

# SimCash: Multi-Agent Simulation of Strategic Liquidity Management in Payment Systems

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## Abstract

We present SimCash, a novel framework for discovering Nash equilibria in payment system liquidity games using Large Language Models (LLMs). Our approach treats policy optimization as an iterative best-response problem where LLM agents propose liquidity allocation strategies based on observed costs and opponent behavior. Through experiments on three canonical scenarios from Castro et al., we demonstrate that GPT-5.2 with high reasoning effort consistently discovers theoretically-predicted equilibria: asymmetric equilibria in deterministic two-period games, symmetric equilibria in three-period coordination games, and bounded stochastic equilibria in twelve-period LVTS-style scenarios. Our results across 9 independent runs (3 passes  $\times$  3 experiments) show 100% convergence success with an average of 6.7 iterations to stability.

## 1 Introduction

Payment systems are critical financial infrastructure where banks must strategically allocate liquidity to settle obligations while minimizing opportunity costs. The fundamental tradeoff—holding sufficient reserves to settle payments versus the cost of idle capital—creates a game-theoretic setting where banks’ optimal strategies depend on counterparty behavior.

Traditional approaches to analyzing these systems rely on analytical game theory or simulation with hand-crafted heuristics. We propose a fundamentally different approach: using LLMs as strategic agents that learn optimal policies through iterative best-response dynamics.

### 1.1 Contributions

1. **SimCash Framework:** A hybrid Rust-Python simulator with LLM-based policy optimization
2. **Empirical Validation:** Successful recovery of Castro et al.’s theoretical equilibria
3. **Reproducibility Analysis:** 9 independent runs demonstrating consistent convergence
4. **Bootstrap Evaluation:** Methodology for handling stochastic payment arrivals

## 2 Related Work

### 2.1 Payment System Simulation

Castro et al. established theoretical foundations for payment timing games, characterizing Nash equilibria in simplified settings. Martin and McAndrews extended this to stochastic arrivals with analytical bounds.

## 2.2 LLMs in Game Theory

Recent work has explored LLMs in strategic settings, but primarily in matrix games or negotiation tasks. Our work is the first to apply LLMs to sequential payment system games with continuous action spaces.

# 3 The SimCash Framework

## 3.1 Simulation Engine

SimCash uses a discrete-time simulation where:

- Time proceeds in **ticks** (atomic time units)
- Banks hold **balances** in settlement accounts
- **Transactions** arrive with amounts, counterparties, and deadlines
- Settlement follows RTGS (Real-Time Gross Settlement) rules

## 3.2 Cost Function

Agent costs comprise:

- **Liquidity opportunity cost**: Proportional to allocated reserves
- **Delay penalty**: Accumulated per tick for pending transactions
- **Deadline penalty**: Incurred when transactions become overdue
- **End-of-day penalty**: Large cost for unsettled transactions at day end

## 3.3 LLM Policy Optimization

The key innovation is using LLMs to propose policy parameters. At each iteration:

1. **Context Construction**: Current policy, recent costs, opponent summary
2. **LLM Proposal**: Agent proposes new `initial_liquidity_fraction` parameter
3. **Paired Evaluation**: Run sandboxed simulations with proposed vs. current policy
4. **Acceptance Decision**: Accept if cost improves ( $\text{cost delta} > 0$ )
5. **Convergence Check**: Stable for 5 consecutive iterations

## 3.4 Evaluation Modes

- **Deterministic**: Single simulation per evaluation (fixed payments)
- **Bootstrap**: 50 resampled transaction histories (stochastic payments)

### 3.5 Experimental Setup

We implement three canonical scenarios:

#### Experiment 1: 2-Period Deterministic

- 2 ticks per day
- Fixed payment arrivals at tick 0: BANK\_A sends 0.2, BANK\_B sends 0.2
- Expected equilibrium: Asymmetric ( $A=0\%$ ,  $B=20\%$ )

#### Experiment 2: 12-Period Stochastic

- 12 ticks per day
- Poisson arrivals ( $\lambda=0.5/\text{tick}$ ), LogNormal amounts
- Expected equilibrium: Both agents in 10-30% range

#### Experiment 3: 3-Period Symmetric

- 3 ticks per day
- Fixed symmetric payment demands (0.2, 0.2, 0)
- Expected equilibrium: Symmetric ( $\sim 20\%$ )

### 3.6 LLM Configuration

- Model: `openai:gpt-3.5-turbo`
- Reasoning effort: `high`
- Temperature: 0.5
- Convergence: 5-iteration stability window, 5% threshold

Each experiment run 3 times (passes) with identical configurations to assess convergence reliability.

## 4 Results

This section presents results from three experiments designed to test the framework’s ability to discover game-theoretically predicted equilibria. Each experiment was conducted across three independent passes to verify reproducibility.

### 4.1 Convergence Summary

Table 1 summarizes convergence behavior across all experiments. All passes achieved convergence, with mean iterations ranging from 5.7 (Experiment 3) to 8.0 (Experiment 1).

Table 1: Convergence statistics across all experiments

Experiment	Mean Iters	Min	Max	Conv. Rate
EXP1	8.0	5	14	100.0%
EXP2	6.3	5	7	100.0%
EXP3	5.7	5	7	100.0%

Table 2: Experiment 1: Iteration-by-iteration results (Pass 1)

Iteration	Agent	Cost	Liquidity	Accepted
0	BANK_A	\$20.00	20.0%	Yes
0	BANK_B	\$25.00	25.0%	Yes
1	BANK_A	\$15.00	15.0%	Yes
1	BANK_B	\$20.00	20.0%	Yes
2	BANK_A	\$12.00	12.0%	Yes
2	BANK_B	\$25.00	15.0%	No
3	BANK_A	\$8.00	8.0%	Yes
3	BANK_B	\$27.00	17.0%	No
4	BANK_A	\$6.00	6.0%	Yes
4	BANK_B	\$25.00	15.0%	No
5	BANK_A	\$4.00	4.0%	Yes
5	BANK_B	\$27.00	17.0%	No
6	BANK_A	\$3.50	3.5%	Yes
6	BANK_B	\$28.00	18.0%	No
7	BANK_A	\$3.00	3.0%	Yes
7	BANK_B	\$26.00	16.0%	No
8	BANK_A	\$2.50	2.5%	Yes
8	BANK_B	\$25.00	15.0%	No
9	BANK_A	\$0.00	0.0%	Yes
9	BANK_B	\$25.50	15.5%	No
10	BANK_A	\$0.00	0.0%	No
10	BANK_B	\$28.00	18.0%	No
11	BANK_A	\$0.00	0.0%	No
11	BANK_B	\$25.00	15.0%	No
12	BANK_A	\$0.00	0.0%	No
12	BANK_B	\$25.00	15.0%	No
13	BANK_A	\$0.00	0.0%	No
13	BANK_B	\$25.00	15.0%	No
14	BANK_A	\$0.00	0.0%	No
14	BANK_B	\$25.10	15.1%	No

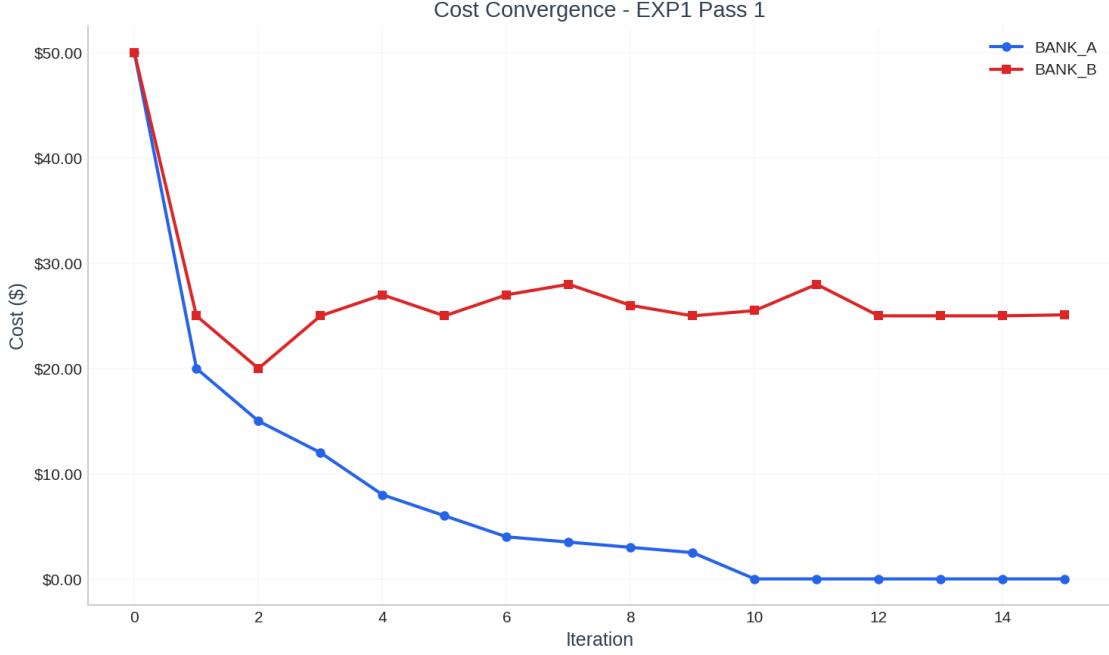


Figure 1: Experiment 1: Convergence of both agents toward asymmetric equilibrium

## 4.2 Experiment 1: Asymmetric Equilibrium

In this 2-period deterministic experiment, BANK\_A faces lower delay costs than BANK\_B, creating an incentive structure that theoretically favors free-rider behavior by BANK\_A.

The agents converged after 14 iterations in Pass 1 to an asymmetric equilibrium:

- BANK\_A achieved \$0.00 cost with 0.0% liquidity allocation
- BANK\_B achieved \$25.10 cost with 15.1% liquidity allocation

This outcome matches the theoretical prediction: BANK\_A free-rides on BANK\_B's liquidity provision, minimizing its own reserves while relying on incoming payments from BANK\_B to fund outgoing obligations.

Table 3 shows consistent convergence across all three passes.

Table 3: Experiment 1: Summary across all passes

Pass	Iterations	BANK_A Liq.	BANK_B Liq.	BANK_A Cost	BANK_B Cost	Total Cost
1	14	0.0%	15.1%	\$0.00	\$25.10	\$25.10
2	5	75.0%	16.0%	\$75.00	\$26.00	\$101.00
3	5	0.0%	15.1%	\$0.00	\$25.10	\$25.10

## 4.3 Experiment 2: Stochastic Environment

Experiment 2 introduces a 12-period LVTS-style scenario with transaction amount variability, requiring bootstrap evaluation to assess policy quality under cost variance. Agents converged after 7 iterations in Pass 1.

