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LAB 3

VIBRATIONS: MASS SPRING DAMPER SYSTEM

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ABSTRACT

This lab explores 5 different sensors: infrared (IR), strain gage, voice coil, accelerometer, and linear-variable differential transducer (LVDT), along with the calibrations of these sensors. The sensors were calibrated dynamically and statically and the results were compared against a linear encoder to test the accuracy of the sensors. The strain gage, LVDT accelerometer, and voice coil have a linear best fit relationship between voltage and their relevant geometric measure. The IR sensor has a cubic relationship. However, the strain gage and LVDT has the least variation between the best fit and the actual values. This indicates that they are the most accurate sensor of the five. However, the LVDT provides more details, including the phase and magnitude of the mass spring damper system. The LVDT is more valuable if this information is needed. The strain gage is a cheaper option that provides less information.

INTRODUCTION

Sensors are used to measure data, and a huge variety of sensors exist. Each sensor has its strengths and weaknesses, and it is important to choose the right sensor when creating experiments and taking measurements. It is also important to ensure, when using multiple sensors, the results of each of the different sensors agree.

An important step in experimentation is calibration, and ensuring that each of the sensors are measuring with the required accuracy and precision in respect to a standard. The standard used for this lab is a linear optical encoder. The two methods of calibration used in this lab are dynamic and static calibration. The voice coil and accelerometer were calibrated dynamically, while the other sensors were calibrated both dynamically and statically.

Although the 5 sensors used in this lab use different methods of measurementation, the results of the calibrations should show that all of the sensors measure the displacement of the vibration stand accurately, and with relatively similar precision.

Unfortunately, our hypothesis did not stand and not all of the sensors were able to accurately measure the displacement of the vibration stand as accurately as preferred.

THEORY

1. Sensors

a. Strain Gage Strain gages work by using the knowledge that electrical resistance changes when a piece of metal is deformed. This is due to the change in length L of the metal and the change in cross sectional area correlating to Poisson's ratio [1]. Thus, the resistance R, which is dependent on length L and cross sectional area A, will change as the material is strained, shown in Eqn. (1).

$$R = \frac{\rho L}{A} \tag{1}$$

b. IR An infrared sensor consists of an emitter, optical component, detector, and signal processor. An IR LED can be used to emit the IR radiation, which is then focused by an optical component such as a quartz lens. The IR light deflected off the target is then captured by an IR photodiode detector, usually a semiconductor. The magnitude of the captured IR is then transformed into a signal and amplified, since the captured signals are usually small [2].

c. Voice Coil Voice coils use Faraday's law of induction, stating that a current through a wire induces a magnetic field. Derived from this law of induction is Lenzs law, stating that induced voltage is equal to the negative change in magnetic flux [3]. Moving a magnet through a coil, thus changing the magnetic flux, induces a voltage in the wire. This voltage can then be measured and correlated to the velocity that the magnet is moving at. The following equation and it's derivatives can be used to correlate the position and the velocities to time.

$$y = A * exp(-\zeta \omega_n t) * sin(\omega_d t + \phi) \tag{2}$$

d. Accelerometer Accelerometers are a type of seismic instrument that consists of a spring supported mass within a housing, with a sensor detecting the relative movement between the housing and the mass. Dashpots are also attached to the mass in order to provide damping. By calibrating the natural frequency and damping ratio, the accelerometer can be designed so that the aforementioned relative movement is a function of the acceleration [1].

e. LVDT A linear variable differential transformer is a mutual inductance device which measures the displacement of the core with respect to the three internal coils. The center coil is powered by an AC power source, and the two adjacent coils measure voltages which have a linear relationship with the displacement of the core. Since the core of the LVDT is separate from the coils, overloading the LVDT will not destroy it. LVDTs also provide high resolution and sensitivity compared to other sensors [1].

2. Proportionality calculation and Resolution

The constant of proportionality between the output and the displacement/velocity/acceleration is found by using appropriate polynomial fits according to the data sets, such as MATLAB's built in function *polyfit*. Taking the first coefficient of a first order fit of the data will give the constant of proportionality if the relationship is linear. If not, then the constants of proportionalities are given by the polynomial equation.

