Source code documentation for:

Phase irregularity: a conceptually simple and efficient approach to characterize electroencephalographic recordings from epilepsy patients

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In this document you will find how to use the source code for the paper Espinoso_2022. Follow the next steps for computing the results for the EEG (Sec. 1) and for the Rössler dynamics (Sec. 2). Some functions are common in both sections for computing the phase-based measures, further explanations in Sec. 0. The code is done using MATLAB and it uses the same notation for mathematical symbols as in the paper. Additional comments can be found throughout the source code. Moreover, the results of the paper are directly available too for the EEG (Sec. 3) and for the Rössler dynamics (Sec. 4). More details can be found below. In case you have any questions, please contact the corresponding author anais.espinoso@gmail.com.

Throughout the document underlined text means name of MATLAB file and **bold text** means name of a variable.

0. Code phase-based measures

A) The source code available here allows you to calculate the univariate phase-based measures in the MATLAB function EA_CoefPhaseVelVar.m:

- Coefficient of phase velocity variation V.
- Mean phase velocity M.
- Phase velocity standard deviation S.

How to call it:

[V,M,S] = EA_CoefPhaseVelVar(data)

where **data** is a vector for which we want to compute V, M and S.

B) Here we also supply the source code for computing the bivariate phase-based measure mean phase coherence R [1]. The MATLAB function is EA_MeanPhaseCoherence.m.

How to call it:

[R] = EA_MeanPhaseCoherence(data)

where **data** is a 2D matrix of 2xN. Each row corresponds to one of the two signals from the pair from which we want to compute R and N the total number of samples.

More details about the phase-based measures can be found in Materials and Methods from the manuscript.

1. Code EEG: EA_EEG_Main.m

The source code for EEG (EA_EEG_Main.m) allows you to calculate the univariate phase-based measures (V, M and S) and the bivariate phase-based measure (R). How to call it:

[V,V_test,M,M_test,S,S_test,R,R_test]=EA_EEG_Main(Data,filt_idx)

where **Data** is a pair of signals from the focal or nonfocal hemisphere. Each variable must enter as dimension 2xN, 2 signals of N samples. The signals used here are from the Bern-Barcelona database, constructed and first analyzed by Andrzejak *et al.* in Ref. [2]. The database can be downloaded in Ref. [3] and Ref. [4] (identical information, two links are provided in case that some web page can migrate in the future).

The input **filt_idx** indicates which kind of filtering we want to apply to the signals:

i) **filt_idx** = 0: low pass filter at 40 Hz and notch filter.

ii) **filt_idx** = 1: filtering of filt_idx = 0 and delta band (0.5-4 Hz).

iii) **filt_idx** = 2: filtering of filt_idx = 0 and theta band (4-8 Hz).

iv) **filt_idx** = 3: filtering of filt_idx = 0 and alpha band (8-12 Hz).

v) **filt_idx** = 4: filtering of filt_idx = 0 and beta band (12-31 Hz).

The **filt_idx** from i) can be used for obtaining the results of Fig. 5 and Fig. 6 from the manuscript. Then, the **filt_idx** from ii) to v) for obtaining the results of Fig. 7 from the manuscript. The filtering of **filt_idx** = 0 uses the functions:

- ASR_Filter.m: general function to apply filters.
- ASR_Butter.m: applies a low pass filter and a notch filter.

Both functions are from the source code available in Ref. [3, 4]. The file ASR_setParameters_Bern.m contains some parameters needed for filtering.

The outputs are the results for the phase-based measures (**V, M, S, R**). The name of the variables are directly related with the symbol for the measures. The results for each measure are obtained for the original signal and for 19 surrogates, *i.e.* dimension 20x1. The iterative amplitude adjusted surrogates are compute with:

- ASR_SurrogateUni.m: computes univariate surrogates.
- ASR_SurrogateMulti.m: computes bivariate surrogates.

Both functions are from the source code available in Ref. [3, 4].

The variables which includes "test" (**V_test, M_test, S_test, R_test**) are the results for the surrogate tests. These outputs are of dimension 1, where:

- 1 is obtained if there is a rejection of the test.

- 0 is obtained if there is a non-rejection of the test.

More details about surrogates and tests can be found in section II.E. and II.F. from the manuscript.

The function EA_EEG_Main.m computes the results for one of the 3750 signal pairs from focal or nonfocal hemisphere available in the Bern-Barcelona database. Be aware that you might need to use a for loop for computing all the results *i. e.* from focal signals, nonfocal signals and all 3750 signal pairs. As the Bern-Barcelona provides signal pairs, for the univariate phase-based measures we only compute the first signal (signal x in the manuscript). If one might want to compute the other signal (signal y in the manuscript), please look for the proper comment in the MATLAB function.

The function EA_EEG_Main.m can be used with other databases different from the Bern-Barcelona database.

