Weighing Time

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Team: The Certainty Principle

Problem Statement

It is commonly known that an hourglass changes its weight (as measured by a scale) while flowing. Investigate this phenomenon.

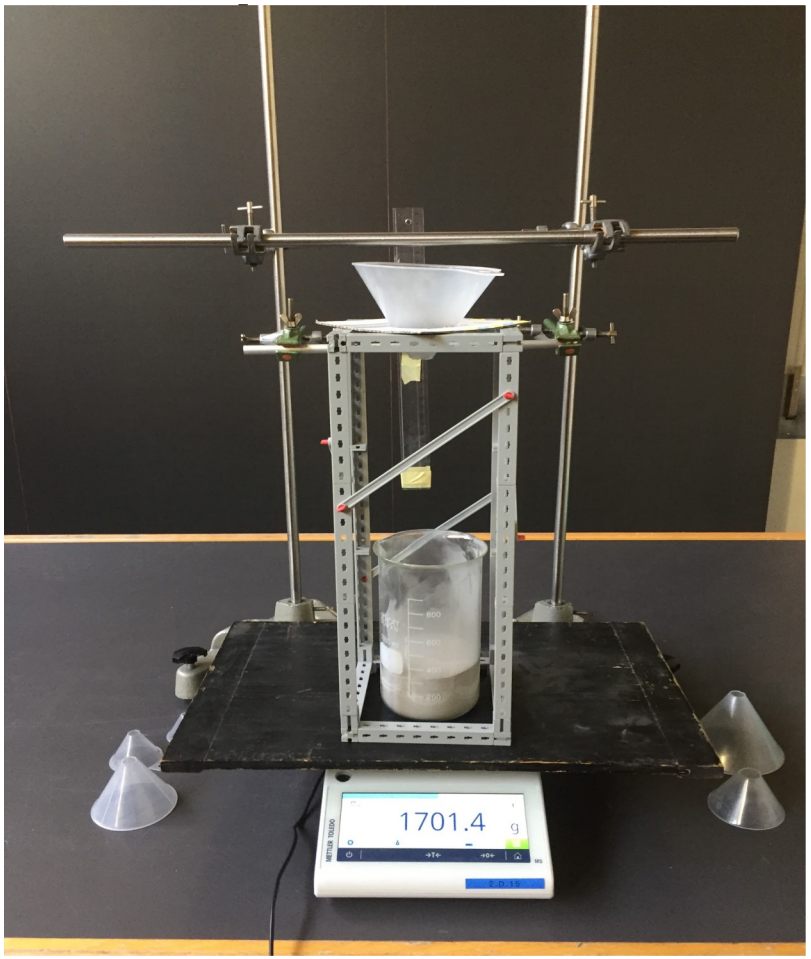
Interpretation

To find the factors influencing the change in Weight of an hourglass and to approximate how the weight changes

Equipment

- Different Sized Funnels
- Clamps
- Beaker
- Weighing Scale
- Stands
- Structure to hold funnel
- Sand
- Mechanism to stop flow of sand

Experimental



Assumptions

- Gravity is the only Force acting on the sand
- The sand does not spray when it impacts at the bottom

Constants and Independent Variable

w = initial weight

L = Height from opening

t = time

Q = Flow rate

r_2 = radius of bottom container

r_1 = radius of opening

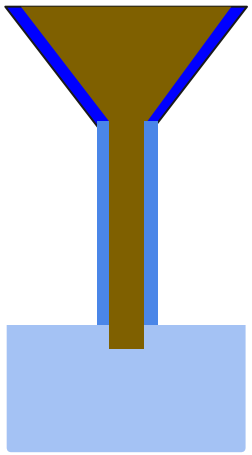
V_1 = Initial Velocity

V_2 = Final Velocity

Governing Equations

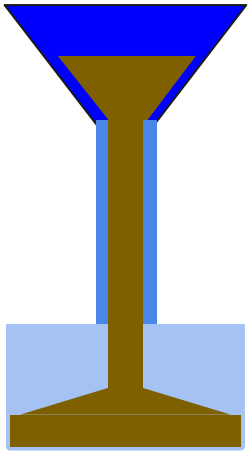
Stage 1: Freefall

- From start to first impact
- Weight constantly decreasing



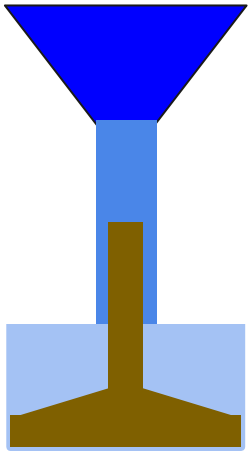
Stage 2: Impact + Freefall

- From first impact to when all particles have left top compartment



Stage 3: Impact + Decreasing Freefall

- From when all particles have left top compartment to all sand at bottom
- Weight is increasing