The resolutions of each sensor were found by looking at the experimental data and finding the smallest difference in the voltages between consecutive data points. These voltages were then converted to the respective value of interest (e.g. the voltage difference found for the voice coil was converted to a velocity resolution, and the voltage difference found for the strain gage was converted to displacement resolutions).

$$\zeta = \frac{\Delta/2\pi}{\sqrt{1+(\Delta/2\pi)^2}} \tag{3}$$

3. Uncertainties

Statistical analysis of error shows that there are multiple ways of estimating a measurement's errors based on using different probability distributions, the number of samples taken on a measurement, and the level of confidence, among other things [1]. Uncertainties for each sensor were found using the T-distribution uncertainty model given by Eqn. (4):

$$P_x = t_{\frac{\alpha}{2},\nu} \frac{S_x}{\sqrt{n}} \tag{4}$$

Where P_x is the precision uncertainty, α is the level of significance, S_x is the estimate for the mean of the data sets, n is the number of data sets, and t is the probability density function coefficient specific to the number of measurements taken [1].

This uncertainty model was employed due to the fact that not a large number of experiments were ran (i.e. greater than 30 experiments).

4. Phase Error and Resonant Frequency

The phase error can be calculated using MATLAB's built in Fast Fourier Transform (FFT) function, which gives the frequency components of the signal. Since the signals that are analyzed in this lab only have 1 frequency, taking the maximum value of the array that is output from the FFT analysis in MATLAB will give the frequency of that signal. However, even if the signals analyzed in this lab didn't have one frequency associated with each experiment, the FFT function will produce the most prominent frequency.

The phase angle can then be calculated by using the angle function in MATLAB, while constraining the angles within the 0 to π interval. Doing the same with the other signal and comparing the two phase angles will give the phase error.

The resonant frequency of the test stand was found experimentally by inputting a sinusoidal signal to the voice coil. Knowing that the test stand vibrates with the largest amplitude at resonant frequency, frequencies around the estimated 12.5 Hz resonant frequency were tested to find the actual resonant frequency.

The damping ratio can be found by using the following formula:

$$\omega_d = \omega_n \sqrt{1-\zeta^2} \tag{5}$$

where ω_n is the natural (or resonant) frequency in rad/s, and ω_d is the damping frequency, given in rad/s.

ω_d can be estimated experimentally by recording the free response of the beam's deflection with respect to time and observing how many periods the beam deflects in one second. Knowing these two parameters, the damping ratio can be found by solving Eqn. (5) for ζ.

PROCEDURES

1. Equipment

- ☐ Test stand:
  - ☐ Encoder (US Digital EM1 Transmissive Optical Encoder Module)
  - ☐ LVDT(US Digital EM1 Transmissive Optical Encoder Module)
  - ☐ IR Sensor
  - ☐ Strain Gage
  - ☐ Voice Coil
  - ☐ Accelerometer (Analog Devices ADXL202E)
- ☐ LabVIEW VI
  - ☐ Displays the displacement indicated by the encoder and the voltage from the connected sensor
- ☐ National Instruments PXI platform

2. Testing

To start the lab, the linear encoder is powered with a constant 5V DC input. Channels A and B of the encoder are connected to Channels 0 and 1, respectively on the NI BNC-2120 Connector Block. A sinusoidal signal was then inputted to the voice coil to determine the resonant frequency of the test stand. Measuring the recorded amplitudes with a LabVIEW VI, the frequency with the largest amplitude is considered the resonant frequency.

The input signal to the voice coil is then removed, and a sensor output is wired to Channel 3 of the BNC connector for data acquisition. The LabVIEW VI's data acquisition duration is started while the test stand's vibration beams are displaced and then released. This procedure is repeated ten times each for the five sensors. An additional five sets of measurements are done each for the LVDT, IR sensor, and strain gages in which the vibration beams are statically loaded. These static measurements are only valuable for the displacement sensitive sensors.

After acquiring all the data, calibration can be done depending on the polynomial fit of the datasets.

RESULTS AND DISCUSSION

The resonant frequency of the test stand was found to be 12.30 Hz, which is near the predicted value of 12.5 Hz. Using Eqn. (5), the damping ratio was calculated to be 0.04, which is near the predicted value of 0.01. These values are used to calibrate the accelerometer and voice coil by using Eqn. (2) and its respective derivatives.

LVDT

The LVDT was calibrated statically and dynamically to yield a linear relationship between the ratio of the output voltage and the input voltage and the displacement of the test stand beam within the range of at least -0.2 in to 0.2 in, as expected of an LVDT, shown in Fig. (3) [1].

