conservation. $n_i(x) \propto v_i(x)^{-1}$
- Energy conservation.
- Combining the conservation laws yields equation for particle density.
- From this the ion current can be derived.

$$\frac{1}{2} m_i \bar{v}_i^2 = \frac{1}{2} m_i v_i(x)^2 + e\phi(x)$$

$$v_i(x) = \sqrt{\bar{v}_i^2 - \frac{2e\phi(x)}{m_i}}$$

$$n_i(x) = \frac{\bar{n}_i}{\sqrt{1 - \frac{2e\phi(x)}{m_i \bar{v}_i^2}}}$$

$$I_i(x) = eld\pi n_i(x) v_i(x)$$

$$\boxed{I_i(x) = eld\pi \bar{n}_i \bar{v}_i}$$

Plasma sheath, negative charge

- Photoelectrons are created by the photoelectric effect.
- Usually solar radiation.
- All photoelectrons reach the plasma.
- i_{ph} depends on energy distribution of the photons and on geometry and material of the S/C.
- A_Φ...illuminated cross section.
- Speed and density as for ions..

$$\boxed{I_{ph} = i_{ph} A_\phi}$$

$$v_{ph}(x) = \sqrt{\bar{v}_{ph}^2 - \frac{2e}{m_e}(V - \phi(x))}$$

$$n_{ph}(x) = \frac{\bar{n}_{ph}}{\sqrt{1 - \frac{2e}{m_e \bar{v}_{ph}^2}(V - \phi(x))}}$$

Plasma sheath, negative charge

- Boundary conditions:

$$\begin{aligned} \phi(0) &= V \\ \phi(\infty) &= 0 \end{aligned}$$

- The currents are summed to zero.
- Quasi neutrality postulated.
- Influence of ions small.
- Sheath thickness can be estimated as the Debye length or one calculates the potential and density distribution across the sheath.

$$V = \frac{\kappa T_e}{e} ln \left[\frac{i_{ph} A_{rel}}{e \bar{n}_e \pi} \sqrt{\frac{2\pi m_e}{\kappa T_e}} \right]$$

$$V = \frac{\kappa T_e}{e} ln \left[\left(\frac{i_{ph} A_{rel}}{e \bar{n}_e \pi} + \bar{v}_i \right) \sqrt{\frac{2\pi m_e}{\kappa T_e}} \right]$$

Plasma sheath, negative charge

- To calculate the potential distribution, Poisson's equation has to be solved:

$$\frac{d^2\phi(x)}{dx^2} = -\frac{e\left(n_i(x) - n_{ph}(x) - n_e(x)\right)}{\epsilon_0}$$

$$\frac{d^2\phi(x)}{dx^2} = -\frac{e}{\epsilon_0}\left(\frac{\bar{n}_i}{\sqrt{1-\frac{2e\phi(x)}{m_i\bar{v}_i^2}}} - \frac{\bar{n}_{ph}}{\sqrt{1-\frac{2e}{m_e\bar{v}_{ph}^2}\left(V-\phi(x)\right)}} - \bar{n}_e e^{\frac{e\phi(x)}{\kappa T_e}}\right)$$

- This equation is non-linear and can not be solved analytically. But it can be linearized if

$$\begin{aligned} e\phi(x) &\ll m_i\bar{v}_i^2 \\ e(V-\phi(x)) &\ll m_e\bar{v}_{ph}^2 \\ e\phi(x) &\ll \kappa T_e \end{aligned}$$

- Then a Taylor approximation can be used:

$$\begin{aligned} \frac{1}{\sqrt{1-\frac{2e\phi(x)}{m_i\bar{v}_i^2}}} &\sim 1+\frac{2e\phi(x)}{m_i\bar{v}_i^2} \\ \frac{1}{\sqrt{1-\frac{2e}{m_e\bar{v}_{ph}^2}\left(V-\phi(x)\right)}} &\sim 1+\frac{2e\left(V-\phi(x)\right)}{m_e\bar{v}_{ph}^2} \\ e^{\frac{e\phi(x)}{\kappa T_e}} &\sim 1+\frac{e\phi(x)}{\kappa T_e} \end{aligned}$$

- Neglecting the photoelectrons and assuming quasi-neutrality, the result is

$$\frac{d^2\phi(x)}{dx^2} = -\frac{e^2\bar{n}_0}{\epsilon_0}\left(\frac{2}{m_i\bar{v}_i^2} - \frac{1}{\kappa T_e}\right)\phi(x)$$

