Downlink Resource Allocation in Cell-Free Massive MIMO Systems

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***Abstract*—Cell-free Massive Multiple-Input Multiple-Output (MIMO) is introduced as a promising network architecture for the upcoming fifth-generation (5G) wireless communication networks. It enhances user experience and improves the overall system performance by providing huge throughput and coverage probability to all users throughout a system. In this paper, a resource allocation problem is studied for downlink cell-free massive MIMO networks, where each access point (AP) serves a cluster of user equipment (UE). Transmit precoding and power allocation are linked to the underlying max-min scheduling to ensure uniform and excellent service throughout the coverage area. Due to the coupled interference among UEs, the resulting max-min resource allocation optimization problem becomes non-convex. We demonstrate the uplink-downlink duality and propose an iterative algorithm which solves the primal downlink problem efficiently. By utilizing the max-min beamformer and taking the channel estimation error into account, we further derive the capacity lower bound of the underlying cell-free massive MIMO network. The performance of the introduced scheme is also investigated through simulations. Numerical results show the efficiency of the introduced algorithm.**

***Index Terms*—Cell-free massive MIMO, resource allocation, max-min beamforming, non-convex optimization**

I. Introduction

Multiple-input multiple-output (MIMO) systems with a large array antenna at the base stations (BSs), also known as Massive MIMO, have been widely studied recently to improve the spectral and energy efficiency of wireless systems with simple signal processing [1]. Due to the promising gains in [1], depicting high performance results with respect to the baseline MIMO system, more attention has been paid to the topic both in the academia [2], [3] and in the industry [4]. Massive MIMO allows a BS to simultaneously serve many number of user equipments (UEs) along with time-frequency resources to improve the overall system performance. In general, depending on the antenna arrays setup at the BSs, massive MIMO can be categorized into the following two architectures: distributed massive MIMO and co-located massive MIMO. While the latter one locates the service antennas in a compact area, the former spreads antennas all over a large area. Although the co-located architecture is attractive due to its low backhaual requirements, the distributed one offers higher coverage probability at the expense of increased infrastructure costs.

A cell-free Massive MIMO network, which has been introduced recently in [5], is a form of distributed massive MIMO. It is considered a promising network architecture for the upcoming 5G wireless networks due to its ability to offer huge throughput and coverage probability to all users throughout a system. This architecture spreads a tremendous number of randomly-located access points (APs) over a large area, to simultaneously serve a much smaller number of single-antenna UEs, as an alternative to a small-cell network. In order to have an efficient resource allocation and interference management among multiple APs, unspecified backhaul links connect all these APs to a central processing unit (CPU) [5]–[7].

The performance of conjugate beamforming (CB) along with a pilot assignment algorithm has been investigated in [5], [6] to combat pilot contamination in a cell-free massive MIMO network. In order to boost system throughput, a max-min power allocation was considered. However, the optimal solution to the power allocation problem involves high computational complexity, due to the non-convexity of the optimization problem. To address this issue, [7] proposed a power allocation algorithm with a trade-off between low complexity and moderate decrease in performance. The authors in [7] further combined the max-min power allocation algorithm with a linear zero-forcing (ZF) precoder to tackle the high inter-user interference in CB technique. These studies restricted their discussion to the simplest linear precoding schemes which are CB and ZF, and did not design an optimal beamformer that ensures uniformly good service over all the coverage area.

