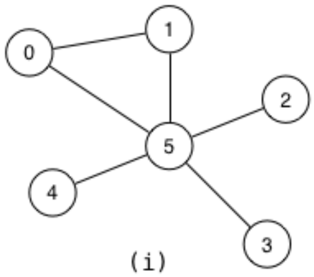
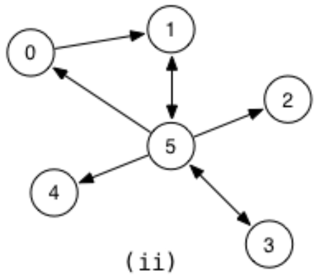
Q1.



(i)



(ii)

First graph:

$$\begin{array}{c|c} & \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

Adj. Matrix

0 → 1, 5
1 → 0, 5
2 → 5
3 → 5
4 → 5
5 → 0, 1, 2, 3, 4

Adj. List

Second graph:

$$\begin{array}{c|c} & \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

Adj. Matrix

0 → 1
1 → 5
2
3 → 5
4
5 → 0, 1, 2, 3, 4

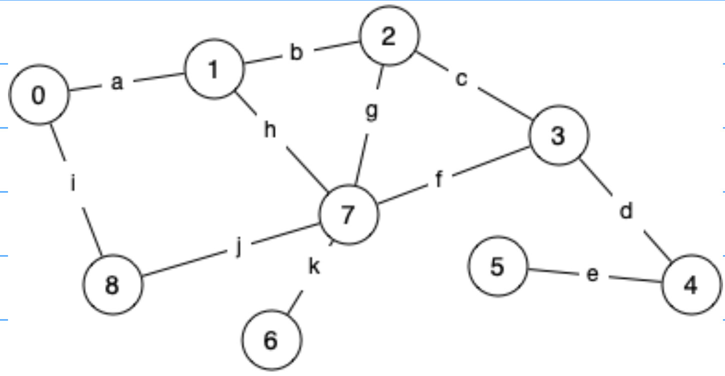
Adj. List

Note the space complexity differences:

Matrix $\Rightarrow O(V^2)$

List $\Rightarrow O(V+E)$

Q2.



Cycle: Path that starts + ends at the same node.
Clique: Complete subgraphs (with $|V| \geq 3$)

# Edges	# Cycles	# Cliques
11	6	2

V	0	1	2	3	4	5	6	7	8
deg(v)	2	3	3	3	2	1	1	5	2

Longest path from $5 \overset{*}{\to} 8$: $5 \to 4 \to 3 \to 2 \to 7 \to 1 \to 0 \to 8$

Q4. Differences between adj. matrices of directed / undirected graphs:

Undirected graphs have symmetric adj. matrices

(M is symmetric $\Rightarrow M[i][j] = M[j][i]$ for all i,j)

Why?

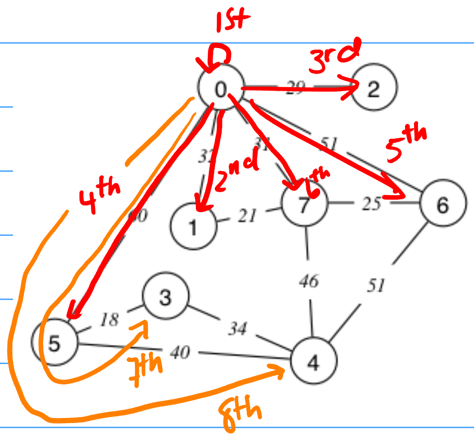
$M[i][j] = 1 \iff$ can go directly from i to j

$\iff$ can go directly from j to i too

$\iff M[j][i] = 1$.

Similarly for $M[i][j] = 0 \iff M[j][i] = 0$

Q6.

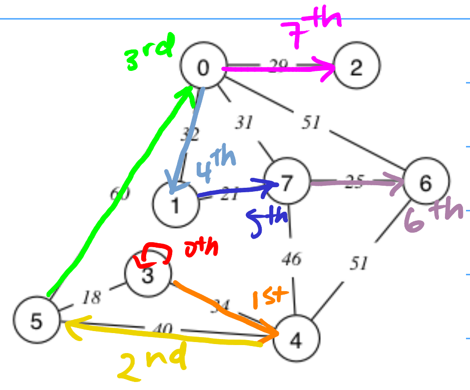


breadthFirst(g, 0);

Iteration	Visited	Queue (head, ..., tail)
0	0 1 2 3 4 5 6 7	0
1	~~0~~ 1 2 3 4 5 6 7	1, 2, 5, 6, 7
2	~~0~~ ~~1~~ 2 3 4 5 6 7	2, 5, 6, 7, 7
3	~~0~~ ~~1~~ ~~2~~ 3 4 5 6 7	5, 6, 7, 7
4	~~0~~ ~~1~~ ~~2~~ 3 4 ~~5~~ 6 7	6, 7, 7, 3, 4
5	~~0~~ ~~1~~ ~~2~~ 3 4 ~~5~~ ~~6~~ 7	7, 7, 3, 4, 4, 7
6	~~0~~ ~~1~~ ~~2~~ 3 4 ~~5~~ ~~6~~ ~~7~~	7, 3, 4, 4, 7, 4
7	~~0~~ ~~1~~ ~~2~~ ~~3~~ 4 ~~5~~ ~~6~~ ~~7~~	4, 4, 7, 4, 4
8	~~0~~ ~~1~~ ~~2~~ ~~3~~ ~~4~~ ~~5~~ ~~6~~ ~~7~~	4, 7, 4, 4
9..12	~~0~~ ~~1~~ ~~2~~ ~~3~~ ~~4~~ ~~5~~ ~~6~~ ~~7~~	(eventually becomes empty)

Output: 0, 1, 2, 5, 6, 7, 3, 4

BFS ⇒ visit neighbours, then neighbours of neighbours, etc.



