Lab 5: Magnetic Levitation

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10 November 2016

Lab Section 4: Thursday 8-11

MEC134/EEC128: Feedback Control Systems
Fall 2016

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1. Purpose

The main objectives of this lab were to design and implement an analog controller for a magnetic levitation (MagLev) system. In order to design the controller, we needed to identify and linearize the model of the plant to be controlled.

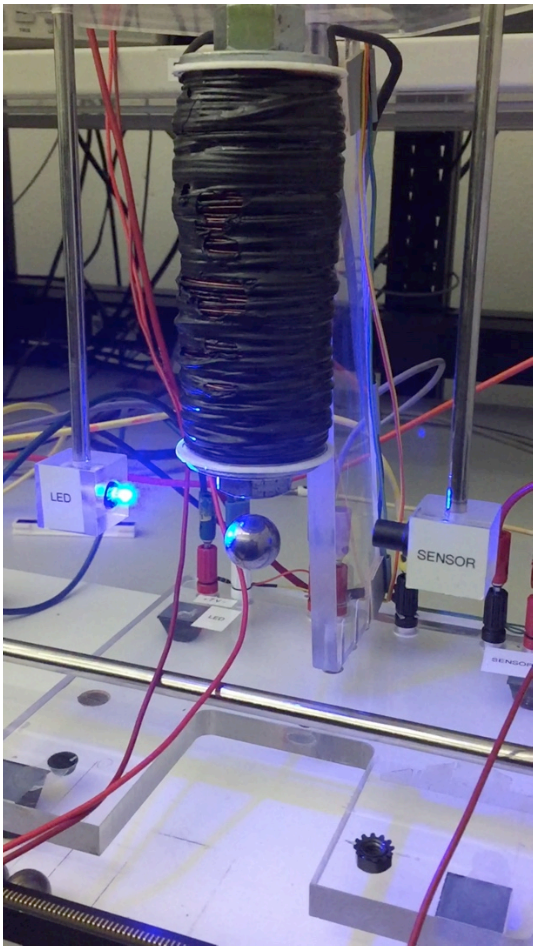

Figure 1: Magnetic levitation system

2. System Identification

a) Pre-Lab:

In order to design and implement the controller to achieve magnetic levitation, we must first perform system identification to determine a linearized plant model. The high level block diagram is shown below.

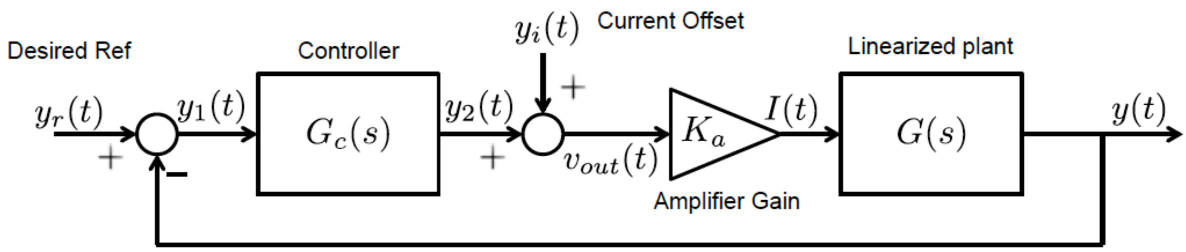

Figure 2: High level block diagram of the magnetic levitation setup

The equations of motion of the ball can be modeled as follows:

$$m\ddot{x} = f(I, x)\text{-}mg \tag{1}$$

$$y = h(x) \tag{2}$$

where x is the vertical position of the ball (in m), I is the current through the coil (in A), and $g = 9.81\frac{m}{s^2}$ is the gravitational constant. The *nonlinear* function $f(I, x)$ describes the magnetic force (in N) on the ball as a function of x and I, and the *nonlinear* function $h(x)$ describes the voltage drop across the photo resistor as a function of x.

