PHYS 128AL Lab Notebook

Æther Zhou

Feb. 2023

Projects & Collaborations

Lab 3 Noise

Collaborator Vivian Liao

Contents

Lab Manuel Questions

Question

What is the refresh rate of the monitor?

60 Hz

Question

Are you looking into the "base" or "emitter" of the amplifier as the resistor? Is the impedance larger or smaller on the other side? Which side of the amplifier then is more important for C as a result?

emitter (higher voltage require higher impedance)

Question

Is there any difference between the band voltage of the Short tail and the amplifier at ground coupling? Do you expect there to be?

No difference. When we shorted the analyzer, we are equalizing the electric potential on both polar, which is the same as amplifier at ground coupling.

Question

Does G converge? How fast? Is it possible to integrate over the whole of the zero to infinity range? Is it sensible to?

The band gain has a Gaussian-like distribution, it should converge as fast as a Gaussian distribution. Hence, it is possible to integrate over the whole domain. However, we don't need to, since the band gain almost dies out around 1-σ. It is more efficient to just integrate within a certain bandwidth.

Question

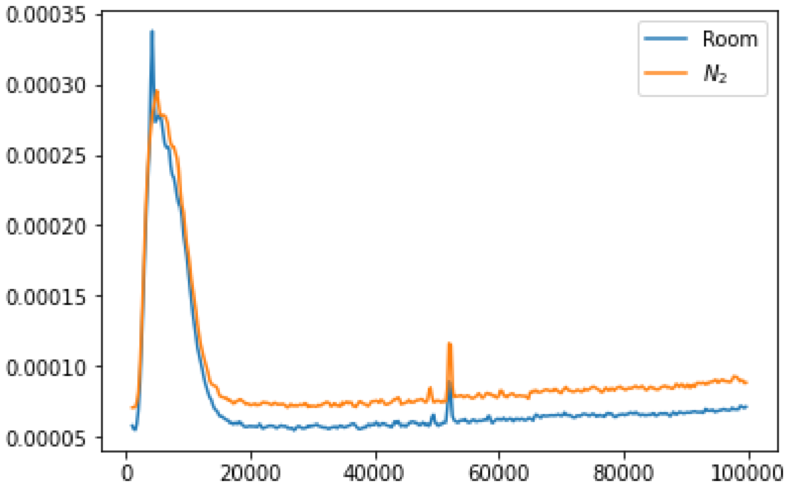
How close is your value of absolute zero? Given your result, is Johnson Noise useful for measuring temperature?

Our final result suggests the absolute zero to be $-286 \pm 16\,°\mathrm{C}$, which is within 5% of the accepted value. So it is useful to measure temperature with Johnson Noise.

Question

Is it necessary to chill the Short tail? Do you expect its band voltage to change?

For an ideally short wire, its resistance should be zero regardless of the temperature. So we expect the result to be the same.



Question

What could be done to improve your findings? What temperatures would you prefer to work in?

Our measurement are affected by the background noise (Systematic error). We'll need to repeat more trails to figure out the noise generated by the analyzer itself, so we can measure the Johnson noise as accurate as possible.

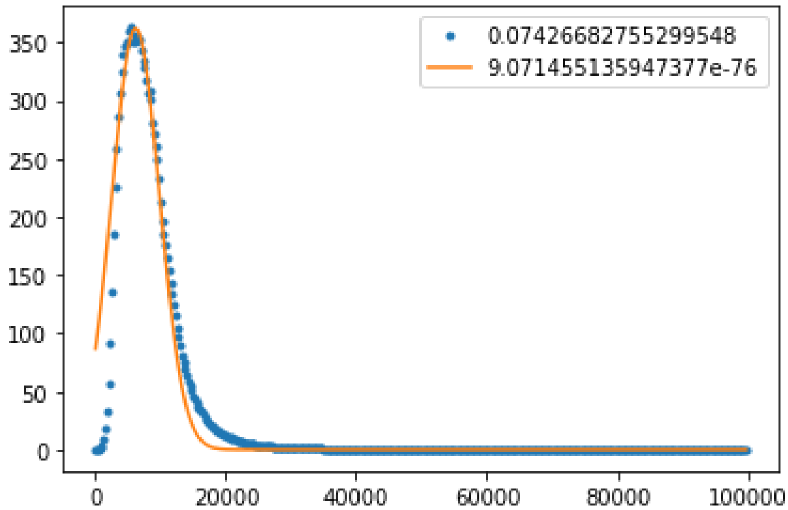
It would be preferable to work with temperature slightly higher than the room temperature. So we can further verify Nyqiust's theorem on Johnson noise.

