Problem 9: Magnetic Levitation

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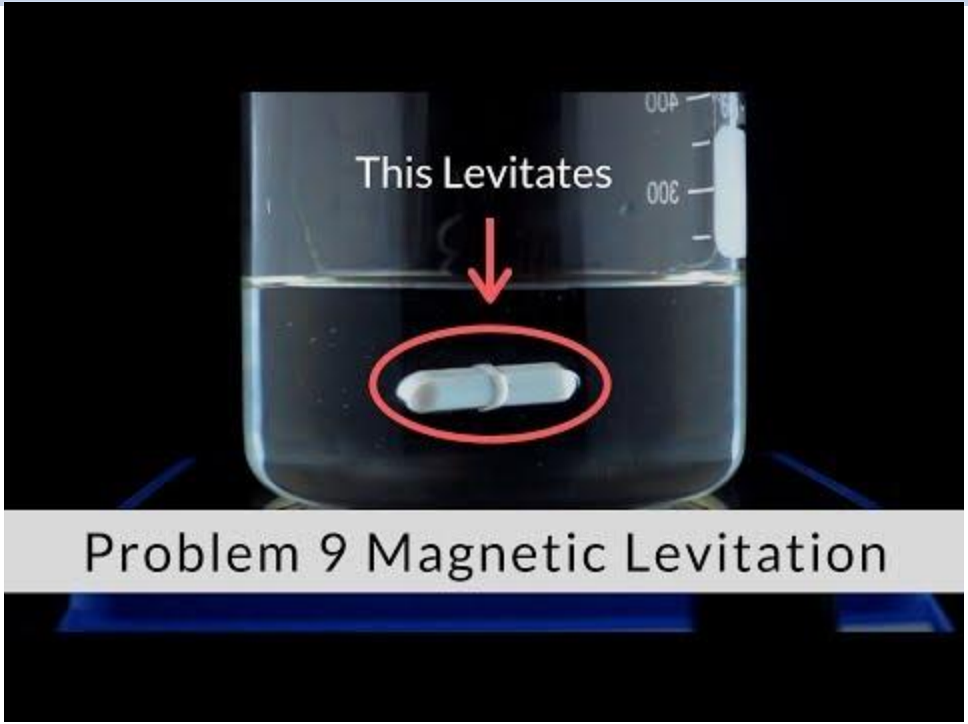
Interpretation of Task

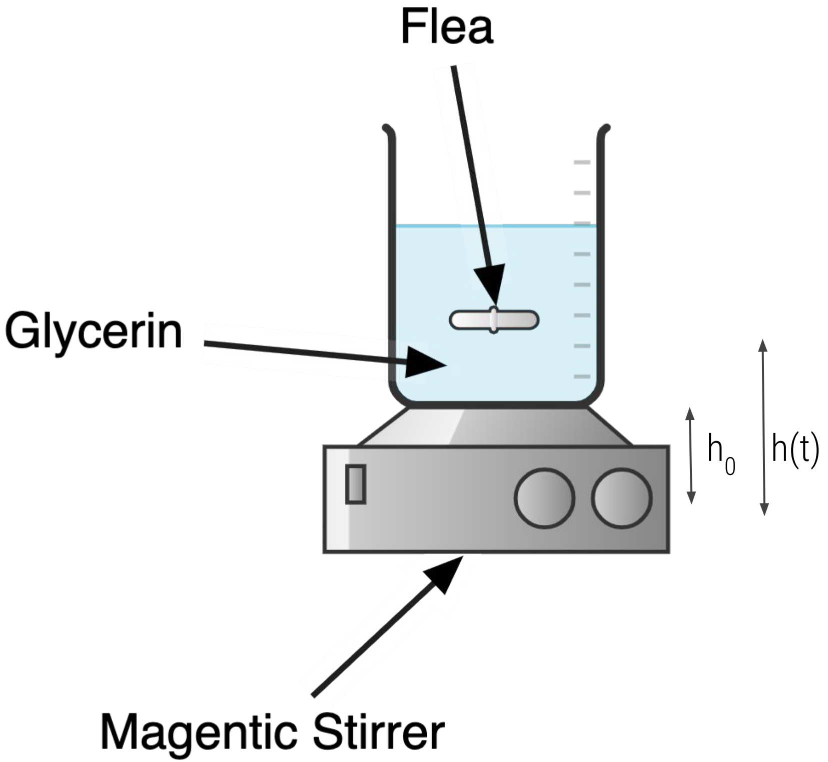
Under certain circumstances, the “flea” of a magnetic stirrer can rise up and levitate stably in a viscous fluid during stirring. Investigate the origins of the dynamic stabilization of the “flea” and how it depends on the relevant parameters.