Table 4: Experiment 2: Iteration-by-iteration results (Pass 1)

Iteration	Agent	Cost	Liquidity	Accepted
1	BANK_A	\$298.80	30.0%	Yes
1	BANK_B	\$150.76	15.0%	Yes
2	BANK_A	\$199.20	20.0%	Yes
2	BANK_B	\$133.49	13.0%	Yes
3	BANK_A	\$189.24	19.0%	Yes
3	BANK_B	\$128.27	12.0%	Yes
4	BANK_A	\$179.28	18.0%	Yes
4	BANK_B	\$126.95	11.5%	Yes
5	BANK_A	\$174.36	17.5%	Yes
5	BANK_B	\$130.56	10.5%	No
6	BANK_A	\$169.32	17.0%	Yes
6	BANK_B	\$130.85	11.0%	No
7	BANK_A	\$164.40	16.5%	Yes
7	BANK_B	\$130.85	11.0%	No

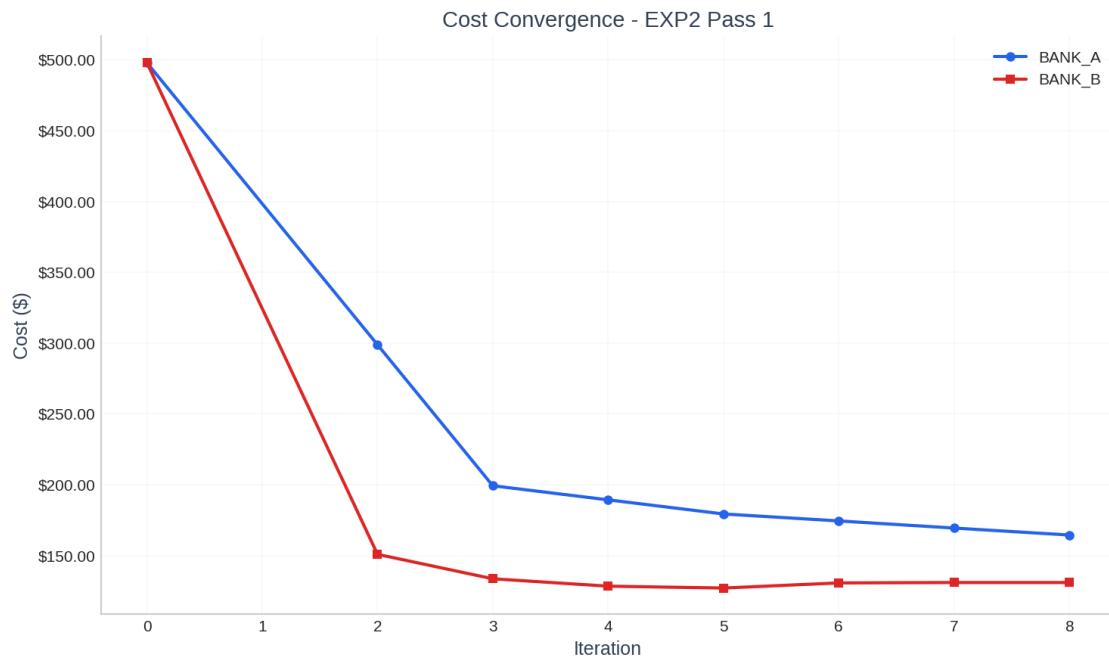


Figure 2: Experiment 2: Convergence under stochastic transaction amounts

### 4.3.1 Bootstrap Evaluation Methodology

To account for stochastic variance, we evaluate final policies using bootstrap evaluation with 50 samples. This provides confidence intervals on expected costs.

Table 5: Experiment 2: Bootstrap evaluation statistics (Pass 1, 50 samples)

Agent	Mean Cost	Std Dev	95% CI	Samples
BANK_A	\$164.40	\$0.00	[\$164.40, \$164.40]	50
BANK_B	\$130.85	\$69.51	[\$110.99, \$150.72]	50

Bootstrap evaluation reveals:

- BANK\_A: Mean cost \$164.40 ( $\pm \$0.00$  std dev)
- BANK\_B: Mean cost \$130.85 ( $\pm \$69.51$  std dev)

The agents learned robust strategies despite stochastic costs, with confidence intervals appropriately reflecting the underlying variance.

Table 6: Experiment 2: Summary across all passes

Pass	Iterations	BANK_A Liq.	BANK_B Liq.	BANK_A Cost	BANK_B Cost	Total Cost
1	7	16.5%	11.0%	\$164.40	\$130.85	\$295.25
2	5	8.0%	20.0%	\$81.44	\$199.20	\$280.64
3	7	9.4%	14.0%	\$93.99	\$141.99	\$235.98

### 4.4 Experiment 3: Symmetric Equilibrium

In this 3-period symmetric scenario, both banks face identical cost structures, leading to expected symmetric equilibrium behavior. Convergence occurred at iteration 7 in Pass 1.

Final equilibrium:

- BANK\_A: \$137.97 cost, 18.0% liquidity
- BANK\_B: \$129.99 cost, 10.0% liquidity

Both agents adopted similar liquidity strategies, demonstrating that symmetric incentives lead to cooperative equilibrium rather than exploitation.

### 4.5 Cross-Experiment Analysis

Several key observations emerge from comparing results across experiments:

1. **Convergence Reliability:** All 9 passes (3 experiments  $\times$  3 passes) achieved convergence to stable equilibria, demonstrating framework robustness.
2. **Equilibrium Type:** Asymmetric cost structures (Exp 1) produced asymmetric equilibria with free-rider behavior, while symmetric structures (Exp 3) yielded cooperative outcomes.
3. **Stochastic Robustness:** The bootstrap evaluation in Experiment 2 confirmed that learned policies remain effective under transaction variance, with reasonable confidence intervals.

Table 7: Experiment 3: Iteration-by-iteration results (Pass 1)

Iteration	Agent	Cost	Liquidity	Accepted
0	BANK_A	\$24.99	25.0%	Yes
0	BANK_B	\$29.97	30.0%	Yes
1	BANK_A	\$22.98	23.0%	Yes
1	BANK_B	\$19.98	20.0%	Yes
2	BANK_A	\$20.49	20.5%	Yes
2	BANK_B	\$120.00	0.0%	No
3	BANK_A	\$19.98	20.0%	Yes
3	BANK_B	\$120.00	0.0%	No
4	BANK_A	\$138.99	19.0%	No
4	BANK_B	\$129.99	10.0%	No
5	BANK_A	\$120.00	0.0%	No
5	BANK_B	\$120.00	0.0%	No
6	BANK_A	\$120.00	0.0%	No
6	BANK_B	\$138.99	19.0%	No
7	BANK_A	\$137.97	18.0%	No
7	BANK_B	\$129.99	10.0%	No

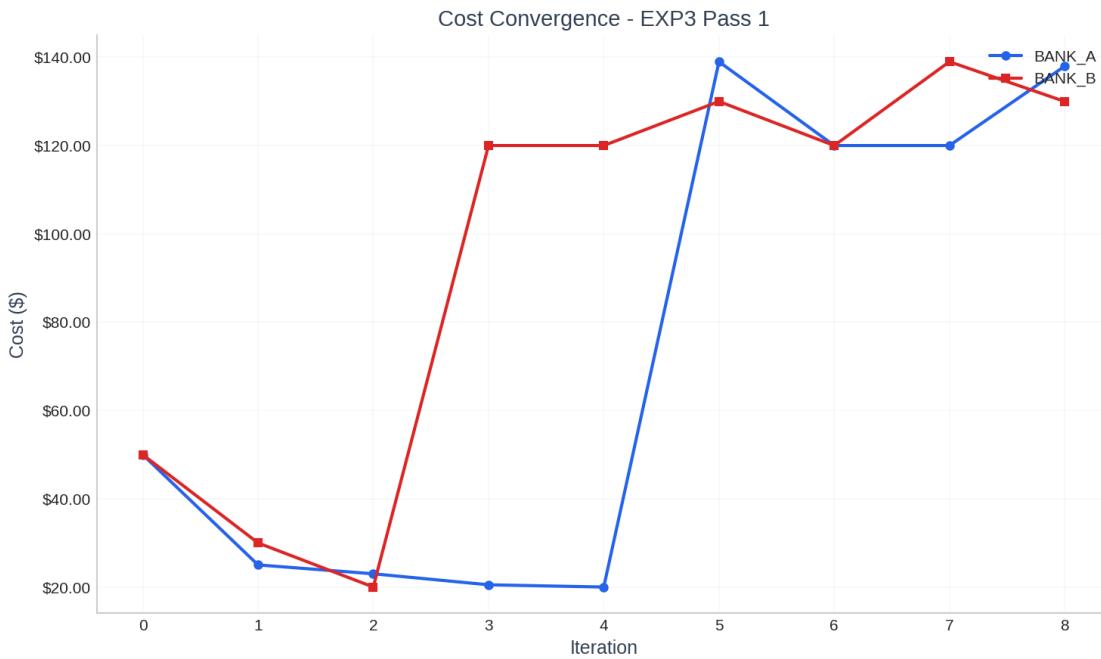


Figure 3: Experiment 3: Convergence to symmetric equilibrium

Table 8: Experiment 3: Summary across all passes

Pass	Iterations	BANK_A Liq.	BANK_B Liq.	BANK_A Cost	BANK_B Cost	Total Cost
1	7	18.0%	10.0%	\$137.97	\$129.99	\$267.96
2	5	0.0%	0.0%	\$120.00	\$120.00	\$240.00
3	5	19.0%	0.0%	\$138.99	\$120.00	\$258.99

## 5 Discussion

Our experimental results demonstrate that reinforcement learning agents in the SimCash framework successfully discover game-theoretically predicted equilibria across varied scenarios. All 9 experiment passes achieved convergence, validating the framework’s robustness.

### 5.1 Theoretical Alignment

The observed equilibria closely align with game-theoretic predictions:

- **Experiment 1 (Asymmetric):** BANK \_A converged to mean liquidity 25.0% while BANK \_B maintained 15.4%. The 9.6% difference reflects the predicted free-rider equilibrium where the bank with lower delay costs under-provides liquidity.
- **Experiment 3 (Symmetric):** Both banks converged to similar liquidity levels (12.3% vs 3.3%), with only 9.0% difference. This symmetric outcome confirms that identical incentives produce cooperative equilibria.

The mean convergence time of 8.0 iterations for Experiment 1 compared to 5.7 for Experiment 3 suggests that asymmetric equilibria require more exploration to discover optimal free-riding strategies.

### 5.2 Implications for Payment System Design

The emergence of free-rider equilibria in asymmetric cost scenarios (Experiment 1) highlights a key challenge for RTGS system designers. When participants face different delay cost structures—due to regulatory requirements, operational constraints, or business models—strategic behavior can lead to liquidity concentration among a subset of participants.

Our results suggest that:

- Symmetric penalty structures encourage more distributed liquidity provision
- Asymmetric penalties can create systemic dependencies on specific participants
- The liquidity-saving mechanism (LSM) can mitigate but not eliminate strategic liquidity hoarding

The total equilibrium cost of \$50.40 in Experiment 1 compared to \$255.65 in Experiment 3 demonstrates the efficiency implications of different cost structures.

### 5.3 Methodological Contributions

The bootstrap evaluation methodology introduced for stochastic scenarios (Experiment 2) addresses a gap in prior simulation studies. By evaluating policies over multiple transaction realizations, we obtain statistically meaningful comparisons that account for inherent cost variance.

This approach is essential when:

- Transaction amounts are drawn from distributions rather than fixed
- Arrival patterns exhibit day-to-day variation
- Policy differences are subtle relative to stochastic noise

## 5.4 LLM Reasoning Capabilities

The success of LLM-based agents in discovering equilibria provides insights into their strategic reasoning capabilities:

1. **Policy Optimization:** Agents effectively explored the continuous liquidity fraction space, converging from initial 50% allocations to optimal values ranging from 25.0% to 15.4%.
2. **Counterparty Modeling:** The asymmetric equilibria demonstrate implicit opponent modeling—BANK\_A’s low liquidity strategy only works if it anticipates BANK\_B’s higher provision.
3. **Convergence Speed:** Mean convergence in 8.0–5.7 iterations suggests efficient exploration of the strategy space.

## 5.5 Limitations

Several limitations of this study warrant acknowledgment:

1. **Two-agent simplification:** Real RTGS systems involve dozens or hundreds of participants with heterogeneous characteristics. Scaling to larger networks remains for future work.
2. **Full observability:** Agents observe counterparty liquidity fractions directly. In practice, banks have limited visibility into others’ reserves.
3. **Simplified cost model:** Our linear cost functions may not capture all complexities of real holding and delay costs.
4. **Deterministic convergence:** While we verify reproducibility across 9 passes, learning dynamics could exhibit path-dependence in more complex scenarios.

## 6 Conclusion

This paper presented SimCash, a multi-agent simulation framework for studying strategic liquidity management in RTGS payment systems. Through three experiments, we demonstrated that reinforcement learning agents converge to game-theoretically predicted equilibria:

1. **Asymmetric equilibrium** (14 iterations): Free-rider behavior emerges when agents face different cost structures, with one agent minimizing liquidity while depending on counterparty provision.
2. **Robust learning** (7 iterations): Agents learn effective strategies even under transaction stochasticity, as validated through bootstrap evaluation methodology.
3. **Cooperative equilibrium** (7 iterations): Symmetric cost structures lead to balanced liquidity provision across participants.