Daily

22 Feb 2023, W

- equipment set up: noise generator, pre-amplifier, filter, spectral analyzer
- Room temperature measurement (20 ± 1 Celsius)
- using floppy disk (bad reading, not saving data)

$$g(f) = \frac{V_{out}(f)}{V_{in}}$$



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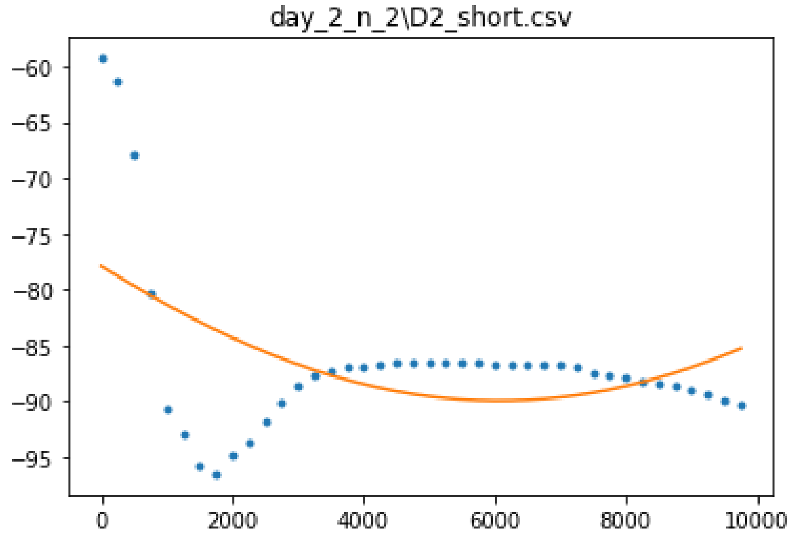
24 Feb 2023, F

- measure resistor
- control temperature using boiling liquid nitrogen (77 Celsius)
- getting negative voltage reading (negative V_{rms} doesn't make sense)



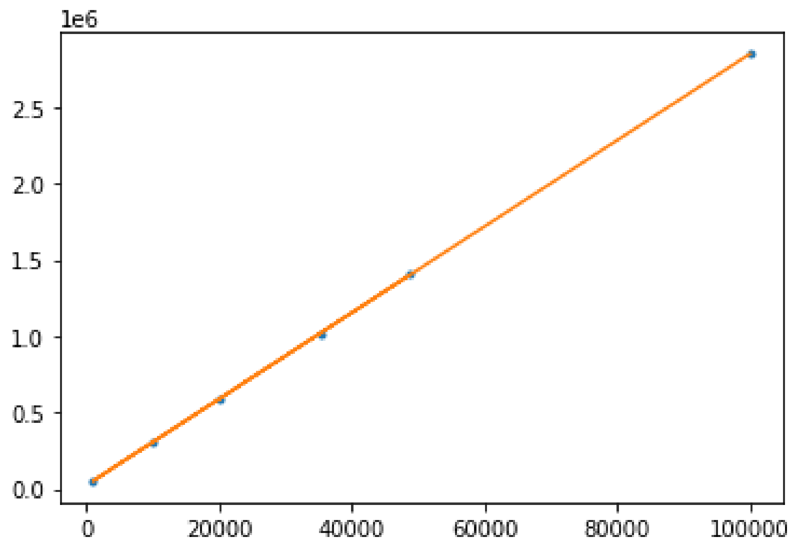
Gain, band gain, temperature

$$G = \int_0^\infty \frac{[g(f)]^2}{1+(2\pi fCR)^2}\, df$$

Found capacitance C for coaxial cable to be 60 pF/ft

$$\text{slope} = T = \frac{V^2}{4k_B GR}$$

Using peak value for V (find max for each data set)



get 28 degree Kelvin, which is wrong

possible mistake: should use rms(V) not max(V)