2. Code Rössler: EA_Rossler_Main.m

The source code for Rössler dynamics (EA_Rossler_Main.m) allows you to calculate the univariate phase-based measures (V, M and S) and the bivariate phase-based measures (R) for a pair of coupled Rössler dynamics. Please, for further details about the differential equations go to Section II.A. from the manuscript.
How to call it:

[V_x,V_y,M_x,S_x,R_all,V_s_x,M_s_x,S_s_x,R_all_s] = EA_Rossler_Main(ii,E,Ey, dt,At,N,npre,dim,Max_noise_X,Max_noise_Y,Min_noise_X,Min_noise_Y,N_noise)

The different inputs are:

i) **ii**: number of realization, can go from 1 to whatever desired value. This values is useful for obtaining the same initial conditions for every value of **ii**.

ii) **E**: coupling strength from system X to Y.

iii) **Ey**: coupling strength from system Y to X.

iv) **dt**: integration time step.

v) **At**: sampling interval.

vi) **N**: number of data points for the analysis of Rössler dynamics.

vii) **npre**: number of preiterations to avoid transients in the dynamics.

viii) **dim**: number of equations, if we have system X and Y and each one comes from 3-dimensional equation **dim** = 6.

ix) **Max_noise_X**: maximum value of noise level applied to system X.

x) **Max_noise_Y**: maximum value of noise level applied to system Y.

xi) **Min_noise_X**: minimum value of noise level applied to system X.

xii) **Min_noise_Y**: minimum value of noise level applied to system Y.

xiii) **N_noise**: number of analysed levels of noise

The function EA_Rossler_Main.m can be called without any input because some suggested inputs are provided:

[V_x,V_y,M_x,S_x,R,V_s_x,M_s_x,S_s_x,R_all_s] = EA_Rossler_Main

Suggested parameters for obtaining similar dynamics as in the example of Fig. 3(d)-Fig. 3(f) for bidirectional coupling with unequal coupling strengths from the manuscript.

i) **ii**: 1

ii) **E**: 2

iii) **Ey**: 1

iv) **dt**: 0.001

v) **At**: 0.3

vi) **N**: 4096

vii) **npre**: 100000

viii) **dim**: 6

ix) **Max_noise_X**: 2

x) **Max_noise_Y**: 2

xi) **Min_noise_X**: 0

xii) **Min_noise_Y**: 0

xiii) **N_noise**: 20

Parameter **ii** indicates the number of realization. So, if you are interested in obtaining different realizations for the same parameters, use this function in a for loop changing this value. Then, the function can be also called only indicating **ii** with the previous indicated parameters:

[V_x,V_y,M_x,S_x,R_all,V_s_x,M_s_x,S_s_x,R_all_s] = EA_Rossler_Main(ii)

The parameters from iv)-xiii) are the same throughout the results analysed in the manuscript. For the different coupling strategies, parameters from ii)-iii) are changed. Be careful for the selection of the parameters from ix)-x), because higher values might lead to destabilization of the dynamics. The parameter from iv) **dt** was selected after pre-analysis because it was suitable for Euler integration when applying dynamical noise. The functions needed for the computation of Rössler dynamics are:

- EA_RosslerODE_bi.m: differentials equations for a pair of coupled Rössler dynamics.
- EA_Integrator_DynamicNoise.m: Euler integrator with dynamical noise in every integration step.

The functions needed for the computation of iterative amplitude adjusted univariate and bivariate surrogates are:

- ASR_SurrogateUni.m: computes univariate surrogates.
- ASR_SurrogateMulti.m: computes bivariate surrogates.

Both functions are from the source code available in Ref. [3, 4].

The outputs **V_x, M_x, S_x, R_all** are the results for the phase-based measures. The name of the variables are directly related with the symbol for the measures. Moreover, each variable is accompanied by "_x" or "_y" if the measures are computed from system X or system Y, respectively. Also is provided the output **V_y** for system Y. However, as in the manuscript the results of the first dimension of system Y are not shown, here we only show how to get this one. If you are interested to obtain other results for system Y, modify the code following this example of the variable **V_y**. These variables are obtained for all noise levels indicated by **Max_noise_X, Max_noise_Y, Min_noise_X, Min_noise_Y and N_noise**. Then, the dimensions for these variables are **N_noise**x**N_noise**.

We also provided the results for surrogates **V_s_x, M_s_x, S_s_x, R_all_s**, when the name of the variable is accompanied by "_s". The dimensions for these variables are **N_noise**x**N_noise**x19, since we compute 19 surrogates. The results for the surrogate tests are not provided, instead, the measures are computed for surrogates. Using the results for original dynamics jointly with surrogates one can obtain the results for the tests.

As commented before, the function EA_EEG_Main.m obtains the results for just one realization indicated by the variable **ii**. To obtain different realizations, you might need to call this function in a loop changing the number for **ii**.