In order to address this issue and improve the system performance along with per-user minimum level of service, we propose a design of the resource allocation in a way that the same quality of service can be provided to all UEs. To enforce such fairness, we maximize (over the resource allocation: precoding vectors and power allocations) the minimum achievable rate among the users in a cell-free massive MIMO network. Due to the coupled interference among UEs, the resulting optimization problem is non-convex and difficult to solve. Therefore, we demonstrate the uplink-downlink duality and propose an iterative algorithm which solves the primal downlink problem efficiently by utilizing the result of the dual uplink. Exploiting proposed precoder and taking the channel estimation error into consideration, we derive the lower bound for the capacity of the underlying cell-free massive MIMO system. Furthermore, unlike [5]–[7] which assumed each single-antenna AP serves all UEs, we consider a cluster of multiple-

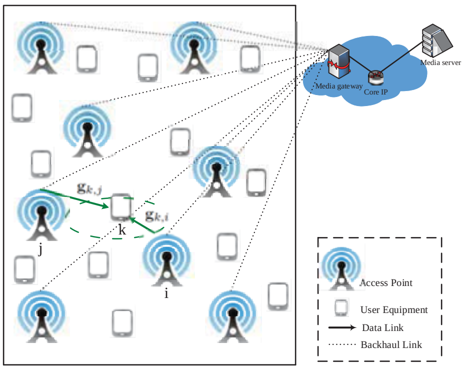

Figure 1. System architecture of a cell-free massive MIMO network.

antenna APs serving each UE. Note that, the strongest APs under coordination may not necessarily be the nearest ones to the UE, due to the shadowing effect.

The remainder of this paper is organized as follows. In Section II, the system model is described, and the main assumptions required for our analysis are introduced. Section III presents the problem formulation and analysis, and demonstrates the designing of resource allocations. Numerical results are shown in Section IV. Finally, in Section V, an overview of the results and some concluding remarks are presented.

Notation: Throughout the paper, normal letters are used for scalars. Boldface capital and lower case letters denote matrices and vectors, respectively. The hermitian of a complex vector $\mathbf{a}$ is denoted by $\mathbf{a}^H$. The trace of a square matrix $\mathbf{A} = [a_{ij}]_{n\times n}$ is defined as $\text{Tr}(\mathbf{A}) = \sum_{i=1}^{n} a_{ii}$. The $\text{vec}(\cdot)$ operator aligns all elements of a matrix into a column vector by stacking column vectors of the matrix, i.e., for $\mathbf{A} \in \mathbb{C}^{M\times N}$ then $\text{vec}(\mathbf{A}) \in \mathbb{C}^{MN\times 1}$. Finally, $\nu_{\max}(\mathbf{A})$ denotes the eigenvector corresponding to the largest eigenvalue of matrix $\mathbf{A}$.

II. System Model and Assumptions

We consider a downlink cell-free massive MIMO network consisting of one central processing unit, M access points, and K user equipments as depicted in Figure 1. The APs and UEs are randomly distributed over a large area. To exchange the network information, all APs are connected to the CPU through error-free infinite-capacity backhaul links.[1] Each AP is equipped with N_t transmit antennas, serves several single-antenna UEs. We assume that each UE is served by a cluster of cooperative APs. To be specific, a cooperating set $\mathcal{B}_k$ is assigned to the k^{th} UE, where $\mathcal{B}_k$ is formed by aggregating all the APs that have knowledge of channels to the k^{th} UE, access to its message, and may also jointly encode the intended message for the k^{th} UE in their transmission [14]. Note that, due to the shadowing effect, the $|\mathcal{B}_k|$ strongest APs under coordination are not necessarily the $|\mathcal{B}_k|$ nearest APs, where $|\mathcal{B}_k|$ denote the sets cardinality. Since each AP may get involved in transmission to more than one UE, the cooperating set of different users may overlap. We denote the set of users served by the i^{th} AP by $\mathcal{K}_i = \{k | i \in \mathcal{B}_k\}$.

