SIE 330R Homework, Spring 2023

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HW 3 (Chapter 2)

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Due: Feb 7

Homework must be readable! Do not just send in numbers or charts. You must explain the homework answers Preferred to receive homework in Word doc format with any excel or Minitab results pasted into word document. You may choose to use pdf which is also OK.

Put answers to all questions in one document NOT in separate documents.

2.3: Suppose that we are testing $H_0: \mu = \mu_0$ vs $H_1: \mu \neq \mu_0$.

Calculate the P-value for the following observed values of the test statistic.

a) $Z_0 = 2.25$

$$P - value = 2 * \left(1 - P(Z < 2.25)\right)$$
$$P - value = 2 * (1\text{-}0.9878)$$
$$P - value = 2 * 0.0122$$
$$P - value = 0.0244$$

b) $Z_0 = 1.55$

$$P - value = 2 * \left(1 - P(Z < 1.55)\right)$$
$$P - value = 2 * (1\text{-}0.9394)$$
$$P - value = 2 * 0.0606$$
$$P - value = 0.1212$$

c) $Z_0 = 2.10$

$$P - value = 2 * \left(1 - P(Z < 2.10)\right)$$
$$P - value = 2 * (1 - 0.9821)$$
$$P - value = 2 * 0.0179$$
$$P - value = 0.0358$$

d) $Z_0 = 1.95$

$$P - value = 2 * \left(1 - P(Z < 1.95)\right)$$
$$P - value = 2 * (1 - 0.9744)$$
$$P - value = 2 * 0.0256$$
$$P - value = 0.0512$$

e) $Z_0 = -0.10$

$$P - value = 2 * \left(1 - P(Z < \text{-}0.10)\right)$$
$$P - value = 2 * (1 - 0.4602)$$

$$P - value = 2 * 0.5398$$
$$P - value = 1.0796$$

2.5: Suppose that we are testing $H_0: \mu_1 = \mu_2$ vs $H_1: \mu_1 \neq \mu_2$ where the sample sizes are $n_1 = n_2 = 12$. Both sample variances are unknown but assumed to be equal. Find bounds on the P-value for the following observed values of the test statistic.

$$df = (n1\text{-}1) + (n2\text{-}1)$$

$$df = (12 - 1) + (12 - 1) = 12 + 12\text{-}2 = 22$$

a) $t_0 = 2.30$

$$P - value = P(t < 2.30)$$

The P-value bounds are 0.02 and 0.05.

b) $t_0 = 3.41$

$$P - value = P(t < 3.41)$$

The P-value bounds are 0.002 and 0.01.

c) $t_0 = 1.95$

$$P - value = P(t < 1.95)$$

The P-value bounds are 0.05 and 0.10.

d) $t_0 = -2.45$

$$P - value = P(t < 2.45)$$

The P-value bounds are 0.02 and 0.05.

2.8. A computer program has produced the following output for the hypothesis testing problem:

Difference in sample means: 2.35

Degrees of freedom: 18

Standard error of the difference in the sample means: ?

Test statistic: $t_o = 2.01$

P-Value = 0.0298

(a) What is the missing value for the standard error?

SE = $\frac{\text{difference in means}}{\text{test statistic}}$

SE = $\frac{2.35}{2.01} = 1.169$

(b) Is this a two-sided or one-sided test?
This is a one-sided test because the given p-value, 0.0298, is between 0.025 to 0.05 whereas two-sided test would have a p-value between 0.05 and 0.10.

(c) If α=0.05, what are your conclusions?
Because the p-value is less than $\alpha = 0.05$, the p-value is statistically significant, and therefore the null hypothesis should be rejected.

(d) Find a 90% two-sided CI on the difference in the means.

2.19S. The shelf life of a carbonated beverage is of interest. Ten bottles are randomly selected and tested, and the following results are obtained:

Days	
108	138
124	163
124	159
106	134
115	139

(a) We would like to demonstrate that the mean shelf life exceeds 120 days. Set up appropriate hypotheses for investigating this claim.

$\mathrm{H_0}: \mu = 120 \qquad \mathrm{H_1}: \mu > 120$

(b) Test these hypotheses using α = 0.01. What are your conclusions?

$\bar{\mathrm{x}} = 131$

$\mathrm{S}^2 = 3438/9 = 382$

$S = \sqrt{382} = 19.54$

$\mathrm{t_0} = \frac{\bar{\mathrm{x}}\text{-}\mu_0}{\mathrm{S}/\sqrt{\mathrm{n}}} = \frac{131}{19.54/\sqrt{10}} = 1.78$

Since $\mathrm{t}_{0.01,9} = 2.821$, do not reject $\mathrm{H_0}$

(c) Find the *P*-value for the test in part (b).
P-value = 0.054

(d) Construct a 99 percent confidence interval on the mean shelf life.

$\bar{y} - t(S/\sqrt{n}) \leq \mu \leq \bar{y} + t(S/\sqrt{n})$ with $\alpha = 0.01$

$131 - (3.250)\left(\frac{1954}{\sqrt{10}}\right) \leq \mu \leq 131 + (3.20)\left(\frac{1954}{\sqrt{10}}\right)$

$110.91 \leq \mu \leq 151.09$

2.21. The time to repair an electronic instrument is a normally distributed random variable measured in hours. The repair time for 16 such instruments chosen at random are as follows:

Hours			
159	280	101	212
224	379	179	264
222	362	168	250
149	260	485	170

(a) You wish to know if the mean repair time exceeds 225 hours. Set up appropriate hypotheses for investigating this issue.

$H_0: \mu = 225 \qquad H_1: \mu > 225$

(b) Test the hypotheses you formulated in part (a). What are your conclusions? Use $\alpha = 0.05$.

$\bar{x} = 241.5$

$S^2 = 146202/(16\text{-}1) = 9746.8$

$S = \sqrt{9746.8} = 98.73$

$t_0 = \frac{\bar{x}\text{-}\mu_0}{S/\sqrt{n}} = \frac{241.5\text{-}225}{98.73/\sqrt{16}} = 0.67$

Since $t_{0.05,15} = 1.753$, do not reject H_0

(c) Find the *P*-value for this test.

P-value = 0.26

(d) Construct a 95 percent confidence interval on mean repair time.

$\bar{y} - t(S/\sqrt{n}) \leq \mu \leq \bar{y} + t(S/\sqrt{n})$ with $\alpha = 0.05$

$241.5 - (2.131)\left(\frac{98.73}{16}\right) \leq \mu \leq 241.5 + (2.131)\left(\frac{98.73}{16}\right)$

$188.9 \leq \mu \leq 294.1$