Schema for Tactical Asset Allocation

A Practical Approach

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Outline

Bio

- Master of Financial Mathematics, expected 2017
  - Understanding, reporting, managing and optimizing of financial risk
  - Coursework consists of mathematics, statistics and finance
  - Manipulating large data sets and programming
- Experience
  - First Light Asset Management - Intern, Quantitative Analyst
  - Whitebox Advisors - Intern - Quantitative Strategy
  - Thrivent Asset Management - Intern - Portfolio Analyst
- Interests
  - Asset Allocation: portfolio optimization, quantifying risk
  - Security Selection: linear factor models

The Problem

Consider the Markowitz mean variance optimization problem:

$$\mathbf{w}^*(v) \equiv \underset{\substack{\mathbf{w} \in C \\ \mathbf{w}'\Sigma\mathbf{w} = v}}{\operatorname{argmax}} \, \mathbf{w}'\mu$$

It is well documented that using sample moments, $(\hat{\mu}, \hat{\Sigma})$ in the **unconstrained problem** above leads to portfolios that:

- are highly concentrated
- are unstable
- take large short positions

Traditional Approaches

The traditional solution is to apply constraints:

- maximum weights
- prohibit shorting
- benchmark-relative allocation

More sophisticated investors focus on the covariance matrix. However, optimization results are still:

- unstable
- unable to outperform out-of-sample

Even naïve (equally-weighted) portfolios have been shown to beat mean-variance and risked-based optimization out-of-sample.

Black-Litterman

Don't try to capture the distribution of the market, instead develop a framework to express your views:

1. The market parameters of the optimal allocation are not known and therefore cannot be implemented
2. Benchmark allocations are your expectation
3. Determine the distribution of your risk drivers
4. Formulate an investment opinion based on an equilibrium model

Items 3 and 4 above will make or break your performance. First you must understand your data. Then to have an investment opinion you must be an expert, have a track record, and update your opinion frequently.

Market Opinion as Conditional Distribution

Investment opinions are normal perturbations to the equilibrium distribution and is an application of Bayes' rule:

$$f_{X|v}(x|v) = \frac{f_{V|P_X}(v|x) f_X(x)}{\int f_{V|P_X}(v|x) f_X(x) dx}$$

where:

$$V|x \equiv V|P_X \qquad X \sim N(P_X, \Omega)$$

and is represented by the respective conditional probability density function:

$$f_{V|P_X}$$

Comment: return opinions must come with a degree of confidence Ω which represents an estimation around an **unknown** mean

The Intuition Behind Black-Litterman

Investors don't have to forecast returns; instead views are modelled as random variables conditioned on the market (relative returns with a degree of confidence).

- No confidence $\implies$ benchmark allocation
- Total confidence $\implies$ conditional model
- Black-Litterman $\implies$ a calibrated place in between

$$\mathrm{X}\sim\mathrm{N}(\mu,\Sigma) \qquad (\Omega \to \infty)$$

$$\mathrm{X}\sim\mathrm{N}(\mu_{BL},\Sigma_{BL}) \quad \nearrow \quad \searrow$$

$$\mathrm{X}\sim\mathrm{N}(\mu_C,\Sigma_C) \qquad (\Omega \to 0)$$

Inputs

- w Equilibrium market capitalization weights
- Σ Matrix of covariances between assets
- rf Risk free rate
- δ Risk aversion coefficient of the market portfolio - market returns and standard deviation of returns
- τ Uncertainty of prior estimate of mean returns
- P Pick matrix, or how market drivers are combined

Canonical Expression of the Model I

The recipe:

I Reverse optimize to create the vector of equilibrium returns Π:

$$\Pi = \delta \Sigma w$$

II Quantify uncertainty by selecting a value for τ ($\tau = 1/\text{n}$)

III Formulate views by specifying P, Q, and Ω, given k views and n assets:

- i P is a k x n matrix where each row sums to 0 (relative view) or 1 (absolute view)
- ii Q is a k x 1 vector of excess returns for each view
- iii Ω is a diagonal k x k matrix expressing error terms from the expressed views, representing the uncertainty of each view

