Multicomponent, multiphase flow in porous media

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1 Governing equations

1.1 Definitions and nomenclature

Phase

Component

Notation and nomenclature

In all following equations, the subscript α refers to the phase, while the superscript κ refers to the component.

P_α	Phase pressure	ϕ	Porosity
T	Temperature	K	Permeability
S_α	Phase saturation	$k_{r\alpha}$	Relative permeability
P_c	Capillary pressure	ρ_α	Phase density
μ_α	Phase viscosity	$\mathbf{g}$	Gravity
X_α^κ	Mass fraction of component κ in phase α		
q_α^κ	Source/sink term of κ in phase α		

1.2 Mass balance equation

Conservation of mass of of each component κ gives the following balance equation for each component (for an isothermal system)

$$\phi \frac{\partial}{\partial t} \left(\sum_\alpha \rho_\alpha X_\alpha^\kappa S_\alpha \right) = \sum_\alpha \nabla \cdot \left\{ \frac{K k_{r\alpha} \rho_\alpha X_\alpha^\kappa}{\mu_\alpha} \left(\nabla P_\alpha - \rho_\alpha \mathbf{g} \right) \right\} + \sum_\alpha q_\alpha^\kappa. \quad (1)$$

The number of unknown variables can be reduced using fundamental relationships and constitutive models. The phase saturations must sum to unity (assuming that the entire pore space is occupied)

$$\sum_\alpha S_\alpha = 1, \quad (2)$$

while the sum of mass fraction components in each phase must be unity by definition

$$\sum_{\kappa} X_{\alpha}^{\kappa} = 1. \tag{3}$$

Relative permeability is calculated as a function of saturation (using a prescribed relationship). The pressure of each phase is related by capillary pressure, defined as

$$P_c = P_n - Pw, \tag{4}$$

where the subscripts n and w refer to the non-wetting and wetting phases, respectively. Like relative permeability, capillary pressure can be calculated as a function of saturation using prescribed relationships.