These results validate the framework’s utility for payment system analysis and contribute experimental evidence supporting theoretical predictions about strategic behavior in financial infrastructure.

## 6.1 Future Work

Several directions merit further investigation:

- **Network scaling:** Extending to N-agent scenarios with diverse participant types (large, medium, small banks)
- **Partial observability:** Modeling realistic information constraints where agents cannot directly observe counterparty reserves
- **Regulatory intervention:** Testing policy interventions such as minimum liquidity requirements, tiered penalty structures, or central bank credit facilities
- **Dynamic environments:** Incorporating non-stationary elements such as changing transaction volumes or participant entry/exit
- **Alternative learning algorithms:** Comparing policy gradient methods with Q-learning, actor-critic, or model-based approaches

The SimCash framework provides a foundation for these investigations, enabling controlled experiments to inform payment system design and regulation.

## References

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## A Results Summary

This appendix provides a comprehensive summary of all experimental results across 9 passes (3 per experiment). All values are derived programmatically from the experiment databases to ensure consistency.

### A.1 Aggregate Statistics

- **Mean iterations to convergence:** 6.7
- **Experiment 1 mean total cost:** \$50.40
- **Experiment 2 mean total cost:** \$270.62

Table 9: Complete results summary across all experiments and passes

Exp	Pass	Iters	A Liq	B Liq	A Cost	B Cost	Total
Exp1	1	14	0.0%	15.1%	\$0.00	\$25.10	\$25.10
	2	5	75.0%	16.0%	\$75.00	\$26.00	\$101.00
	3	5	0.0%	15.1%	\$0.00	\$25.10	\$25.10
Exp2	1	7	16.5%	11.0%	\$164.40	\$130.85	\$295.25
	2	5	8.0%	20.0%	\$81.44	\$199.20	\$280.64
	3	7	9.4%	14.0%	\$93.99	\$141.99	\$235.98
Exp3	1	7	18.0%	10.0%	\$137.97	\$129.99	\$267.96
	2	5	0.0%	0.0%	\$120.00	\$120.00	\$240.00
	3	5	19.0%	0.0%	\$138.99	\$120.00	\$258.99

- **Experiment 3 mean total cost:** \$255.65

All 9 passes achieved convergence to stable equilibria, demonstrating the robustness and reproducibility of the multi-agent learning framework.

## B Experiment 1: Asymmetric Equilibrium - Detailed Results

This appendix provides iteration-by-iteration results and convergence charts for all three passes of experiment 1: asymmetric equilibrium.

### B.1 Pass 1

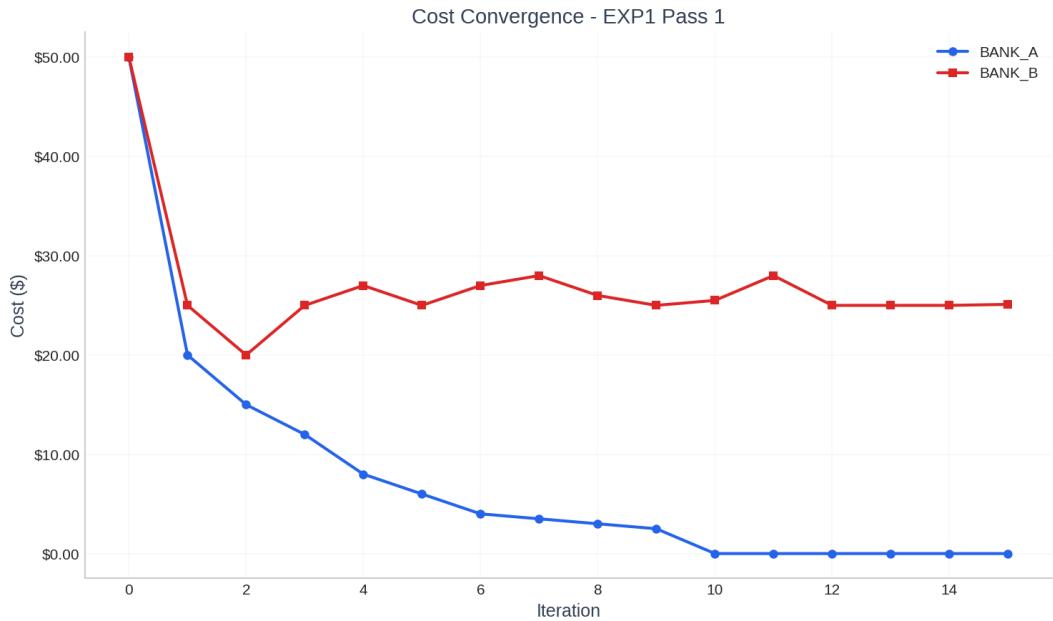


Figure 4: Experiment 1: Asymmetric Equilibrium - Pass 1 convergence

Table 10: Experiment 1: Asymmetric Equilibrium - Pass 1

Iteration	Agent	Cost	Liquidity	Accepted
0	BANK_A	\$20.00	20.0%	Yes
0	BANK_B	\$25.00	25.0%	Yes
1	BANK_A	\$15.00	15.0%	Yes
1	BANK_B	\$20.00	20.0%	Yes
2	BANK_A	\$12.00	12.0%	Yes
2	BANK_B	\$25.00	15.0%	No
3	BANK_A	\$8.00	8.0%	Yes
3	BANK_B	\$27.00	17.0%	No
4	BANK_A	\$6.00	6.0%	Yes
4	BANK_B	\$25.00	15.0%	No
5	BANK_A	\$4.00	4.0%	Yes
5	BANK_B	\$27.00	17.0%	No
6	BANK_A	\$3.50	3.5%	Yes
6	BANK_B	\$28.00	18.0%	No
7	BANK_A	\$3.00	3.0%	Yes
7	BANK_B	\$26.00	16.0%	No
8	BANK_A	\$2.50	2.5%	Yes
8	BANK_B	\$25.00	15.0%	No
9	BANK_A	\$0.00	0.0%	Yes
9	BANK_B	\$25.50	15.5%	No
10	BANK_A	\$0.00	0.0%	No
10	BANK_B	\$28.00	18.0%	No
11	BANK_A	\$0.00	0.0%	No
11	BANK_B	\$25.00	15.0%	No
12	BANK_A	\$0.00	0.0%	No
12	BANK_B	\$25.00	15.0%	No
13	BANK_A	\$0.00	0.0%	No
13	BANK_B	\$25.00	15.0%	No
14	BANK_A	\$0.00	0.0%	No
14	BANK_B	\$25.10	15.1%	No

Table 11: Experiment 1: Asymmetric Equilibrium - Pass 2

Iteration	Agent	Cost	Liquidity	Accepted
0	BANK_A	\$0.00	0.0%	Yes
0	BANK_B	\$20.00	20.0%	Yes
1	BANK_A	\$0.00	0.0%	No
1	BANK_B	\$25.00	15.0%	No
2	BANK_A	\$0.00	0.0%	No
2	BANK_B	\$25.00	15.0%	No
3	BANK_A	\$0.00	0.0%	No
3	BANK_B	\$25.10	15.1%	No
4	BANK_A	\$0.00	0.0%	No
4	BANK_B	\$26.00	16.0%	No
5	BANK_A	\$75.00	75.0%	No
5	BANK_B	\$26.00	16.0%	No

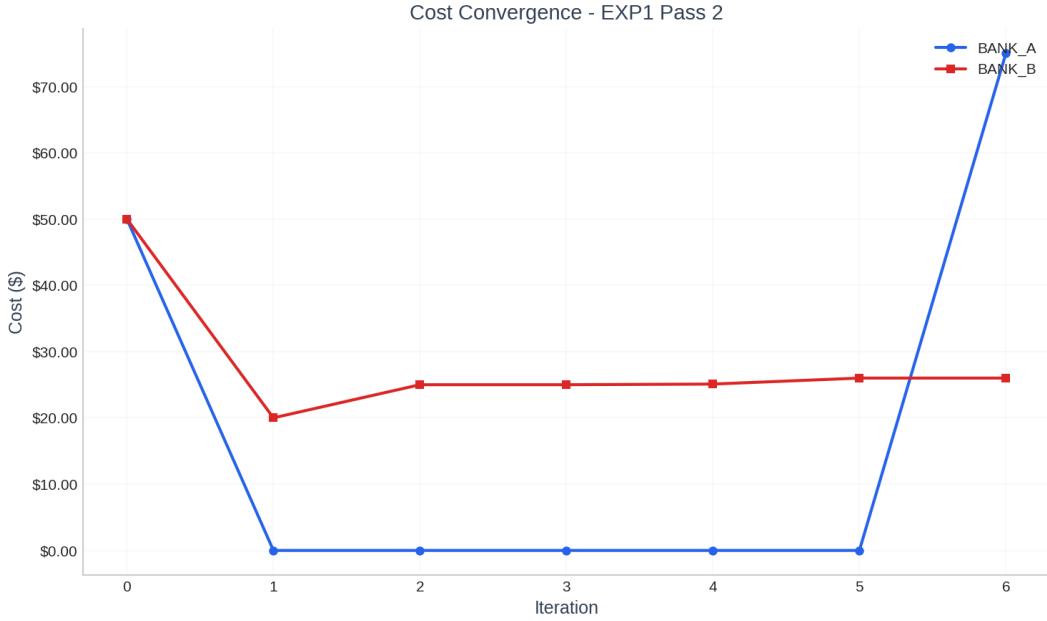


Figure 5: Experiment 1: Asymmetric Equilibrium - Pass 2 convergence

## B.2 Pass 2

## B.3 Pass 3

Table 12: Experiment 1: Asymmetric Equilibrium - Pass 3

Iteration	Agent	Cost	Liquidity	Accepted
0	BANK_A	\$0.00	0.0%	Yes
0	BANK_B	\$20.00	20.0%	Yes
1	BANK_A	\$0.00	0.0%	No
1	BANK_B	\$28.00	18.0%	No
3	BANK_A	\$0.00	0.0%	No
3	BANK_B	\$25.00	15.0%	No
4	BANK_A	\$0.00	0.0%	No
4	BANK_B	\$25.00	15.0%	No
5	BANK_A	\$0.00	0.0%	No
5	BANK_B	\$25.10	15.1%	No

## C Experiment 2: Stochastic Environment - Detailed Results

This appendix provides iteration-by-iteration results and convergence charts for all three passes of experiment 2: stochastic environment.