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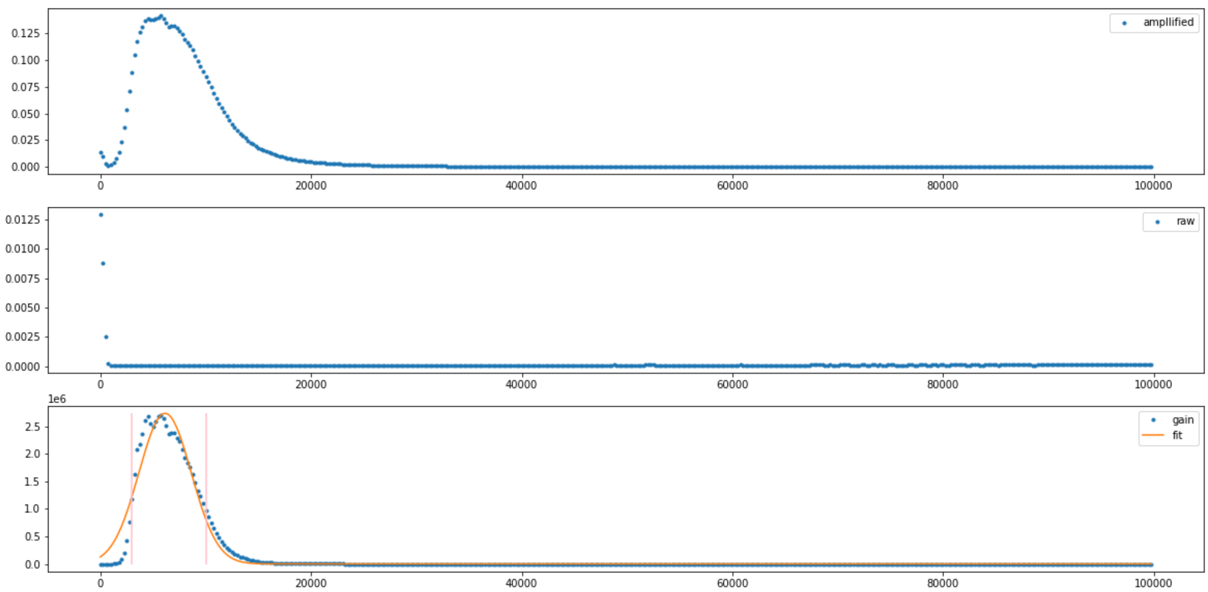
27 Feb 2023, M

Getting temperature offed by a factor of 4 measured capacitance (38 pF/m)

With scipy.curve_fit() Fitted data to get gain function (Gaussian fit)



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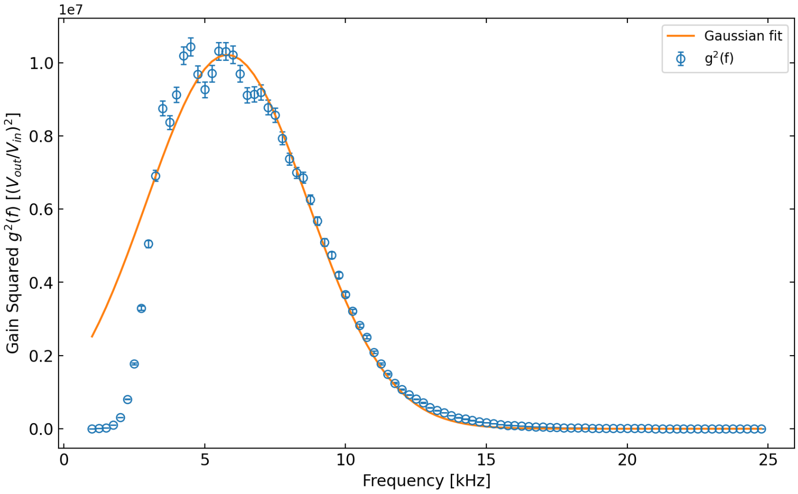
1 Mar 2023, W