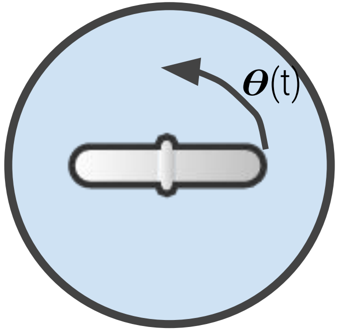
Phenomenon



- Angular Motion
- Vertical Oscillation
- Dynamic Stabilization

Apparatus





h_0 = *initial vertical displacement*

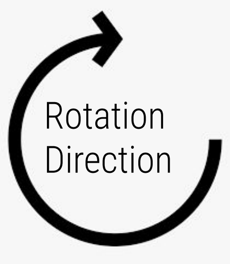
$h(t)$ = *vertical displacement*

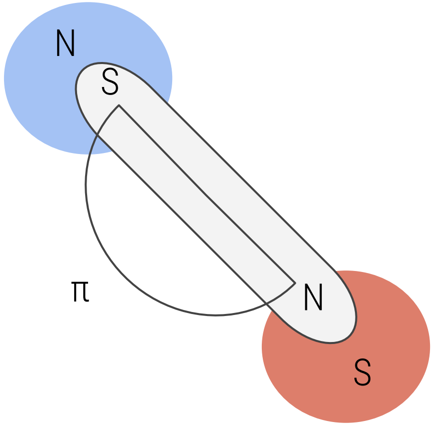
$\theta(t)$ = *angular displacement*

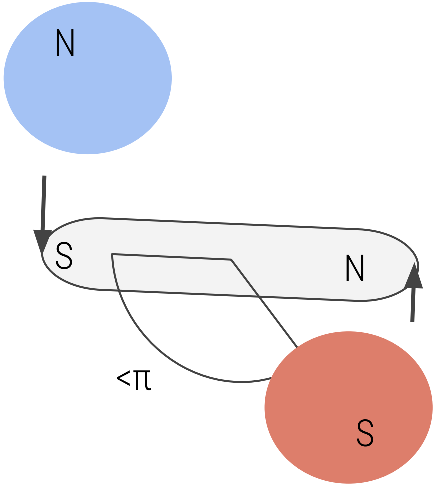
l = *length of flea*

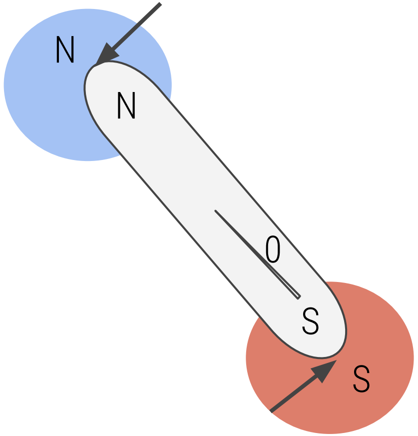
η = *viscosity of solution*

Initial Cause



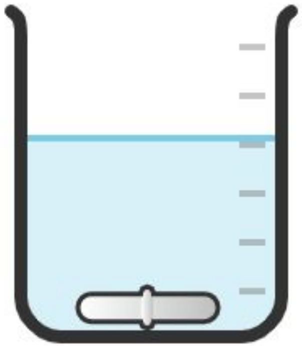




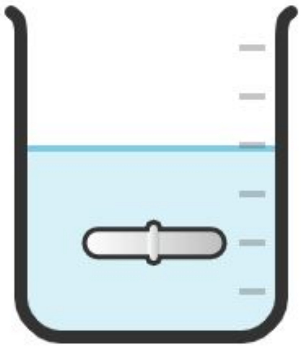


Types of Motion

Chaotic Hopping



Grounded
Asynchronous Motion



Levitating
Asynchronous Motion

Regimes

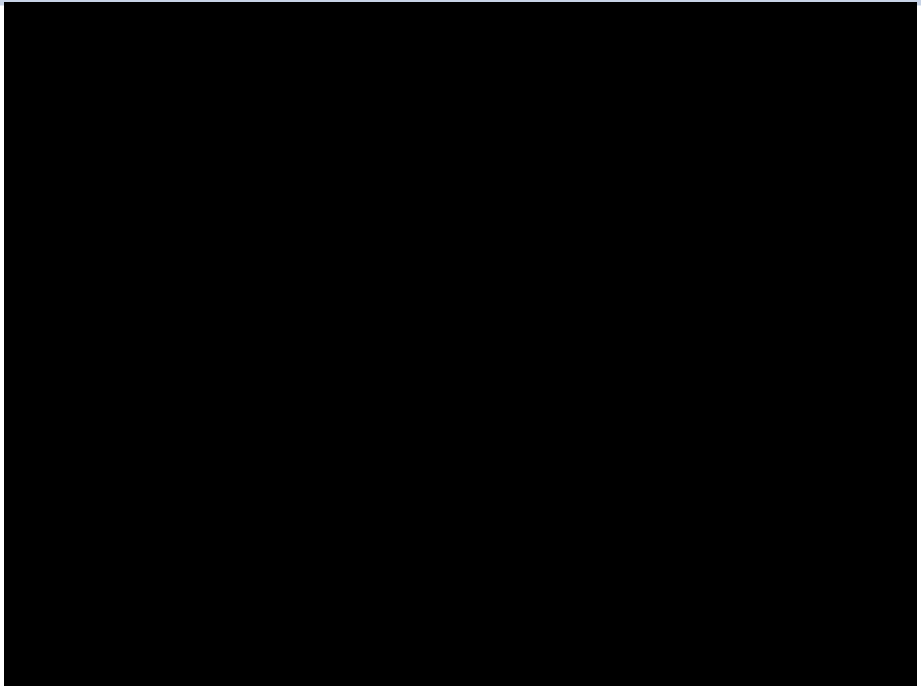
Parameters	Viscosity less than critical value ($\eta < \eta_c$)	Viscosity greater than critical value ($\eta > \eta_c$)