- The solution: $\phi(x) = V e^{-\frac{x}{\lambda_{sh}}}$

$$\lambda_{sh} = \sqrt{\frac{\epsilon_0}{e^2\bar{n}_0}\left(\frac{2}{m_i\bar{v}_i^2} - \frac{1}{\kappa T_e}\right)^{-1}}$$

Plasma sheath, negative charge

- Sheath resistance is the gradient of the V-I curve.

$$R_s = \frac{\partial V}{\partial I}$$

- Explicit expression by [Gurnett 2000]

$$R_s = \frac{\kappa T_e}{e(I_{ph} + I_i)}$$

- At high frequencies the resistance can be neglected.

- For the capacity I use the formula of a cylindrical capacitor.

$$C_s = l_a \frac{2\pi\epsilon_0}{\ln\left(\frac{\delta}{r_a}\right)}$$

Plasma sheath, positive charge

- Photo-electrons dominate.
- Again, the currents sum up to zero.
- →S/C is charged positively. Number of photoelectrons reaching the plasma (only a few %) == number of thermal electron reaching the surface.
- Most photoelectrons have too low energy and fall back to the surface from which they are attracted.
- They form an electron-sheath.
- They are not part of the photo-electron current.

Plasma sheath, positive charge

- All thermal electrons reach the surface → no Boltzmann

$$I_e = -e n_e d l \pi \sqrt{\frac{\kappa T_e}{2\pi m_e}}$$

- Only photoelectrons which are energetic enough reach the plasma.
- Photo-electrons nearly Maxwell-distributed [Grard et. al].

$$I_{ph} = A_{rel} i_{ph} l d e^{-\frac{eV}{\kappa T_{ph}}}$$

- Photo-electron backflow:

$$I_{ph,back} = A_{rel} i_{ph} l d (1 - e^{-\frac{eV}{\kappa T_{ph}}})$$

- Photo-electron current == thermal electron-current.
- → total backflow:

$$I_{back} = I_{ph,back} + I_e \sim A_{rel} i_{ph} l d$$

- Potential:

$$V = -\frac{\kappa T_{ph}}{e} \ln\left[\frac{e n_e \pi}{A_{rel} i_{ph}} \sqrt{\frac{\kappa T_e}{2\pi m_e}}\right]$$

Plasma sheath, positive charge

- Poisson equation, 1D over flat surface:

$$\frac{d^2\phi(x)}{dx^2} = -\frac{e}{\epsilon_0}\left(\bar{n}_i - n_{ph}(x) - n_e(x)\right)$$

- Or

$$\frac{d^2\phi(x)}{dx^2} - \frac{2en_{ph}(0)}{\epsilon_0}e^{-\frac{e(V-\phi(x))}{\kappa T_{ph}}} - \frac{e\bar{n}_e}{\epsilon_0\sqrt{1+\frac{2e\phi(x)}{m_e\bar{v}_e^2}}} = -\frac{e\bar{n}_i}{\epsilon_0}$$

- Linearizion:

$$\frac{1}{\sqrt{1+\frac{2e\phi(x)}{m_e\bar{v}_e^2}}} \sim 1 - \frac{2e\phi(x)}{m_e\bar{v}_e^2}$$

$$e^{\frac{e\phi(x)}{kT_{eph}}} \sim 1 + \frac{e\phi(x)}{\kappa T_{ph}}$$

- With assumed quasi-neutrality:

$$\frac{d^2\phi(x)}{dx^2} - \frac{2e^2}{\epsilon_0}\left(\frac{n_{ph}(0)}{\kappa T_{ph}}e^{-\frac{eV}{\kappa T_{ph}}} - \frac{\bar{n}_0}{m_e\bar{v}_e^2}\right)\phi(x) = \frac{2en_{ph}(0)}{\epsilon_0}e^{-\frac{eV}{\kappa T_{ph}}}$$

- Which gives:

$$\lambda_{\text{sh}} = \left[\frac{2e^2}{\epsilon_0}\left(\frac{n_{ph}(0)}{\kappa T_{ph}}e^{-\frac{eV}{\kappa T_{ph}}} - \frac{\bar{n}_0}{m_e\bar{v}_e^2}\right)\right]^{-\frac{1}{2}}$$