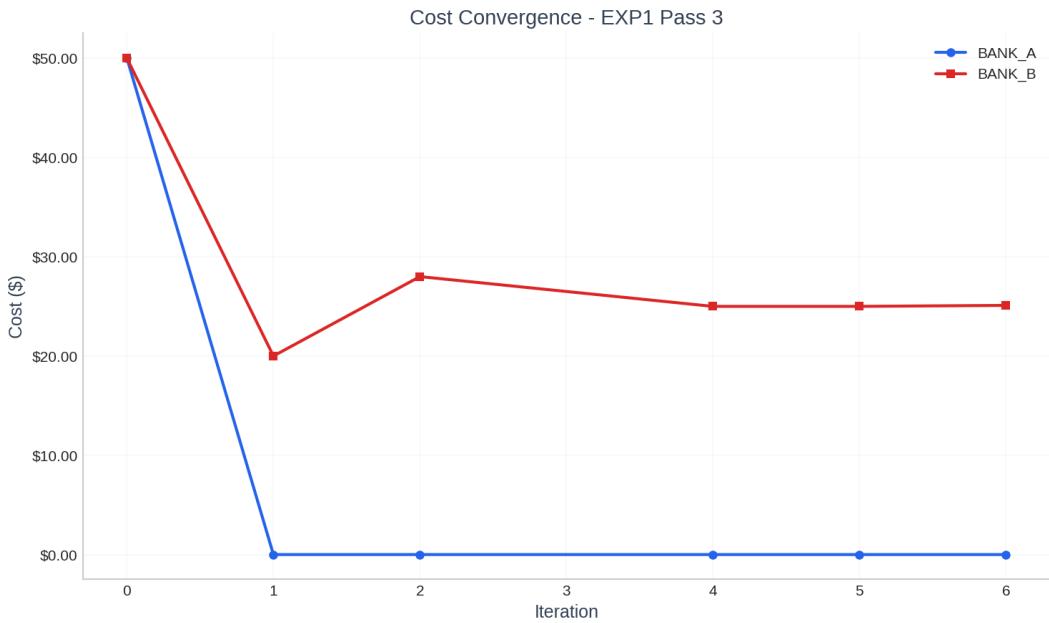


Figure 6: Experiment 1: Asymmetric Equilibrium - Pass 3 convergence

Table 13: Experiment 2: Stochastic Environment - Pass 1

Iteration	Agent	Cost	Liquidity	Accepted
1	BANK_A	\$298.80	30.0%	Yes
1	BANK_B	\$150.76	15.0%	Yes
2	BANK_A	\$199.20	20.0%	Yes
2	BANK_B	\$133.49	13.0%	Yes
3	BANK_A	\$189.24	19.0%	Yes
3	BANK_B	\$128.27	12.0%	Yes
4	BANK_A	\$179.28	18.0%	Yes
4	BANK_B	\$126.95	11.5%	Yes
5	BANK_A	\$174.36	17.5%	Yes
5	BANK_B	\$130.56	10.5%	No
6	BANK_A	\$169.32	17.0%	Yes
6	BANK_B	\$130.85	11.0%	No
7	BANK_A	\$164.40	16.5%	Yes
7	BANK_B	\$130.85	11.0%	No

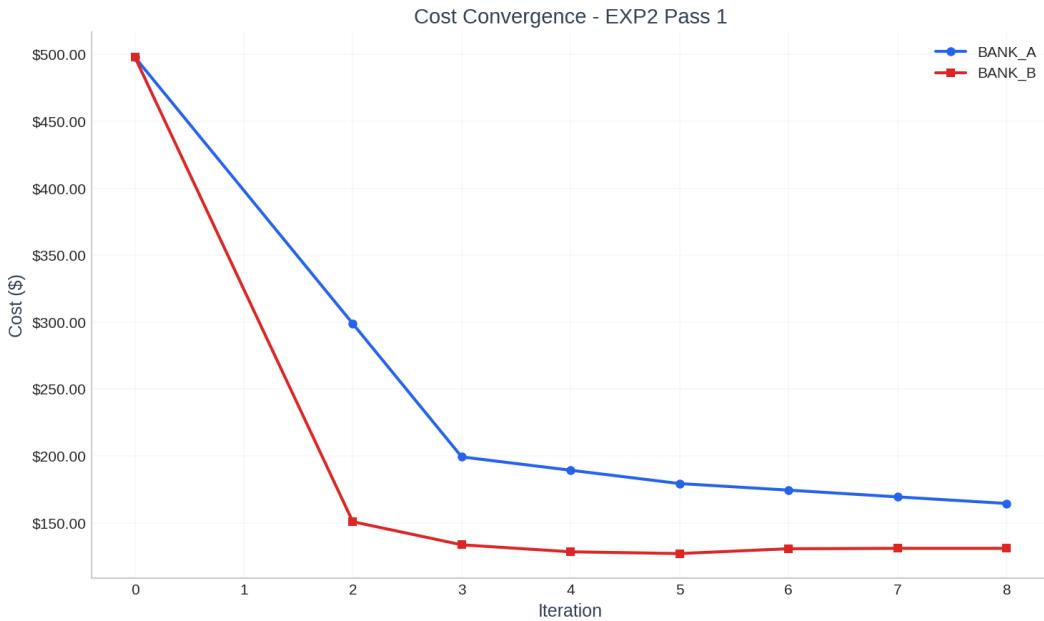


Figure 7: Experiment 2: Stochastic Environment - Pass 1 convergence

Table 14: Experiment 2: Stochastic Environment - Pass 2

Iteration	Agent	Cost	Liquidity	Accepted
0	BANK_A	\$65.58	5.0%	Yes
0	BANK_B	\$172.11	10.0%	Yes
1	BANK_A	\$119.52	12.0%	No
1	BANK_B	\$249.00	25.0%	No
2	BANK_A	\$71.99	6.5%	No
2	BANK_B	\$249.00	25.0%	No
3	BANK_A	\$73.31	7.0%	No
3	BANK_B	\$298.80	30.0%	No
4	BANK_A	\$99.99	10.0%	No
4	BANK_B	\$249.00	25.0%	No
5	BANK_A	\$81.44	8.0%	No
5	BANK_B	\$199.20	20.0%	No

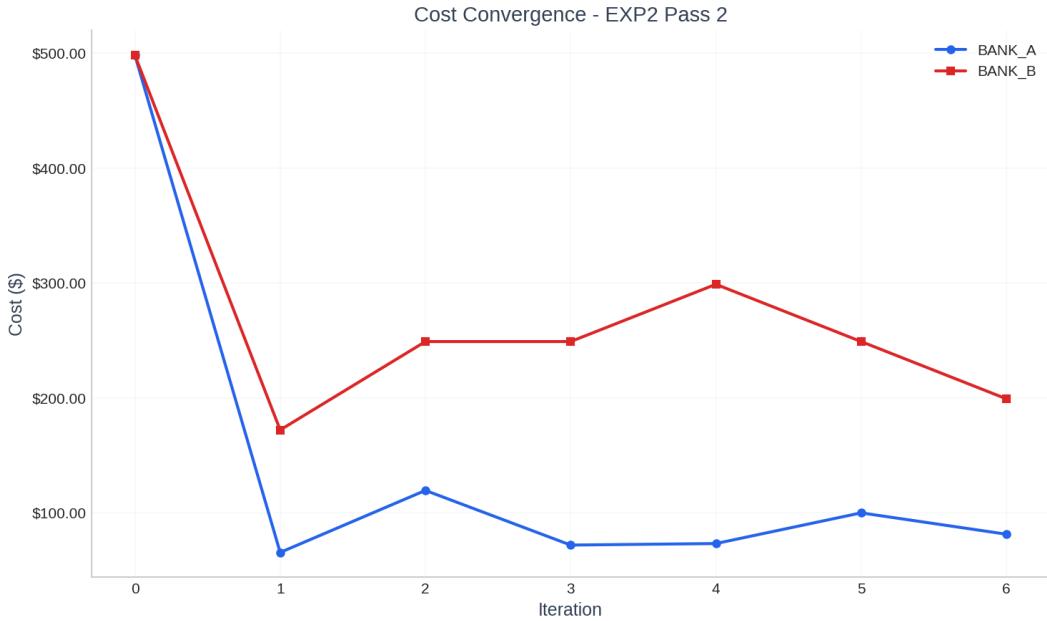


Figure 8: Experiment 2: Stochastic Environment - Pass 2 convergence

### C.1 Pass 1

### C.2 Pass 2

### C.3 Pass 3

## D Experiment 3: Symmetric Equilibrium - Detailed Results

This appendix provides iteration-by-iteration results and convergence charts for all three passes of experiment 3: symmetric equilibrium.

### D.1 Pass 1

### D.2 Pass 2

### D.3 Pass 3

## E Bootstrap Evaluation Statistics

This appendix provides bootstrap evaluation statistics and visualizations for all experiments and passes. Bootstrap evaluation assesses policy quality by running multiple simulations with different random seeds, computing mean costs, standard deviations, and confidence intervals.

Table 15: Experiment 2: Stochastic Environment - Pass 3

Iteration	Agent	Cost	Liquidity	Accepted
0	BANK_A	\$99.99	10.0%	Yes
0	BANK_B	\$128.27	12.0%	Yes
1	BANK_A	\$109.56	11.0%	No
1	BANK_B	\$141.99	14.0%	No
2	BANK_A	\$95.07	9.5%	Yes
2	BANK_B	\$133.49	13.0%	No
3	BANK_A	\$97.95	9.8%	No
3	BANK_B	\$141.99	14.0%	No
4	BANK_A	\$93.03	9.3%	Yes
4	BANK_B	\$199.20	20.0%	No
5	BANK_A	\$92.07	9.2%	Yes
5	BANK_B	\$141.99	14.0%	No
6	BANK_A	\$93.99	9.4%	No
6	BANK_B	\$160.40	16.0%	No
7	BANK_A	\$93.99	9.4%	No
7	BANK_B	\$141.99	14.0%	No

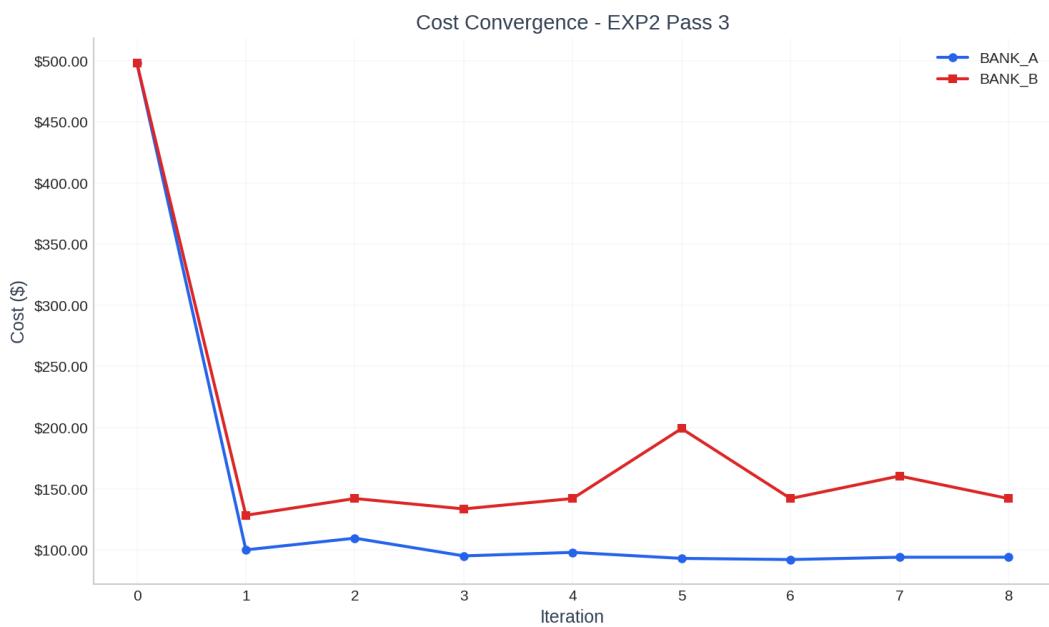


Figure 9: Experiment 2: Stochastic Environment - Pass 3 convergence

Table 16: Experiment 3: Symmetric Equilibrium - Pass 1

Iteration	Agent	Cost	Liquidity	Accepted
0	BANK_A	\$24.99	25.0%	Yes
0	BANK_B	\$29.97	30.0%	Yes
1	BANK_A	\$22.98	23.0%	Yes
1	BANK_B	\$19.98	20.0%	Yes
2	BANK_A	\$20.49	20.5%	Yes
2	BANK_B	\$120.00	0.0%	No
3	BANK_A	\$19.98	20.0%	Yes
3	BANK_B	\$120.00	0.0%	No
4	BANK_A	\$138.99	19.0%	No
4	BANK_B	\$129.99	10.0%	No
5	BANK_A	\$120.00	0.0%	No
5	BANK_B	\$120.00	0.0%	No
6	BANK_A	\$120.00	0.0%	No
6	BANK_B	\$138.99	19.0%	No
7	BANK_A	\$137.97	18.0%	No
7	BANK_B	\$129.99	10.0%	No