Height less than critical value ($h_0 < h_c$)	**Chaotic Hopping**	**Levitating Asynchronous Motion**
Height greater than critical value ($h_0 > h_c$)	**Chaotic Hopping**	**Grounded Asynchronous Motion**

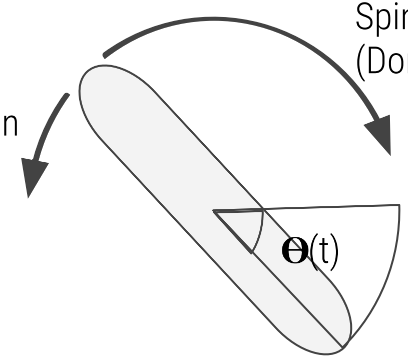
h_c = 37 ± 1 mm

η_c = 0.34 ± 0.09 Pa s

Angular Motion

Phenomenon




Spin motion
(Dominating motion)
Waggle Motion
θ(t)
Angular
Displacement

Net Force

I = moment of inertia
m = mass
l = length of stirrer
D = drag constant
C = geometric factor
μ = viscosity
M = magnetic coupling
μ = magnetic constant
a = magnetic dipole moment

Inertia (I)

$$= I \times \theta''$$

$$= [\tfrac{1}{12} \times m \times l^2] \times \theta''$$

$$= [2.45 \cdot 10^{-7}] \times \theta''$$

Drag Force (D)

$$= D \times \theta'$$

$$= [8\pi \times C \times \eta \times (\tfrac{l}{2})^3] \times \theta'$$

$$= [1.03 \cdot 10^{-5}] \times \theta'$$

Magnetic Force (M)

