SIE 330R Homework, Spring 2023

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HW 10 (Chapter 11)

4/11/2023

Homework must be readable! Do not just send in numbers or charts. You must explain the homework answers Preferred to receive homework in Word doc format with any excel or Minitab results pasted into word document. You may choose to use pdf which is also OK.

Problems 11.4, 11.6, 11.7

11.4S. The region of experimentation for two factors are temperature $\left(100 \le T \le 300°\mathrm{F}\right)$ and catalyst feed rate $\left(10 \le C \le 30\mathrm{lb/h}\right)$. A first order model in the usual $\pm$ coded variables has been fit to a molecular weight response, yielding the following model. *Note:* x1 is Temperature, and x2 is catalyst feed rate.

$$\hat{y} = 2000 + 125x_1 + 40x_2$$

(a) Find the path of steepest ascent.

The path of steepest ascent can be calculated by solving for the gradient vector which points in the direction of the steepest ascent. The gradient of y, ∇y = <125, 40>. To find the direction of the steepest ascent, the unit vector must be obtained. ||∇y|| = sqrt(125^2 + 40^2) = 131.91. Therefore, the direction of the steepest ascent is <125/131.91, 40/131.91>. This is further simplified to <0.95, 0.30>.

(b) It is desired to move to a region where molecular weights are above 2500. Based on the information you have from the experimentation in this region, about how many steps along the path of steepest ascent might be required to move to the region of interest?

To solve this problem, the model equation must be updated to move to a region above 2500:

$2000 + 125x_1 + 40x_2 > 2500$

Dividing both sides of the equation by the unit vector, solved for in the problem above, gives us the estimated number of steps required to reach the molecular weight region specified in the problem statement, above 2500.

$x_1/1.056 + x_2/3.298 = 3.79 \approx 4$

Therefore, in order to move to a region where molecular weights are above 2500, approximately 4 steps along the path of steepest ascent are required.

11.6. The data shown in Table P11.2 were collected in an experiment to optimize crystal growth as a function of three variables x_1, x_2, and x_3. Large values of *y* (yield in grams) are desirable. Fit a second order model and analyze the fitted surface. Under what set of conditions is maximum growth achieved?

(Note: since RSM files can be tricky to create, the minitab file for this question is loaded into the class D2L page under Content/Minitab Files from class/ at hw_11_6.mtw.)

Table P11.2

x_1	x_2	x_3	y
-1	-1	-1	66
-1	-1	1	70
-1	1	-1	78
-1	1	1	60
1	-1	-1	80
1	-1	1	70
1	1	-1	100
1	1	1	75
-1.682	0	0	100
1.682	0	0	80
0	-1.682	0	68
0	1.682	0	63
0	0	-1.682	65
0	0	1.682	82
0	0	0	113
0	0	0	100
0	0	0	118
0	0	0	88
0	0	0	100
0	0	0	85

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	9	3662.00	406.89	2.19	0.119
Linear	3	77.89	25.96	0.14	0.934
X1	1	22.08	22.08	0.12	0.738
x2	1	25.31	25.31	0.14	0.720
x3	1	30.50	30.50	0.16	0.694
Square	3	3291.74	1097.25	5.90	0.014
X1*X1	1	204.55	204.55	1.10	0.319
x2*x2	1	2226.45	2226.45	11.96	0.006
x3*x3	1	1328.46	1328.46	7.14	0.023
2-Way Interaction	3	292.38	97.46	0.52	0.676
X1*x2	1	66.13	66.13	0.36	0.564
X1*x3	1	55.12	55.12	0.30	0.598
x2*x3	1	171.13	171.13	0.92	0.360
Error	10	1860.95	186.09		
Lack-of-Fit	5	1001.61	200.32	1.17	0.435
Pure Error	5	859.33	171.87		
Total	19	5522.95			