IV The Black-Litterman 'master equation' gives you a posterior estimate of returns:

$$\hat{\Pi} = \Pi + \tau P'[(P\tau\Sigma P') + \Omega]^{-1}[Q - P\Pi]$$

Canonical Expression of the Model II

V. Next calculate the second moment of this estimated mean about the unknown mean:

$$M = \tau\Sigma - \tau\Sigma P'(P\tau\Sigma P' + \Omega)^{-1}P\tau\Sigma$$

VI. Covariance of returns about the estimated mean:

$$\Sigma_p = \Sigma + M$$

VII. Optimal portfolio weights (unconstrained):

$$w = (\delta\Sigma_p)^{-1}\hat{\Pi}$$

Implementation I

delta	Risk aversion coefficient (δ)
weq	Equilibirum market capitalization weights (w)
V	Matrix of covariances between assets (Σ)
tau	Uncertainty of prior estimate of mean returns (τ)
P	pick matrix of views (P)
Q	vector of view returns (Q)
Omega	matrix of view error terms (Ω)

Implementation II

```
def blacklitterman(delta, weq, sigma, tau, P, Q, Omega):
    # Reverse optimize and back out equilibrium returns
    pi = weq.dot(sigma * delta)

    # To simplify calculations:
    ts = tau * sigma

    middle = linalg.inv(np.dot(np.dot(P, ts), P.T) + Omega

    er = np.expand_dims(pi, axis=0).T + np.dot(np.dot(np.dot(ts, P.T), middle),...
        (Q - np.expand_dims(np.dot(P,pi.T), axis=1)))

    # Posterior estimate of the uncertainty in the mean
    posteriorSigma = sigma + ts - ts.dot(P.T).dot(middle).dot(P).dot(ts)

    # Posterior weights based on uncertainty in mean
    w = er.T.dot(linalg.inv(delta * posteriorSigma)).T

    # Compute lambda
    lmbda = np.dot(linalg.pinv(P).T,(w.T * (1 + tau) - weq).T)

    return [er, w, lmbda]
```

Output

Sector	w_{eq}	P0	mu	w*
Pharma	35.95	-50	5.106	17.933
Biotech	22.56	50	10.637	37.795
Med Eq and Svcs	23.94	50	6.613	39.101
Hlth Care Svcs	17.55	-50	5.823	0.410
q	4.05			
omega/tau	0.01130			
lambda	0.34239			

Table: Optimization results

The outcome is counter intuitive: Pharma shares covariance with the outperforming sectors

Applications

- You can cook this at home
  - All you need are equilibrium weights, expected returns, and a covariance matrix
  - Outcomes are reasonable and will express your opinions
  - No preposterous academic assumptions...except for Ω
- Applicable across a variety of asset management contexts
  - Various asset classes
  - Strategic or tactical allocations
  - Fundamental "bottom up" portfolios
  - Even sector portfolios

Next Steps

- Covariance is your "garbage in" problem
  - Overlap parameter
  - Maximum-likelihood estimation
  - Shrinkage estimators
  - GARCH
  - Volatility clustering
- What factors will inform your "Pick Matrix"
  - Linear Factor Models
    - Regression
    - Cross-Sectional
- Adjustments to model
  - Determine optimization constraints
  - Alternative versions may be explored

Appendix: Bayes' Rule

Conditional distribution as the probability density function:

$$f_{X_{B|x_A}}(x_B) = \frac{f_X(x_A, x_B)}{\int f_X(x_A, x_B)dx_B} = \frac{f_X(x_A, x_B)}{f_{X_A}(x_A)}$$

In words, the conditional probability of B given A is the joint pdf of A and B divided by the marginal pdf of A evaluated at A

Bayes' Rule states:

$$f_{X_{A|x_B}}(x_A) = \frac{f_X(x_A, x_B)}{\int f_X(x_A, x_B)dx_A}$$

$$= \frac{f_{X_{B|x_A}}(x_B) f_{X_A}(x_A)}{\int f_{X_{B|x_A}}(x_B) f_{X_A}(x_A) d_{x_A}}$$

In words, the conditional distribution of A given B can be expressed in terms of the distribution of B given A and the marginal distribution of A

Appendix: Tau

Black-Litterman applies a Mixed Estimation model. $\tau\Sigma$ is the error in the estimated mean, or the standard error of the prior estimate.

The blue band demonstrates the sample error of the estimated mean. i.e. $\tau\Sigma$ where τ is $1/\mathrm{n}$.

Intuitively, τ is much smaller than one, meaning that the uncertainty of the mean of the distribution is much smaller than the variance in returns.