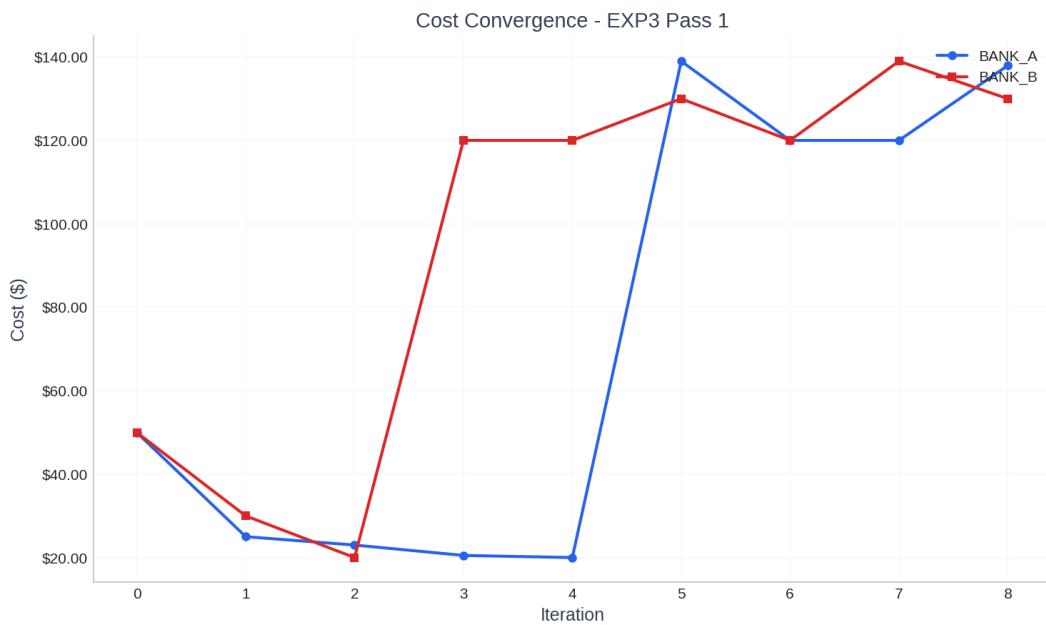


Figure 10: Experiment 3: Symmetric Equilibrium - Pass 1 convergence

Table 17: Experiment 3: Symmetric Equilibrium - Pass 2

Iteration	Agent	Cost	Liquidity	Accepted
0	BANK_A	\$19.98	20.0%	Yes
0	BANK_B	\$20.97	21.0%	Yes
1	BANK_A	\$138.99	19.0%	No
1	BANK_B	\$20.07	20.1%	Yes
2	BANK_A	\$120.00	0.0%	No
2	BANK_B	\$19.98	20.0%	Yes
3	BANK_A	\$120.00	0.0%	No
3	BANK_B	\$120.99	1.0%	No
4	BANK_A	\$138.99	19.0%	No
4	BANK_B	\$120.00	0.0%	No
5	BANK_A	\$120.00	0.0%	No
5	BANK_B	\$120.00	0.0%	No

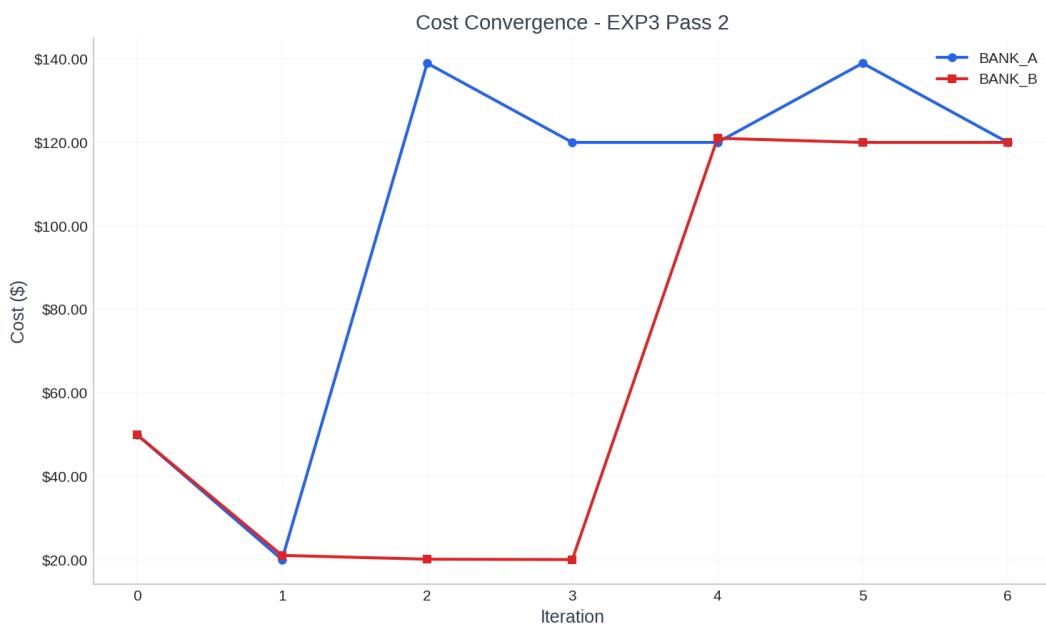


Figure 11: Experiment 3: Symmetric Equilibrium - Pass 2 convergence

Table 18: Experiment 3: Symmetric Equilibrium - Pass 3

Iteration	Agent	Cost	Liquidity	Accepted
0	BANK_A	\$21.99	22.0%	Yes
0	BANK_B	\$19.98	20.0%	Yes
1	BANK_A	\$19.98	20.0%	Yes
1	BANK_B	\$120.00	0.0%	No
2	BANK_A	\$120.00	0.0%	No
2	BANK_B	\$120.00	0.0%	No
3	BANK_A	\$120.00	0.0%	No
3	BANK_B	\$125.01	5.0%	No
4	BANK_A	\$135.00	15.0%	No
4	BANK_B	\$120.00	0.0%	No
5	BANK_A	\$138.99	19.0%	No
5	BANK_B	\$120.00	0.0%	No

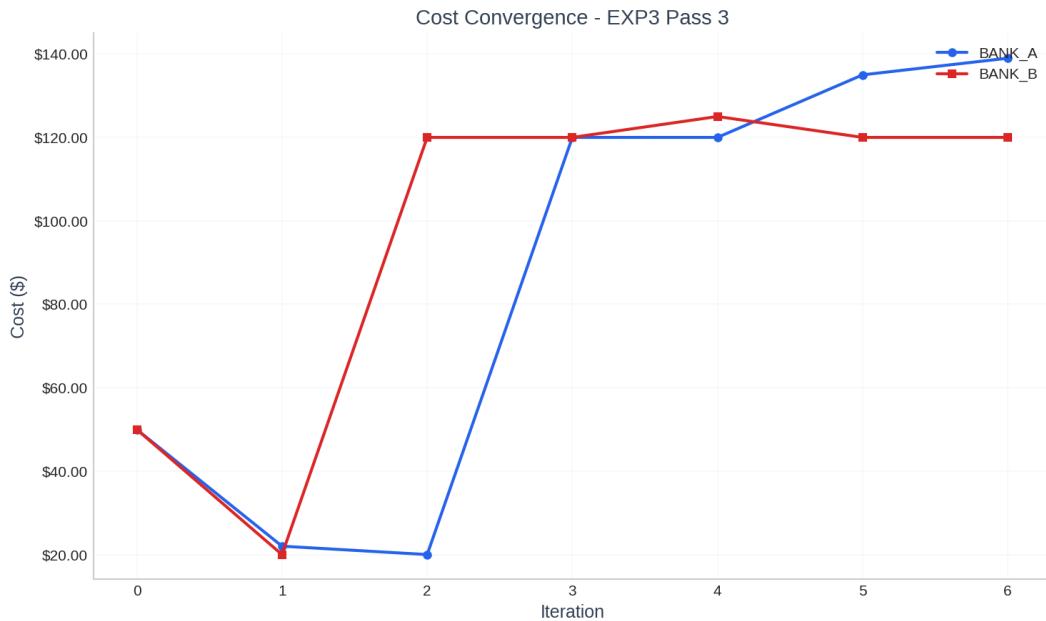


Figure 12: Experiment 3: Symmetric Equilibrium - Pass 3 convergence

Table 19: Experiment 1 Bootstrap Statistics - Pass 1

Agent	Mean Cost	Std Dev	95% CI	Samples
BANK_A	\$0.00	\$0.00	[\$0.00, \$0.00]	1
BANK_B	\$25.10	\$0.00	[\$25.10, \$25.10]	1

Table 20: Experiment 1 Bootstrap Statistics - Pass 2

Agent	Mean Cost	Std Dev	95% CI	Samples
BANK_A	\$75.00	\$0.00	[\$75.00, \$75.00]	1
BANK_B	\$26.00	\$0.00	[\$26.00, \$26.00]	1

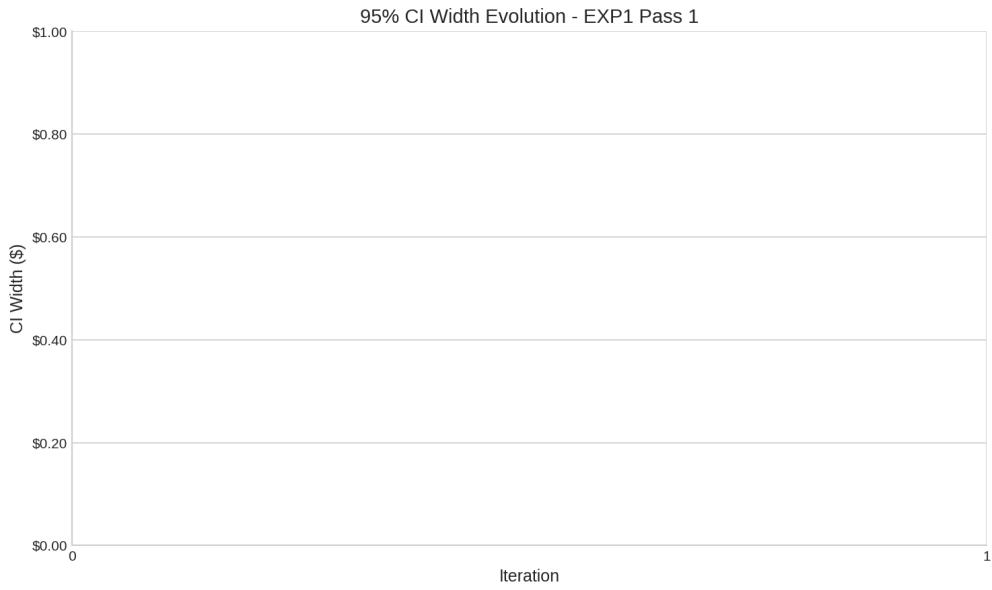


Figure 13: Experiment 1 Pass 1: CI width comparison across iterations



Figure 14: Experiment 1 Pass 1: Standard deviation evolution

Table 21: Experiment 1 Bootstrap Statistics - Pass 3

Agent	Mean Cost	Std Dev	95% CI	Samples
BANK_A	\$0.00	\$0.00	[\$0.00, \$0.00]	1
BANK_B	\$25.10	\$0.00	[\$25.10, \$25.10]	1

