14. Looping Pendulum

Kedar Krishnan

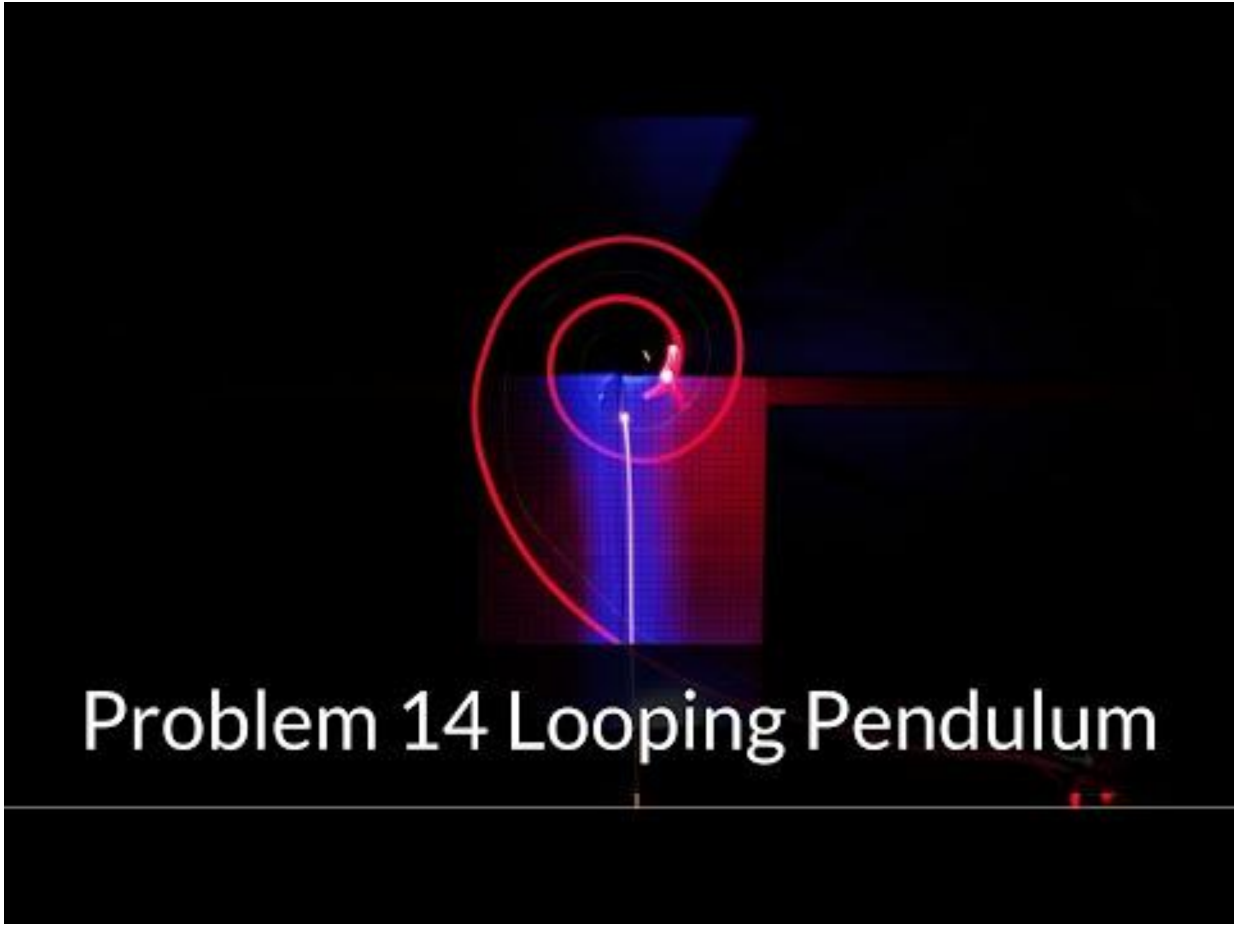
Interpretation of Task

Connect two loads, one heavy and one light, with a string over a horizontal rod and lift up the heavy load by pulling down the light one. Release the light load and it will sweep around the rod, keeping the heavy load from falling to the ground. Investigate this phenomenon.

Build a Looping Pendulum → Explain the phenomenon → Investigate relevant parameters

Phenomenon

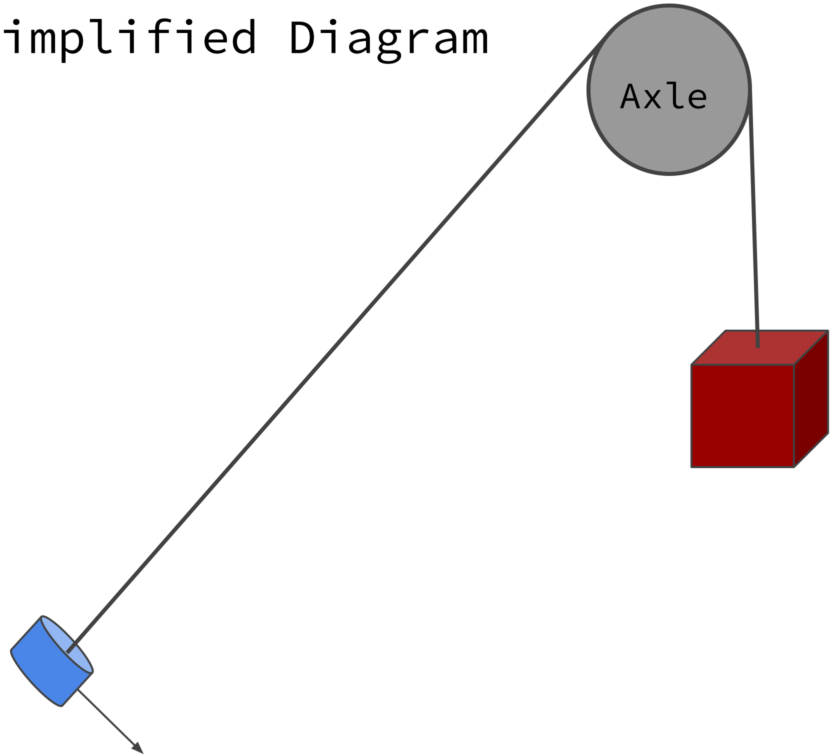
Connect two loads, one heavy and one light, with a string over a horizontal rod and lift up the heavy load by pulling down the light one. Release the light load and it will sweep around the rod, keeping the heavy load from falling to the ground. Investigate this phenomenon.



