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1. M/M/1 queueing system problem

$W = 60$ mins
$L = 5$ customers

$\lambda = \frac{L}{W}$

$\lambda = \frac{5}{60} = \frac{1}{12}$ customers/min

Expected time spent in a system:

$W = \frac{1}{\mu - \lambda}$

$\mu - \lambda = \frac{1}{W}$

$\mu = \frac{1}{W} + \lambda$

$\mu = \frac{1}{60} + \frac{1}{12} = \frac{1}{10}$

$P(X > t) = e^{-\mu t}$

$P(X \geq 15) = e^{-\frac{1}{10}(15)}$

$= e^{-\frac{15}{10}}$

$\boxed{P(X > 15) = 0.22 = 22\%}$

∴ the probability that a customer's service time exceeds 15 mins is approximately 22%.

2. Gas station problem

M/M/1 queueing system

arrival rate: $\lambda = 6$ customers/hr
service rate: $\mu = 15$ customers/hr

traffic intensity: $\rho = \frac{\lambda}{\mu} = \frac{6}{15} = 0.4$

probability that an arriving customer has to wait:

$$P(W>0) = \rho = 0.4$$

probability that an arriving customer doesn't wait:

$$1 - P(W>0) = 1 - 0.4 = 0.6$$

Expected price of gas/gal:

$$E[\text{price}] = 0.4(\$2.7) + 0.6(\$3.5)$$

$$\boxed{E[\text{price}] = \$3.18}$$

∴ the expected price of gasoline per gallon is approximately \$3.18

3. Friendly Neighbor Grocery Store problem

arrival rate: $\lambda = 20$ customers/hr
service rate: $\mu = 60/2.5 = 24$ customers/hr

$\lambda = 20$, $\mu = 24$

traffic intensity: $\rho = \frac{\lambda}{\mu} = \frac{20}{24} = 0.833$

- Expected wait time in the system:

$$\boxed{L = \frac{1}{\mu - \lambda} = \frac{1}{24-20} = 0.25 \text{ hrs} = 15 \text{ mins}}$$

- Expected wait time in queue:

$$\boxed{L_q = L - \frac{1}{\mu} = 0.25 - \frac{1}{24} = 0.208 \text{ hr} = 12.5 \text{ mins}}$$

Probability 0 customers (P_0)

$$\boxed{P_0 = 1 - \rho = 1 - 0.833 = 0.167}$$

Probability 1 customer (P_1)

$$\boxed{P_1 = \rho P_0 = 0.833(0.167) = 0.139}$$

Probability 2 customers (P_2)

$$\boxed{P_2 = \rho P_1 = 0.833(0.139) = 0.116}$$

Alternative calculations:

$\lambda = 20$ customers/hr

$\mu = 40$ customers/hr

$\rho = \frac{\lambda}{\mu} = \frac{20}{40} = 0.5$

$$\boxed{L = \frac{\lambda}{\mu - \lambda} = \frac{20}{(40-20)} = 1.5}$$

$$\boxed{L_q = \rho L = 0.5(1.5) = 0.75}$$

$$\boxed{W = \frac{1}{\mu - \lambda} = \frac{1}{40-20} = 0.05}$$

$$\boxed{W_q = W - \frac{1}{\mu} = 0.05 - \frac{1}{40} = 0.025 \text{ hrs} = 1.5 \text{ mins}}$$

$$\boxed{P_0 = 1 - \rho = 1 - 0.5 = 0.5}$$

$$\boxed{P_1 = \rho P_0 = 0.5(0.5) = 0.25}$$

$$\boxed{P_2 = \rho P_1 = 0.5(0.25) = 0.125}$$

4. Centerville Int. Airport problem

M/M/1 queue system

$\lambda = 15$ airplanes/hr

$\mu = 30$ airplanes/hr

a) Evaluate how well the first criterion is being satisfied.

Average # of airplanes waiting for clearance to land.

$$\boxed{L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{15^2}{30(30-15)} = 7.5 > 1}$$

∴ The first criterion is not satisfied

Evaluate how well the second criterion is being satisfied.

Probability # of airplanes waiting to receive clearance to land exceeds 2.

$$P(n>2) = 1 - (1 - P(0) - P(1) - P(2))$$

$$P(n) = (1-\rho)\rho^n \qquad \rho = \frac{\lambda}{\mu} = \frac{15}{30} = 0.5$$

$$P(0) = 1 - 0.5 = 0.5$$

$$P(1) = (1-0.5)0.5^1 = 0.25$$

$$P(2) = (1-0.5)0.5^2 = 0.125$$

$$P(n>2) = 1 - (1 - 0.5 - 0.25 - 0.125)$$

$$= 1 - 0.125$$

$$\boxed{P(n>2) = 0.875}$$

Since this exceeds 0.05, the second criterion is not being met.

b) Second runway is added and the mean arrival rate increases to 20 airplanes/hr.

M/M/2 queue system

$\lambda = 20$ airplanes/hr

$\mu = 30$ airplanes/hr

$$L_q = \left(\frac{\lambda}{\mu}\right)^2 \left(\frac{\rho}{2(1-\rho)^2 + \frac{\rho}{1-\rho}}\right) \qquad \rho = \frac{\lambda}{2\mu}$$

$$\rho = \frac{20}{2(30)} = \frac{20}{60} = \frac{1}{3}$$

$L_q = 0.135$

Since L_q does not exceed 1, the first criterion would be satisfied.

Evaluate second criterion:

$P(n > 2) = 0.0123$

$P(n > 2)$ does not exceed 0.05.

$\therefore$ the second criterion would also be satisfied if a second runway for landings is added and the arrival rate is increased to 20 airplanes per hour.

5 The Security & Trust Bank problem
M/M/3 queueing system

a) $\lambda = 2$ customers/min

$$\mu = \frac{1}{0.5} = 2 \text{ customers/min}$$

$$\mu = 3 \cdot 2 = 6 \text{ customers/min}$$

$$\rho = \frac{\lambda}{\mu} = \frac{2}{6} = \frac{1}{3}$$

$$L_q = \frac{\rho^2}{1-\rho}\left(\frac{c}{c-\rho}\right)$$

$$L_q = \frac{\left(\frac{1}{3}\right)^2}{\left(\frac{2}{3}\right)}\left(\frac{3}{3-\frac{1}{3}}\right)$$

$$\boxed{L_q = \frac{1}{6}}$$

$$P(N>n) = \frac{c\rho^c}{c!(1-\rho)}\left(\frac{1}{1+\left(\frac{c\rho}{c-\rho}\right)^{n+1-c}}\right)$$

$$P(N>5) = \frac{3\left(\frac{1}{3}\right)^3}{3!\left(1-\frac{1}{3}\right)}\left(\frac{1}{1+\frac{3\left(\frac{1}{3}\right)}{\left(3-\frac{1}{3}\right)^{5+1-3}}}\right) = 0.001$$

Because $P(N>5) = 0.001 < 0.005$ and the total number of customers waiting in line is less than 1, both guidelines are being satisfied.

b) In one year, arrival rate will increase to 5 customers/min

$\lambda = 5$ customers/min
$\mu = 6$ customers/min
$\rho = \frac{5}{6}$

$$L_q = \frac{(5/6)^2}{(1-5/6)}\left(\frac{3}{3-5/6}\right) = \frac{125}{6} = 20.83$$

$$P(N>5) = \frac{3(5/6)^3}{3!(1-5/6)}\left(\frac{1}{1+\left(\frac{3(5/6)}{3-5/6}\right)}\right) = 0.999 \approx 1$$

Because $P(N>5)$ approximately equals 1 neither guideline will be satisfied if no change is made.