The channel (propagation) coefficient between the i^{th} AP and the k^{th} UE form channel vector $\mathbf{g}_{k,i} = \sqrt{\beta_{k,i}}\mathbf{h}_{k,i} \in \mathbb{C}^{N_t}$ where $\beta_{k,i}$ and $\mathbf{h}_{k,i}$ indicate the large-scale and small-scale fading coefficients, respectively. The large-scale fading coefficient depends upon the shadowing and distance between the corresponding UE and AP, and is denoted by $\beta_{k,i} = \psi_{k,i} d_{k,i}^{-\alpha}$ where $d_{k,i}$ is the distance between the k^{th} user and the i^{th} AP; α is the path-loss exponent; and $\psi_{k,i}$ is a log-normal random variable, i.e., the quantity $10\log_{10}(\psi_{k,i})$ is distributed zero-mean Gaussian with a standard deviation of $\sigma_{\text{shadowing}}$. Here, we employ the same three slope path-loss model as in [6], [9], [10]. The path loss from the k^{th} user to the i^{th} AP is modeled as COST-231 Hata propagation model [11] and can be expressed in dB as

$$PL_{k,i} = \begin{cases} -L - 35\log_{10}(d_{k,i}), & \text{if } d_{k,i} > d_1 \\ -L - 10\log_{10}(d_{k,i}^2 d_1^{1.5}), & \text{if } d_0 < d_{k,i} \le d_1 \\ -L - 10\log_{10}(d_0^2 d_1^{1.5}), & \text{if } d_{k,i} \le d_0 \end{cases}$$

where

$$L = 46.3 + 33.9\log_{10}(f) - 13.82\log_{10}(h_{\text{AP}}) \\ - \big(1.11\log_{10}(f) - 0.7\big)h_{\text{UE}} + 1.56\log_{10}(f) - 0.8$$

where f is the carrier frequency in MHz, and h_{AP} and h_{UE} indicate the effective transmitter and receiver antenna heights in meters, respectively. Furthermore, we assume that the shadow fading random variables are correlated, since in a practical scenario the APs and UEs may be located close by, and therefore be all around the common obstacles. To model this correlation, we use the two-component shadowing correlation model, as in [6], [9], [12],

$$z_{k,i} = \sqrt{\delta}a_i + \sqrt{1-\delta}b_k$$

where $0 \le \delta \le 1$ depicts the cross-correlation at the AP while $(1-\delta)$ denotes the cross-correlation at the UE, and $a_i \sim \mathcal{N}(0,1)\ \forall i = 1,\dots,M$ and $b_k \sim \mathcal{N}(0,1)\ \forall k = 1,\dots,K$ are independent random variables with $\mathbb{E}\{a_i a_j^*\} = 2^{-d_{\text{AP}}(i,j)/d_{\text{decorr}}}$ and $\mathbb{E}\{b_k b_\ell^*\} = 2^{-d_{\text{UE}}(k,\ell)/d_{\text{decorr}}}$ where $d_{\text{AP}}(i,j)(d_{\text{UE}}(k,\ell))$ denotes the geographical distance between the i^{th} and j^{th} (k^{th} and ℓ^{th}) APs (UEs), and d_{decorr} is the decorrelation distance. Therefore, the large-scale fading coefficients can be modified as $\beta_{k,i} = 10^{PL_{k,i}/10} \cdot 10^{z_{k,i}\sigma_{\text{shadowing}}/10}$. The small-scale fading coefficients, i.e., elements of $\mathbf{h}_{k,i}$, are modeled as i.i.d. complex Gaussian variables with zero-mean and unit-variance [5]. We further assume a block fading model, where small-scale channels are constant over a few time slots, with respect to channel estimation and channel state information (CSI) feedback procedures. Similarly, we assume that large-scale fading coefficients stay constant during large-scale coherence blocks. The small-scale and large-scale fading coefficients in different coherence blocks are assumed to be independent.