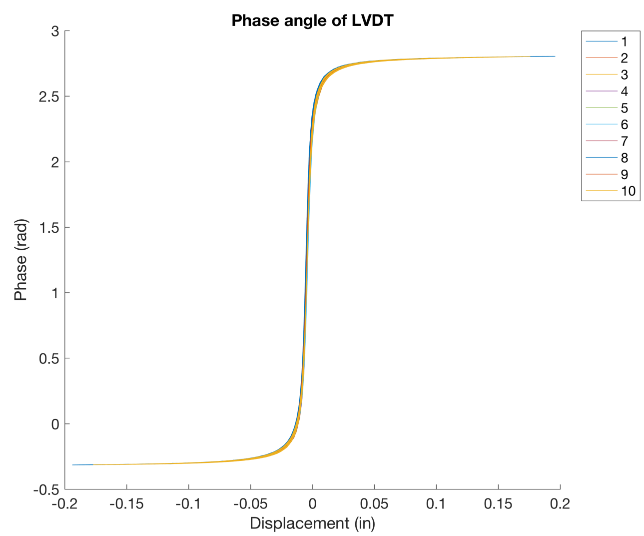

FIGURE 1: LVDT Phase Angle

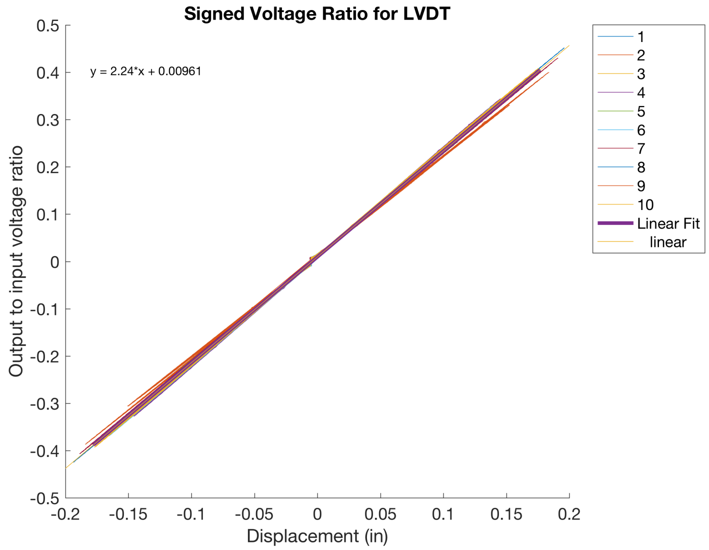

FIGURE 2: LVDT Signed Voltages Ratio

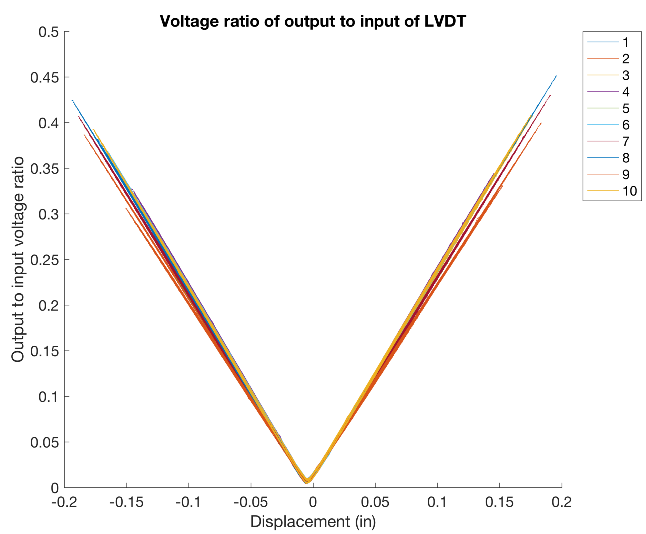

FIGURE 3: LVDT Voltage Ratio

Fig. (1) and Fig. (3) show the sinusoidal amplitude and phase relative to the position of the core within the body of the device. A sharp phase angle change is displayed around a displacement of zero in.

The overall calibration of the LVDT is shown in Fig. (2) as the signed voltage ratio of the LVDT. The linear relationship is given by:

$$y = 2.24x + 0.00961 \tag{6}$$

where the first coefficient, 2.24 is the sensitivity, given in the units of [V / V/in].