Plasma sheath, positive charge

- In cylindrical coordinates:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\phi(r)}{dr}\right) = -\frac{e}{\epsilon_0}\left(\bar{n}_i - n_{ph}(r) - n_e(r)\right)$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\phi(r)}{dr}\right) - \frac{2en_{ph}(0)}{\epsilon_0}e^{-\frac{e(V-\phi(r))}{\kappa T_{ph}}} = -\frac{e\bar{n}_i}{\epsilon_0}$$

- Linearized:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\phi(r)}{dr}\right) - \frac{2e^2 n_{ph}(0)}{\kappa T_{ph}\epsilon_0}e^{-\frac{eV}{\kappa T_{ph}}}\phi(r) = \frac{e}{\epsilon_0}\left(2n_{ph}(0)e^{-\frac{eV}{\kappa T_{ph}}} - \bar{n}_i\right)$$

- Can be brought into the form of a Sturm Luiville problem:

$$\frac{d}{dr}\left(r\frac{d\phi(r)}{dr}\right) - r\frac{2e^2 n_{ph}(0)}{\kappa T_{ph}\epsilon_0}e^{-\frac{eV}{\kappa T_{ph}}}\phi(r) = 0$$

- With the solution:

$$\phi(\sqrt{C}r) = \Phi(R) = AK_0(R) = AK_0(\lambda_{sh}r)$$

- and

$$\begin{aligned} R &= \lambda_{sh} r \\ \Phi(R) &= \phi(r) \\ \lambda_{sh} &= \sqrt{\frac{2e^2 n_{ph}(0)}{\kappa T_{ph}\epsilon_0}e^{-\frac{eV}{\kappa T_{ph}}}} \end{aligned}$$

Plasma sheath, positive charge

- The sheath resistivity:

$$R_s = \frac{dV}{dI_{ph}}$$

- Using just the photoelectrons, one can write:

$$V = -\frac{\kappa T_{ph}}{e} \ln \frac{I_{ph}}{A_{rel} i_{ph} l d}$$

- Then

$$R_s = -\frac{\kappa T_{ph}}{e I_{ph}}$$

- As before

$$C_s = l_a \frac{2\pi\epsilon_0 \bar{\epsilon}_r}{\ln\left(\frac{\delta}{r_a}\right)}$$

- But now using the mean permittivity:

$$\bar{\epsilon}_r = \frac{1}{\delta} \int_{r'=0}^{r'=\delta} \left(1 - \frac{\omega_p(r')^2}{\omega^2}\right) dr'$$

$$\bar{\epsilon}_r = 1 - \frac{\kappa T_{ph} n_e(0) e}{V m_e \epsilon_0 \omega^2} \left(1 - e^{-\frac{eV}{\kappa T_{ph}}}\right)$$

Plasma sheath, STEREO

- Using the equations presented before:

$$I_{ph} = A_{rel} i_{ph} l d \sim 7.6 \cdot 10^{-6} A$$

$$I_e = -e n_e d \pi \sqrt{\frac{\kappa T_e}{2\pi m_e}} \sim -2 \cdot 10^{-7} A$$

$$V \sim 5.5V$$

- the S/C is positively charged and 5.5V is in accordance with experience of similar missions.

- The Debye length of the photoelectrons:

$$\lambda_{ph} \sim 0.6m$$

- Using the derived formula:

$$\lambda_{sh} \sim 0.4m$$

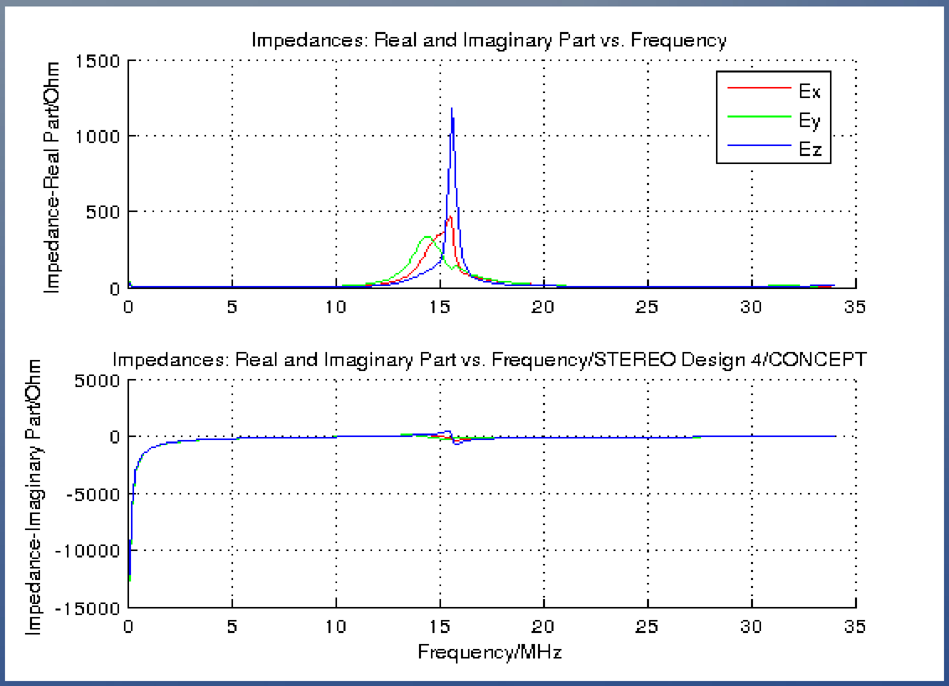
- This gives:

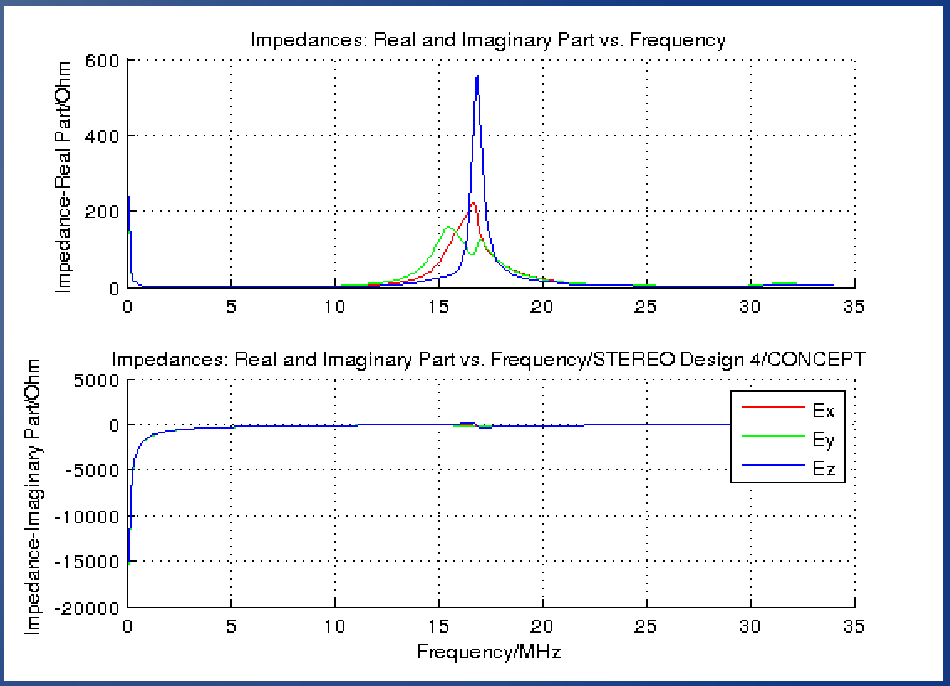
$$R_s = 0.2M\Omega$$

$$C_s = 87pF$$

- Bale gives a capacity of 40pF

Plasma sheath, STEREO





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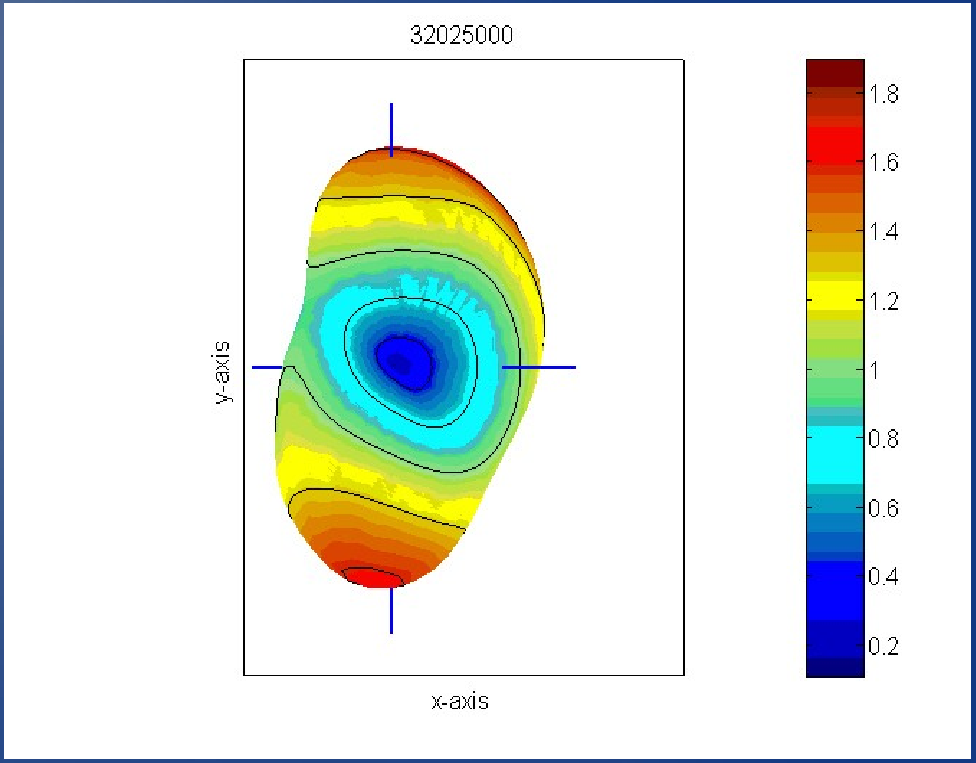