Time-division duplexing (TDD) mode is assumed in this system. In the uplink training phase, the users send pilot sequences to the AP synchronously. Each AP estimates the

[1] However, in a practical scenario the backhaul links are subject to practical constraints and investigating the effect of these constraints would be an important topic for future work.

channel to all users based on the received pilot signals. Assuming $\beta_{k,i}$ is known, as in [5], the i^{th} AP computes the minimum mean squared error (MMSE) estimate of the channel vectore $\mathbf{g}_{k,i}$ as $\hat{\mathbf{g}}_{k,i}$. Considering the channel estimation error as $\tilde{\mathbf{g}}_{k,i} = \mathbf{g}_{k,i} - \hat{\mathbf{g}}_{k,i}$, it is easy to show that $\tilde{\mathbf{g}}_{k,i}$ and $\hat{\mathbf{g}}_{k,i}$ are uncorrelated [13]. In the downlink phase, the channel estimates $\hat{\mathbf{g}}_{k,i}$ are regarded as true channels by APs, i.e., APs rely on channel hardening, and use these channel estimates to precode data to UEs. Each AP, say $i \in \{1, 2, \ldots, M\}$, plans to communicate a symbol vector $\mathbf{s}_i = [s_{i_1}, \ldots, s_{i_{|\mathcal{K}_i|}}]^T \in \mathbb{C}^{|\mathcal{K}_i|}$ to its associated receivers, where s_{i_k} is the transmit symbol from the i^{th} AP to the k^{th} receiver with unit power of $E\{|s_{i_k}|^2\} = 1$. Prior to transmitting, the i^{th} AP linearly precodes its symbol vector $\mathbf{x}_i = \sum_{k=1}^{|\mathcal{K}_i|} \mathbf{f}_{i_k} s_{i_k}$ where $\mathbf{f}_{i_k}$ denotes the precoding vector that the i^{th} AP uses to transmit the signal s_{i_k} to the k^{th} UE and $||\mathbf{f}_{i_k}||^2$ is the allocated downlink transmit power. Each AP i is under a transmit power constraint of $P_{i,\max}$ and so, the transmit power at the i^{th} AP is computed as $p_i = E\{||\mathbf{x}_i||^2\} \leq P_{i,\max}$ [14], [15]. In order to transmit data to the k^{th} UE, coordinated joint transmission is used to coherently combine the received signals from APs that serve the k^{th} UE. Since there are $|\mathcal{B}_k|$ APs participating in the cooperating data transmission to user k, at the same frequency and time, we denote $s_k \in \mathbb{C}$ as the complex data symbol for the k^{th} UE. Hence, over each symbol duration time, the cooperating APs transmit the same symbol s_k. The received signal at the k^{th} receiver can be written as:

$$y_k = \sum_{i \in \mathcal{B}_k} \mathbf{g}_{k,i} \mathbf{f}_{i_k} s_k + \sum_{\substack{j=1 \\ j \notin \mathcal{B}_k}}^{M} \sum_{\substack{\ell=1 \\ \ell \neq k}}^{K} \mathbf{g}_{k,j} \mathbf{f}_{j_\ell} s_\ell + n_k \tag{1}$$

where the first term on the right-hand side of (1) depicts the received useful signal, the second term denotes the multi-user interference, and $n_k \sim \mathcal{CN}(0,1)$ is the additive white Gaussian noise (AWGN) at the k^{th} UE. Recall that the signal-to-interference-plus-noise ratio (SINR) is defined as the ratio of the received signal power at the desired user to the interference power plus the noise power, the SINR at the k^{th} UE can be expressed as