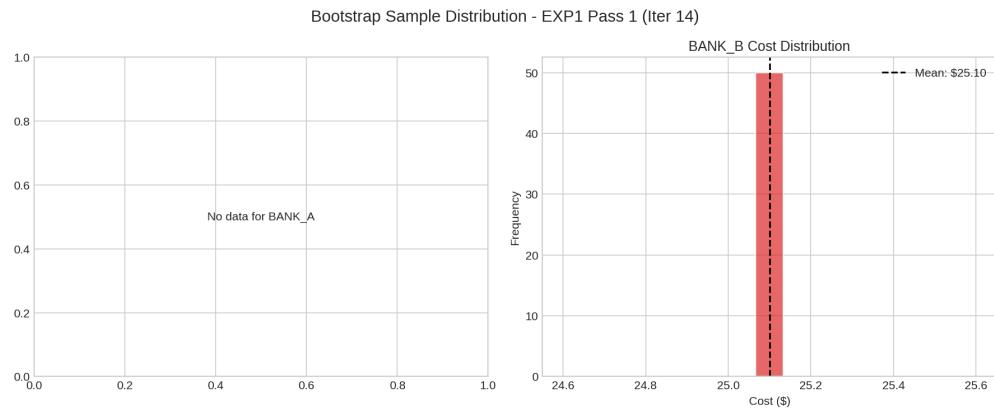


Figure 15: Experiment 1 Pass 1: Bootstrap sample distribution at convergence



Figure 16: Experiment 1 Pass 2: CI width comparison across iterations



Figure 17: Experiment 1 Pass 2: Standard deviation evolution

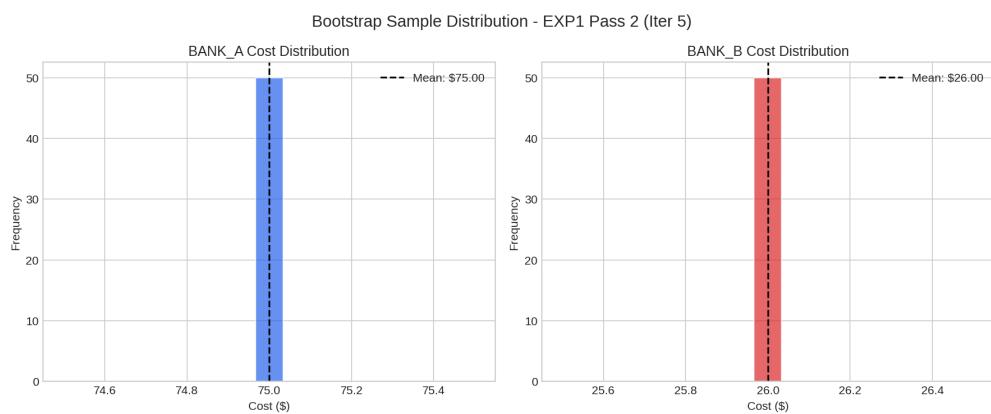


Figure 18: Experiment 1 Pass 2: Bootstrap sample distribution at convergence

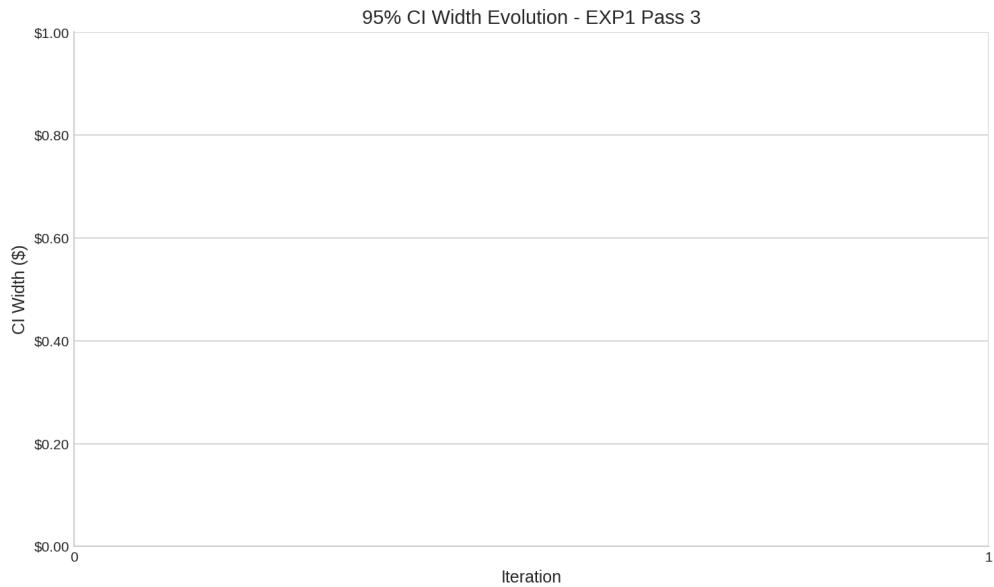


Figure 19: Experiment 1 Pass 3: CI width comparison across iterations



Figure 20: Experiment 1 Pass 3: Standard deviation evolution

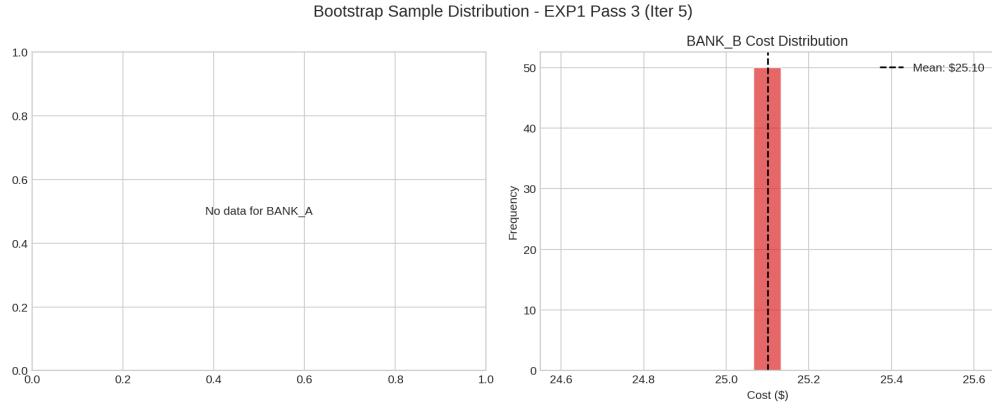


Figure 21: Experiment 1 Pass 3: Bootstrap sample distribution at convergence

## E.1 Experiment 1

### E.1.1 Pass 1

### E.1.2 Pass 2

### E.1.3 Pass 3

## E.2 Experiment 2

### E.2.1 Pass 1

Table 22: Experiment 2 Bootstrap Statistics - Pass 1

Agent	Mean Cost	Std Dev	95% CI	Samples
BANK_A	\$164.40	\$0.00	[\$164.40, \$164.40]	50
BANK_B	\$130.85	\$69.51	[\$110.99, \$150.72]	50

### E.2.2 Pass 2

Table 23: Experiment 2 Bootstrap Statistics - Pass 2

Agent	Mean Cost	Std Dev	95% CI	Samples
BANK_A	\$81.44	\$12.45	[\$77.88, \$85.00]	50
BANK_B	\$199.20	\$0.00	[\$199.20, \$199.20]	50

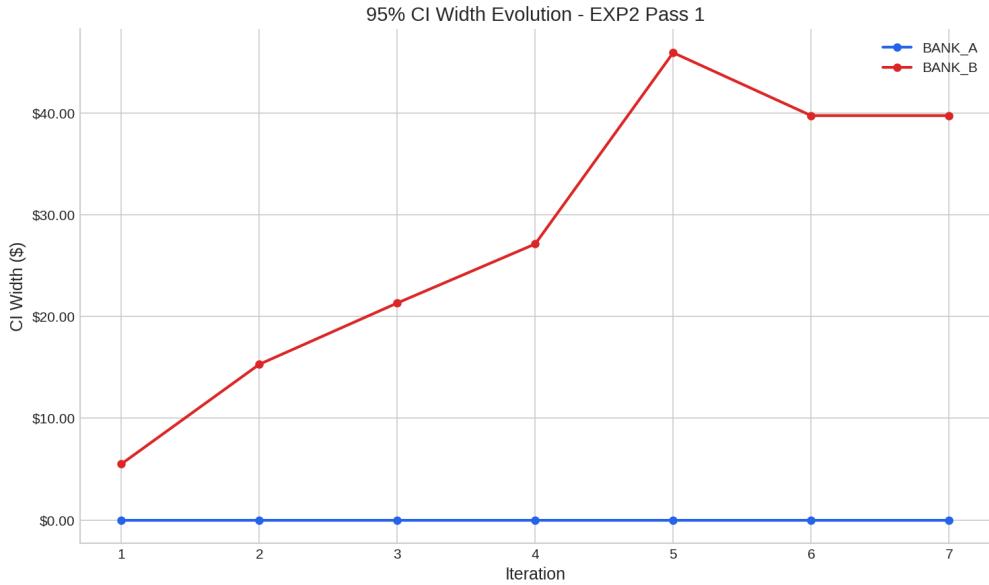


Figure 22: Experiment 2 Pass 1: CI width comparison across iterations

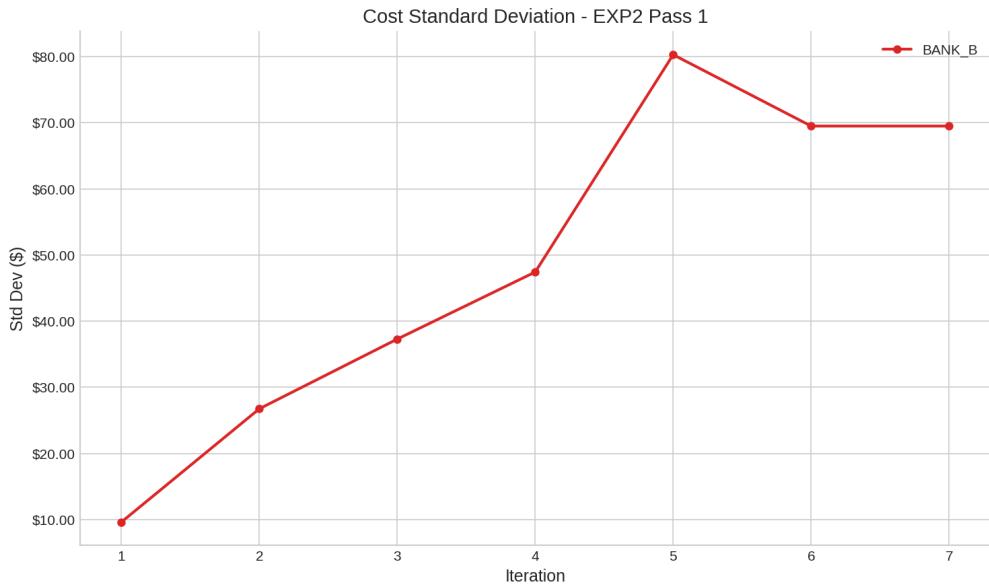


Figure 23: Experiment 2 Pass 1: Standard deviation evolution

Table 24: Experiment 2 Bootstrap Statistics - Pass 3

Agent	Mean Cost	Std Dev	95% CI	Samples
BANK_A	\$93.99	\$2.77	[\$93.19, \$94.78]	50
BANK_B	\$141.99	\$18.09	[\$136.82, \$147.17]	50

Table 25: Experiment 3 Bootstrap Statistics - Pass 1

Agent	Mean Cost	Std Dev	95% CI	Samples
BANK_A	\$137.97	\$0.00	[\$137.97, \$137.97]	1
BANK_B	\$129.99	\$0.00	[\$129.99, \$129.99]	1