$$= M \times sin(\theta - \omega_d t)$$

$$= [\frac{\mu \times a}{4\pi h(t)^3}] \times sin(\theta - \omega_d t)$$

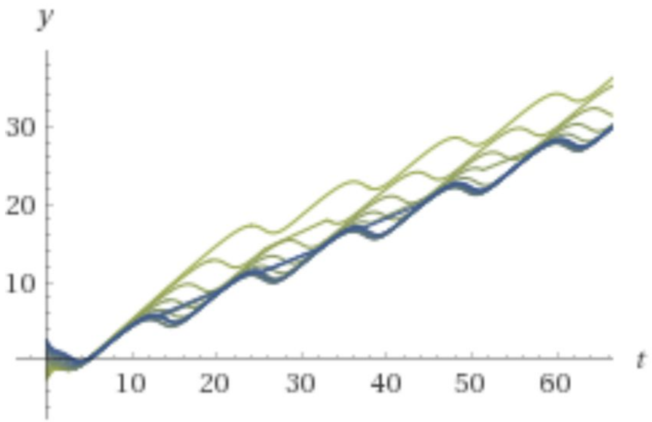
$$= \frac{2.5 \cdot 10^{-8}}{h^3} \times sin(\theta - \omega_d t)$$

D. Kong, W. Lin, Y. Pan, and K. Zhang, Front. Microbiol. 5, 41 (2014).

Theoretical Equation

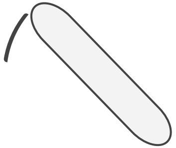
$$(2.45 \cdot 10^{-7})\,\theta'' + (1.03 \cdot 10^{-5})\,\theta' - \left(\frac{2.5 \cdot 10^{-8}}{h^3} \mathit{sin}(\theta - \omega_d t)\right) = 0$$

Sample solution family:



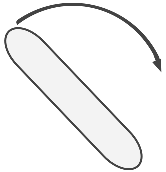
(sampling $y(0)$ and $y'(0)$)