$$\begin{aligned}\mathrm{SINR}_k &= \frac{\sum_{i \in \mathcal{B}_k} \beta_{k,i} \mathrm{Tr}(\mathbf{f}_{i_k}^H \mathbf{h}_{k,i}^H \mathbf{h}_{k,i} \mathbf{f}_{i_k})}{\sigma_k^2 + \sum_{\substack{\ell=1 \\ \ell \neq k}}^{K} \sum_{j \in \mathcal{B}_\ell} \beta_{\ell,j} \mathrm{Tr}(\mathbf{f}_{j_\ell}^H \mathbf{h}_{k,j}^H \mathbf{h}_{k,j} \mathbf{f}_{j_\ell})} \\ &= \frac{\sum_{i=1}^{M} \beta_{k,i} \mathrm{Tr}(\mathbf{f}_{i_k}^H \mathbf{h}_{k,i}^H \mathbf{h}_{k,i} \mathbf{f}_{i_k})}{\sigma_k^2 + \sum_{\substack{j=1 \\ j \neq i}}^{M} \sum_{\substack{\ell=1 \\ \ell \neq k}}^{K} \beta_{\ell,j} \mathrm{Tr}(\mathbf{f}_{j_\ell}^H \mathbf{h}_{k,j}^H \mathbf{h}_{k,j} \mathbf{f}_{j_\ell})}\end{aligned}$$

where the second equality holds under the assumption that $\mathbf{f}_{i_k} = \mathbf{0}$ for $i \notin \mathcal{B}_k$. Therefore, the downlink achievable transmission rate for the k^{th} UE can be expressed as $R_k = \log_2(1 + \mathrm{SINR}_k)$.

III. A Novel Algorithm for Downlink Resource Allocation

In order to assure a uniform user experience across the network along with the improved system performance, the problem of interest in this paper is to maximize (over the resource allocation: precoding vectors and power allocations) the minimal performance of each user in a cell-free massive MIMO network. Precisely, we are interested in the max-min SINR problem, over the transmit precoder and power control, given by

$$\max_{\mathbf{f}_{i_k}} \; \min_{k} \quad \mathrm{SINR}_k \tag{2}$$

which is subject to $\sum_{k \in \mathcal{K}_i} \mathbf{f}_{i_k}^H \mathbf{f}_{i_k} < P_{i,\max}$.

Due to the interference present, the optimization problem (2) is non-convex and hence, finding the global optimum is challenging. Therefore, in order to efficiently solve this optimization problem we take advantage of the fact that in contrast to the downlink SINR which is coupled with both precoders and transmit powers; the uplink SINR is only accompanied by transmit power, thereby, it provides an easier optimization problem to solve. Consequently, to exploit network duality, we need to reformulate the optimization problem (2), in terms of precoders. We use the following theorem for this purpose, which establishes the uplink-downlink duality for the downlink beamforming in a cell-free massive MIMO network.

Theorem 1. *Let p_k be the uplink transmit power of user k and p_{i_k} stands for the allocated downlink transmit power to user k from the i-th AP. Then, the uplink and downlink SINR will be equal if the downlink power allocation satisfies the same power constraint as in the uplink, i.e., $\sum_{i=1}^{M} \sum_{k \in \mathcal{K}_i} p_{i_k} = \sum_{k=1}^{K} p_k$.*

Proof. The proof can be achieved by following the procedures incorporated by Theorem 3 in [17]. ■

Using Lagrangian duality presented in Theorem 1, the primal downlink problem (2) can be solved by exploiting the result of the dual uplink problem. Therefore, the optimal beamforming vector in (2) can be acquired using the following theorem.

Theorem 2. *The optimal precoder $\mathbf{f}_{i_k}$, which the i-th AP uses to transmit the signal s_{i_k} to the k-th UE, is given by $\mathbf{f}_{i_k}^* = \sqrt{p_{i_k}} \mathbf{u}_k^H / ||\mathbf{u}_k||^2$ where $\mathbf{u}_k$ is the optimal receive beamformer of the Lagrangian dual uplink problem of (2) and can be expressed as $\mathbf{u}_k = \mathbf{u}_{opt}[(k-1)N_t + 1 : kN_t]$ where $\mathbf{u}_{opt} = \nu_{\max}(\mathbf{B}_i^{-1} \mathbf{A}_i)$ and $\mathbf{A}_i$ and $\mathbf{B}_i$ are defined as follows*

$$\mathbf{A}_i = \begin{pmatrix} \mathbf{R}_{i,1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_{i,2} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{R}_{i,|\mathcal{K}_i|} \end{pmatrix}, \quad \mathbf{B}_i = \mathbf{I} + \sum_{j \neq i} \mathbf{A}_j$$

where $\mathbf{R}_{i,k} = p_k \mathbf{g}_{i,k}^H \mathbf{g}_{i,k} / \sigma_i^2$.