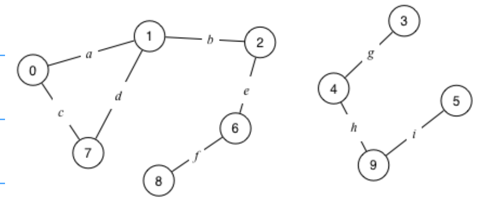
depthFirst(g, 3);

Iteration	Visited	Stack (top, ..., bottom)
0	0 1 2 3 4 5 6 7	3
1	0 1 2 ~~3~~ 4 5 6 7	4, 5
2	0 1 2 ~~3~~ ~~4~~ 5 6 7	5, 6, 7, 5
3	0 1 2 ~~3~~ ~~4~~ ~~5~~ 6 7	0, 6, 7, 5
4	~~0~~ 1 2 ~~3~~ ~~4~~ ~~5~~ 6 7	1, 2, 6, 7, 6, 7, 5
5	~~0~~ ~~1~~ 2 ~~3~~ ~~4~~ ~~5~~ 6 7	7, 2, 6, 7, 6, 7, 5
6	~~0~~ ~~1~~ 2 ~~3~~ ~~4~~ ~~5~~ 6 ~~7~~	6, 2, 6, 7, 6, 7, 5
7	~~0~~ ~~1~~ 2 ~~3~~ ~~4~~ ~~5~~ ~~6~~ ~~7~~	2, 6, 7, 6, 7, 5
8	~~0~~ ~~1~~ ~~2~~ ~~3~~ ~~4~~ ~~5~~ ~~6~~ ~~7~~	6, 7, 6, 7, 5
9..12	~~0~~ ~~1~~ ~~2~~ ~~3~~ ~~4~~ ~~5~~ ~~6~~ ~~7~~	(eventually becomes empty)

Output: 3, 4, 5, 0, 1, 7, 6, 2

DFS ⇒ Follow a path as far as you can, then backtrack to find the next path to explore

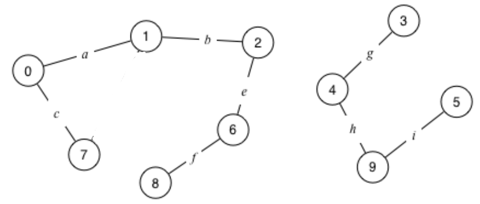
Q8.



Component 0 Component 1

Bridges: e, f, g, h, i

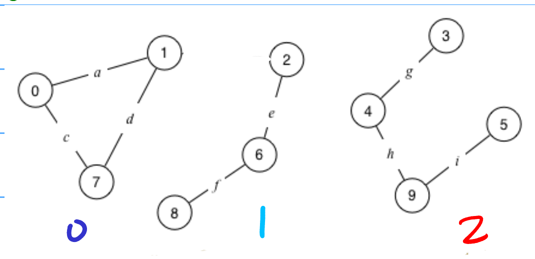
Edge d removed:



Component 0 Component 1

Connected components unchanged

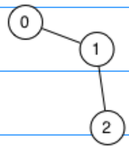
Edge b removed:



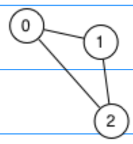
$$\begin{array}{ccccccccccc} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ cc[\,] = & \{0, & 0, & 1, & 2, & 2, & 2, & 1, & 0, & 1, & 2\} \end{array}$$

(1 more connected component)

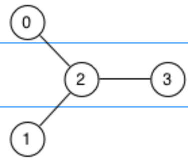
Q9.



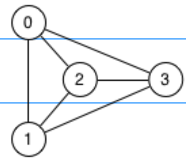
Graph 1



Graph 2



Graph 3



Graph 4

Euler path: visit each edge exactly once
Hamiltonian path: visit each vertex exactly once

E for Edge and Euler

Same thing for Euler | Hamilton circuits except you must end up where you started.

Lemmas:

(1) G has an Euler path
$\iff$ There are exactly 2 vertices with odd degree
(2) G has an Euler circuit
$\iff$ Every vertex has even degree

No similar lemmas for Hamiltonian paths/circuits. ῀

	Euler path	Euler circuit	Hamiltonian path	Hamiltonian circuit
Graph 1	✓	✗	✓	✗
Graph 2	✓	✓	✓	✓
Graph 3	✗	✗	✗	✗
Graph 4	✗	✗	✓	✓