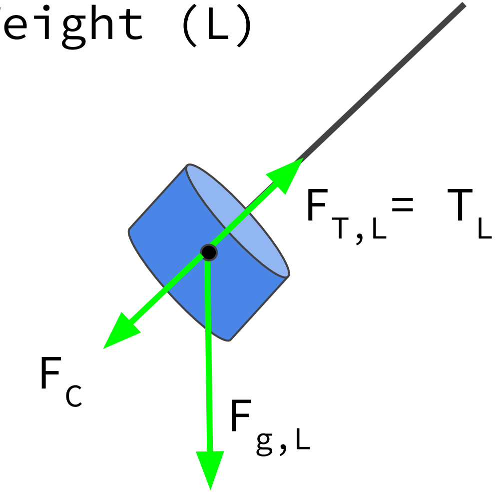
The Theory

Basic Forces

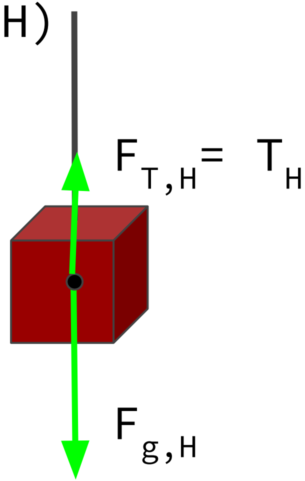
Simplified Diagram



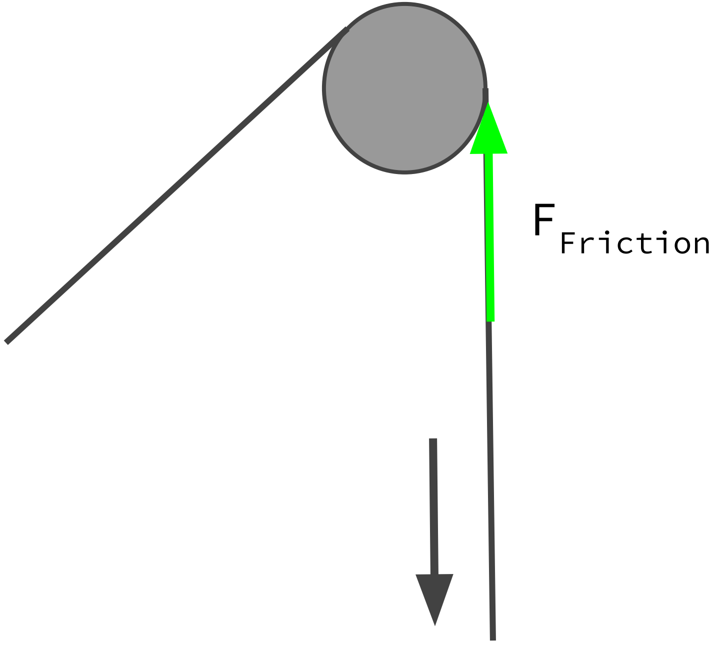
Lighter Weight (L)

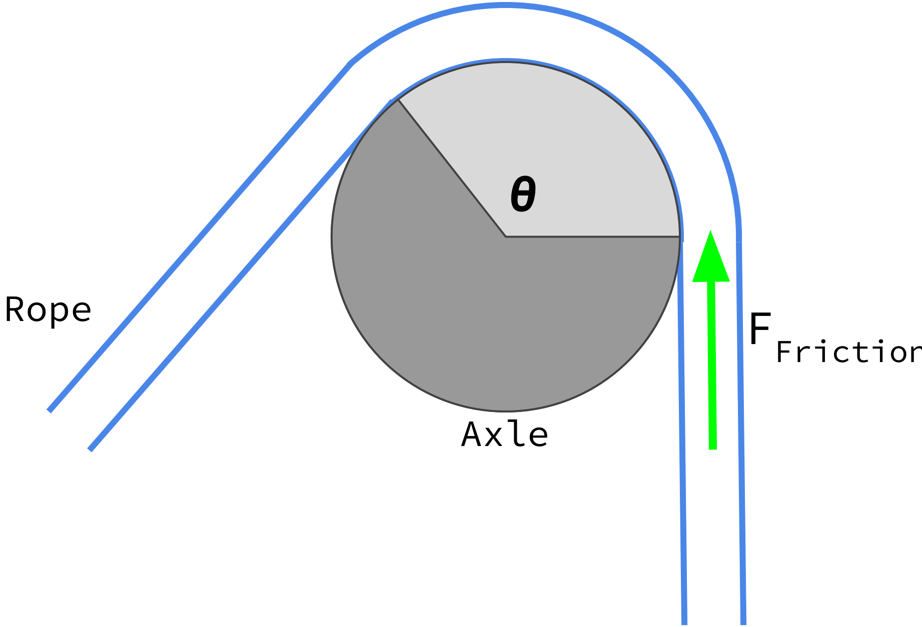


Heavier Weight (H)



Frictional Forces



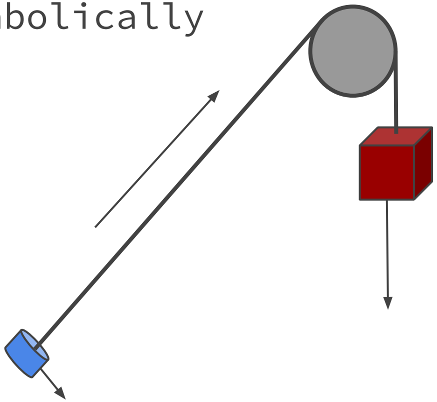


- Angle of Contact (**θ**)
- Coefficient of Kinetic Friction (μ)

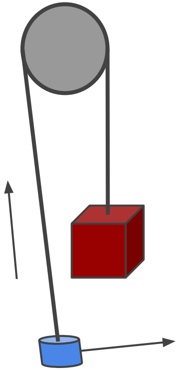
Theoretical Model

The Two Phases

- Heavier Weight moves downwards
- Lighter Weight increases angular velocity
- Radius of string attached to lighter weight decreasing parabolically

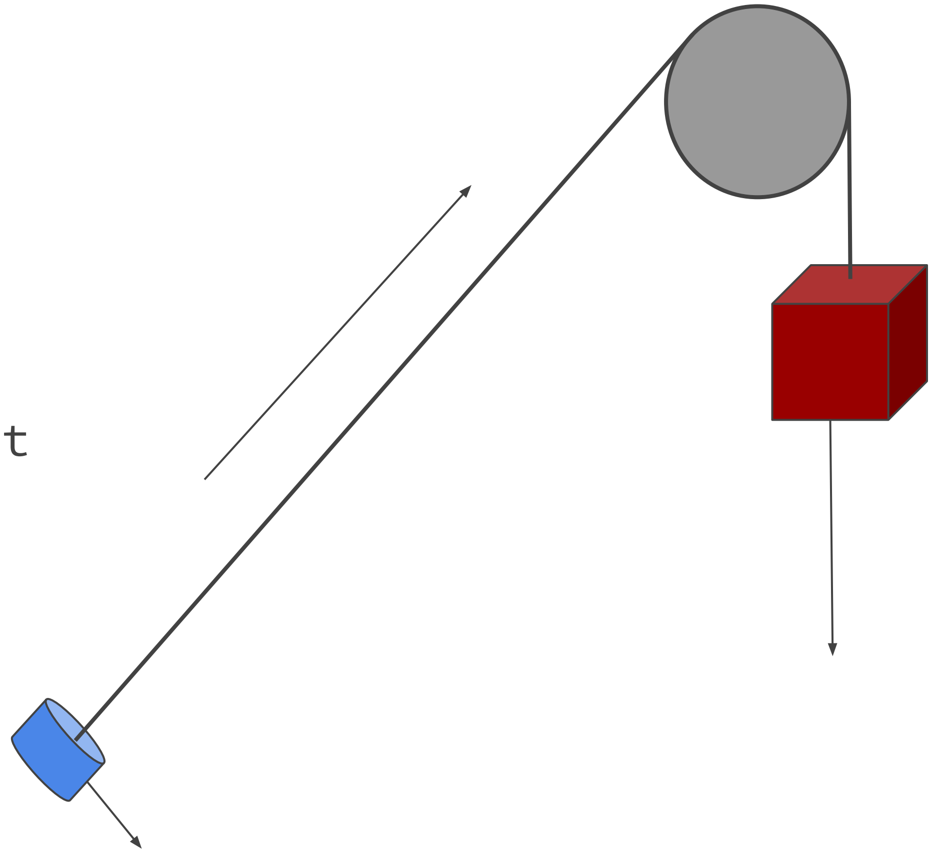


- Heavier Weight stops moving
- Lighter Weight continues to increase angular velocity
- Radius of String decreases linearly forming an Archimedes Spiral