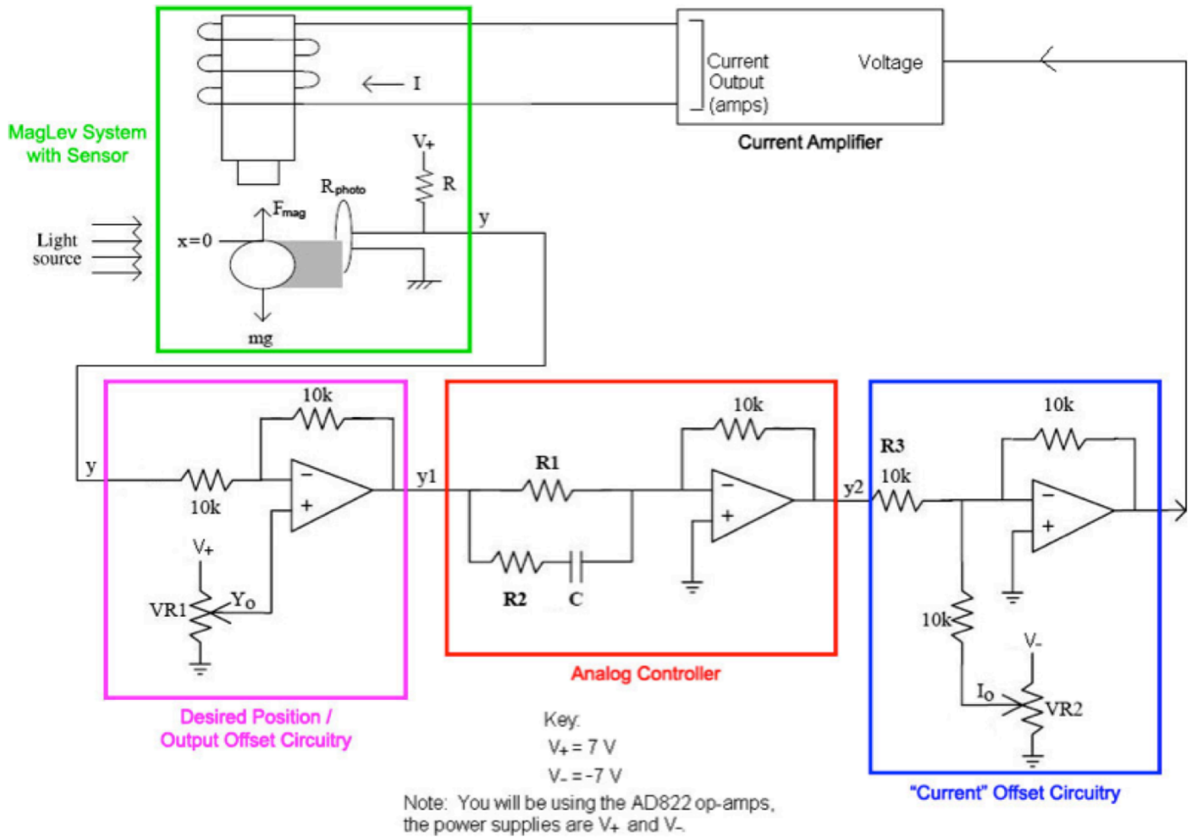

Figure 3: Block diagram of the magnetic levitation setup with circuit level details

i) Let $y_{ref} := Y_0$ in the left box in Figure 3. This is the reference voltage that we will adjust using a potentiometer. To find a relationship between the signal y_1, the voltage y from the photoresistor, and the reference voltage y_{ref}:

$$\frac{y\text{-}y_{ref}}{10^3} = \frac{y_{ref}\text{-}y_1}{10^3}$$
$$y\text{-}y_{ref} = y_{ref}\text{-}y_1$$
$$y_1 = 2y_{ref}\text{-}y \tag{3}$$

ii) To find the transfer function of the analog controller in the red box of Figure 3:

$$\frac{y_1}{Z_1} = -\frac{y_2}{10^4}$$

where $Z_1 = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2+\frac{1}{Cs}}} = \frac{R_1\left(R_2+\frac{1}{Cs}\right)}{R_2+\frac{1}{Cs}+R_1} = \frac{R_1(R_2Cs+1)}{R_2Cs+1+R_1Cs}$

$$\frac{y_2}{y_1} = -\frac{10^4}{R_1}\frac{(R_1+R_2)Cs+1}{R_2Cs+1} \tag{4}$$

iii) To find the output of the op amp in the current offset circuitry:

$$\frac{y_2}{10^4} = -\frac{y_i}{10^4}-\frac{V_{out}}{10^4}$$
$$V_{out} = -(y_2+y_i) \tag{5}$$