Proof. The Lagrangian dual uplink problem of the underlaying downlink precoding problem (2) can be written as

$$\begin{aligned} \max_{\mathbf{u}_k} \quad & \frac{\sum_{k \in \mathcal{K}_i} \mathbf{u}_k^H \mathbf{R}_{i,k} \mathbf{u}_k}{\mathbf{u}_k^H \big(\mathbf{I} + \sum_{j \neq i} \sum_{\ell \in \mathcal{K}_j} \mathbf{R}_{i,\ell}\big) \mathbf{u}_k} \\ \text{s.t.} \quad & \sum_{k \in \mathcal{K}_i} p_k < P_{\max} \end{aligned}$$

The above optimization problem can be rearranged as

$$\max_{\mathbf{u}} \quad \frac{\mathrm{Tr}(\mathbf{u}^H \mathbf{A}_i \mathbf{u})}{\mathrm{Tr}(\mathbf{u}^H \mathbf{B}_i \mathbf{u})} \tag{3}$$

where $\mathbf{u} \triangleq \mathrm{vec}(\mathbf{u}_1^T \ \mathbf{u}_2^T \ \ldots \ \mathbf{u}_{|\mathcal{K}_i|}^T)$ and the power constraint will be satisfied with $||\mathbf{u}||^2 \leq P_{\max}$. It can be shown that the optimum beamformer $\mathbf{u}$ at problem (3) can be obtained by generalized eigenvalue decomposition as $\mathbf{u}_{\text{opt}} = \nu_{\max}(\mathbf{B}_i^{-1}\mathbf{A}_i)$. ■

Utilizing the optimal precoder $\mathbf{f}_{i_k}^*$ as given in Theorem 2, it can be shown easily that the optimal downlink transmit power at each AP can be given using the following corollary.

Corollary 1. *Using the optimal beamformer $\mathbf{f}_{i_k}^*$ and assuming the perfect channel state information is available, the optimal downlink transmit power at the i^{th} AP can be expressed as follows:*

$$\begin{aligned}\mathbf{p}_i^* &= [p_{i_1}^*, \ldots, p_{i_k}^*, \ldots, p_{i_{|\mathcal{K}_i|}}^*]^T \\ &= \xi_i \sigma_{i_k}^2 (\mathbf{I}_{|\mathcal{K}_i|} - \xi_i \mathbf{\Gamma}\mathbf{\Upsilon})^{-1}\mathbf{\Gamma}\mathbf{1}_{|\mathcal{K}_i|}\end{aligned}$$

where $p_{i_k}^*$ denotes the optimal allocated downlink transmit power to the k^{th} user from the i^{th} AP, $\mathbf{\Gamma}$ is a diagonal matrix where the k^{th} nonzero elements in the diagonal is given by $\|\mathbf{f}_{i_k}^*\|^2/|\mathbf{g}_{i,k}^H\mathbf{f}_{i_k}^*|^2$ while all its non-diagonal elements are zero, the entries outside the main diagonal of matrix $\mathbf{\Upsilon}$ are given by $[\mathbf{\Upsilon}]_{k,i} = |\mathbf{g}_{i,k}^H\mathbf{f}_{i_k}^*|^2/\|\mathbf{f}_{i_k}^*\|^2$ while all its diagonal elements are zero, and ξ_i can be expressed as

$$\xi_i = \frac{P_{i,\max}}{\sum_{k=1}^{|\mathcal{K}_i|}(\mathbf{g}_{i,k}^H\mathbf{f}_{i_k}^*)^{-1}}.$$

Taking into consideration the existence of channel estimation error, the following theorem provides the lower bound on UE achievable rate in the underlying cell-free massive MIMO network.