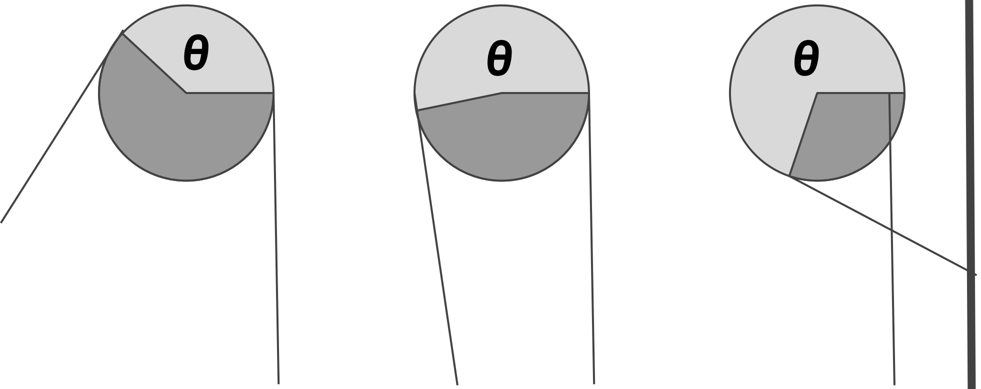
First Phase

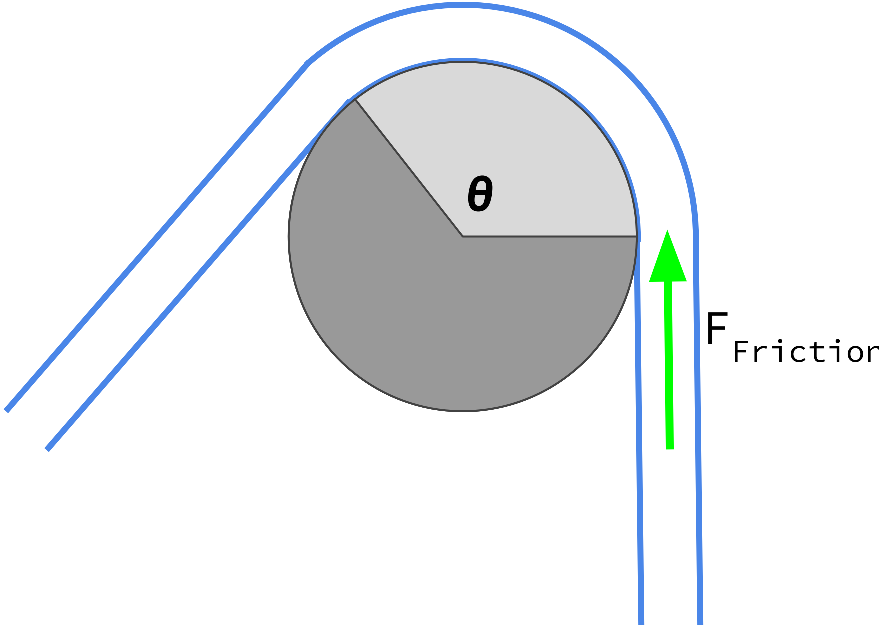
- Angular Velocity of the lighter weight increases
- Angle of Contact increases
- Tension of the heavy weight increases



Capstan Equation

$$T_H = T_L * e^{\mu\boldsymbol{\theta}}$$





- Angle of Contact (**$\boldsymbol{\theta}$**)
- Coefficient of Kinetic Friction (μ)