Time Frames

w = initial weight

L = Height from opening

t = time

Q = Flow rate

r_2 = radius of bottom container

r_1 = radius of opening

V_1 = Initial Velocity

V_2 = Final Velocity

Stage 1: Freefall

When first particle hits bottom

$$d = v_1 t + \frac{1}{2} g t^2$$

$$0 < t < \sqrt{\frac{L}{\left(\frac{g}{(2)}\right)}}$$

Stage 2: Impact + Freefall

When the last particle passes through the opening

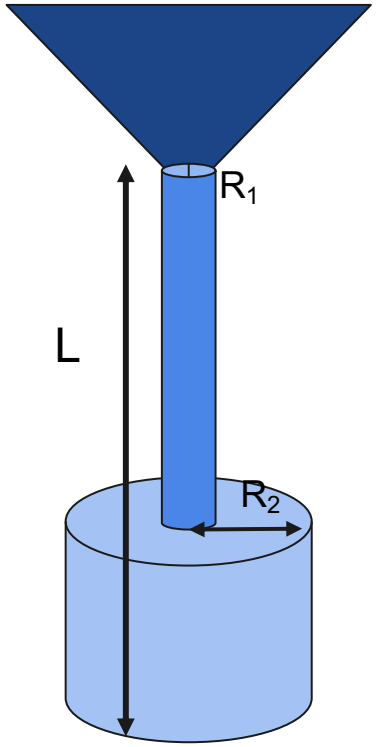
$$\sqrt{\frac{L}{\left(\frac{g}{(2)}\right)}} < t < \frac{w}{Q}$$