Theorem 3. *Considering the channel estimation error and by utilizing max-min precoder and power control, the achievable rate lower bound of underlying cell-free massive MIMO network can be expressed as*

$$\log_2\left(1 + \frac{\mathbb{E}\{|\sum_{i\in\mathcal{B}_k}\hat{\mathbf{g}}_{k,i}\mathbf{f}_{i_k}s_k|^2\}}{\mathbb{E}\{|\sum_{\substack{j=1\\ j\notin\mathcal{B}_k}}^{M}\sum_{\substack{\ell=1\\ \ell\neq k}}^{K}\hat{\mathbf{g}}_{k,i}\mathbf{f}_{j_\ell}s_\ell|^2 + |\sum_{j=1}^{M}\sum_{\ell=1}^{K}\tilde{\mathbf{g}}_{k,j}\mathbf{f}_{j_\ell}s_\ell|^2\} + 1}\right)$$

Proof. Taking the channel estimation error $\tilde{\mathbf{g}}_{k,i}$ into consideration, the received signal in (1) can be re-evaluated as

$$y_k = \underbrace{\sum_{i\in\mathcal{B}_k}\hat{\mathbf{g}}_{k,i}\mathbf{f}_{i_k}s_k}_{\text{desired signal}} + \underbrace{\sum_{\substack{j=1\\ j\notin\mathcal{B}_k}}^{M}\sum_{\substack{\ell=1\\ \ell\neq k}}^{K}\hat{\mathbf{g}}_{k,i}\mathbf{f}_{j_\ell}s_\ell}_{\text{multi-user interference}} + \underbrace{\sum_{j=1}^{M}\sum_{\ell=1}^{K}\tilde{\mathbf{g}}_{k,j}\mathbf{f}_{j_\ell}s_\ell}_{\text{channel estimation error}} + n_k$$

All the terms in the right hand side of the above equation are mutually uncorrelated due to the assumption that the signals are different for distinctive users, and n_k is independent from the channel coefficients and data symbols. In order to acquire the lower bound of the achievable rate, we take the worst case noise into account. It is shown in [18, Theorem 1] that the worst case noise is a Gaussian additive noise with the variance equal to the accumulation of variance of noise, multi-user interference, and channel estimation error. Therefore, the SINR can be written as

$$\mathrm{SINR}_k = \frac{\mathbb{E}\{|\sum_{i\in\mathcal{B}_k}\hat{\mathbf{g}}_{k,i}\mathbf{f}_{i_k}s_k|^2\}}{\mathbb{E}\{|\sum_{\substack{j=1\\ j\notin\mathcal{B}_k}}^{M}\sum_{\substack{\ell=1\\ \ell\neq k}}^{K}\hat{\mathbf{g}}_{k,i}\mathbf{f}_{j_\ell}s_\ell|^2 + |\sum_{j=1}^{M}\sum_{\ell=1}^{K}\tilde{\mathbf{g}}_{k,j}\mathbf{f}_{j_\ell}s_\ell|^2\} + 1}$$

and the achievable rate lower bound of underlying cell-free massive MIMO network can be expressed as $\log_2(1+\mathrm{SINR}_k)$. ■

IV. SIMULATION EVALUATIONS

In this preliminary experimental evaluation, we evaluate the performance of the proposed schemes for a downlink cell-free massive MIMO network. The setup of our experiments is in accordance with the ones in [5], [6], [9] and given as follows. We simulated a cell-free massive MIMO network in which M APs and K single-antenna UEs are randomly distributed over a square dense urban area of size $1000 \times 1000\ m^2$. We employ a three-slope path-loss model along with an uncorrelated shadowing model as discussed in Section II. The transmission is subject to interference from all APs in the area. We assume that all the pilot sequences have the same length as the number of UEs, i.e., $\tau = K$. All deployments and channel model parameters are listed in Table I. The performance is measured in terms of the power consumption (dBm) and the rate (bit/s) per each UE. Moreover, we considered the following schemes for comparison under simulation: CB with optimal power allocation [5] and ZF with optimal power allocation [7].