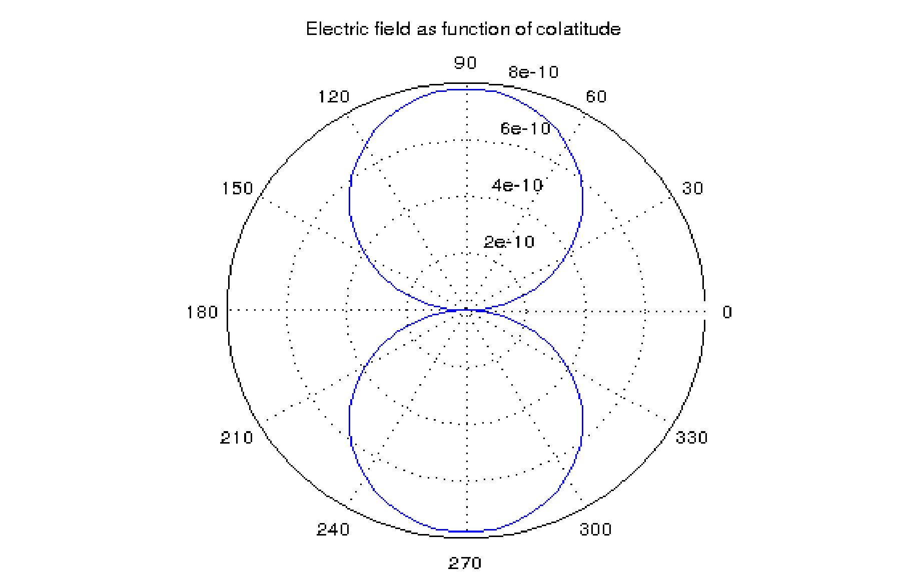
Outlook

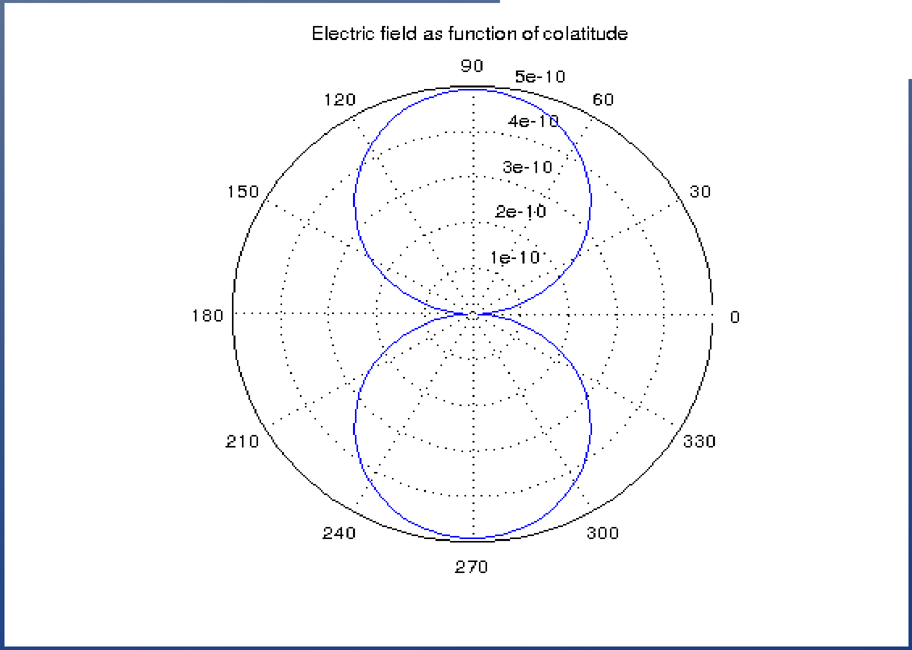
- Only the most simple situation has been treated, cold isotropic plasma.
- It has been shown that kinetic treatment is not necessary.
- Including the effect of magnetized plasma would be interesting, because S/C typically operate in such an environment.
- Using the FDTD method, it would be easier to implement anisotropy.
- Also isotropic results could be verified.
- But there is probably another way to implement anisotropic plasma in MoM simulations without having to use the dielectric tensor.
- In this simple dielectric plasma model, plasma manifests itself as a change in wavelength and therefore phase speed.
- By scaling the model, the same results can be achieved.

