SIE 321 - Probabilistic Operations Research

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1. A cube has six sides colored red, white, blue, green, yellow, & violet. Assume six sides equally likely to show when the cube is tossed. The cube is tossed once.

a) sample space:

$$\boxed{S = \{R, W, B, G, Y, V\}}$$

b) $\boxed{P(X=0) = \frac{2}{6} = \frac{1}{3}}$

$\boxed{P(X=1) = 2/6 = 1/3}$

$\boxed{P(X=2) = 2/6 = 1/3}$

X	0	1	2
$P(X=x)$	1/3	1/3	1/3

2. Suppose sample space S consists of four points

$$S = \{w_1, w_2, w_3, w_4\}$$

and the associated prob. over the events: $P(w_1) = 0.2$, $P(w_2) = 0.3$, $P(w_3 = 0.1$, $P(w_4) = 0.4$

Define the RV X_1 by

$X_1(w_1) = 1$,
$X_1(w_2) = 4$,
$X_1(w_3) = 2$,
$X_1(w_4) = 1$,

a)

X_1	1	2	4
$P(X_1)$	0.2 + 0.4 = 0.6	0.1	0.3

and the RV X_2 by

$X_2(w_1) = 1$,
$X_2(w_2) = 5$,
$X_2(w_3) = 1$,
$X_2(w_4) = 1$,

b)

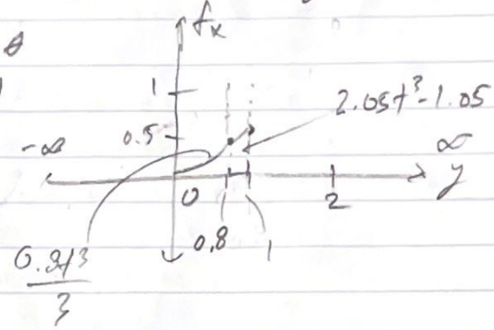
$X_1 + X_2$		X_2: 1	5
X_1	1	0.6(0.7) = 0.42	0.6(0.3) = 0.18
	2	0.1(0.7) = 0.07	0.1(0.3) = 0.03
	4	0.3(0.7) = 0.21	0.3(0.3) = 0.09

$P(X_1 + X_2) = 0.42 + 0.07 + 0.21 + 0.18 + 0.03 + 0.09$

$$\boxed{P(X_1 + X_2) = 1}$$

3. The random Variable (RV) X has density function f given by:

$$f_X(y) = \begin{cases} \theta y^2, & \text{for } 0<y\le\theta \\ ky^2, & \text{for } \theta<y\le 1 \\ 0, & \text{elsewhere} \end{cases}$$



a) Assume $\theta = 0.8$, determine k:

$$\int_{-\infty}^{\infty} f(y)\,dy = 1$$

$$\int_{0.8}^{1} ky^2\,dy = 1$$

$$k\int_{0.8}^{1} y^2\,dy = 1$$

$$k\left[\frac{y^3}{3}\right]\Big|_{0.8}^{1} = 1$$

$$k\left[\frac{1^3}{3} - \frac{0.8^3}{3}\right] = 1$$

$$k\left[\frac{1}{3} - \frac{0.512}{3}\right] = 1 \;;\; \frac{0.488}{3}k = 1$$

$$\boxed{k = 6.15} \qquad k = \frac{3}{0.488} \approx 6.15$$

b) Find $F_X(t)$, the cummulative dist. funct. (cdf) of X:

$$F_X(t) = \int_{-\infty}^{t} f_X(y)\,dy \quad -\infty < t \le 0$$

$$F_X(t) = \int_{-\infty}^{t} 0\,dy = 0 \quad -\infty < t \le 0$$

$$F_X(t) = \int_{0}^{t} f_X(y)\,dy \quad 0 < t \le [\theta = 0.8]$$

$$F_X(t) = 0.8\int_0^t y^2\,dy = 0.8\left[\frac{y^3}{3}\right]_0^t = 0.8\left[\frac{t^3}{3}\right] = \frac{0.8t^3}{3}$$

$$F_X(t) = \int_{0.8}^{t} 6.15y^2\,dy \quad 0.8 < t \le 1$$

$$F_X(t) = 6.15\int_{0.8}^{t} y^2\,dy$$

$$= 6.15\left[\frac{y^3}{3}\right]_{0.8}^{t}$$

$$= 6.15\left[\frac{t^3}{3} - \frac{0.8^3}{3}\right] = 2.05t^3 - 1.05$$

$$F_x(t) = \begin{cases} 0 & -\infty < t \le 0 \\ \frac{0.8t^3}{3} & 0 < t \le (\theta = 0.8) \\ 2.05t^3 - 1.05 & (\theta = 0.8) < t \le 1 \\ 0 & 1 < t \le \infty \end{cases}$$

c) $P(0.4 \le x \le 0.8)$

$$\int_{0.4}^{0.8} 0.8y^2\,dy = 0.8\int_{0.4}^{0.8} y^2\,dy$$

$$= 0.8\left[\frac{y^2}{3}\right]_{0.4}^{0.8}$$

$$\boxed{P(0.4 \le X \le 0.8) = 1.024}$$

4. Machine turns 2 items during the course of a day (morn. & aft.).
Quality of each item:
Good (G), Mediocre (M), Bad (B)

$P(G) = 0.3$
$P(M) = 0.4$
$P(B) = 0.3$

$S = \{GG, GM, MG, GB, BG, BB, MB, BM\}$

Profit: G = \$4, M = \$1, B = \$0

Calc. prob. of each sample point:	Calc profit:
$P(GG) = 0.3(0.3) = 0.09$	$GG = 4+4 = 8$
$P(GM) = 0.3(0.4) = 0.12$	$GM = 4+1 = 5$
$P(MG) = 0.4(0.3) = 0.12$	$MG = 1+4 = 5$
$P(GB) = 0.3(0.3) = 0.09$	$GB = 4+0 = 4$
$P(BG) = 0.3(0.3) = 0.09$	$BG = 0+4 = 4$
$P(BB) = 0.3(0.3) = 0.09$	$BB = 0+0 = 0$
$P(MB) = 0.4(0.3) = 0.12$	$MB = 1+0 = 1$
$P(BM) = 0.3(0.4) = 0.12$	$BM = 0+1 = 1$

Total profit $= P(X) \cdot X = 0.09(8) + 0.12(5) + 0.12(5) + 0.09(4) + 0.09(4) + 0.09(0) + 0.12(1) + 0.12(1)$
$= 2.88$

Prob. Profit

$\boxed{\text{Total Profit/day} = \$2.88}$