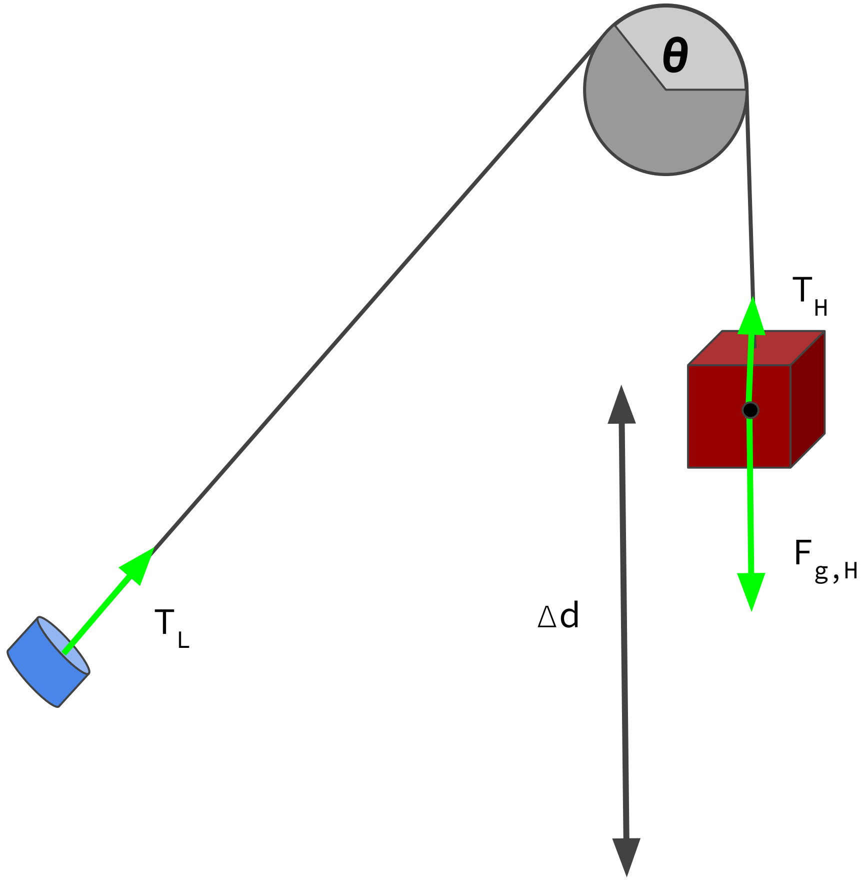
Derivation

Theoretical Model

$$\Delta d = 0.5a(\Delta t)^2$$

$$a = \frac{F_{g,H} - T_H}{m_H}$$

$$T_H = T_L \times e^{\mu\theta}$$

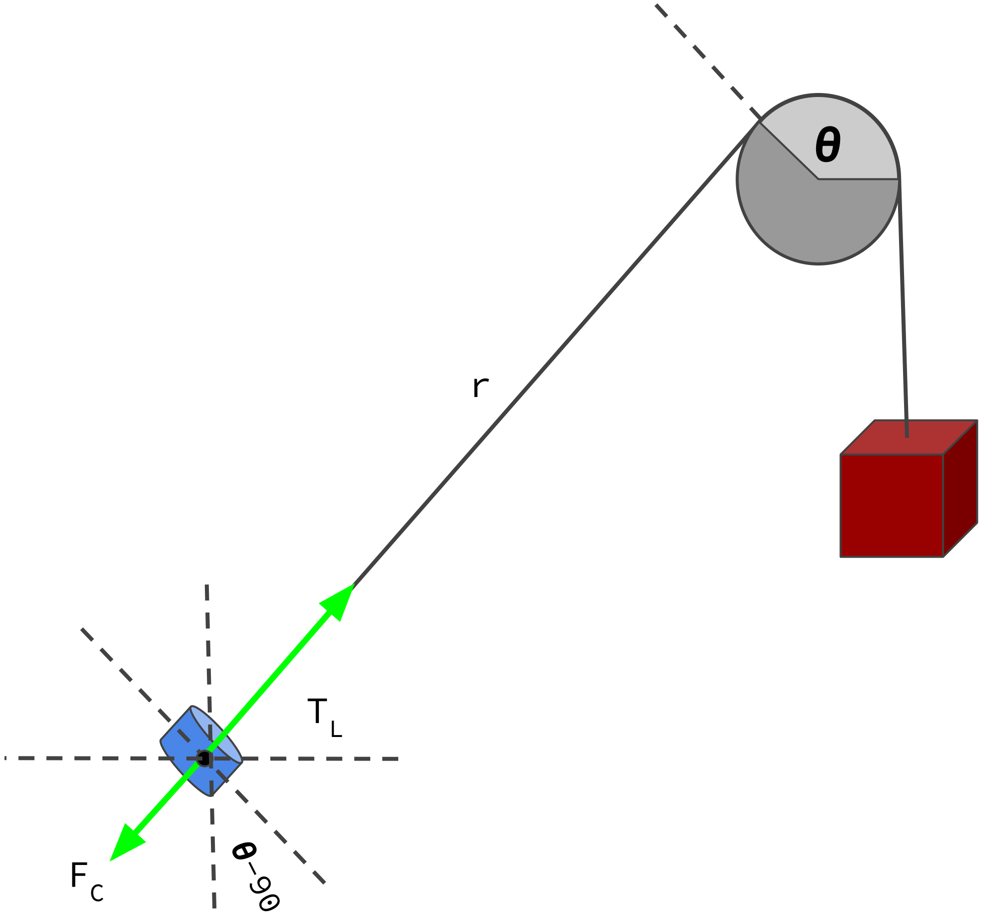


Theoretical Model (pt.2)

$$T_L = F_C - F_{g,L}cos(\theta)$$

$$F_C = m\omega^2(r)$$

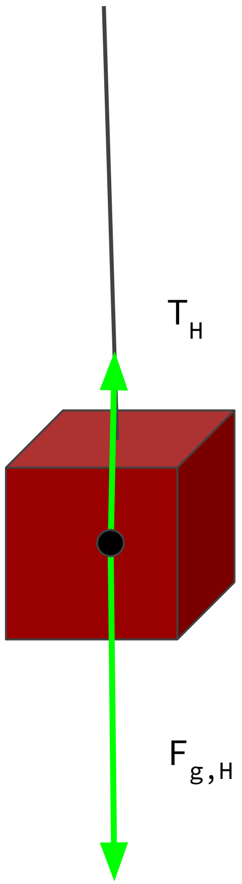
$$\omega = \frac{gsin(\theta)}{r}(\Delta t)$$



Theoretical Model (completed)

$$\Delta d = 0.5\left(\frac{F_{g,H} - T_H}{m_H}\right)(\Delta t)^2$$

$$F_{g,H} = e^{\mu\theta}\left(\left(\frac{(0.49)\sin^2(\theta)}{r}\right)(\Delta t)^2 - F_{g,L}\cos(\theta)\right)$$



Experimental Method

Materials Used

Cotton String

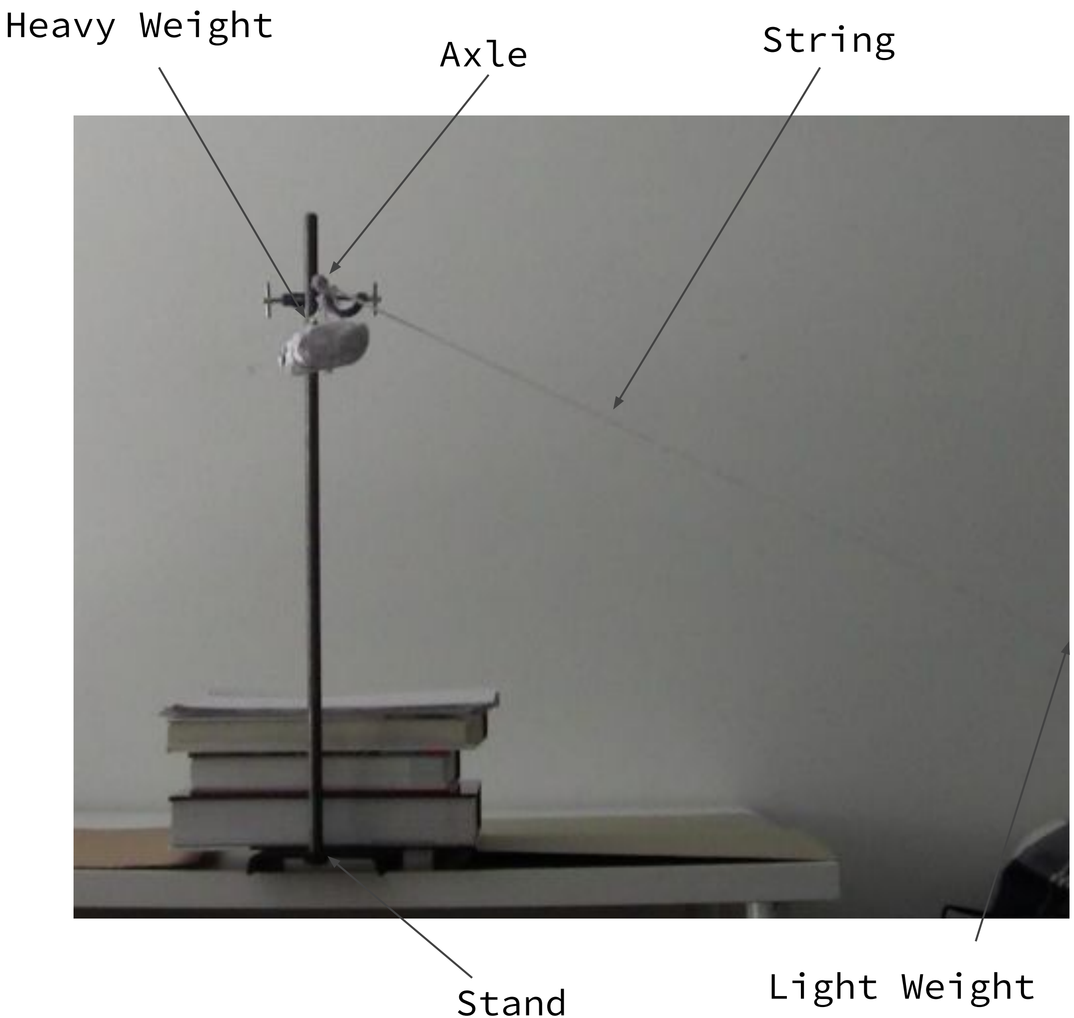
Weights

Stand with Axel

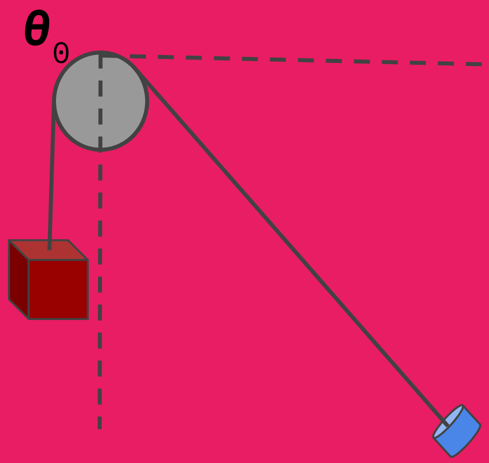
Experimental Setup

Parameters Manipulated:

- Angle of Release
- Length of the string
- Ratio of Weights



Angle of Release

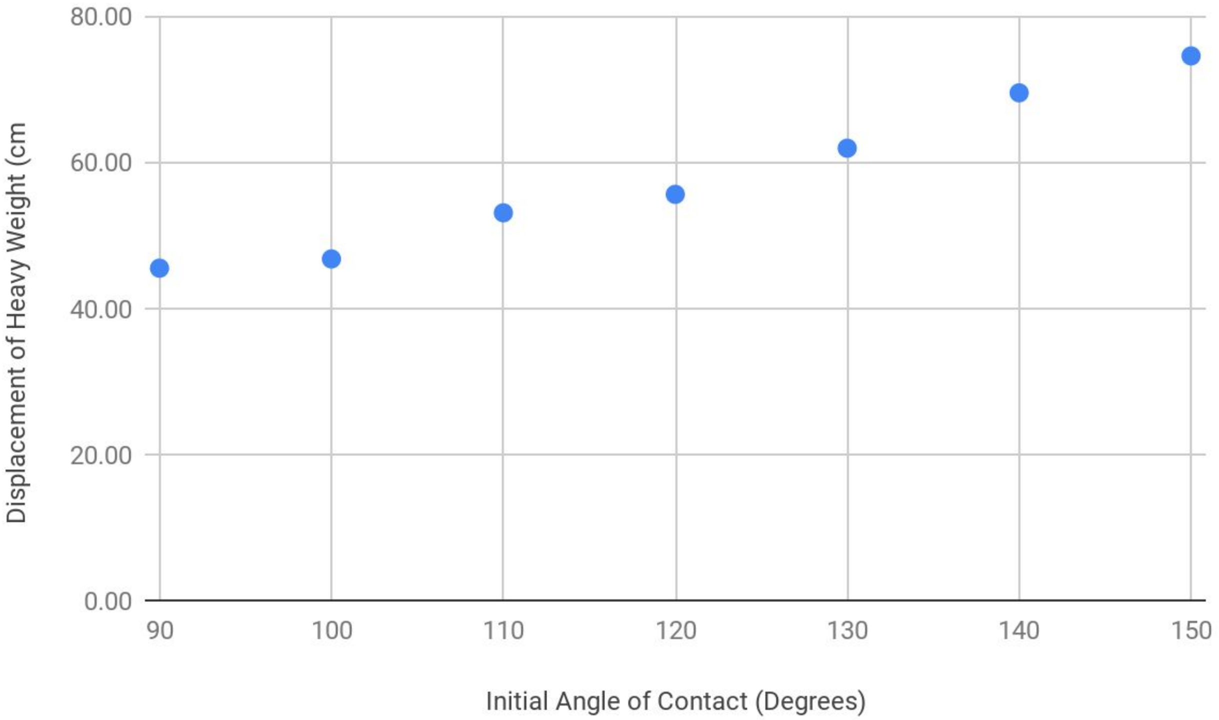


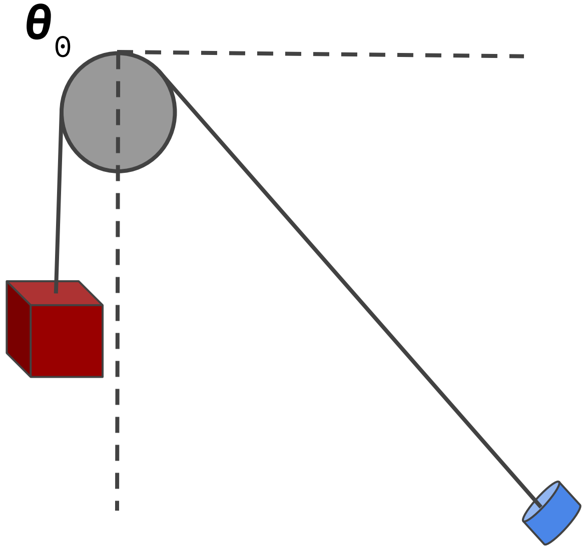
0° → 90°

$$F_{g,H} = e^{\mu\boxed{\theta}}\left(\left(\overbrace{\frac{(0.49)\sin^2(\boxed{\theta})}{r}}^{\text{Angular Velocity}}\right)(\Delta t)^2 - F_{g,L}\cos(\theta)\right)$$

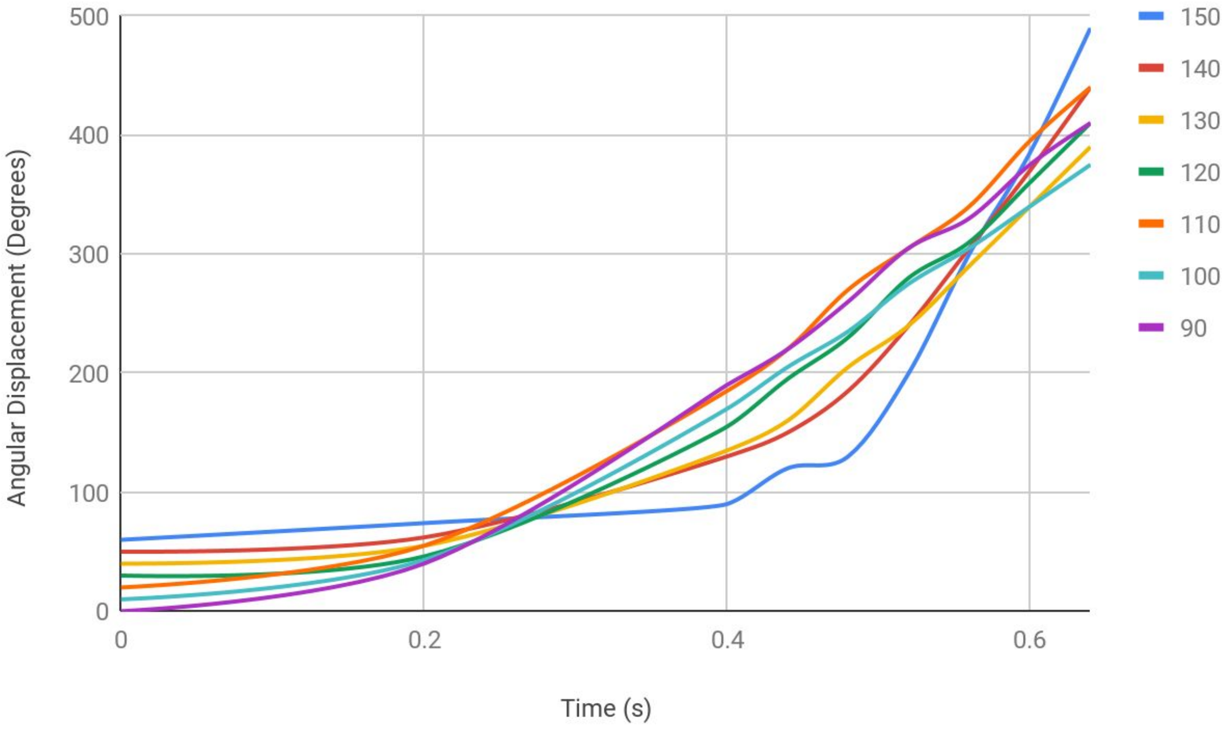
$$\Delta d = 0.5\left(\frac{F_{g,H} - T_H}{m_H}\right)(\Delta t)^2$$