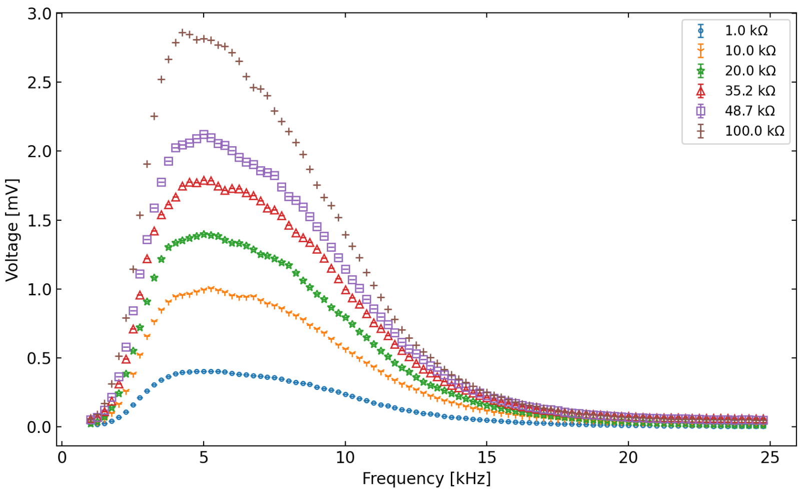
- measured R to be check the 1% error (checked)
- using band analyze function to get V^2

R [kΩ]	$V_{rms,\ 20°C}$ [mV]	$V_{rms,\ -195.79°C}$ [mV]
1	1.318	1.097
10	3.281	2.065
20	4.569	2.615
35.2	5.878	3.329
48.7	6.804	3.895
100	9.021	4.967
short	0.9291	0.9589
$\frac{\delta R}{R} = 1\%$	$\frac{\delta V_{rms}}{V_{rms}} = 0.02\%$	$\frac{\delta V_{rms}}{V_{rms}} = 0.02\%$

Raw data of the band RMS voltage over the frequency band from 3 kHz to 10 kHz of different resistors under two different temperatures, $T = 20$ °C and $T = -195.79$ °C.



Fitted gain function using the signal from white noise generator. Since $g(f)$ is independent with the signal sources, We will use this to infer the actual magnitude of Johnson noise. We only care about the region between 3 kHz and 10 kHz.



Raw Data of the RMS voltage of different resistors under room temperature $T = 20$ °C. Different colored symbols represent the data point of different resistances. Error bars are presented.

Theory and Setup

Set up and Procedure

Noise signal are pre-amplified, filtered to band of 3 kHz to 10 kHz, and fed into the spectral analyzer (FFT)

So we can see the physical property of Johnson noise.

To calculate temperature or k_B, we need first restore our data to its original amplitude. So we need the gain function and the band gain.

$$g(f) = \frac{V_{out}(f)}{V_{in}}$$

$$G = \int_0^\infty \frac{[g(f)]^2}{1 + (2\pi f C R)^2} df$$

By Nyquist

$$V^2 = 4 k_B T G R$$

Hence, by fixing k_B, we can give a measurement of temperature, and to derive the absolute zero in Celsius; and by fixing T (using boiling liquid nitrogen), we can estimate the value of k_B.

