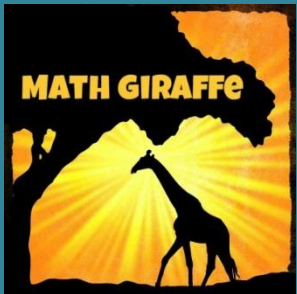


# PROOFS



TWO-COLUMN  
PROOFS  
FULL UNIT:  
PRESENTATION

# Terms of Use



Thanks so much for downloading this file!

This resource is for personal use in your own classroom only. The copyright is owned by Math Giraffe and all rights are reserved. Permission is granted to copy pages for your own students.

**Placing any part of this file anywhere online is a violation of the Digital Millennium Copyright Act.**

## **You may:**

- Print this file and distribute it to your students
- Use it in your own classroom

## **You may not:**

- Upload it on the internet to another site in any form
- Sell it, sell any part of it, or sell anything based upon it
  - Share it without permission

I hope that you and your students enjoy this!

# *Note to Teacher:*

THIS UNIT WILL GUIDE YOU AND YOUR STUDENTS THROUGH THE ENTIRE CONCEPT OF GEOMETRY PROOFS FROM BEGINNING TO END. THE UNIT BEGINS WITH THE BASIC BUILDING BLOCKS OF PROOFS. STUDENTS LEARN THE PROPERTIES AND POSTULATES THAT WILL BECOME THE JUSTIFICATIONS FOR PROOFS. THEY BEGIN WITH JUSTIFYING ALGEBRA PROBLEMS THAT THEY ARE FAMILIAR WITH AND EASE INTO THE GEOMETRY PROOFS. THE UNIT USES THE TWO-COLUMN STRUCTURE FOR PROOF WRITING.

THIS PRESENTATION SET IS ACCOMPANIED BY A SET OF PRINTABLE RESOURCES THAT YOU CAN USE AS PRACTICE AND ASSESSMENT AS YOU GO THROUGH THE UNIT.

# *Suggested Pacing Guide*

	Regular Class Periods	Block Schedule
Day 1	Properties of Equality and Congruence (presentation p.5-18, printables p.2 -6)	Properties of Equality and Congruence, Postulates (presentation p. 5-24, printables 2-10)
Day 2	Properties Warm-Up (Printables p.7-8) Lesson on Postulates (presentation p.19-24, printables p.9-10)	The Two-Column Proof Structure, Algebraic Proofs (presentation p.25-34, printables p.11-16)
Day 3	The Two-Column Proof Structure, Algebraic Proofs (Presentation p.25-34, printables p.11-14)	Proofs Using Definitions and Postulates (presentation p. 35-51, printables p. 17-25), Review of Special Angle Pairs (presentation p. 52, 53)
Day 4	Algebra Proofs Warm-Up (printables p. 15-16) Lesson on Proofs Using Definitions and Postulates (presentation p. 35-51, printables p.17-25)	Quiz 1 (printables p.26-29), Proofs Using Special Angle Pairs (presentation p. 54-59, printables p.30-37)
Day 5	Quiz 1 (printables p.26-29) Review of Special Angle Pairs (presentation p. 52, 53)	Proofs Using Triangles (presentation p.60, printables p.38-41)
Day 6	Proofs Using Special Angle Pairs (presentation p. 54-59, printables p.30-31)	Quiz 2 (printables p.42-45)
Day 7	Warm-Up using Special Angle Pairs (printables p.32-33), More Proofs Using Special Angle Pairs (Transversals)(printables p. 34-37)	
Day 8	Proofs Using Triangles (presentation p.60, printables p.38-41)	
Day 9	Quiz 2 (printables p.42-45)	

# ***Introduction to Proofs***

**When writing a proof, your job is to complete a puzzle. You start with the given information and have to reach the “goal” statement. You must show your logic to prove that you can get the desired result. To do this, you need to justify every statement that you make. Your statements and justifications link together to form the proof.**

**This unit will focus on a two-column style proof. Before we begin writing the formal proofs, we need a few building blocks of knowledge that will become our justifications. Justifications can be properties, postulates, and theorems that have already been proven or accepted as truth.**



# PROPERTIES OF EQUALITY

## *Addition Property of Equality*

**Adding the same number to both sides of an equation results in an equivalent equation.**

**If  $a = b$  is true,**

**Then  $a + m = b + m$  is also true.**

## *Subtraction Property of Equality*

**Subtracting the same number from both sides of an equation results in an equivalent equation.**

**If  $a = b$  is true,**

**Then  $a - m = b - m$  is also true.**



# PROPERTIES OF EQUALITY

## *Multiplication Property of Equality*

**Multiplying both sides of an equation by the same nonzero number results in an equivalent equation.**

If  $a = b$  is true,  
Then  $am = bm$  is also true.