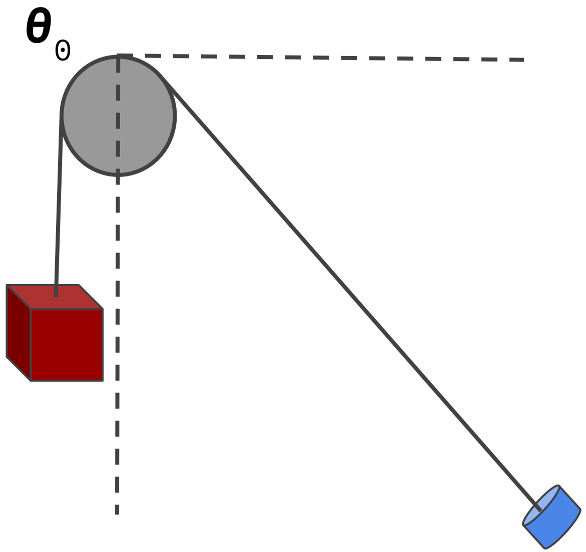
Effect of Angle of Release on Heavy Weight Displacement



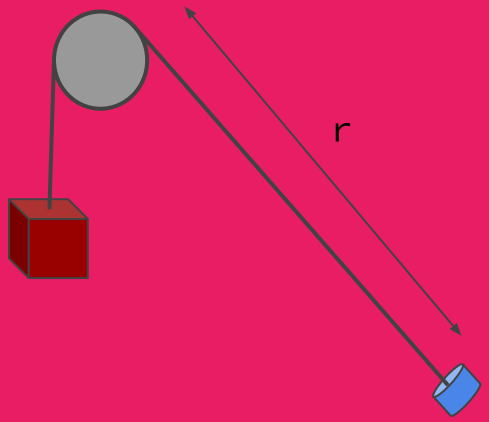


Effect of Angle of Release on Angular Displacement (Velocity)





Length of String



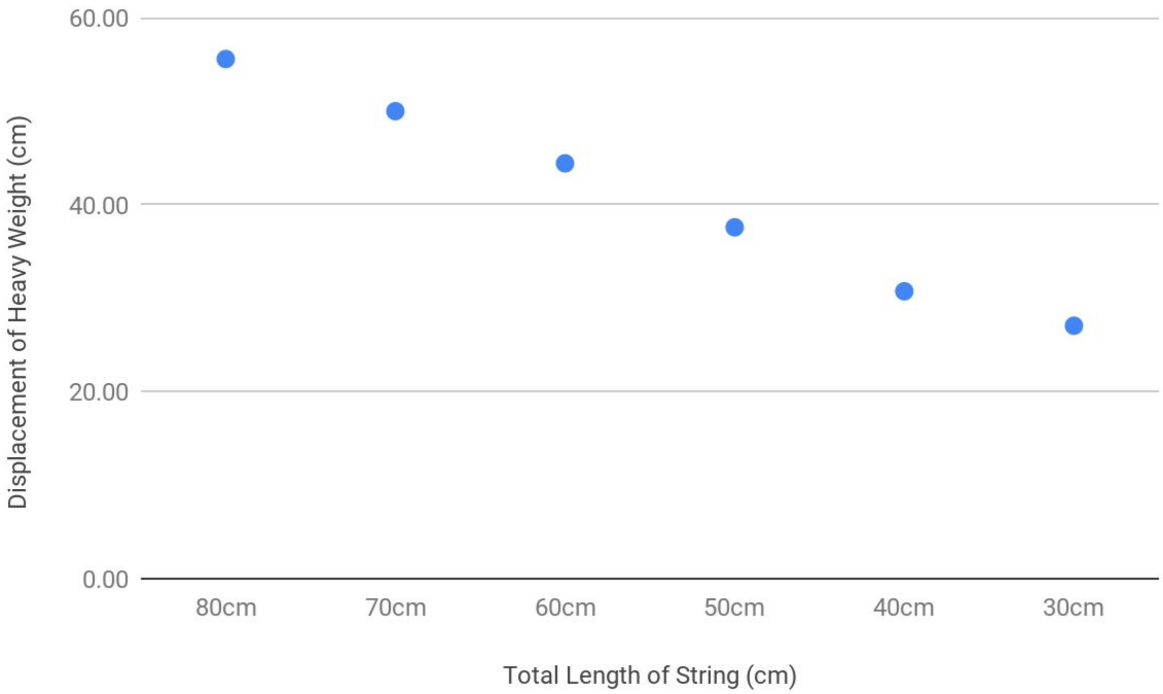
10cm → 80cm

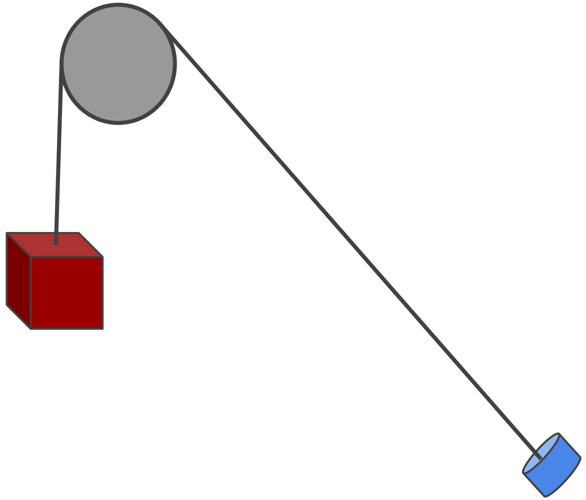
Angular Velocity

$$F_{g,H} = e^{\mu\theta}\Big(\underbrace{\Big(\frac{(0.49)sin^2(\theta)}{\boxed{r}}\Big)(\Delta t)^2}_{\text{Angular Velocity}} - F_{g,L}cos(\theta)\Big)$$

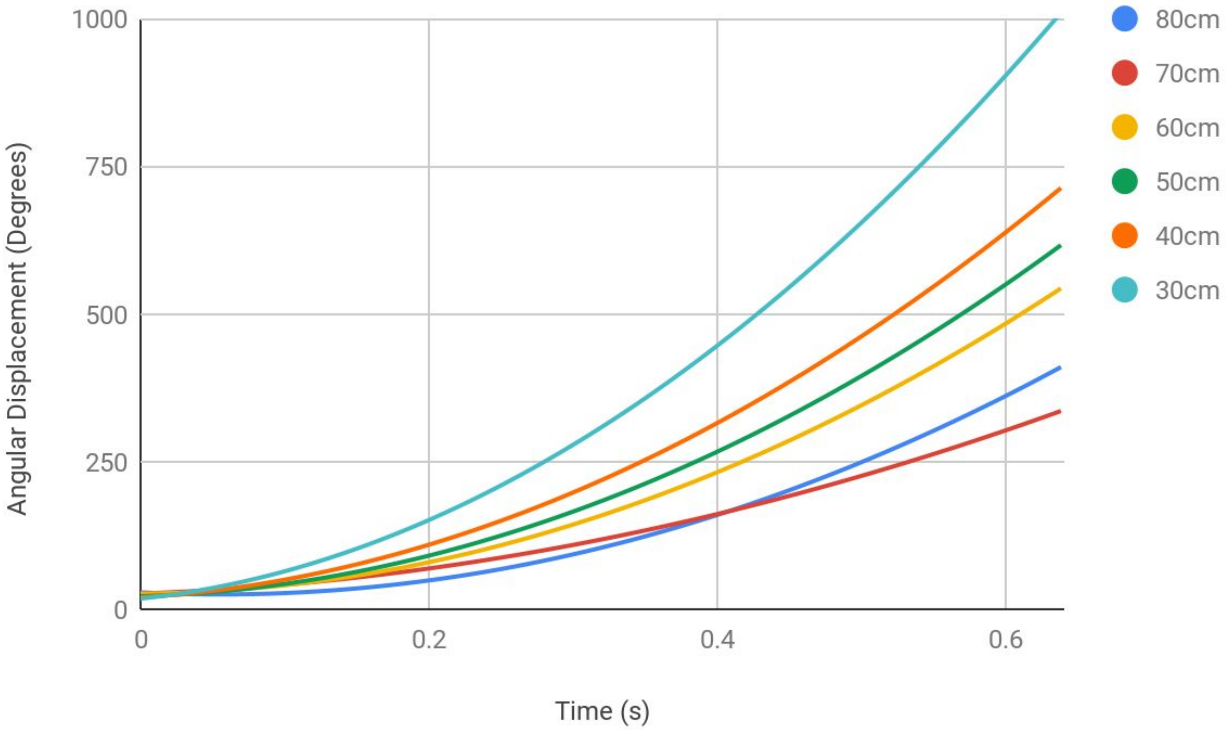
$$\Delta d = 0.5\Big(\frac{F_{g,H} - T_H}{m_H}\Big)(\Delta t)^2$$