iv) The linearized model of the plant is given by:

$$m\ddot{x} = K_i\delta I + K_x\delta x \tag{6}$$
$$y = a\delta x \tag{7}$$

To find the transfer function of the linearized plant:

$$ms^2X = K_iI + K_xX$$
$$Y = aX$$
$$\Rightarrow X = \frac{K_iI}{ms^2\text{-}K_x}, \quad Y = \frac{aK_iI}{ms^2\text{-}K_x}$$
$$G(s) = \frac{Y(s)}{I(s)} = \frac{aK_iI}{ms^2\text{-}K_x} \tag{8}$$

b) Lab

i) In order to linearize the plant, an equilibrium height must be set such that the photoresistor only sees about half the light emitted from the LED source. This height was found to be at 4.5mm.

ii) The resistance of the photoresistor was then found by changing the amount of light the sensor receives from fully covered to uncovered by the ball. The resistance measured ranged from $3.25k\Omega$ and $0.784k\Omega$. In order to choose a resistance value R we had to use the fact that the supply voltage was 7V and that the photo resistor must not exceed a power dissipation of 250mW. We found the value $R = 1k\Omega$ to satisfy these specifications.

iii) *Linearizing h(x)*

To find a linear relationship between the position of the ball and the output voltage, we varied the position of the ball away from the equilibrium point and recorded the corresponding output voltages. Linear regression was then used in MATLAB to find the slope of the linearized relationship, a, and the optimal equilibrium voltage of y. The slope, a, was found to be 682.33 V/m and the optimal voltage was found to be 4.24V.

Deviation from equilibrium height (m)	**Voltage of y (V)**
-0.002	3.1
-0.0015	3.1
-0.001	3.38
-0.0005	3.84
0	4.06
0.0005	4.72
0.0010	5.22
0.0015	5.37
0.0020	5.41

Table 1: Values of the output voltage for varying heights

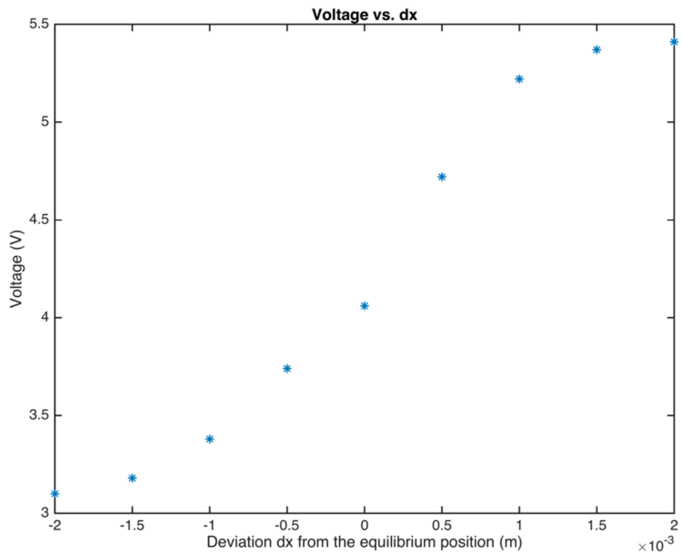

Figure 4: Voltage outputs for varying heights

iv) *Determining Ki (N/A)*

To find K_i, measurements of the mass were taken at the equilibrium height as the voltage of the current amplifier changed. Thus, we can plot the weight against the deviation of the equilibrium current. The slope was found to be $K_i = 0.0931$.

Amplifier current deviation from equilibrium current (A)	**Mass (g)**
0	0.7
0.2467	3.4
0.2867	4.4
0.600	7.4
0.733	8.5
0.9333	10.5
1.1333	12.0
1.4267	14.3