General Equation



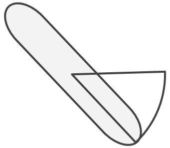
Amplitude

Spin Speed

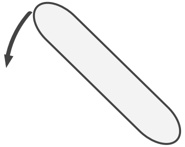


$$\theta(t) = A\, sin(\omega_w t) + \omega_s t$$

Angular Displacement



Waggle Speed



Frequency = 13 radians/second

$0.41\sin(11.46x)+1.79x$

Frequency = 14 radians/second

$0.38\sin(14.4x)+1.47x$

Frequency = 15 radians/second

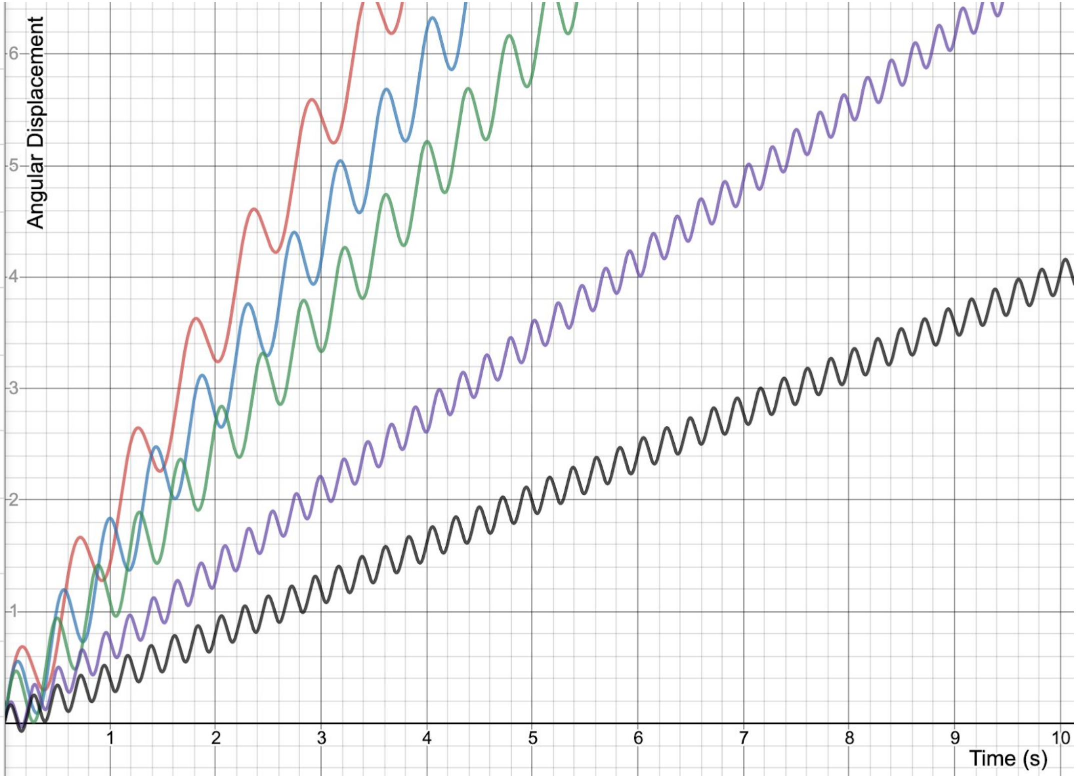
$0.34\sin(16.14x)+1.22x$

Frequency = 17 radians/second

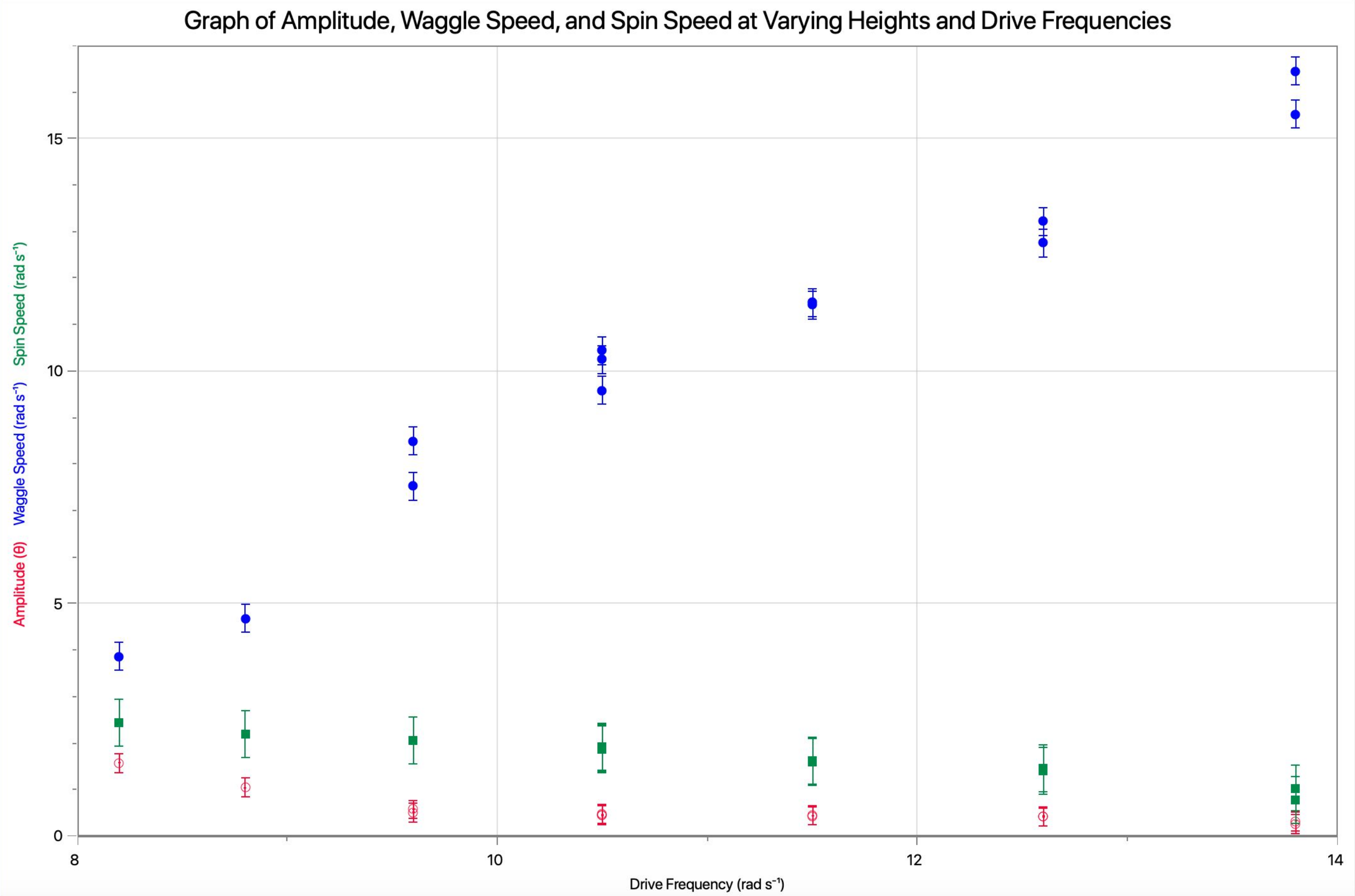