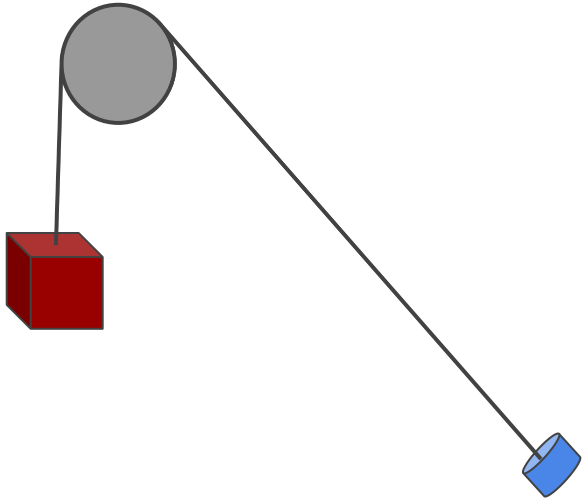
Effect of String Length on Heavy Weight Displacement



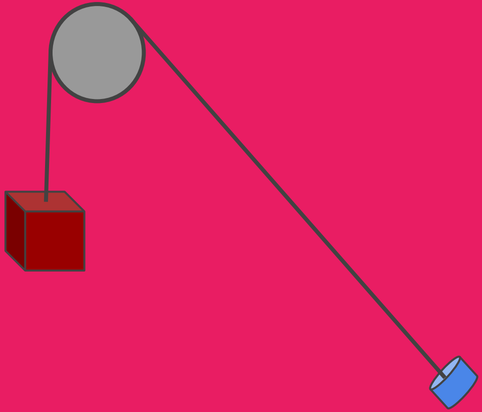


Effect of String Length on Angular Displacement (Velocity)





Ratio of Weights

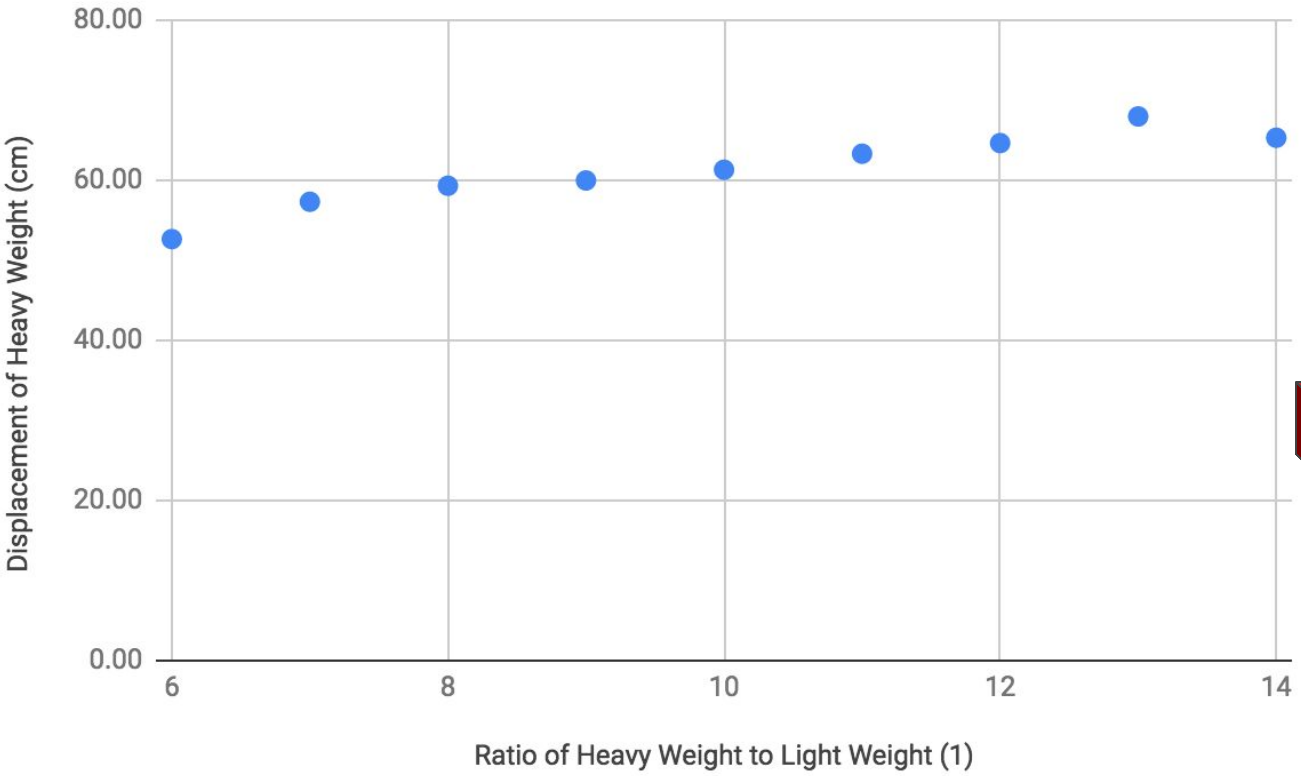


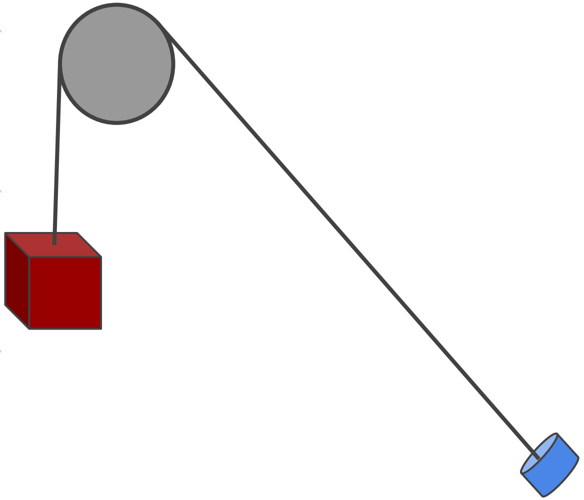
1:7 → 1:14

$$\boxed{F_{g,H}} = e^{\mu\theta}\left(\overbrace{\left(\frac{(0.49)sin^2(\theta)}{r}\right)}^{\text{Angular Velocity}}(\Delta t)^2 - F_{g,L}cos(\theta)\right)$$