Regression Equation in Uncoded Units

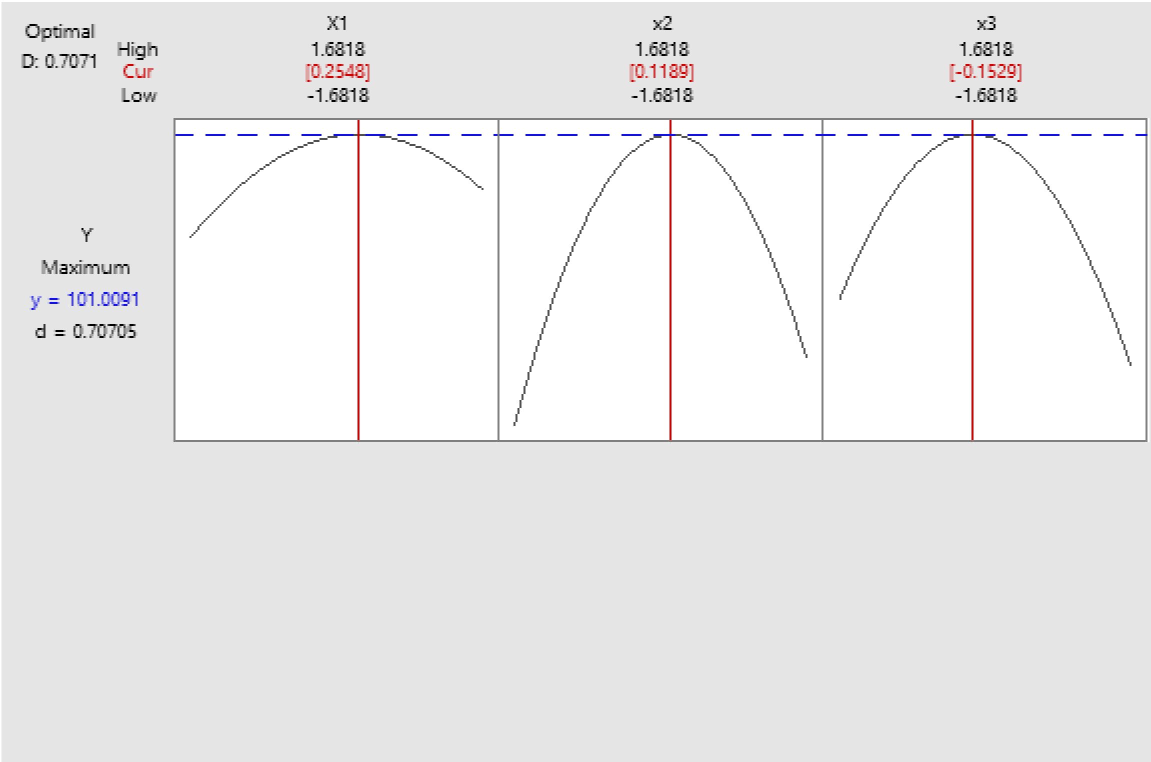
$$Y = 100.67 + 1.27\,X1 + 1.36\,x2 - 1.49\,x3 - 3.77\,X1*X1 - 12.43\,x2*x2 - 9.60\,x3*x3 + 2.88\,X1*x2 - 2.62\,X1*x3 - 4.63\,x2*x3$$

Based on the Response Surface and Response Optimization provided above, the set of conditions necessary to achieve a maximum yield, Y = 101.01, include the following variable definitions: X1 = 0.25, X2 = 0.12, X3 = -0.15. These settings are also provided in the Multiple Response Prediction table and Response Optimization graph below.

Multiple Response Prediction

Variable	Setting
X1	0.254817
x2	0.118915
x3	-0.15289

Response	Fit	SE Fit	95% CI	95% PI
Y	101.01	5.51	(88.74, 113.28)	(68.23, 133.79)



11.7. The data in Table P11.3 were collected by a chemical engineer. The response y is filtration time, x_1 is temperature, and x_2 is pressure. Fit a second-order model.

(Note: since RSM files can be tricky to create, the minitab file for this question is loaded into the class D2L page under Content/Minitab Files from class/ at hw_11_7.mtw.)

Table P11.3

x_1	x_2	y
-1	-1	54
-1	1	45
1	-1	32
1	1	47
-1.414	0	50
1.414	0	53
0	-1.414	47
0	1.414	51
0	0	41
0	0	39
0	0	44
0	0	42

0	0	40

Regression Equation

Y = 45.00 - 1.97 X1 + 1.46 X2

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	48.02	24.011	0.57	0.583
X1	1	31.04	31.037	0.74	0.411
X2	1	16.99	16.985	0.40	0.540
Error	10	421.98	42.198		
Lack-of-Fit	6	407.18	67.863	18.34	0.007
Pure Error	4	14.80	3.700		
Total	12	470.00			

Coded Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	41.20	2.10	19.62	0.000	
X1	-1.97	1.66	-1.19	0.274	1.00
X2	1.46	1.66	0.88	0.409	1.00
X1*X1	3.71	1.78	2.08	0.076	1.02
X2*X2	2.46	1.78	1.38	0.209	1.02
X1*X2	6.00	2.35	2.56	0.038	1.00

