Agustin Espinoza SIE321 HW3 2/8/23

1. Let X be discrete random var. (DRV), with probability dist:

$$P(X=x_1)=0.65,\ P(X=x_2)=0.35$$

Determine x_1, x_2 such that $E[X]=0$ and $\sigma^2(x)=8$.

Expectation $E[X]$:
- Continuous Var.
$$E[Y]=\int_{-\infty}^{\infty} y f_Y(y)\,dy$$
- Discrete Var.
$$E[Y]=\sum_{i=1,2,\dots} y_i P(Y=y_i)$$

Variance $\sigma^2(Y)$:
$$\sigma^2(Y)=E[(Y-E[y])^2]$$
$$=E[Y^2]-E[Y]^2$$

$$E[X]=0=\sum_{i=1,2} x_i P(X=x_i)$$
$$0=x_1(0.65)+x_2(0.35)$$

$$\sigma^2(X)=8=E[X^2]-E[X]^2$$
$$8=E[X^2]-0$$
$$E[X^2]=8$$
$$E[X^2]=8=\sum_{i=1,2} x_i^2 P(X=x_i)$$
$$8=x_1^2(0.65)+x_2^2(0.35)$$
$$0=x_1^2(0.65)^2+x_2^2(0.35)^2$$

$$\begin{cases} 8=0.65x_1^2+0.35x_2^2 \\ 0=0.4225x_1^2+0.1225x_2^2 \end{cases}$$

$$\boxed{x_1=\pm\frac{2\sqrt{14}}{\sqrt{13}}=\pm2.08},\ \boxed{x_2=\pm\frac{2\sqrt{26}}{\sqrt{7}}=\pm3.85}$$

2. The life X, in hours, of a certain device, has a PDF

$$f_x(t) = \begin{cases} \frac{1800}{t^3}, & t \geq 30 \\ 0, & t < 30 \end{cases}$$

a) $P(110) = \int_{30}^{\infty} \frac{1800}{t^3} dt = 1800 \int_{30}^{\infty} t^{-3} dt$

$= 1800 \left[\frac{t^{-2}}{-2} \right]_{30}^{\infty}$

$= 1800 \left[\frac{1}{-2t^2} \right]_{30}^{\infty}$

$= 1800 \left[0 - \left(-\frac{1}{1800} \right) \right]$

$= 1800 \left(\frac{1}{1800} \right)$

$\boxed{P(110) = 1 = 100\%}$

4. a) $P(X \geq 25) = 1 - P(X \leq 24)$ $\quad P(x) = \frac{e^{-\lambda} \lambda^x}{x!}$

$\lambda = 25$

$P(X \leq 24) = P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6) + P(7) + P(8) + P(9) + P(10) + P(11) + P(12) + P(13) + P(14) + P(15) + P(16) + P(17) + P(18) + P(19) + P(20) + P(21) + P(22) + P(23) + P(24)$

$= \sum_{x=0}^{24} \frac{e^{-\lambda} \lambda^x}{x!}$

$= 0.394$

$P(X \geq 25) = 1 - P(X \leq 24)$

$= 1 - 0.394$

$= 0.606 = \boxed{60.6\%}$

$\lambda = 25$

b) $P(X=25) = \frac{25^{25} e^{-25}}{25!} = 0.0795 = \boxed{7.95\%}$