$$\Delta d = 0.5\left(\frac{F_{g,H} - T_H}{m_H}\right)(\Delta t)^2$$

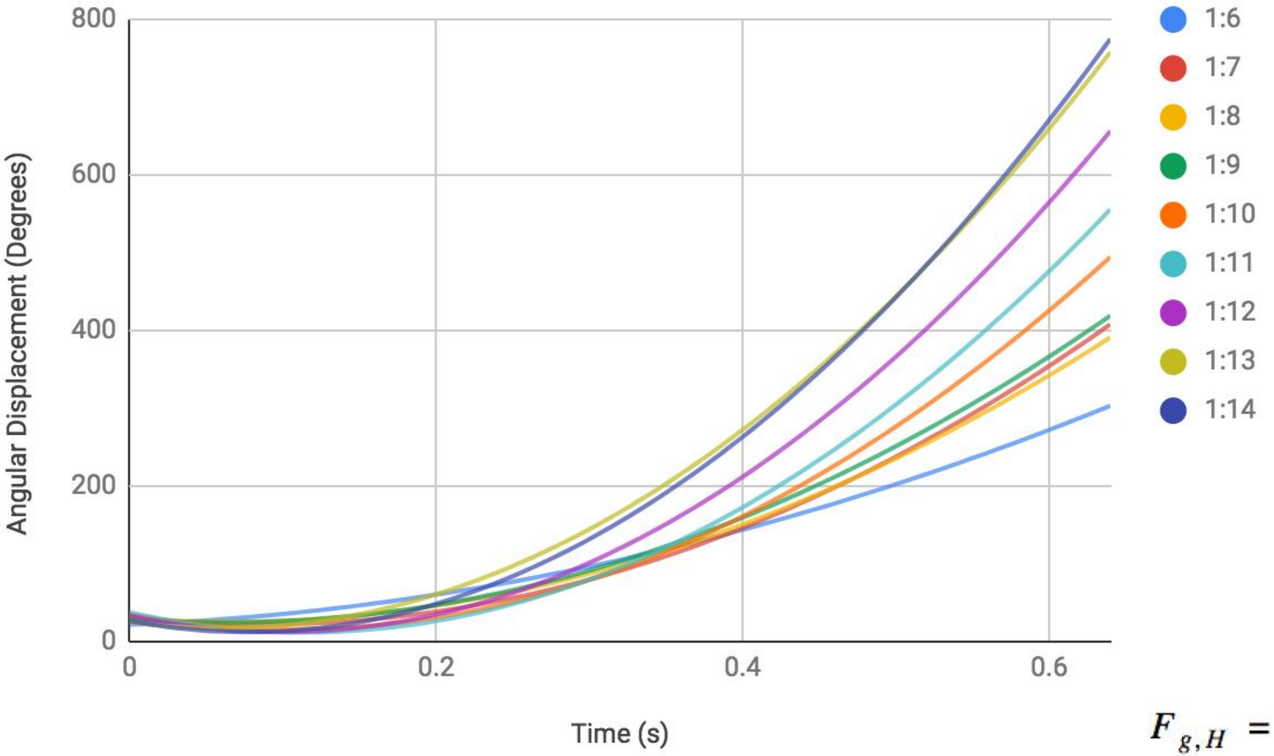
Effect of Weight Ratios on Heavy Weight Displacement

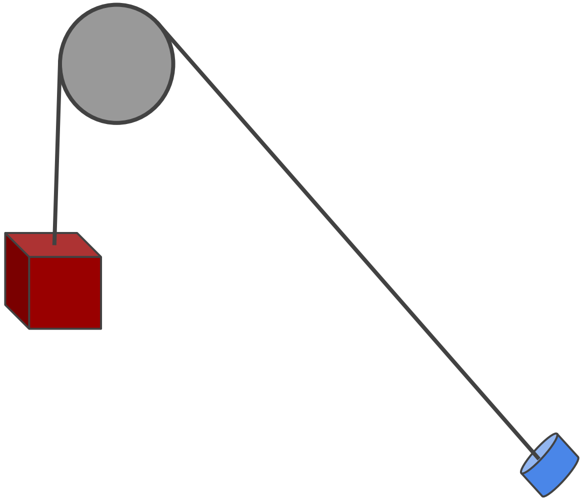




$$\Delta d = 0.5\left(\frac{F_{g,H} - T_H}{m_H}\right)(\Delta t)^2$$

Effect of Weight Ratios on Angular Displacement (Velocity)





$$F_{g,H} = e^{\mu\theta}\left(\left(\frac{(0.49)sin^2(\theta)}{r}\right)(\Delta t)^2 - F_{g,L}cos(\theta)\right)$$

Conclusion

Conclusion

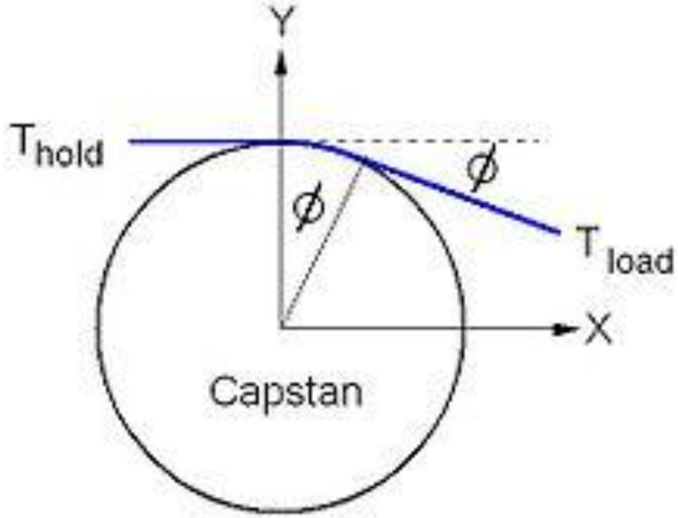
- There are two phases of the motion:
    - Non-linear string length decrease
    - Linear string length decrease

- The distance the heavy load moves can be expressed using time, the angle of release, the length of the string and the friction coefficient, and the two weights

- The ideal values for stopping the heavier weights acceleration earlier are 90°, 1:6 ratio, and 30cm string length

Additional Resources

Derivation of Capstan Equation

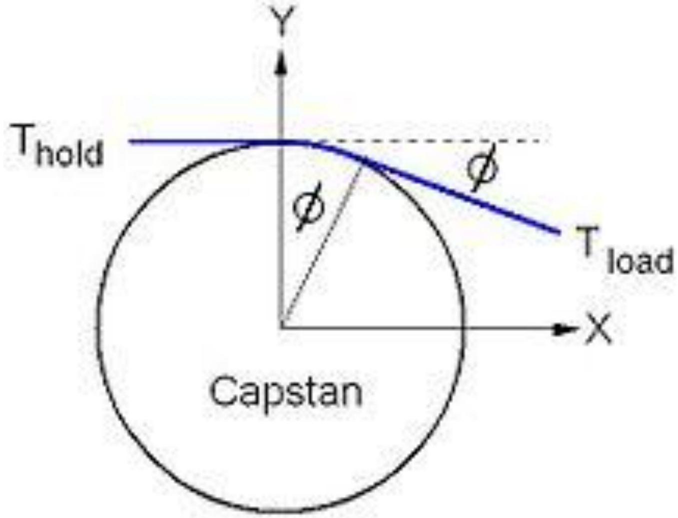


$$F = T_L \sin(\phi)$$

$$F = T_L d\phi \qquad \lim_{\phi \to 0} \sin(d\phi) = d\phi \qquad T_H = T_L$$

$$\mu F = \mu T_L d\phi$$

Derivation of Capstan Equation



$$dT = \mu T_L d\phi$$

$$\frac{1}{T} dT = \mu d\phi$$

$$\int_{T_L}^{T_H} \frac{1}{T} dT = \int_0^{\phi} \mu d\phi$$

$$ln(T_H) - ln(T_L) = \mu\phi$$

$$ln\left(\frac{T_H}{T_L}\right) = \mu\phi$$

$$T_H = T_L e^{\mu\phi}$$