Stage 3: Impact + Decreasing Freefall

$$d = v_1 t + \frac{1}{2} g t^2$$

$$L - H(t) = V_1\left(t - \frac{w}{Q}\right) + \left(\frac{1}{2}\right)(g)t^2$$

$$\frac{w}{Q} < \text{"InterSection"}$$



w = initial weight

L = Height from opening

t = time

Q = Flow rate

r_2 = radius of bottom container

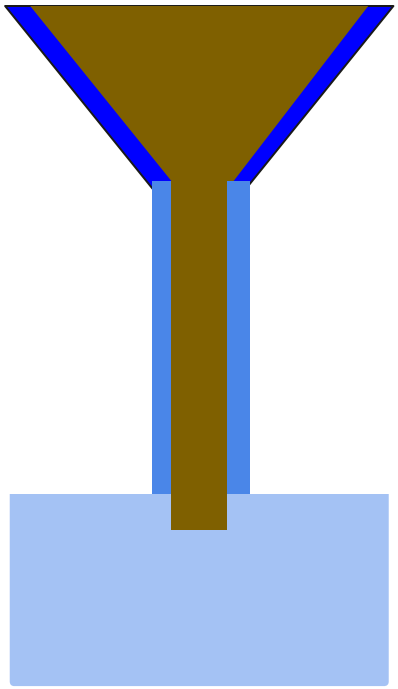
r_1 = radius of opening

V_1 = Initial Velocity

V_2 = Final Velocity

Stage 1: Freefall

$$W(t) = w - (Q * t)$$



w = initial weight L = Height from opening

t = time Q = Flow rate

r_2 = radius of bottom container r_1 = radius of opening

V_1 = Initial Velocity V_2 = Final Velocity

Stage 2: Height of pile

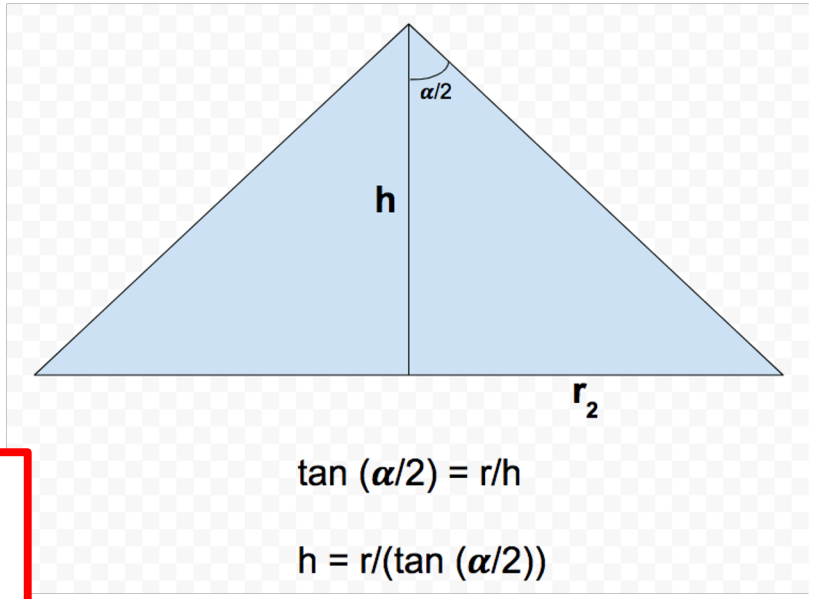
$$Q \cdot \left(t - \sqrt{\frac{L}{\left(\frac{g}{2}\right)}} \right) = Volume \cdot \rho$$

Volume = $\frac{1}{3}\pi r_2^2 h$

Volume = $\frac{1}{3}\pi r_2^2 \left(\frac{r}{\tan\left(\frac{\alpha}{2}\right)} \right) + \pi r_2^2 h$

H(t)= $\frac{3Q\left(t - \sqrt{\frac{L}{490}}\right)}{\pi\left(r_2^2\right)d}$

$\frac{Q\left(t - \sqrt{\frac{L}{490}}\right)}{\pi r_2^2 d} + \frac{2\left(\frac{r_2}{\tan\left(\frac{\alpha}{2}\right)}\right)}{3}$



tan (α/2) = r/h

h = r/(tan (α/2))

Stage 2: Impact

w = initial weight

L = Height from opening

t = time

Q = Flow rate

r_2 = radius of bottom container

r_1 = radius of opening

V_1 = Initial Velocity

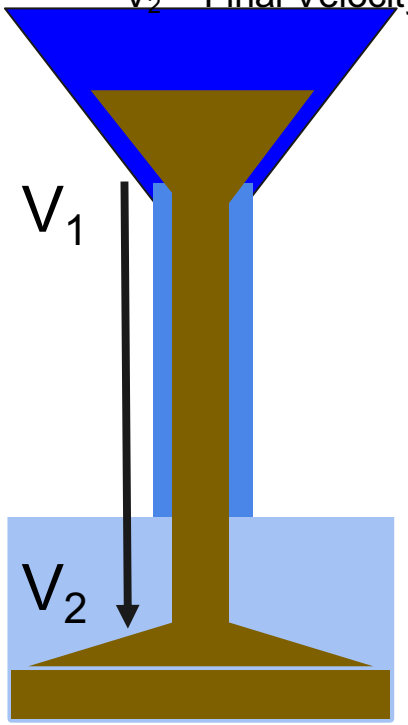
V_2 = Final Velocity

F = m * a

Impact mass = Q * v_2 /g

$$V_2^2 = V_1^2 + 2gd$$

$$Q \cdot \frac{\sqrt{(V_1)^2 + 2(g)(L - H(t))}}{g}$$



w = initial weight

L = Height from opening

t = time

Q = Flow rate

r_2 = radius of bottom container

r_1 = radius of opening

V_1 = Initial Velocity

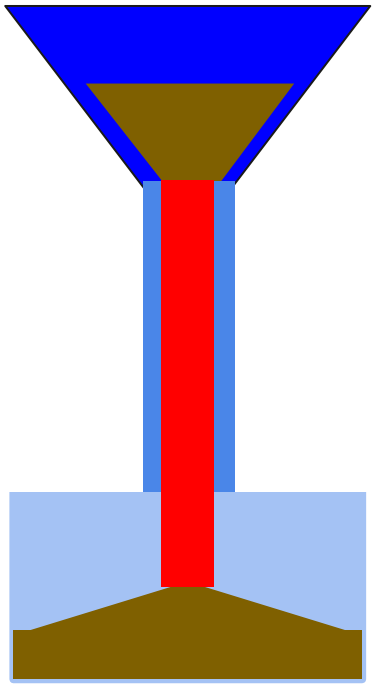
V_2 = Final Velocity

Stage 2: Freefall

Amount in Freefall = Q * time for sand to travel

$$d = v_1 t + \frac{1}{2} g t^2$$

$$Q \cdot \frac{\left(-V_1 + \sqrt{V_1^2 + 2g\left(L - H(t)\right)}\right)}{g}$$



Stage 2: Impact + Freefall

$$W(t) = w + \cdot Q \cdot \frac{\sqrt{(V_1)^2 + 2(g)(L - H(t))}}{g} \text{ ---- } Q \cdot \frac{\left(-V_1 + \sqrt{V_1^2 + 2g(L - H(t))}\right)}{g}$$