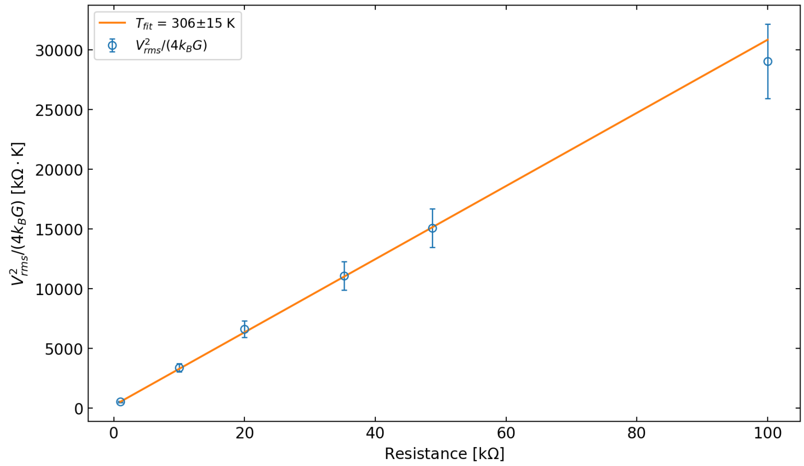
The hard part is to calculate a clean band gain. Due to the high sensitivity, the spectral analyzer tends to take up lots of irrelevant background noise.

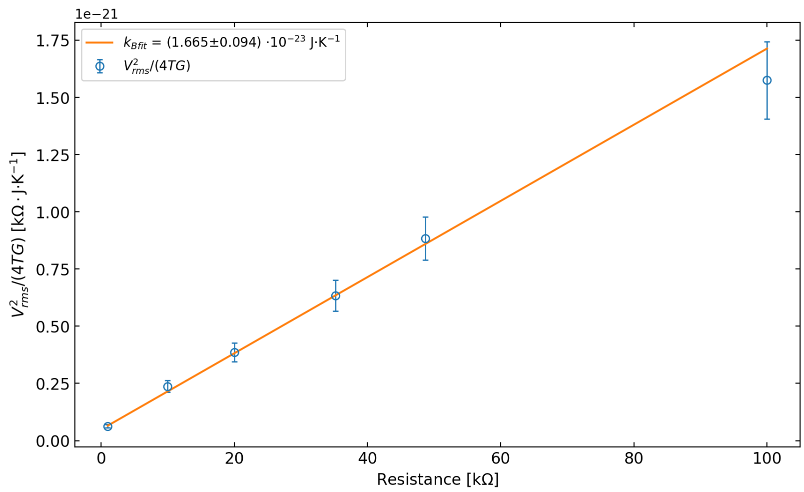
In our experiment, we took several data for background noise of the analyzer itself (when nothing is attached to it). We tried analyze that to get a descriptive V_b, so we can minus it from the other signal data we took, and get the clean signal with background noise blocked.

Result and Analysis

Experiment Result

By measuring the thermal noises of a series of shorted resistors with a filtered amplifier, we estimated absolute zero temperature to be $-286 \pm 16\,^\circ\mathrm{C}$, in agreement with the accepted value $-273.15\,^\circ\mathrm{C}$. We also calculated the Boltzmann constant by controlling the temperature of the resistors. However, our calculated value $(1.665 \pm 0.094) \times 10^{-23}\,\mathrm{J} \cdot \mathrm{K}^{-1}$ deviates from the accepted value $1.381 \times 10^{-23}\,\mathrm{J} \cdot \mathrm{K}^{-1}$ by 21%. A possible cause for this discrepancy is the variation in background noise, which we will explore further in the Error Analysis section.