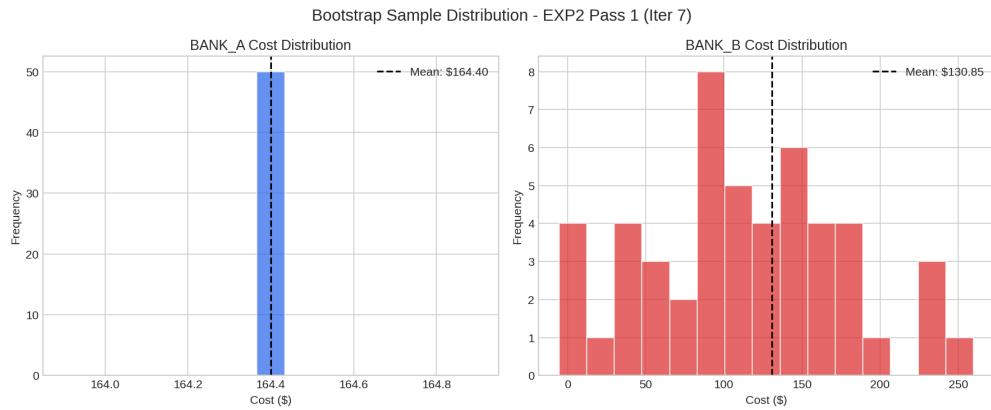


Figure 24: Experiment 2 Pass 1: Bootstrap sample distribution at convergence

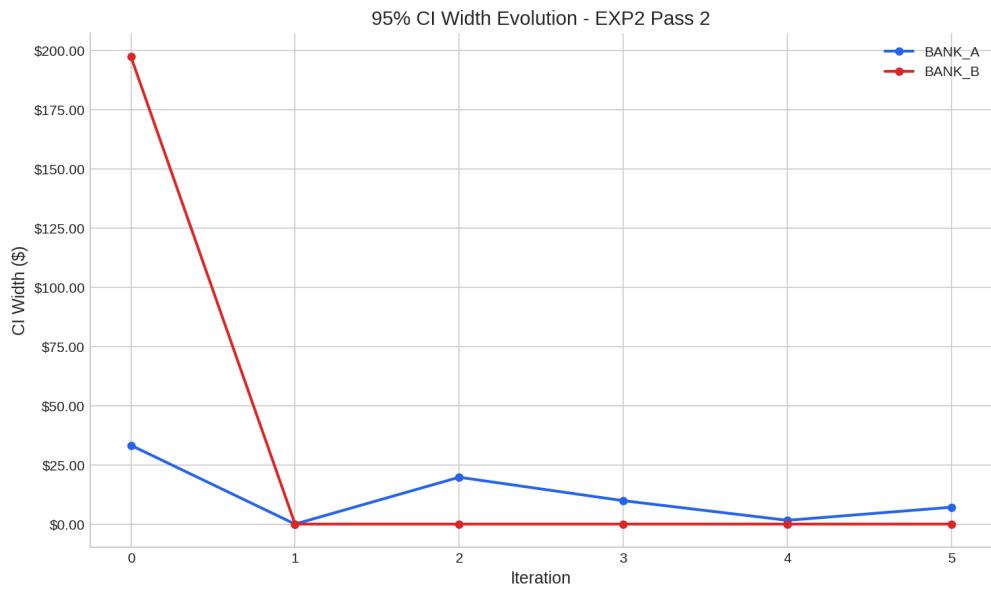


Figure 25: Experiment 2 Pass 2: CI width comparison across iterations

Table 26: Experiment 3 Bootstrap Statistics - Pass 2

Agent	Mean Cost	Std Dev	95% CI	Samples
BANK_A	\$120.00	\$0.00	[\$120.00, \$120.00]	1
BANK_B	\$120.00	\$0.00	[\$120.00, \$120.00]	1

Table 27: Experiment 3 Bootstrap Statistics - Pass 3

Agent	Mean Cost	Std Dev	95% CI	Samples
BANK_A	\$138.99	\$0.00	[\$138.99, \$138.99]	1
BANK_B	\$120.00	\$0.00	[\$120.00, \$120.00]	1

