CS 331: Algorithms and Complexity (Spring 2015)

Unique numbers: 51825, 51830, 51835, 51840

Take Home Exam 1

Due: Wednesday, February 25^{th}, 2015 by 11.59pm

INSTRUCTIONS

- Each student must work on his/her own. No communication with anyone is allowed, not even high level ideas.
- You are free to use the office hours to get clarifications from the TAs, proctors or the professor. But no hints will be given.
- In total 30 points are available. Points for each question are given in brackets.
- Read the questions carefully to know exactly what you need to answer.
- Submissions must be done through canvas. PDF, doc and txt formats are allowed.

1. (10 points) You are given the following recurrences. Solve them using Master theorem or recursion tree to produce explicit forms for each. Show your work. Finally, sort them in increasing order of complexity (fastest to slowest) for sufficiently large n.

- $T_1(n) = 6T_1(n/4) + n^2$
- $T_2(n) = 4T_2(n/2) + n^2$
- $T_3(n) = \sqrt{n}T_3(\sqrt{n}) + n$

2. (7 points) Let us define a **maximum spanning tree** as a spanning tree with weight greater than or equal to the weight of every other spanning tree. We have seen greedy algorithms for finding the minimum spanning tree of a weighted undirected graph. It is possible to compute the **maximum spanning tree** by using a simple tweak. Describe how you would tweak the inputs and/or the algorithm to find the **maximum spanning tree**. Prove the correctness and runtime of your algorithm.

[Note that you can use any results/theorems related to the minimum spanning tree algorithms, but must show that your tweaks does not violate the conditions required by those theorems.]

3. (5 points) Let $G = (V, E)$ be a connected undirected graph where each edge has weight 10 or 11. Let W^* denote the weight of a minimum spanning tree of G, and let T be a spanning tree of G with weight W where $W > W^*$. Prove or disprove: The graph G has a spanning tree with weight equal to $W - 1$.

4. (3 points) Give an example of a directed graph that has five strongly connected components (SCCs), and such that the number of SCCs can be reduced to one by adding only a single directed edge between two existing vertices.

4. (5 points) Let s be a vertex of a connected undirected graph G. Let $T^B_{G,s}$ and $T^D_{G,s}$ respectively be the trees obtained by applying BFS and DFS on graph G starting at node s. Prove or disprove: the depth of $T^D_{G,s}$ is at least as big as the depth of $T^B_{G,s}$.