Error Analysis

Error propagation follows these equations:

$$\delta g = \sqrt{\left(\frac{\partial g}{\partial V_{in}}\right)^2 (\delta V_{in})^2 + \left(\frac{\partial g}{\partial V_{out}}\right)^2 (\delta V_{out})^2} \tag{1}$$

$$\delta G = \sqrt{\left(\frac{\partial G}{\partial g}\right)^2 (\delta g)^2 + \left(\frac{\partial G}{\partial R}\right)^2 (\delta R)^2 + \left(\frac{\partial G}{\partial C}\right)^2 (\delta C)^2} \tag{2}$$

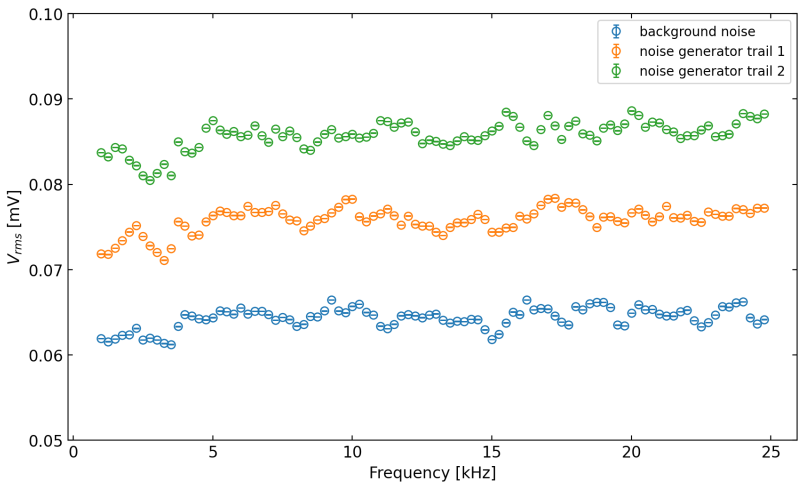
$$\delta T = \sqrt{\left(\frac{\partial T}{\partial G}\right)^2 (\delta G)^2 + \left(\frac{\partial T}{\partial R}\right)^2 (\delta R)^2 + \left(\frac{\partial T}{\partial V}\right)^2 (\delta V)^2} \tag{3}$$

$$\delta k_B = \sqrt{\left(\frac{\partial k_B}{\partial G}\right)^2 (\delta G)^2 + \left(\frac{\partial k_B}{\partial R}\right)^2 (\delta R)^2 + \left(\frac{\partial k_B}{\partial V}\right)^2 (\delta V)^2} \tag{4}$$

To measure absolute zero temperature, we used Equation 3 to compensate for the difference between our measured results and the accepted results. However, when estimating the Boltzmann constant, the discrepancy exceeded our propagated error bars.

We deduced that the uncertainty in capacitance (C) cannot account for the upper shift in our results. Since our circuit and coaxial cables are connected in series, the total effective capacitance can only decrease, leading to an increase in gain and a decrease in the slope of the linear fit used to measure k_B. Furthermore, the uncertainty in resistance (R) is not significant enough to explain the deviation in our measurement.

Thus, we speculate that the calculated gain function may be inaccurate due to the presence of ambient background noise from the equipment. Notably, the background noise we measured was comparable to the un-amplified signal from the noise generator. Additionally, the measurement of white noise showed significant deviation before and after we conducted other measurements.



Measurement of background noise of the spectral analyzer, and unprocessed signal from the random noise generator. We conducted trail 1 before measuring Johnson noise of the resistors, and trail 2 after we finished them. This plot suggests that our measurement of gain might not be stable, and the background noise will affect our system significantly.

Discussion

The calculated Boltzmann constant in our experiment exhibits a significant discrepancy compared to the accepted value, with a large error. After careful consideration and experimentation, we have eliminated possible error sources from capacitance and resistance measurements. Our analysis points to the gain function $g(f)$ as the main contributor to the large error. As described above, the gain is calculated by dividing the output voltage V_{out} by the input voltage V_{in}. We can measure V_{out} with good accuracy, as its value is sufficiently large. However, V_{in} is too small in comparison to the background noise, leading to inaccurate measurements and resulting in the observed large error. One possible solution to this issue is to increase the output voltage of the white noise generator, thereby decreasing the gain of the pre-amplifier. This would allow the V_{in} signal to increase, making it easier to separate from the background noise.