The average total power consumption per UE in dBm is modeled as in [19], [20] and demonstrated in Figure 2. We compare the proposed resource allocation scheme with the co-located massive MIMO, the cell-free ZF with optimal power allocation, and the cell-free CB with optimal power allocation. It is shown that although the cell-free ZF algorithm with optimal power allocation causes an improved energy

Table I
SYSTEM PARAMETERS

Parameters	Values
Shadowing standard deviation	8 dB
Transmitted power of each UE	20 dBm
AP radiated power	23 dBm
Carrier frequency	1.9 GHz
Bandwidth	20 MHz
Noise figure	9 dB
Thermal noise level	-174 dBm/Hz
AP antenna height	15 m
UE antenna height	1.65 m
d_1	50 m
d_0	10 m
d_{decorr}	100 m
δ	0.5

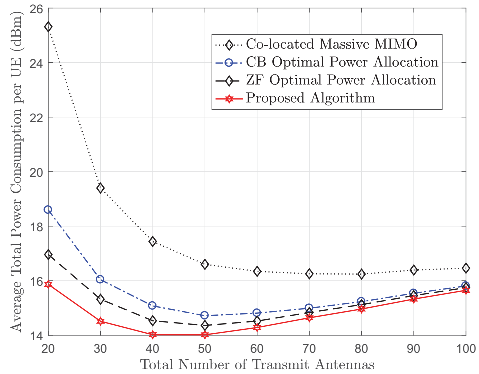

Figure 2. Average total power consumption versus total number of transmit antennas in a cell-free massive MIMO network.

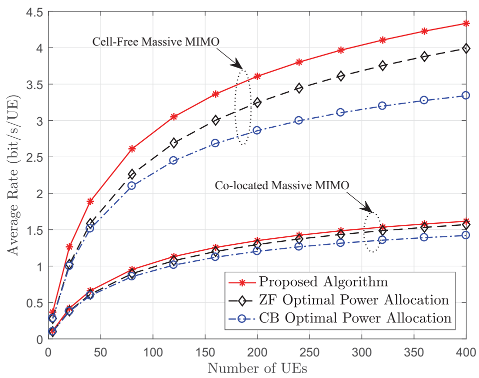

Figure 3. Downlink average rate per UE versus total number of UEs in both cell-free and co-located massive MIMO networks.

efficiency in compare with cell-free CB and co-located massive MIMO architecture, by designing the beamforming and power allocation vectors the proposed algorithm provides better improvements in terms of energy efficiency.

Figure 3 depicts the average rate per UE versus the number of UEs in the underlying network, for both cell-free and co-located massive MIMO architectures. It is demonstrated that the cell-free massive MIMO architecture outperforms the co-located one for different number of UEs as well as resource allocation schemes. The results indicate that for smaller number of UEs the proposed algorithm provides slightly higher achievable rate than other schemes. However, it is notable that with an increase in the number of UEs, the achievable rate is significantly higher.

V. CONCLUSION

We have studied the downlink resource allocation in a cell-free massive MIMO network and proposed an algorithm for maximizing (over the resource allocation: precoding vectors and power allocations) the minimum achievable rate among the UEs in the network. The optimal solution of this problem was obtained by utilizing the uplink-downlink duality. Exploiting proposed beamformers and taking the effects of channel estimation error into consideration, we analyzed the performance of underlying cell-free massive MIMO network and derived the capacity lower bound. Through the system level evaluations, the results showed that the proposed algorithm significantly outperforms the conventional resource allocation schemes in practical environments.

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