Table 2: Values of mass for varying current

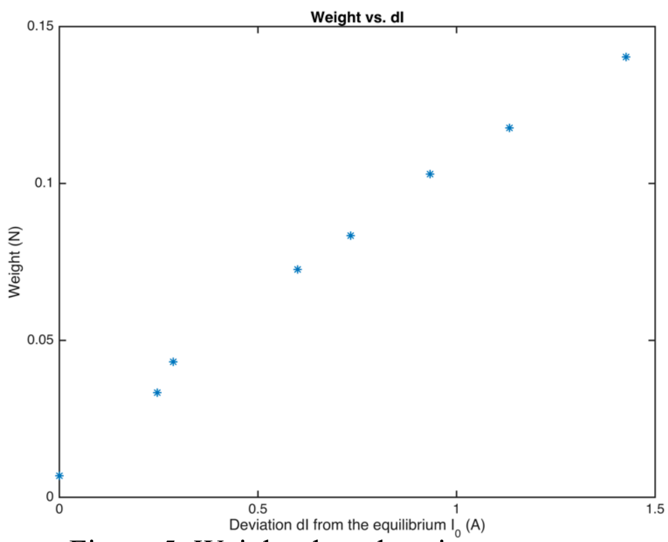

Figure 5: Weight plotted against current

v) *Determining Kx (N/m)*

To determine K_x, measurements of the mass was recorded as the height of the ball changed from equilibrium height to decreasing heights at the equilibrium current. The value of K_x was found to 15.9763 N/m.

Deviation from equilibrium height (m)	**Mass (g)**
0	1
0.0005	2
0.001	2.9
0.0015	3.9
0.002	4.6
0.0025	5.2
0.003	5.9

Table 3: Values of mass for varying heights

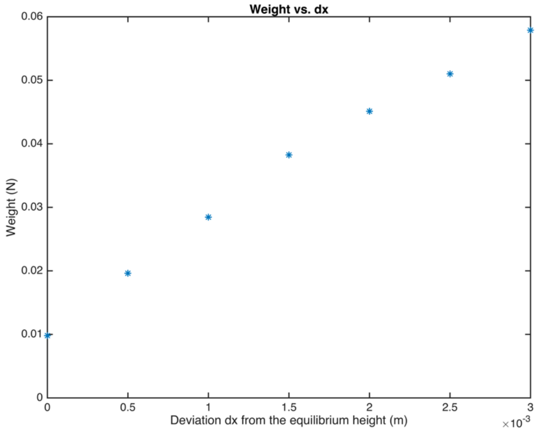

Figure 6: Weight vs position

vi) The DC gain of the system is given by $DC\ gain = aK_cK_a$
If we want the DC gain of our circuit to be 1000 A/m and we assume $K_a = 2A/V$, we find that K_c is 0.7328.

Our linearized system, as described in equations 6 and 7, are given by:

$$0.0161\ddot{x} = 0.0931\delta I + 15.9763\delta x \quad (9)$$

$$y = 682.33\delta x \quad (10)$$

3. Controller design and implementation

a) Pre-Lab:

i) Using the linearized plant model (equations 9 and 10), we can observe the system's performance and behavior. One way to do it is to look at the root locus and frequency response.

