

Part (i)

$$\begin{bmatrix} 1 & 1 \\ 10^{-k} & 0 \\ 0 & 10^{-k} \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -10^{-k} \\ 1+10^{-k} \\ 1-10^{-k} \end{bmatrix}$$

A \underline{x} \underline{b}

QR factorization of A :

$$Q = \begin{bmatrix} -1 & 7.0711 \times 10^{-6} & -7.0711 \times 10^{-6} \\ -10^{-5} & -0.7071 & 0.7071 \\ 0 & 0.7071 & 0.7071 \end{bmatrix}$$

$$R = \begin{bmatrix} -1 & -1 \\ 0 & 1.4142 \times 10^{-5} \end{bmatrix}$$

$$\underline{c} = Q^T \underline{b} = \begin{bmatrix} 10^{-5} \\ -1.4142 \times 10^{-5} \\ 1.4142 \end{bmatrix}$$

$$R \underline{\hat{x}} = \underline{c} \Rightarrow \underline{\hat{x}} = R^{-1} \underline{c}$$

$$R^{-1} = \frac{1}{(-1)(1.4142 \times 10^{-5}) - (-1)(0)} \begin{bmatrix} -1.4142 \times 10^{-5} & 1 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -7.0711 \times 10^4 \\ 0 & 7.0711 \times 10^4 \end{bmatrix}$$

$$\Rightarrow \underline{\hat{x}} = R^{-1} \underline{c} = \begin{bmatrix} -10^{-5} \\ -1.7071 \end{bmatrix}$$