Agustin Espinoza SIE321 HW6 3/3/23

1. American families: urban, rural, surburban location

a) $P(\text{stay in urban}) = 1 - 0.06 - 0.12 = 0.82$
$P(\text{move to suburban}) = 6\% = 0.06$
$P(\text{move to rural}) = 12\% = 0.12$

$P(\text{stay in urban 2yrs}) = P(\text{urban yr1}) \cdot P(\text{urban in yr2} \mid \text{stay in urban yr1})$
$= 0.82(1 - 0.04 - 0.08)$
$\boxed{P(\text{stay in urban 2yrs}) = 0.7544}$

$P(\text{surburban in 2yrs}) = P(\text{move to suburban}) \cdot P(\text{stay in suburban yr2} \mid \text{move to suburban in yr1})$
$= 0.06(1 - 0.05 - 0.08) + 0.94(0.04)$
$\boxed{P(\text{suburban in 2yrs}) = 0.0804}$

$P(\text{rural in 2yrs}) = P(\text{move to rural in yr1}) \cdot P(\text{move to rural in yr2} \mid \text{stay in rural yr1})$
$= 0.12(1 - 0.06 - 0.05) + 0.88(0.06)$
$\boxed{P(\text{rural in 2yrs}) = 0.1056}$

b) Suppose at present:

$P(\text{urban}) = 0.30$, $P(\text{suburban}) = 0.45$, $P(\text{rural}) = 0.25$
$P(\text{urban in 3yrs}) = P(\text{urban after 2yrs}) \cdot P(\text{urban after 1yr})$

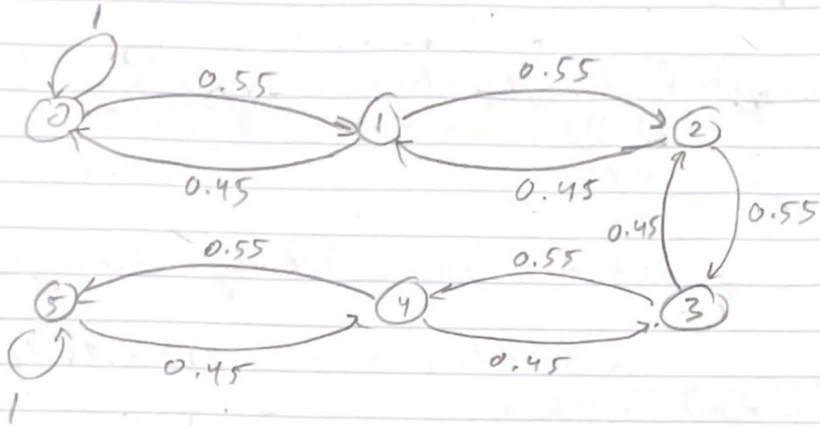
$P(\text{urban after 1yr}) = 0.82(0.88) = 0.7216$
$P(\text{urban after 2yrs}) = 0.7216(0.7216) = 0.5206$

$P(\text{urban in 3yrs}) = 0.3(0.5206) + 0.45(0.1404) + 0.25(0.1123)$
$\boxed{P(\text{urban in 3yrs}) = 0.2677}$

c) The problem with using this model to predict the future population distribution of the U.S. is that the probabilities are held constant as time progresses, and in real-life, this may not be the case. There are many external factors (economic, social, political) that may affect these outcomes.

2.

a)



state space:
$M = 5$

Probability Transition Matrix (PTM)

$$\begin{array}{c|cccccc} & 0 & 1 & 2 & 3 & 4 & 5 \\ \hline 0 & 1 & 0.55 & 0 & 0 & 0 & 0 \\ 1 & 0.45 & 0 & 0.55 & 0 & 0 & 0 \\ 2 & 0 & 0.45 & 0 & 0.55 & 0 & 0 \\ 3 & 0 & 0 & 0.45 & 0 & 0.55 & 0 \\ 4 & 0 & 0 & 0 & 0.45 & 0 & 0.55 \\ 5 & 0 & 0 & 0 & 0 & 0.45 & 1 \end{array}$$

b) $P^{(3)}_{1,0} = \sum_{k=0}^{5} P^{(1)}_{1,k} P^{(2)}_{k,0}$; $P^{(2)}_{2,0} = P^{(1)}_{2,0} P^{(1)}_{0,0} + P^{(1)}_{2,1} P^{(1)}_{1,0} + P^{(1)}_{2,3} P^{(1)}_{3,0}$