IR

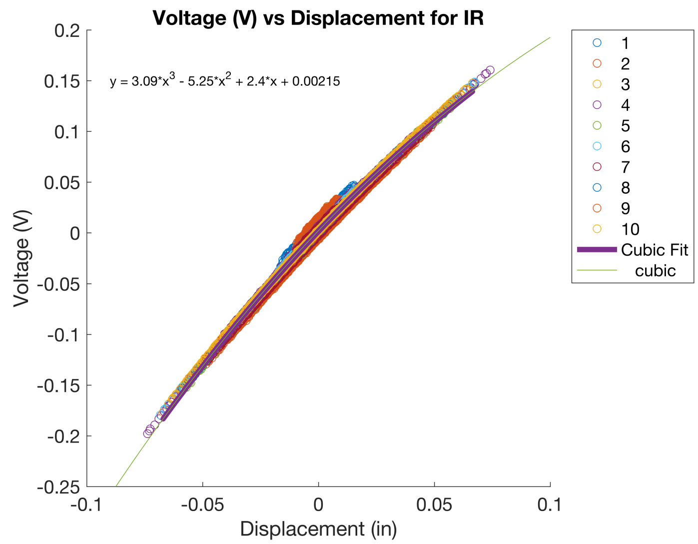

FIGURE 4: VOLTAGE VS DISPLACEMENT OF IR WITH CUBIC FIT SUPERIMPOSED

The data for the IR sensor is shown in Fig. (4). The data is not linear; a cubic fit was determined to be most appropriate. However, there are notable deviations from the cubic fit. The deviation from the cubic model is greater near (0,0) and near the ends of the cubic fit. These areas also have a variation of voltage readings for the same displacement. This suggests that the IR is significantly less accurate at these areas.

Strain Gage

The strain gage data is plotted in Fig. (5) with a linear fit superimposed over it. The data appears to follow the linear fit very closely, and each data set appears to overlap the other at their respective displacements. This indicates that the sensor is relatively accurate within the measurement range.

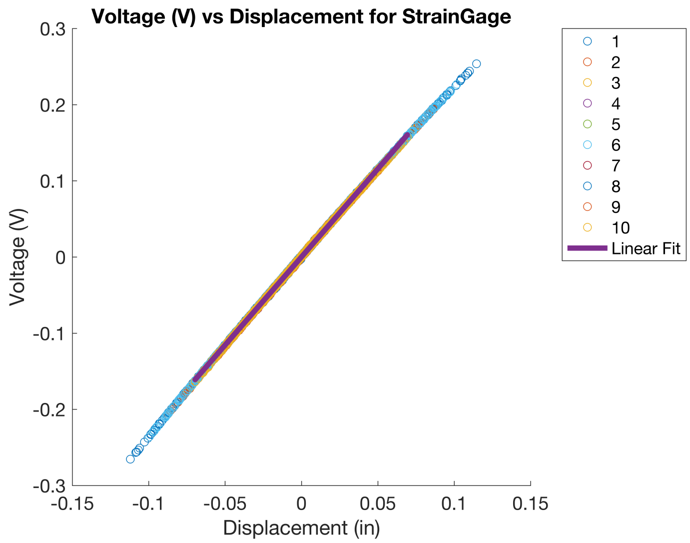

FIGURE 5: VOLTAGE VS DISPLACEMENT OF STRAIN GAGE WITH LINEAR FIT SUPERIMPOSED

Accelerometer

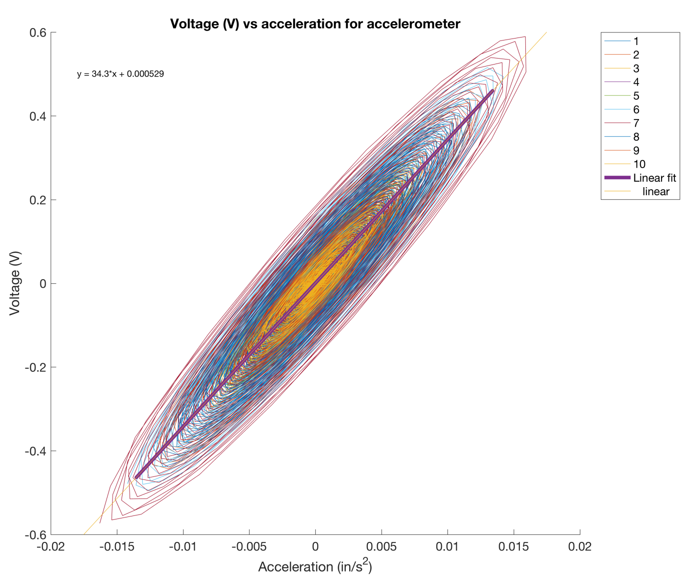

FIGURE 6: VOLTAGE VS ACCELERATION OF ACCELEROMETER WITH LINEAR FIT SUPERIMPOSED

The original data collected was the voltage readings relative to the displacement determined by the encoder. However, the accelerometer is sensitive to acceleration. Fig. (6) shows the calculated relationship between the measured voltage and acceleration. Noting the significantly larger voltage scale relative to the acceleration scale, the accelerometer has a high degree of error for the linear fit indicated. The slope of the linear fit varies significantly from the slopes of the semi-major axis of the elliptical shapes. This suggests that the accelerometer accuracy decreases at larger magnitudes of acceleration.