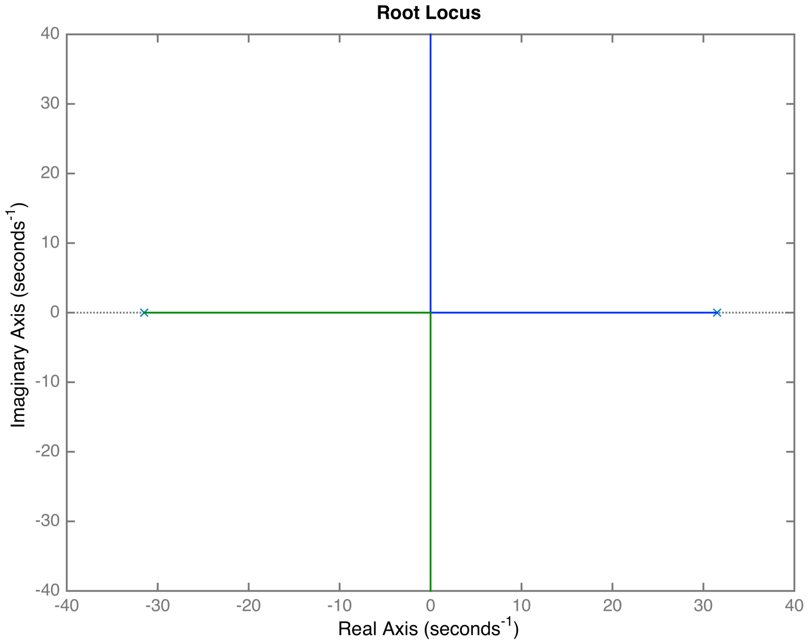

Figure 7: Root locus of the plant

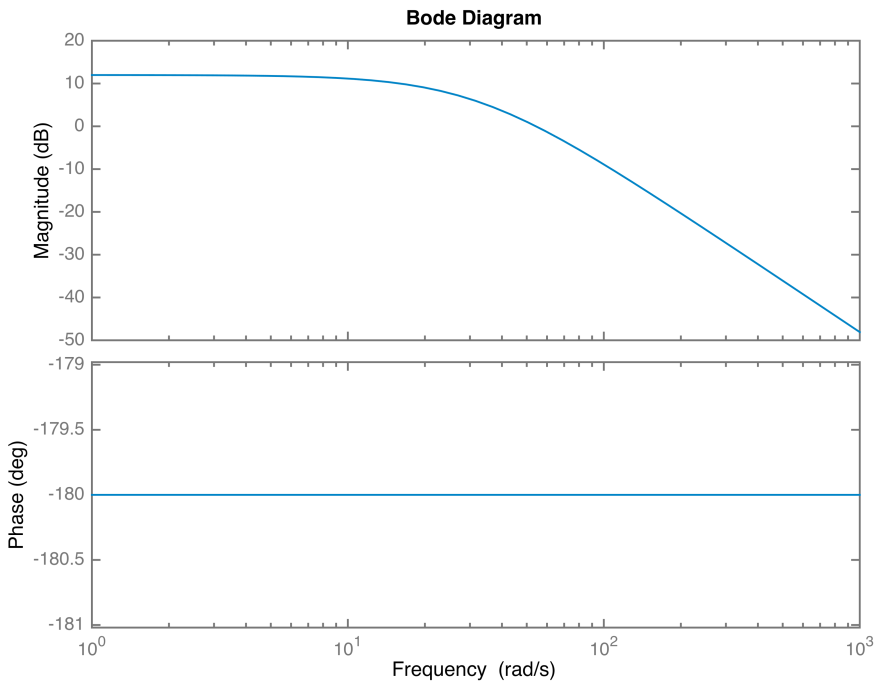

Figure 8: Frequency response of the plant

We see that the open loop system is unstable as the plant has one pole in the RHP. We also see that the root locus plot is symmetric about the imaginary axis, so for gains that are high, all poles will be on the imaginary axis, causing the system to be only marginally stable. From the frequency response, we see that the phase is always at -180º for any frequency, so the system will be unstable for unity gain at 0dB.

ii) We will add a controller of the form $G(s) = K_c \frac{1+\frac{s}{z_c}}{1+\frac{s}{p_c}}$ to improve the performance of our system. We will pick the location of z_c and p_c such that the overall open loop system has a DC gain of 2 and a pole/zero ratio of 20. To do this, we will use MATLAB's sisotool function

