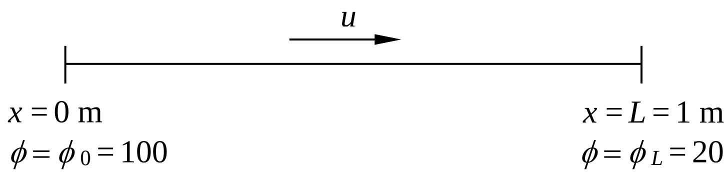
Computational Fluid Dynamics (ME4 & MSc)

Project Assignment 1

A property ϕ is transported by means of convection and diffusion through the 1-D domain of length L shown in **Fig. 1**:



The distribution of ϕ should be determined by the steady state convection-diffusion equation:

$$\frac{d}{dx}(\rho u \phi) = \frac{d}{dx}\left(\Gamma_\phi \frac{d\phi}{dx}\right) \quad \textbf{(1)}$$

where u is the fluid velocity, ρ is the fluid density and Γ_ϕ is the diffusion coefficient. In real-life terms, this problem could be the modelling of temperature distribution in an insulated pipe of length L carrying fluid from a hot reservoir at constant $T = T_0$ to a cold reservoir at constant $T = T_L$. Equation **(1)** should be discretised by the finite-volume method like in your lecture notes, using the following discretisation schemes for the convective term:

- Central Differencing
- Upwind Differencing
- Power-Law Differencing

The resulting set of equations should be solved using the Tri-Diagonal Matrix Algorithm (TDMA). The numerical results of this problem should be produced by using a code written by you in a programming language of your choice (like `FORTRAN`, `C`, `Python`, *etc.*), or using software like `MATLAB`.

The diffusion coefficient Γ_ϕ should be held constant at a value of your choice (*e.g.* pick a value between 0.1 and 1.0 for all test cases). The fluid's density ρ should be held constant at a value of your choice (*e.g.* pick a value between 0.1 and 1.0 for all test cases). The fluid's velocity u should be assumed constant along the full length of the domain. However, for each discretisation scheme, solve the convection-diffusion equation for different values of fluid velocity to assess the advantages and limitations of each method for different values of convective flux. In each case the result should be compared with the analytical solution given by:

$$\phi = \phi_0 + \frac{exp(\rho u x / \Gamma_\phi) - 1}{exp(\rho u L / \Gamma_\phi) - 1}(\phi_L - \phi_0) \quad \textbf{(2)}$$

The total number of grid points should also be varied to assess the effect of grid-point spacing δx. For a fixed discretisation scheme and constant values of convection and diffusion flux, plot the accuracy of your model solution against grid-point spacing δx. For this exercise, the overall accuracy of the numerical solution should be assessed as follows:

$$\text{Error}[\%] = 100 \times n^{-1} \sum \left| \frac{\phi_i - \tilde{\phi}_i}{\tilde{\phi}_i} \right| \tag{3}$$

where ϕ_i is the modelled value at node i, $\tilde{\phi}_i$ the analytical value at i and n the total number of nodes.

Guidelines: You are advised to use graphs to present and discuss your findings, *e.g.* distribution of the dependent variable along x, error plots, *etc.* Please do not build long tables with large amounts of data; volumes of tabulated data are neither useful to read nor easy to digest. The lecture notes of the course are self-contained and sufficient for you to complete this project assignment. If you feel that your programming skills are not sufficient to complete the assignment, you may collaborate with one (*only one*) of your colleagues to develop the code. However, afterwards, you should use the code *on your own* to run *your own* test cases and write *your own* report. On such occasion you *must* mention in your report the name of the colleague whom you collaborated with to prepare the code, otherwise your report will be treated as plagiarism.

Submission: Please submit your study in a technical report of up to 3,500 words by 12:00 pm on Thursday January 13, 2022. The report should be structured as a full technical document, *i.e.* include Abstract, Contents List, Introduction and Objectives, Methodology, Results and Discussion, Conclusions, References, Appendix, *etc.* The source code should be provided in an Appendix of the report. Please submit your report by uploading it on Blackboard. The report should be in `.pdf` format and named strictly as shown below (including your surname and CID):

`CFD21-22_Project1_Report_Surname_CID.pdf`

The source code should also be uploaded on Blackboard in electronic copy in `.zip` format. This format is requested so that you upload in a single `.zip` file one or multiple versions of your code (if you elect to write separate codes for different numerical schemes). The electronic file of the code should be named strictly as follows:

`CFD21-22_Project1_Code_Surname_CID.zip`

Note: All codes will be run and their results will be checked upon marking by the Course Leader. This is a time-consuming process for each report, therefore, feedback usually takes longer than normal and should be expected in 5–6 weeks' time after submission.