$$W(t) = w + \frac{(Q \cdot V_1)}{g}$$

Flow rate > 0

Initial Velocity > 0

$g \geq 0$

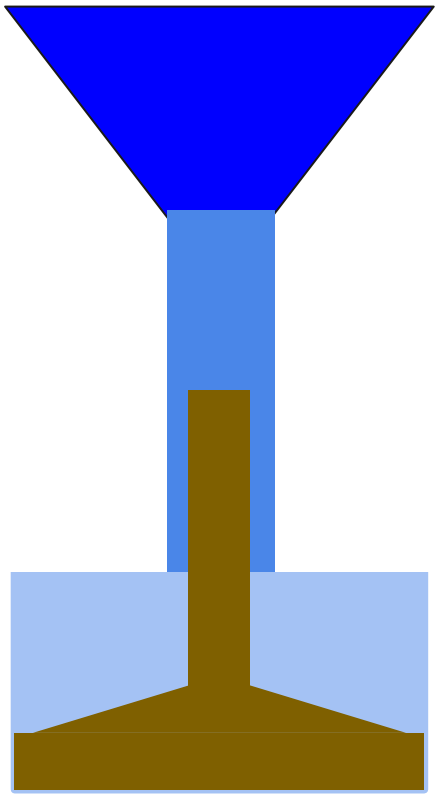
Stage 3: Impact + Decreasing Freefall

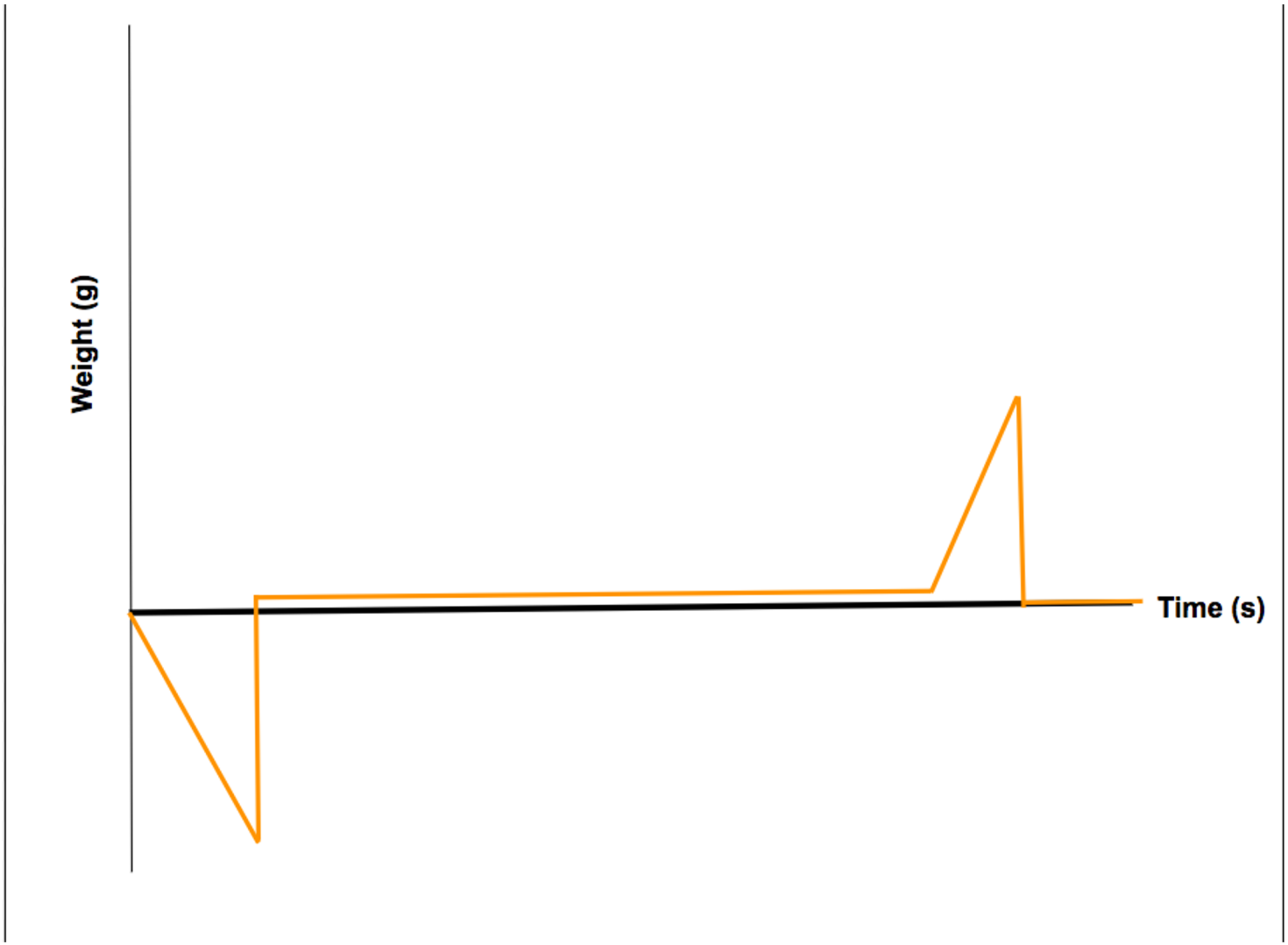
The highest point of weight is when the final particle hits the bottom