3. Results EEG

IMPORTANT REMARK: if you run the analysis using EA_EEG_Main.m, you will not get identical results as provided here for the EEG. The reason is that all tests are based on surrogates which are random signals. Every time that you generate a surrogate, you will get different results for each realization. Therefore, in some cases you might obtain a rejection of a test while in our results might appear as non-rejected. However, overall you will obtain the same statistically results.
The exact results for the EEG signals from the Bern-Barcelona database [3,4] from the manuscript are provided here:

i) EEG_measures.mat: results from the phase-based measures. Results from Fig. 5 and Fig. 6 of the manuscript.

ii) EEG_tests.mat: results from the phase-based tests. Results from Fig. 5 and Fig. 6 of the manuscript.

iii) EEG_measures_freq.mat: results from the phase-based measures for filtering in the classical frequency bands. Results from Fig. 7 of the manuscript.

iv) EEG_tests_freq.mat: results from the phase-based tests for filtering in the classical frequency bands. Results from Fig. 7 of the manuscript.

Results from i) and ii) are from all univariate (V, M and S) and bivariate (R) phase-based measures. In contrast, results from iii) and iv) are only from M and R. The name of the variables are directly related with the symbol for the measures. Moreover, each variable is accompanied by "_F" or "_N", if the measures are computed from a focal or nonfocal signal, respectively. The dimensions of the variables are (following the order of the previous enumeration):

i) 20x3750 *i.e.* 1 original + 19 surrogates x 3750 signals.

ii) 3750x1 *i.e.* 3750 signals.

iii) 20x4x3750 *i.e.* 1 original + 19 surrogates x 4 frequency bands x 3750 signals.

iv) 3750x4 *i.e.* 3750 signals x 4 frequency bands.

The name of the variables from ii) and iv) are accompanied by "test" as they are the results from the phase-based tests. It is a binary result, where:

- 1 is obtained if there is a rejection of the test.
- 0 is obtained if there is a non-rejection of the test.

The name of the variables from iii) and iv) are accompanied by "freq" as they are the results after applying a band-pass filtering for 4 classical frequency-bands (δ, θ, α and β).

4. Results Rössler

IMPORTANT REMARK: if you run the analysis using EA_Rossler_Main.m, you will not get identical results as provided here for the Rössler dynamics. The reason is that all tests are based on surrogates which are random signals. Every time that you generate a surrogate, you will get different results for each realization. Therefore, in some cases you might obtain a rejection of a test while in our results might appear as non-rejected. However, overall you will obtain the same statistically results.
The exact results for the analysis of Rössler dynamics from the manuscript are provided here:

i) Fig3_a-c_measures.mat: V from system X and system Y and R between system X and Y for bidirectional coupling with equal coupling strengths. Results from Fig. 3(a)-Fig. 3(c).

ii) Fig3_d-f_measures.mat: V from system X and system Y and R between system X and Y for bidirectional coupling with unequal coupling strengths. Results from Fig. 3(d)-Fig. 3(f).

iii) Fig3_g-i_measures.mat: V from system X and system Y and R between system X and Y for unidirectional coupling. Results from Fig. 3(g)-Fig. 3(i).

iv) Fig4_a-c_measures.mat: V, M and S from system X for bidirectional coupling with unequal coupling strengths. Results from Fig. 4(a)-Fig. 4(c).

v) Fig4_d-f_tests.mat: results of the phase-based tests from V, M and S. Results from Fig. 4(d)-Fig. 4(f).

vi) Fig4_surrogates.mat: results of the phase-based measures V, M and S for univariate surrogates. These results jointly with Fig4_a-c_measures.mat are used to obtain the results from Fig4_d-f_tests.mat.

The name of the variables are directly related with the symbol for the measures. Moreover, each variable is accompanied (if needed) by "_x" or "_y", if the measures are computed for system X or system Y, respectively. The dimensions of the variables are (following the order of the previous enumeration):

i) 20x20x50 *i.e.* 20 levels of noise in system X x 20 levels of noise in system Y x 50 realizations.

ii) 20x20x50 *i.e.* 20 levels of noise in system X x 20 levels of noise in system Y x 50 realizations.

iii) 20x20x50 *i.e.* 20 levels of noise in system X x 20 levels of noise in system Y x 50 realizations.

iv) 20x20x50 *i.e.* 20 levels of noise in system X x 20 levels of noise in system Y x 50 realizations.

v) 20x20 *i.e.* 20 levels of noise in system X x 20 levels of noise in system Y.

vi) 20x20x19 *i.e.* 20 levels of noise in system X x 20 levels of noise in system Y x 19 surrogates.

The name of the variables from v) are accompanied by "test" as they are the results from the univariate phase-based tests. It is a binary result, where:

- 1 is obtained if there is a rejection of the test.
- 0 is obtained if there is a non-rejection of the test.

References

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[3] R. G. Andrzejak, K. Schindler, and C. Rummel, "Bern-Barcelona database," 2012. Available: `https://www.upf.edu/web/ntsa/downloads`.

[4] R. G. Andrzejak, K. Schindler, and C. Rummel, "Bern-Barcelona database," 2012. Available: `https://repositori.upf.edu/handle/10230/42829`.