$= 0.45(1) + 0.45(0.45) + 0$

Probability of going broke after 3 rounds of play $= \boxed{0.6525}$

3. Communication network transmits binary digits, 0 or 1, where each digit is transmission 10 times in succession

a) Construct (one-step) transition matrix:

$$\boxed{P = \begin{array}{c|cc} & 0 & 1 \\ \hline 0 & 0.995 & 0.005 \\ 1 & 0.005 & 0.995 \end{array}}$$

b) Find (10-step) transition matrix:

$$[\pi_0 \ \ \pi_1] \begin{bmatrix} 0.995 & 0.005 \\ 0.005 & 0.995 \end{bmatrix} = \begin{bmatrix} \pi_0 \\ \pi_1 \end{bmatrix}$$

$$\boxed{P^{10} = \begin{bmatrix} 0.9646 & 0.0354 \\ 0.0354 & 0.9646 \end{bmatrix}}$$

$\pi_0 + \pi_1 = 1$

$\pi_0 = \pi_1 = (1,1)\left[P + \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right]^{-1}$

$\pi_0 = 0.951\pi_1 + 0.049\pi_2$

$\pi_1 = 0.049\pi_1 + 0.951\pi_2$ $\boxed{(\pi_0 = \pi_1) = 0.524}$

The probability that a digit entering the network will be recorded accurately after the last transmission is 52.4%

c) Repeat part (b) with single transmission accuracy of 0.998 instead of 0.995.

- (one-step) transition matrix

$$P = \begin{bmatrix} 0.998 & 0.002 \\ 0.002 & 0.998 \end{bmatrix}$$

- (10-step) transition matrix

$$P^{10} = \begin{bmatrix} 0.9804 & 0.0196 \\ 0.0196 & 0.9804 \end{bmatrix}$$

$0.967\pi_0 + 0.033\pi_1 = \pi_0$; $0.934\pi_0 + 0.033\pi_1 = 0$

$0.033\pi_0 + 0.967\pi_1 = \pi_1$; $0.033\pi_0 + 0.934\pi_1 = 0$

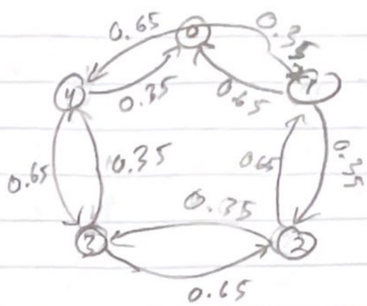
$\boxed{\pi_0 = \pi_1 = 0.969}$

4. Particle moves on a circle through points that have been marked 0, 1, 2, 3, 4 (in a clockwise order).

P(one point cw) = 0.35
P(one point ccw) = 0.65

a) Construct the (one-step) transition matrix for X_n.

$$P = \begin{array}{c|ccccc} & 0 & 1 & 2 & 3 & 4 \\ \hline 0 & 0 & 0.35 & 0 & 0 & 0.65 \\ 1 & 0.65 & 0 & 0.35 & 0 & 0 \\ 2 & 0 & 0.65 & 0 & 0.35 & 0 \\ 3 & 0 & 0 & 0.65 & 0 & 0.35 \\ 4 & 0.35 & 0 & 0 & 0.65 & 0 \end{array}$$



b) Determine the n-step transitions for n=5, 10, 20:

n=5

$$P^5 = \begin{bmatrix} 0.121 & 0.181 & 0.312 & 0.049 & 0.336 \\ 0.336 & 0.121 & 0.181 & 0.312 & 0.049 \\ 0.049 & 0.336 & 0.121 & 0.181 & 0.312 \\ 0.312 & 0.049 & 0.336 & 0.121 & 0.181 \\ 0.181 & 0.312 & 0.049 & 0.336 & 0.121 \end{bmatrix}$$

n=10

$$P^{10} = \begin{bmatrix} 0.167 & 0.257 & 0.141 & 0.238 & 0.197 \\ 0.197 & 0.167 & 0.257 & 0.141 & 0.238 \\ 0.238 & 0.197 & 0.167 & 0.257 & 0.141 \\ 0.141 & 0.238 & 0.197 & 0.167 & 0.257 \\ 0.257 & 0.141 & 0.238 & 0.197 & 0.167 \end{bmatrix}$$

$n = 20$

$$P^{20} = \begin{bmatrix} 0.196 & 0.198 & 0.207 & 0.191 & 0.208 \\ 0.208 & 0.196 & 0.198 & 0.207 & 0.191 \\ 0.191 & 0.208 & 0.196 & 0.198 & 0.207 \\ 0.207 & 0.191 & 0.208 & 0.196 & 0.198 \\ 0.198 & 0.207 & 0.191 & 0.208 & 0.196 \end{bmatrix}$$