Topics Covered

"Installing" the Code

There is no installing per se, but one does need Numpy, Scipy, and the emcee package. The emcee package contains an implementation of Goodman & Weare's Affine Invariant Markov chain Monte Carlo (MCMC) Ensemble sampler, which is necessary for the MCMC portion of the code. Installation instructions can be found at `dan.iel.fm/emcee/current/`.

Running DMBayesian.py will run the main method, executing the commands placed within.

Likelihood of Getting the Data in One Bin

The likelihood of obtaining the observed data is key to computing the posterior for the dark matter signal strength η. For a single bin in the analysis, the likelihood of obtaining the observed counts is

$$p(\eta) = \frac{1}{C}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\eta)\text{Poisson}[\lambda_1; n_1]\text{N}[0,1;x_s]\,\mathrm{d}x_s\text{N}[0,1;x_b]\,\mathrm{d}x_b. \tag{1}$$

where

$$\lambda_1 = \eta s_1 + b_1 \tag{2}$$

$$s_1 = \bar{s_1}\left(1 + \frac{\sigma_{s,1}}{\bar{s_1}}\right)^{x_s} \tag{3}$$

$$b_1 = \bar{b_1}\left(1 + \frac{\sigma_{b,1}}{\bar{b_1}}\right)^{x_b} \tag{4}$$

and

$$\text{Poisson}[\lambda_1; n_1] = \frac{\lambda_1^{n_1}e^{-\lambda_1}}{n_1!} \tag{5}$$

$$\text{N}[0,1;x_s] = \frac{1}{\sqrt{2\pi}}e^{-\frac{x_s^2}{2}} \tag{6}$$

$$\text{N}[0,1;x_b] = \frac{1}{\sqrt{2\pi}}e^{-\frac{x_b^2}{2}}, \tag{7}$$

C is a normalizing factor, and $f(\eta)$ is a prior.

Likelihood of Getting the Data in All Bins

$$p(\eta) = \frac{1}{C}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\eta)\prod_{i=1}^{N_{bins}}\{\text{Poisson}[\lambda_i; n_i]\}\,\text{N}[0,1;x_s]\,\mathrm{d}x_s\text{N}[0,1;x_b]\,\mathrm{d}x_b. \tag{8}$$

Likelihood of Getting the Data in All Bins continued on next page...

where

$$\lambda_i = \eta s_i + b_i \tag{9}$$

$$s_i = \bar{s_i}\left(1 + \frac{\sigma_{s,i}}{\bar{s_i}}\right)^{x_s} \tag{10}$$

$$b_i = \bar{b_i}\left(1 + \frac{\sigma_{b,i}}{\bar{b_i}}\right)^{x_b} \tag{11}$$

and

$$\text{Poisson}[\lambda_1; n_i] = \frac{\lambda_i^{n_i} e^{-\lambda_i}}{n_i!} \tag{12}$$

$$\text{N}[0,1; x_s] = \frac{1}{\sqrt{2\pi}} e^{-\frac{x_s^2}{2}} \tag{13}$$

$$\text{N}[0,1; x_b] = \frac{1}{\sqrt{2\pi}} e^{-\frac{x_b^2}{2}}, \tag{14}$$

C is a normalizing factor, and $f(\eta)$ is a prior.

To use a Markov Chain Monte Carlo (MCMC) approach, we recognize that the integrals are marginalizing over the variables x_s and x_b. So, we consider a three dimensional problem with likelihood

$$p(\eta, x_s, x_b) = f(\eta) \prod_{i=1}^{N_{bins}} \{\text{Poisson}[\lambda_i; n_i]\} \, \text{N}[0,1; x_s]\text{N}[0,1; x_b] \tag{15}$$

and simply flatten the chain on η to get the same posterior that we would get from performing the integrals.

Jeffreys Prior

The Jeffreys Prior for this likelihood is

$$f(\eta) \propto \sqrt{\sum_{i=1}^{N_{bins}} \frac{s_i^2}{\lambda_i}} \tag{16}$$

and it is derived in the accompanying document.