$$F = m \cdot a = Q \cdot V_2$$

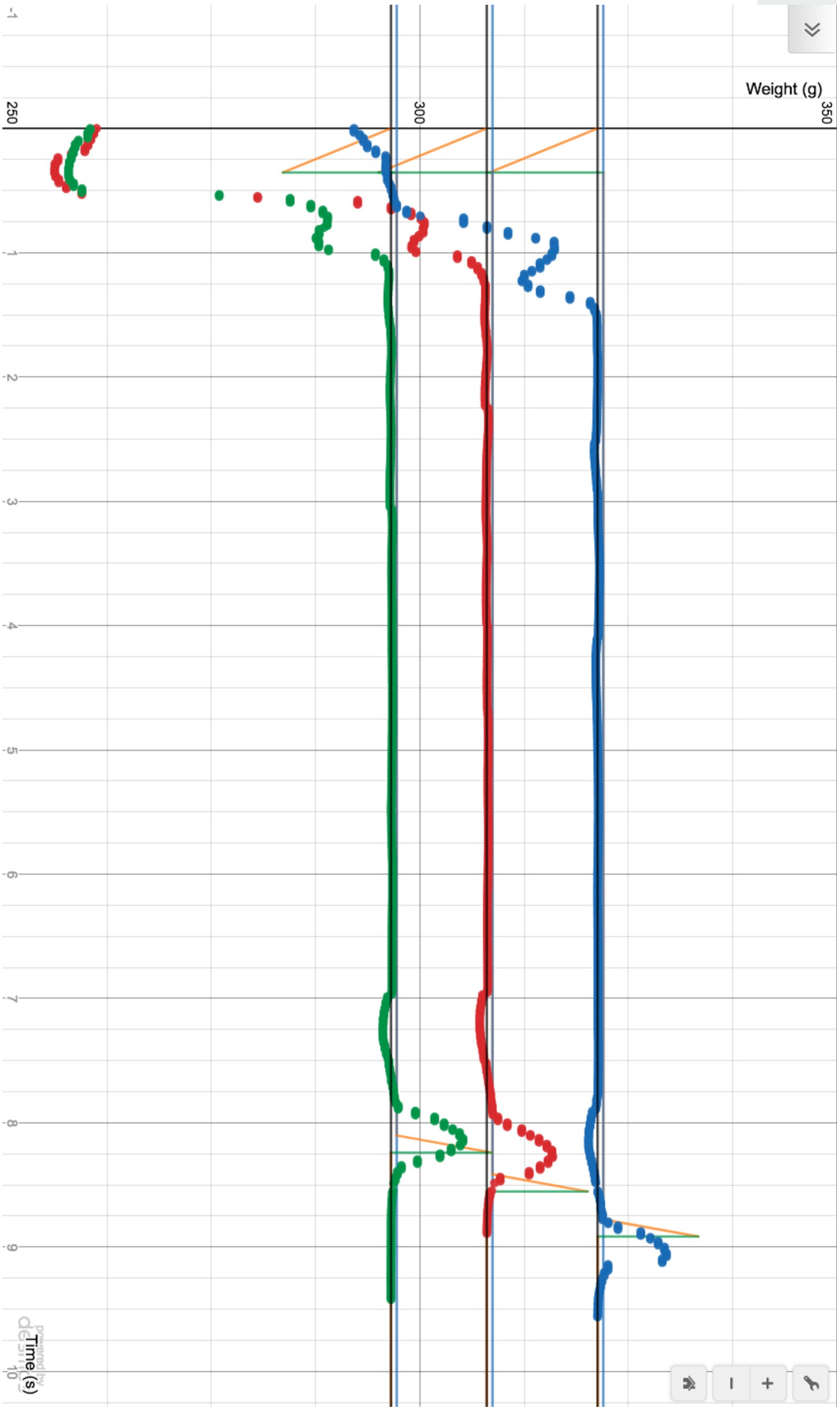
$$V_2^2 = V_1^2 + 2gd$$

$$F = \frac{\left(\sqrt{V_1^2 + 2g\left(L - H(\text{"Final time"})\right)} \cdot Q\right)}{980} + w$$