$0.15\sin(27.87x)+0.69x$

Frequency = 19 radians/second

$0.14\sin(28.30x)+0.40x$



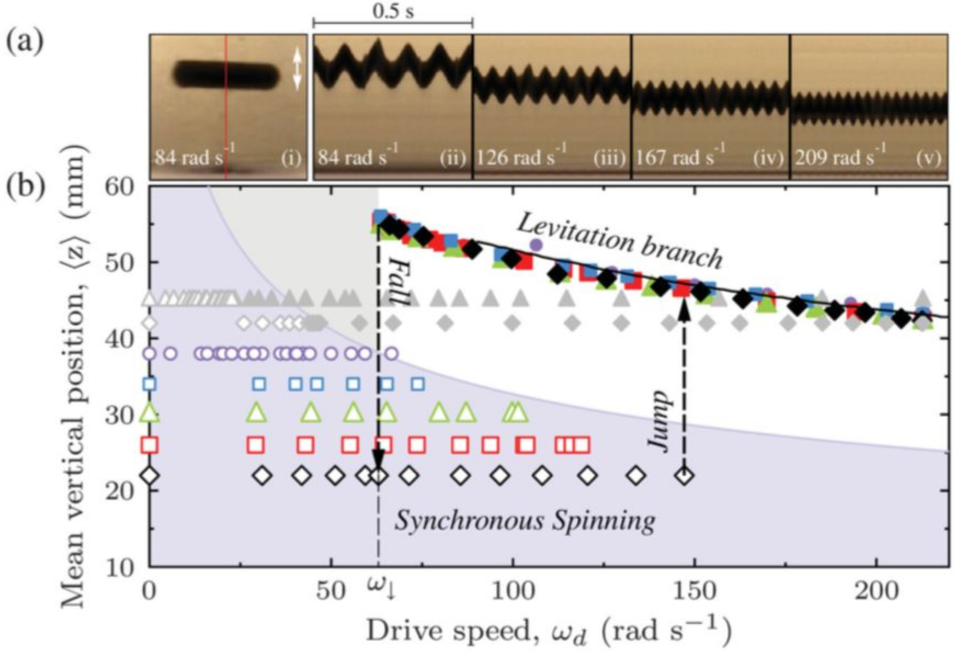
Graph of Angular Motion at Varying Frequencies


Graph of Amplitude, Waggle Speed, and Spin Speed at Varying Heights and Drive Frequencies
Amplitude (θ)
Waggle Speed (rad s⁻¹)
Spin Speed (rad s⁻¹)
0
5
10
15
8
10
12
14
Drive Frequency (rad s⁻¹)

Vertical Oscillation

Mean Vertical Height

- Oscillation caused by drive magnet overtaking flea
- As Drive speed increases mean height decreases