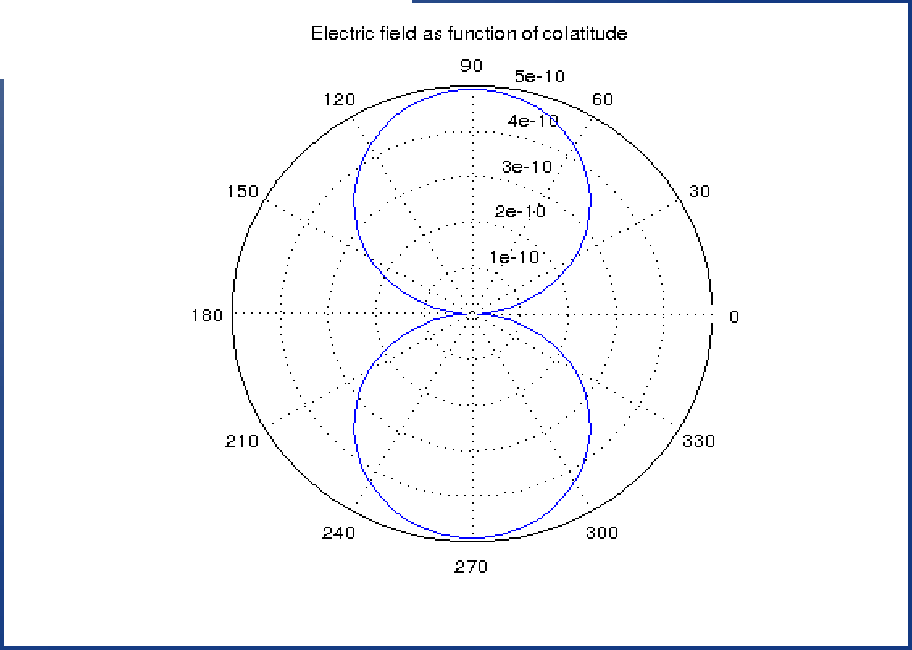
Outlook

Calculation	Impedance
Dipole, 6m, ε=1, r=2mm	6.882736497964838e-03 - 7.112228243779676e+04i
Dipole, 6m, ε=0.5, r=2mm	3.441273366066885e-03 - 1.005881770038659e+05i
Dipole, 6m *sqrt(ε), ε=1, r=2mm*sqrt(ε)	3.441273374398452e-03 - 1.005881770038794e+05i

Outlook


Electric field as function of colatitude
90
8e-10
6e-10
4e-10
2e-10
60
30
0
330
300
270
240
210
180
150
120


Electric field as function of colatitude
90
5e-10
4e-10
3e-10
2e-10
1e-10
60
30
0
330
300
270
240
210
180
150
120


Electric field as function of colatitude
90
5e-10
4e-10
3e-10
2e-10
1e-10
60
30
0
330
300
270
240
210
180
150
120

Outlook

- Unfortunately this does not work with all solvers. But it is easier to write a solver which adheres to the invariance of Maxwell's equations at scaling, than to write one which can cope with dielectric tensors.
- Magnetized plasma could be modeled by scaling the model an-isotropically
- It remains to analyze how the imaginary tensor elements are to be dealt with.
- The plasma sheath theory should be refined and synchronized with existing results and ongoing research.
- It is possible to simulate the sheath realistically by using particle simulations or particle-in-cell simulations.

Thank you for your attention !