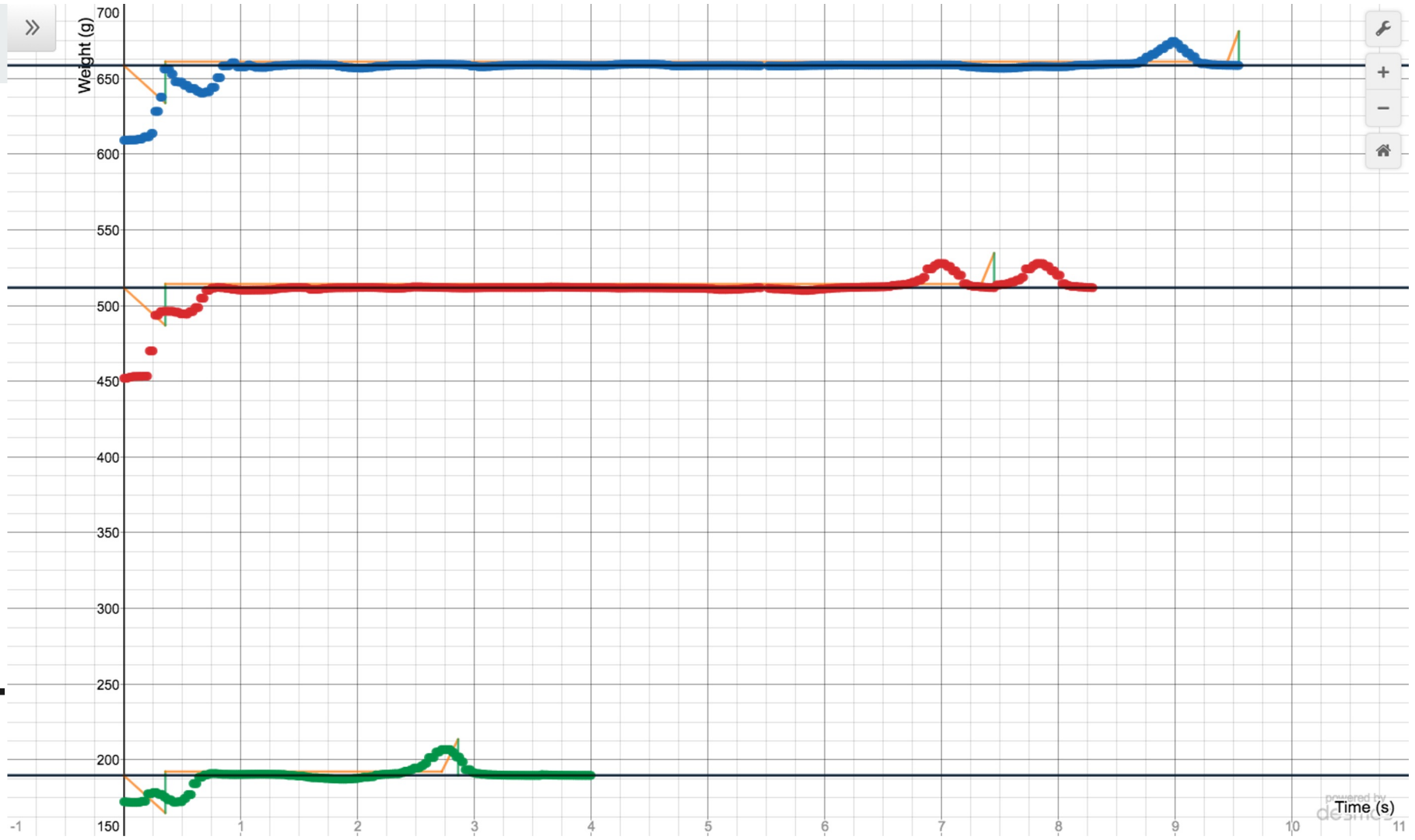



Weight (g)
Time (s)