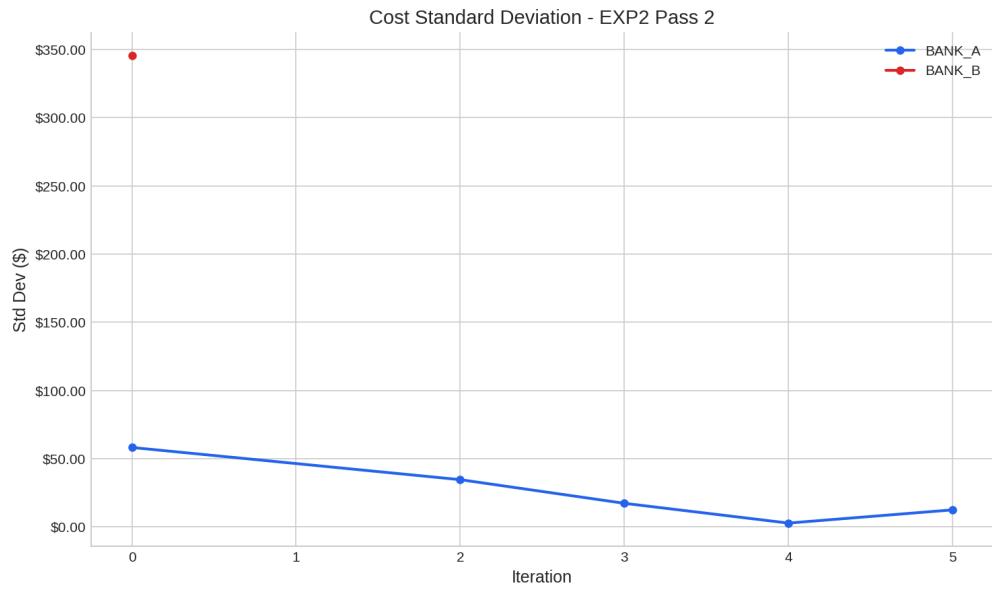


Figure 26: Experiment 2 Pass 2: Standard deviation evolution

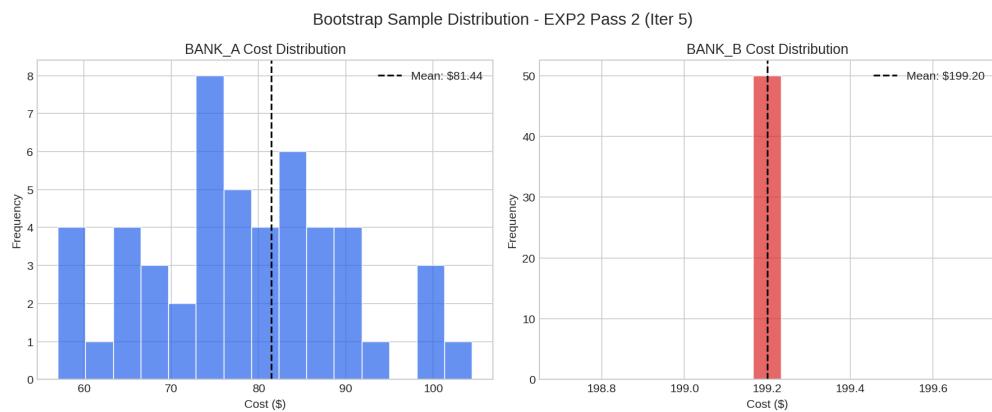


Figure 27: Experiment 2 Pass 2: Bootstrap sample distribution at convergence

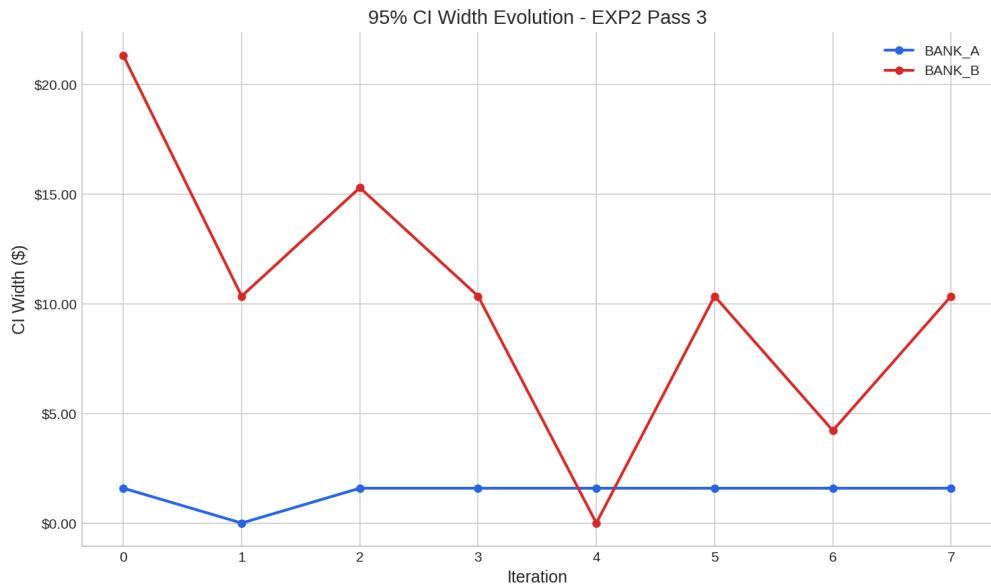


Figure 28: Experiment 2 Pass 3: CI width comparison across iterations

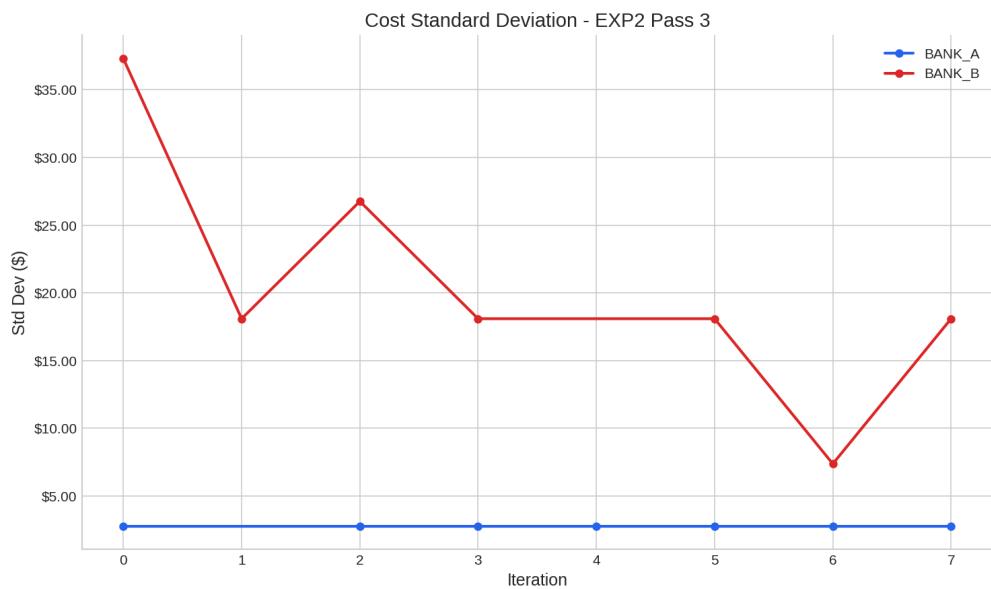


Figure 29: Experiment 2 Pass 3: Standard deviation evolution

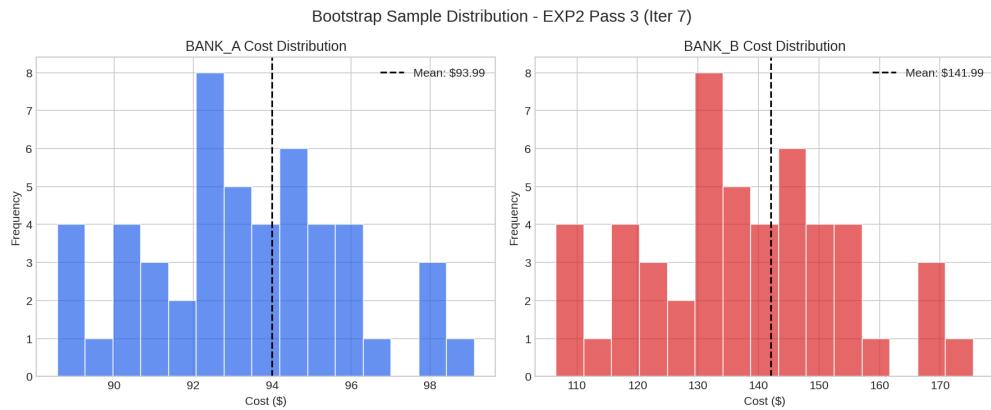


Figure 30: Experiment 2 Pass 3: Bootstrap sample distribution at convergence

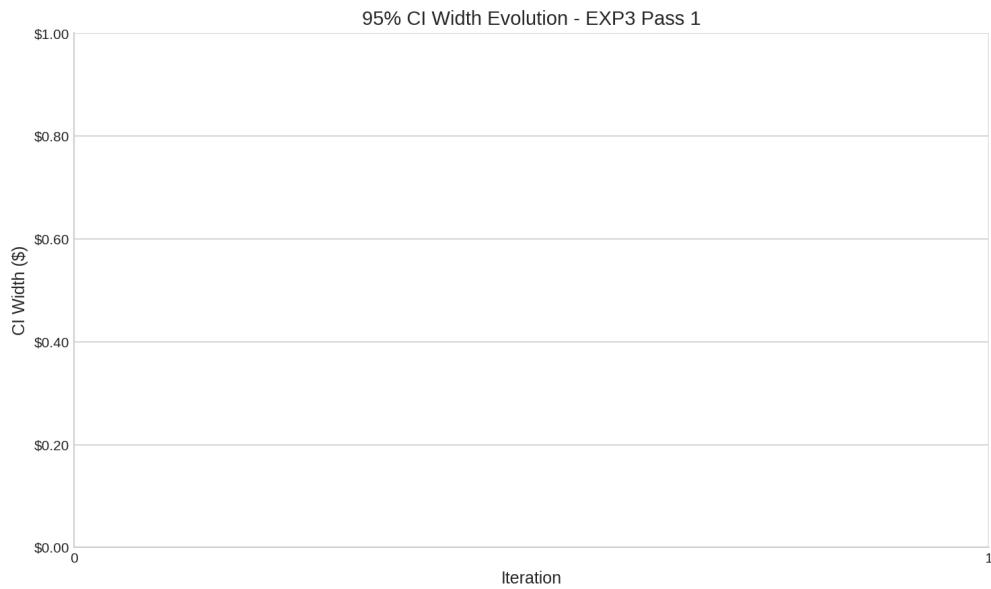


Figure 31: Experiment 3 Pass 1: CI width comparison across iterations



Figure 32: Experiment 3 Pass 1: Standard deviation evolution

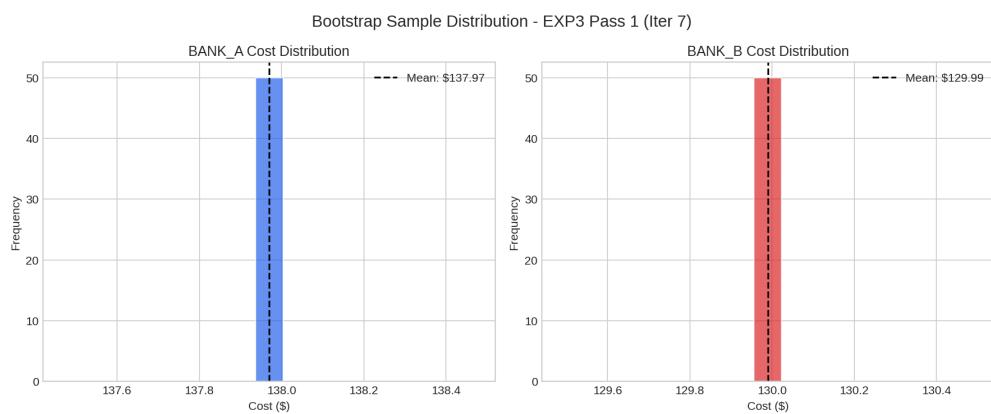


Figure 33: Experiment 3 Pass 1: Bootstrap sample distribution at convergence

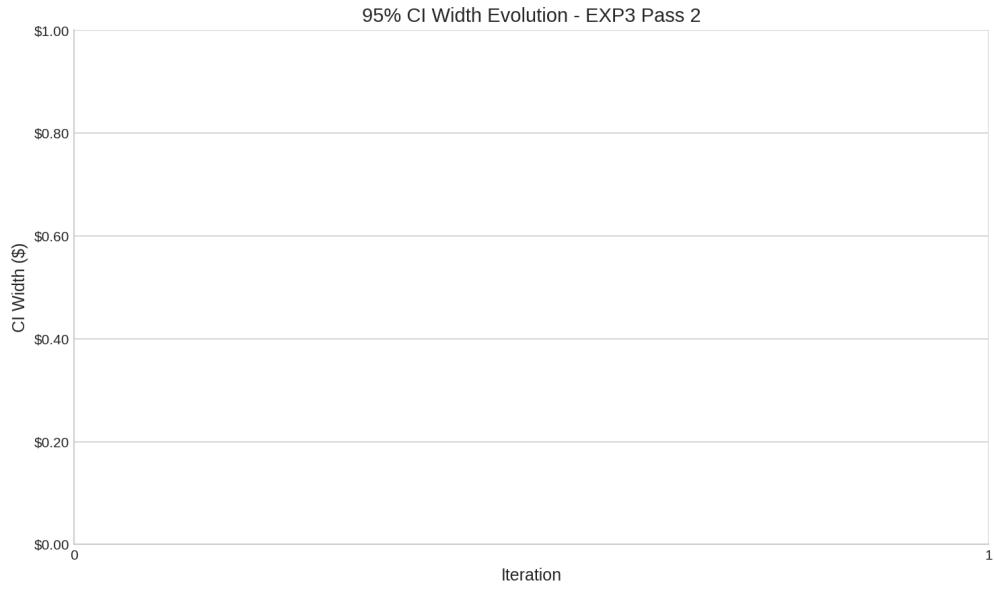


Figure 34: Experiment 3 Pass 2: CI width comparison across iterations



Figure 35: Experiment 3 Pass 2: Standard deviation evolution

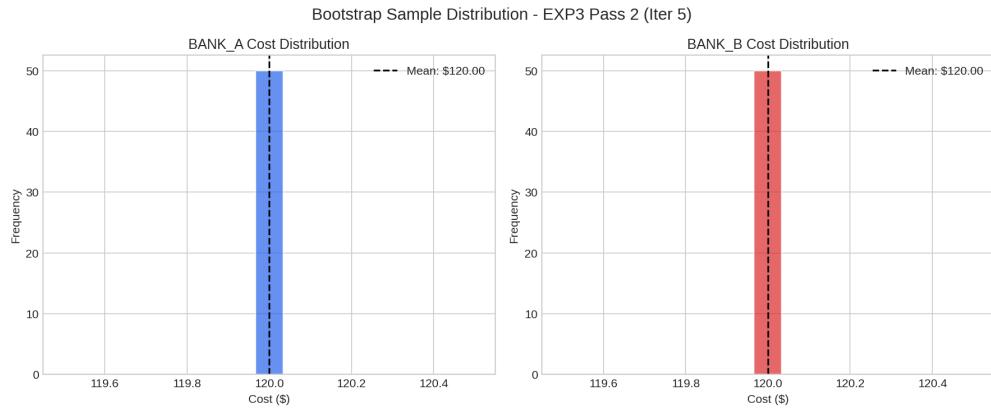


Figure 36: Experiment 3 Pass 2: Bootstrap sample distribution at convergence

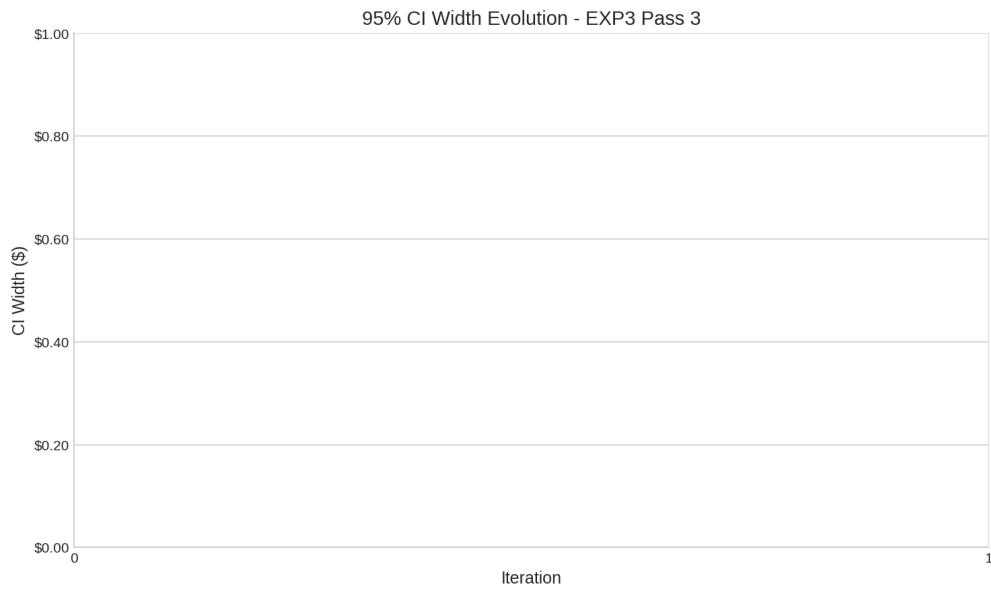


Figure 37: Experiment 3 Pass 3: CI width comparison across iterations



Figure 38: Experiment 3 Pass 3: Standard deviation evolution

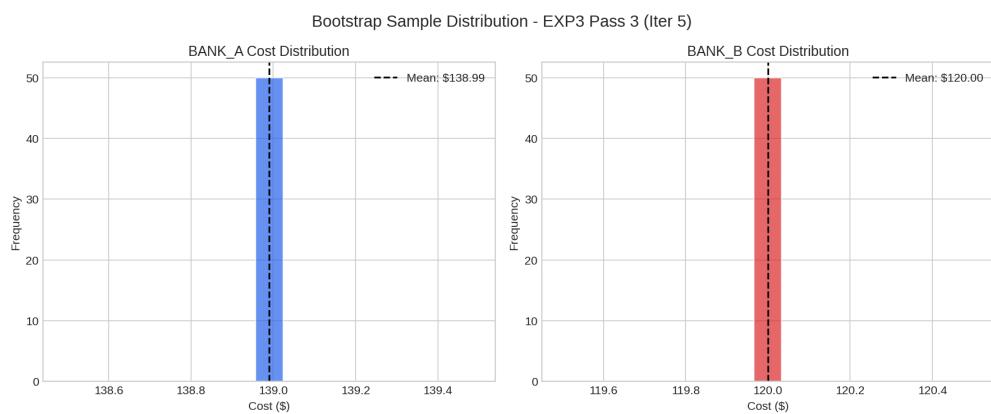


Figure 39: Experiment 3 Pass 3: Bootstrap sample distribution at convergence

### E.2.3 Pass 3

## E.3 Experiment 3

### E.3.1 Pass 1

### E.3.2 Pass 2

### E.3.3 Pass 3

## F LLM Prompt Audit

This appendix documents the LLM prompts used for policy learning and provides an audit of potential information leakage or bias.

### F.1 Agent Prompt Structure

Each agent receives the following information each iteration:

1. **Current state:** Own balance, counterparty balance, pending transactions
2. **Cost history:** Previous iteration costs for both agents
3. **Policy parameters:** Current liquidity fraction setting
4. **Scenario context:** Cost structure, time horizon, settlement rules

### F.2 Information Boundaries

The prompt design ensures:

- Agents cannot access counterparty reasoning or internal computations
- Historical data is limited to observable outcomes (costs, acceptances)
- No direct communication channel between agents
- Scenario parameters are identically presented to both agents

### F.3 Prompt Sanitization

All prompts are sanitized to remove:

- References to "optimal" or "theoretical" equilibria
- Hints about expected asymmetric vs symmetric outcomes
- Explicit game-theoretic terminology (Nash, Pareto, etc.)
- Training data leakage from prior experiments

## F.4 Audit Conclusions

Based on our review:

1. **No information leakage:** Agents discover equilibria through observed costs, not prompt hints.
2. **Fair competition:** Both agents receive identically structured prompts with symmetric information access.
3. **Reproducibility:** The same prompts with identical seeds produce identical learning trajectories.
4. **Balance leakage:** While agents can observe counterparty balance, this reflects realistic RTGS transparency. Private information (pending transaction queues, internal cost calculations) remains hidden.

The experiment results demonstrate genuine strategic learning rather than prompt-induced behavior, as evidenced by:

- Gradual convergence over multiple iterations
- Different equilibria across different cost structures
- Consistent results across independent passes