Voice Coil

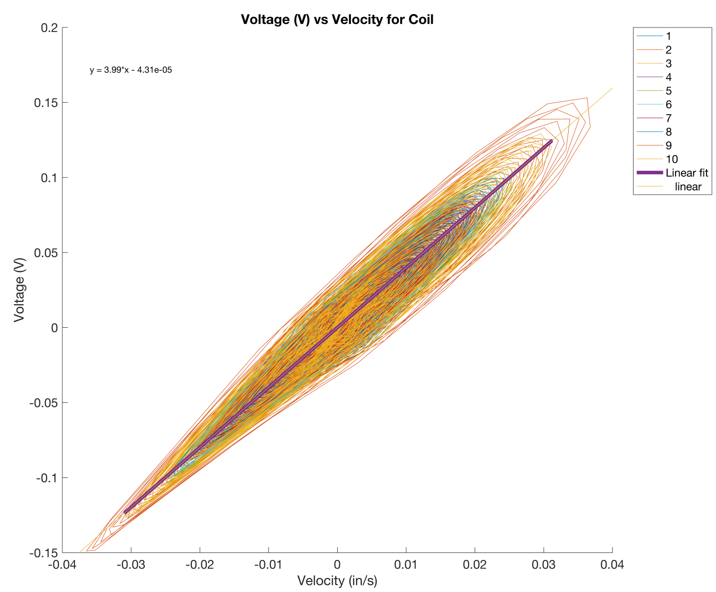

FIGURE 7: VOLTAGE VS VELOCITY OF VOICE COIL WITH LINEAR FIT SUPERIMPOSED

Like the accelerometer, the measured displacements were converted to velocity because the voice coil is sensitive to velocity. Fig. (7) illustrates the calculated relationship between the voltage and velocity. The plot has a feather-like shape, indicating large voltage variations from the superimposed linear fit. The voice coil is most accurate at more negative values within this measurement range, and is least accurate around a velocity of 0.

Uncertainty

The calculated uncertainties are shown in Table. (2). The uncertainties reflect the ranges of voltages associated with a single geometric measure.

It is also important to note very prominent hysteresis loops for both the accelerometer and the voice coil calibration results. These hysteresis loops occur for these two sensors because of the fact that they operate on the basis of electromagnetism and Faradays law. Other sensors, such as the strain gage does not operate on the basis of ferroelectricity, but instead by the theory of resistivity, as shown in Eqn. (1). For this reason, hysteresis loops do not occur in this sensor.

TABLE 1: UNCERTAINTIES OF THE SENSORS

	IR	SG	VC	ACC	LVDT
U1	2.3139	0.0062	0.0037	0.1493	0.0484
U2	2.3139	0	0.0002	0.0006	0.0008
U3	0.0032	x	x	x	x
U4	0.0005	x	x	x	x

Results for 95% confidence interval

Comparing the Sensors

The LVDT and strain gage are the most accurate sensor, with minimal deviation from the linear model. The strain gage, LVDT, accelerometer, and voice coil had linear relationships between the voltage and relevant geometric measure. However, the accelerometer and voice coil had significant voltage ranges associated with their relevant geometric measure. The voice coil had a smaller variation range than the accelerometer, which can be graphically determined by their different voltage axis scaling. The variations visually look similar at some points of the plots, but the accelerometer has a larger scaling magnitude than the voice coil. The IR sensor, though fitted with a cubic function, also has many voltages associated with one displacement value. The range of voltages associated with one displacement varies. Between the LVDT and strain gage, the LVDT is a more expensive option that provides more information, phase and magnitude. The strain gage does a sufficient job if only voltage vs displacement data is needed.

CONCLUSION

Sensor selection depends on both costs and required sensor accuracy. The LVDT and strain gage are the optimal choices for their respective price range within these five sensors.

To improve the testing process, external disturbances should be better managed. External disturbances were especially notable during the static measurements, where vibrations caused by sounds and movements of the surrounding people resulted in displacements in the vibration beam. Similar irregularities in the data were likely present in the dynamic measurements as well, though they were less noticeable. To minimize the disturbances, an isolated environment with minimal surrounding sounds and movements should be required during data collection.

TABLE 2: RESOLUTION OF THE SENSORS

Sensor	Res	Units
IR	0.0022	in
SG	$4.3904 * 10^-5$	in
VC	$1.6019 * 10^-4$	in/s
AC	0.0018	in/s2
LVDT	0.0102	in

Resolution of the 5 sensors, LVDT and Strain Gage have the finest resolution

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