# Fourier series

Will Parker

## 1 Introduction

This page is used to investigate how the number of terms used by a Fourier series approximation affects the accuracy by which the series approximates the given function.

## 2 The Triangle wave

The first investigation will be with the Triangle wave which will display by selecting the appropriate function from the dropdown box on the left. One period of the function should display in the graph. It should be observed that the triangle wave function is continuous. It is defined by

$$f\left(x\right) = 1 - 4|\frac{1}{2} - frac\left(\frac{1}{2}x + \frac{1}{4}\right)|,$$

where $frac$ is defined as the fractional part of $x$.

Now select the checkbox Include Fourier approximation, leaving the slider set to n = 1. The Fourier series for the triangle wave is given as

$$f\left(x\right) = \frac{8}{\pi^2}\sum_{n=1,3,5,\ldots}^{\infty}\frac{(-1)^{(n-1)/2}}{n^2}\sin\left(\frac{n\pi x}{L}\right). \tag{1}$$

A sine wave line should appear on the graph, as can be seen it is however a poor approximation of the triangle wave. Slowly increase the number of terms n and see how this affects the approximation of the triangle wave. It should be found that rather quickly the fourier series approximates the triangle wave function fairly quickly.

## 3 The square wave

Now change the dropdown to the square wave and deselect the fourier checkbox. As before a single period of a square wave should display. It should be noted that this function has a jump discontinuity. The function is defined by

$$f(x) = A(-1)^{floor(2(x-x_0)/L)},$$

where $floor$ is the floor function, $A$ is the amplitude, $L$ is the period and $x_0$ is the offset. The Fourier series for the function is given by

$$f(x) = \frac{4}{\pi} \sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi x}{L}\right).$$

By following the process previously used for the triangle wave investigate how changing the number of terms affects the accuracy of the approximation of the function. Do you notice anything unusual near the jump discontinuity? What causes this?

## 4 The Saw tooth wave

The saw tooth wave is defined as

$$f(x) = A frac\left(\frac{x}{L} + \phi\right),$$

where $A$ is the amplitude and $L$ is the period with phase constant $\phi$ and its fourier approximation is

$$f(x) = \frac{1}{2} - \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi x}{L}\right)$$

For a third (and final) time change the dropdown to the saw tooth wave and in the same way as before investigate how changing the number of terms used to approximate the function affects the accuracy of the approximation.