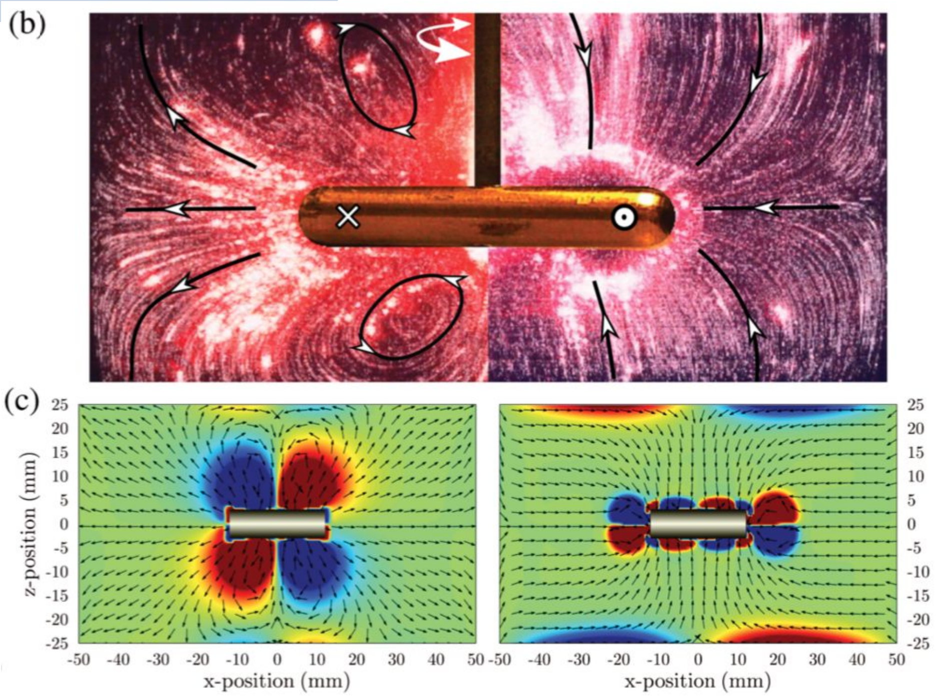


Baldwin, et al. "Magnetic Levitation Stabilized by Streaming Fluid Flows." *Phys. Rev. Lett.*, vol. 121, no. 6, 8 Aug. 2018, DOI:10.1103/ PhysRevLett.121.064502. Accessed 25 Mar. 2020.

Dynamic Stabilization

Stabilization

- Investigation of direction of force of waggle based flows
- $\mathrm{Re}_s = 2A^2 l^2 \rho \omega_w / \eta,$
- Re_s=11.7±0.4 and 400±12



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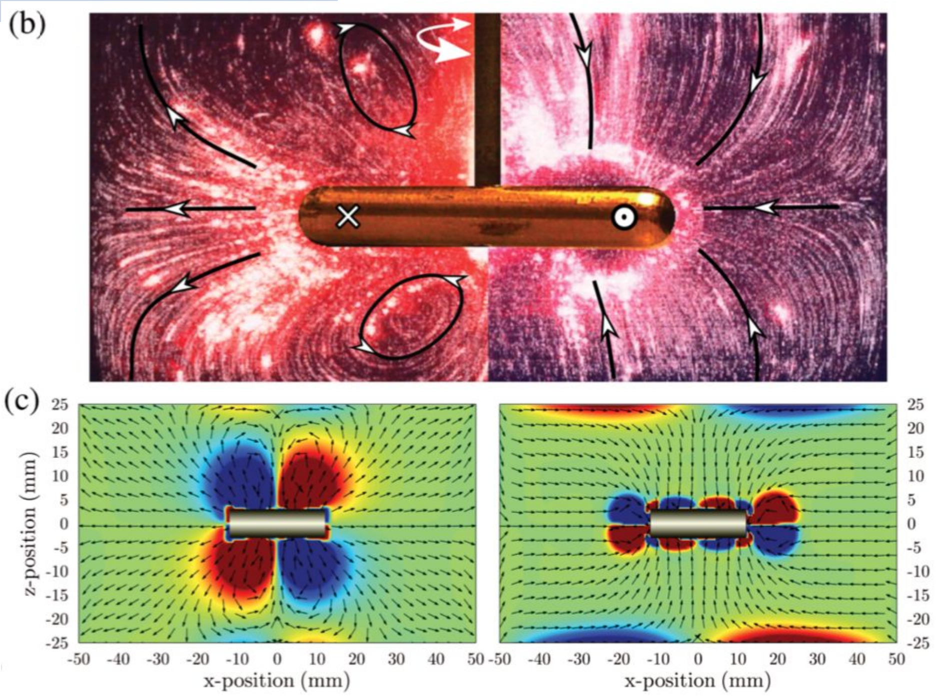
Stabilization

- At low Re_s fluid flows outward (Force = -0.58mN)
- At high Re_s fluid flows inward (Force = 0.26mN)



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Conclusion

- Theoretical Equation using net force
- General Trends shown by experimental data
- Mean Height decreases as drive speed increases
- Stabilization caused by outwards flows at low Re_s

“ Thank You for Listening