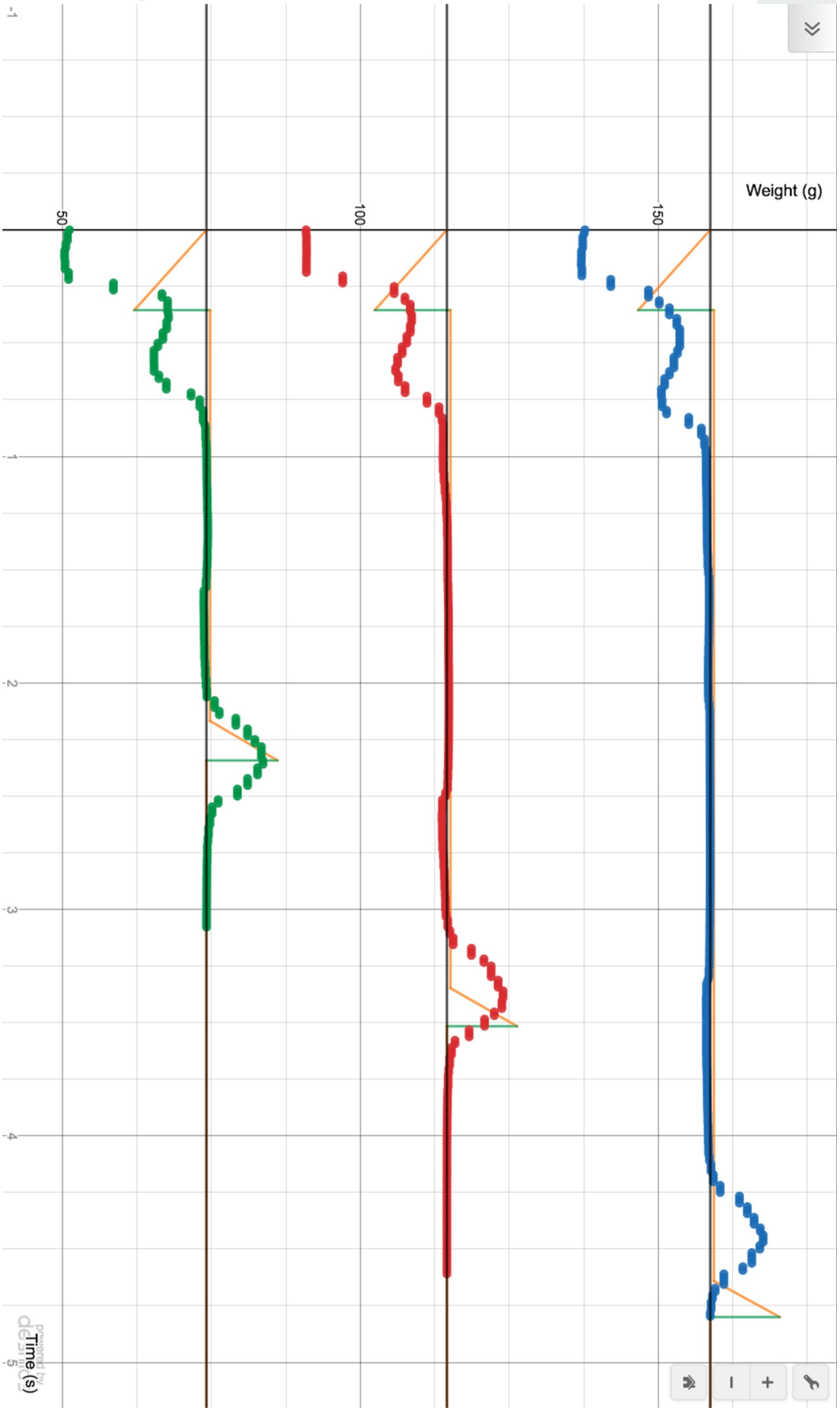
Comparison: Funnel 13mm



Comparison : Funnel 16mm

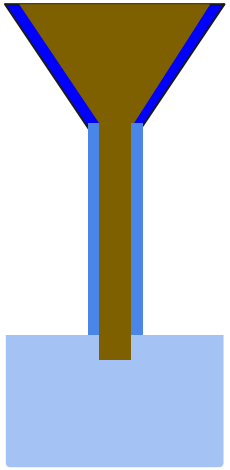


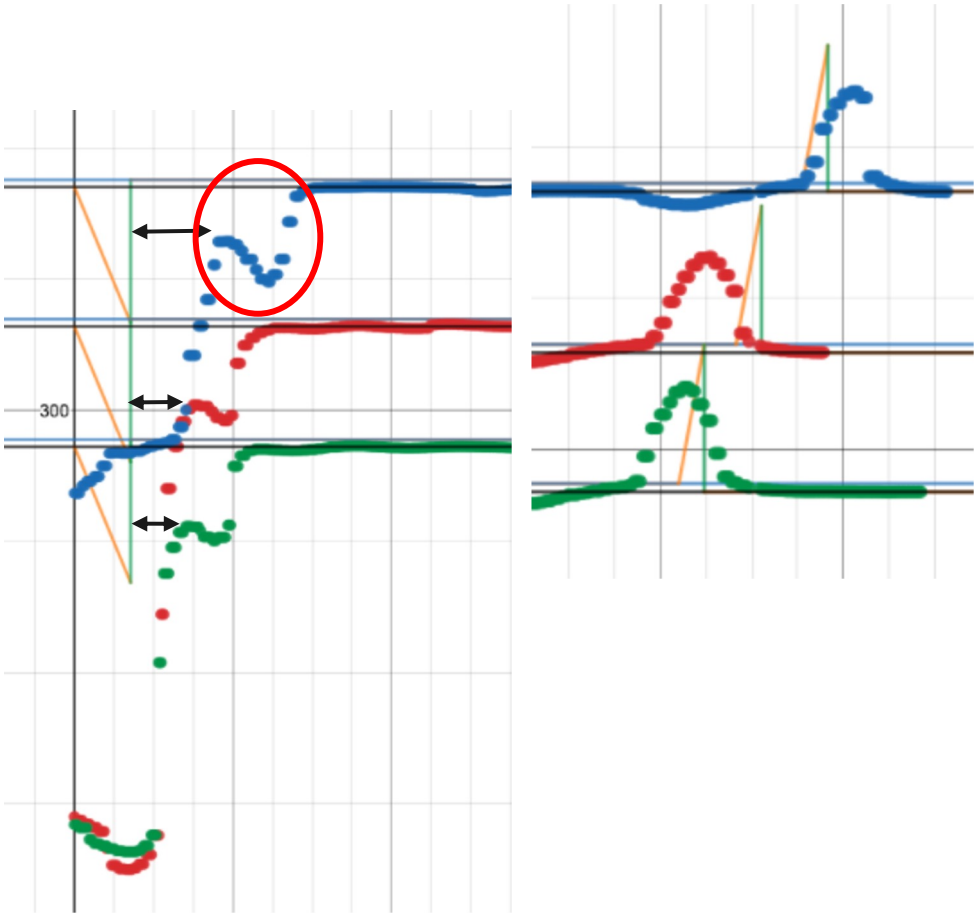
Comparison : Funnel 12mm



Comparison

- Offset in Data
- Fluctuation during start
- Initial Weight
- Difference in Stage 3



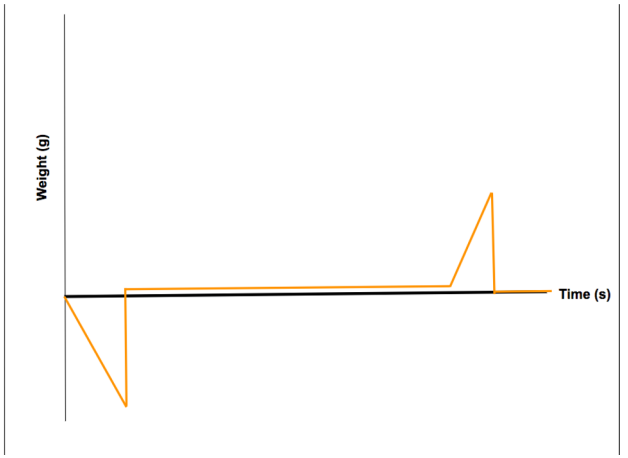


Summary

The results can be split into 3 sections based on factors influencing the weight.

The total weight of the system does indeed vary from its original weight

Crucial factors that influence how much it varies are: Density of Sand, Radius of opening and base, height of column, and the Flow Rate



Bibliography

Sack, Achim, and Thorsten Pöschel. “Weight of an Hourglass—Theory and Experiment in Quantitative Comparison Featured.” *American Journal of Physics*, Jan. 2017. *American Journal of Physics*, doi:10.1119/1.4973527.

Solid Chechen Wood Hourglass With Smooth Spindles. *Just Hourglasses*, www.justhourglasses.com/products/solid-chechen-wood-hourglass-with-smooth-spindles?variant=22334476483.

Thank You for your attention