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	5	315.60	63.119	2.86	0.102
Linear	2	48.02	24.011	1.09	0.388
X1	1	31.04	31.037	1.41	0.274
X2	1	16.99	16.985	0.77	0.409
Square	2	123.58	61.788	2.80	0.128
X1*X1	1	95.88	95.879	4.35	0.076
X2*X2	1	42.18	42.184	1.91	0.209
2-Way Interaction	1	144.00	144.000	6.53	0.038
X1*X2	1	144.00	144.000	6.53	0.038
Error	7	154.40	22.058		
Lack-of-Fit	3	139.60	46.534	12.58	0.017
Pure Error	4	14.80	3.700		
Total	12	470.00			

Regression Equation in Uncoded Units

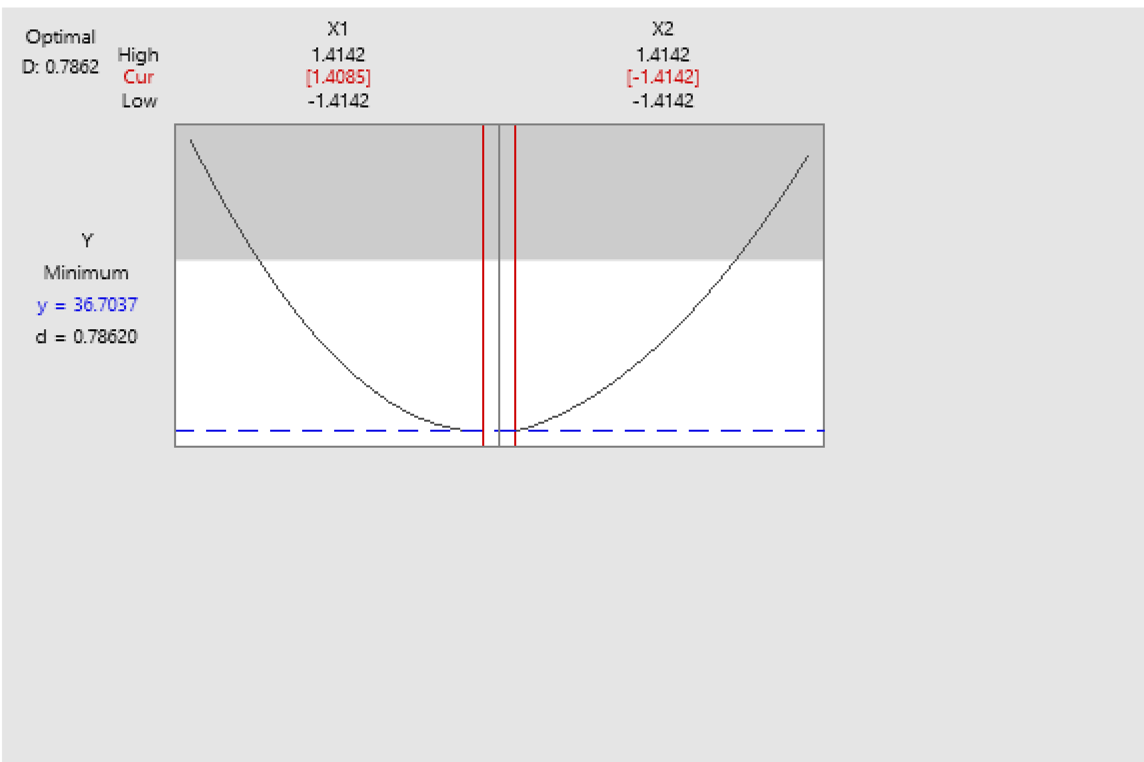
Y = 41.20 - 1.97 X1 + 1.46 X2 + 3.71 X1*X1 + 2.46 X2*X2 + 6.00 X1*X2

(a) What operating conditions would you recommend if the objective is to minimize the filtration time?

To obtain the minimum filtration time, Y = 36.70, the following operating conditions must be set: X1 = 1.41, X2 = -1.41. The solution is provided below along with a graphical representation of the Response Optimization.

Solution

Solution	X1	X2	Y Fit	Composite Desirability
1	1.40846	-1.41421	36.7037	0.786197



(b) What operating conditions would you recommend if the objective is to operate the process at a mean filtration time very close to 46?

To obtain a mean filtration time close to 46, the following operating conditions must be set: X1 = 1.02, X2 = 0.35. The solution and a graphical representation of the Response Optimization are provided below.

Solution

Solution	X1	X2	Y Fit	Composite Desirability
1	1.01618	0.352404	46.0000	1.00000