Parseval's Theorem

Will Parker

1 Parseval's theorem

Parseval's theorem for the Fourier series states that

$$\frac{1}{2L}\int_{-L}^{L}|f(x)|^2dx=\sum_{n=-\infty}^{\infty}|c_n|^2.$$

Parsevals theorem can be used to carry out infinite sums that would be very difficult to evaluate. By taking the Fourier series of the square wave we can evaluate $|f(x)|^2$ to find

$$\begin{aligned}\frac{1}{2L}\int_{-L}^{L}|f(x)|^2dx &= \frac{1}{2L}\left[\int_{-L}^{0}|-1|^2dx+\int_{0}^{L}|1|^2dx\right]\\ &= \frac{L+L}{2L}\\ &= 1.\end{aligned}$$

Using the exponential form of the square wave coefficients Parseval's theoren gives

$$\begin{aligned}1 &= \sum_{n=-\infty}^{\infty}|c_n|^2\\ &= \sum_{nodd}\left|\frac{-2i}{n\pi}\right|^2\\ &= 2\sum_{n=1,3,\dots}\frac{4}{(n\pi)^2}\end{aligned} \tag{1}$$

Rearranging gives

$$\sum_{n=1,3,\dots}\frac{1}{n^2}=\frac{\pi^2}{8}.$$

It would be difficult to carry out this sum by other means. Rearranging this final result also allows for a formula which can be used to approximate the value of π. Investigate how changing the number of terms used to approximate π affects the computed value by moving the slider above.