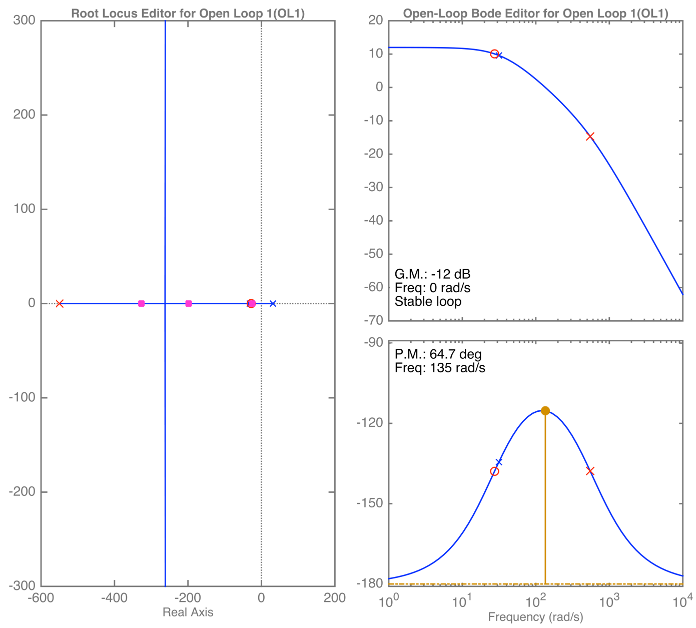

Figure 9: MATLAB's sisotool lead compensator plot

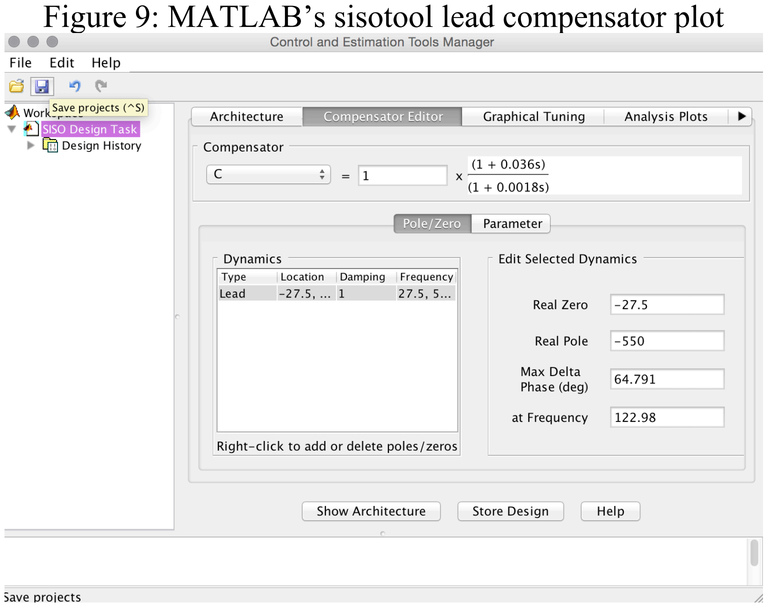

Figure 10: MATLAB's sisotool session interface showing locations of pole and zero

As shown in Figure 10, the locations of the pole and zero for the lead compensator controller was selected at -550 and -27.5, respectively.

iii) Using the derivations from the system identification part of the lab, we are able to find relationships between the unknown resistor and capacitor values:

$$R_1 = 10^4/K_c$$

$$C = \frac{1}{R_1}\left(\frac{1}{Z_c} - \frac{1}{P_c}\right)$$

$$R_2 = \frac{1}{CP_c}$$

With our values of $Z_c = -27.5$, $P_c = -550$, and $K_c = 1$, we get the following values:

$$R_1 = 10k\Omega, R_2 = 526\Omega, and\ C = 1.5\mu F$$

b) Lab:

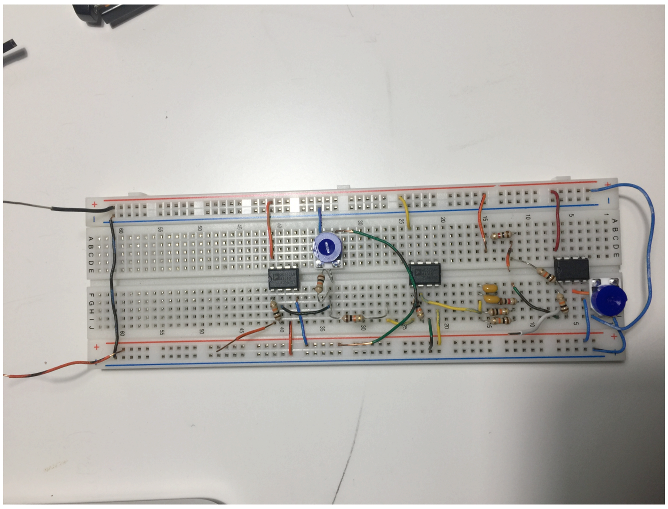

Figure 11: Breadboard with complete circuit

i) Implementation

The circuit was first implemented using the values found in part a of the second part of the lab. However, using these values yielded poor results in the sense that the controller wasn't aggressive enough, so system identification was performed again. Similar values were found, so the method of observing how the controller behaved under different resistor values was followed. It was found that decreasing the value of R_1 made the system more aggressive (i.e. more responsive in our case). Therefore, actual implementation of the circuit used $R_1 = 5k\Omega, R_2 = 500\Omega, and\ C = 1.5\mu F$. Figure 11 shows the complete circuit with the found values and Figure 1 shows the magnetic levitation system working. A video of the system working can be found at: https://www.youtube.com/watch?v=1TiKO9B2M_s