## *Division Property of Equality*

**Dividing both sides of an equation by the same nonzero number results in an equivalent equation.**

If  $a = b$  is true,  
Then  $\frac{a}{m} = \frac{b}{m}$  is also true (for  $m \neq 0$ ).



## REFLEXIVE PROPERTY OF EQUALITY

A number is equal to itself.

$$x = x$$

## REFLEXIVE PROPERTY OF CONGRUENCE

A figure is congruent to itself.

$$\angle A \cong \angle A$$



## SYMMETRIC PROPERTY OF EQUALITY

The sides of an equation can be switched.

If  $a = b$ , then  $b = a$

## SYMMETRIC PROPERTY OF CONGRUENCE

The sides of a congruency statement can be switched.

If  $\angle A \cong \angle B$ , then  $\angle B \cong \angle A$

## TRANSITIVE PROPERTY OF EQUALITY

If two numbers are equal to the same number,  
then they are equal to each other.

If  $a = b$  and  $b = c$ , then  $a = c$

## TRANSITIVE PROPERTY OF CONGRUENCE

If two figures are congruent to the same figure,  
then they are congruent to each other.

If  $\angle A \cong \angle B$ , and  $\angle B \cong \angle C$ , then  $\angle A \cong \angle C$

**Identify which property was used to get from column 1 to column 2.**

<b>1</b>	<b><math>6n + 9 = 15</math></b>	<b><math>6n + 9 - 9 = 15 - 9</math> or <math>6n = 6</math></b>	
<b>2</b>	<b><math>\overline{JK} \cong \overline{GH}</math></b>	<b><math>\overline{GH} \cong \overline{JK}</math></b>	
<b>3</b>	<b><math>\angle QRS \cong \angle UVW</math> and <math>\angle UVW \cong \angle XYZ</math></b>	<b><math>\angle QRS \cong \angle XYZ</math></b>	
<b>4</b>	<b><math>m\angle 4 = m\angle 9</math></b>	<b><math>m\angle 4 + m\angle 1 = m\angle 9</math> <math>+ m\angle 1</math></b>	

**Write a new congruency statement using the transitive property.**

<b>5</b>	<b><math>\angle DEF \cong \angle ABC</math> and <math>\angle JKL \cong \angle DEF</math></b>	
<b>6</b>	<b><math>\overline{RS} \cong \overline{VW}</math> and <math>\overline{RS} \cong \overline{XY}</math></b>	



**Identify which property was used to get from column 1 to column 2.**

<b>1</b>	$6n + 9 = 15$	$6n + 9 - 9 = 15 - 9$ or $6n = 6$	<b>Subtraction Property of Equality</b>
<b>2</b>	$\overline{JK} \cong \overline{GH}$	$\overline{GH} \cong \overline{JK}$	<b>Symmetric Property of Congruence</b>
<b>3</b>	$\angle QRS \cong \angle UVW$ and $\angle UVW \cong \angle XYZ$	$\angle QRS \cong \angle XYZ$	<b>Transitive Property of Congruence</b>
<b>4</b>	$m\angle 4 = m\angle 9$	$m\angle 4 + m\angle 1 = m\angle 9$ $+ m\angle 1$	<b>Addition Property of Equality</b>

**Write a new congruency statement using the transitive property.**

<b>5</b>	$\angle DEF \cong \angle ABC$ and $\angle JKL \cong \angle DEF$	$\angle ABC \cong \angle JKL$
<b>6</b>	$\overline{RS} \cong \overline{VW}$ and $\overline{RS} \cong \overline{XY}$	$\overline{VW} \cong \overline{XY}$



# *Substitution*

## *Property*

SUBSTITUTION  
PROPERTY OF  
EQUALITY:

IF TWO NUMBERS ARE  
EQUAL, ONE CAN  
REPLACE THE OTHER IN  
AN EQUATION.

SUBSTITUTION  
PROPERTY OF  
CONGRUENCE:

IF TWO FIGURES ARE  
CONGRUENT, ONE CAN  
REPLACE THE OTHER IN  
A CONGRUENCY  
STATEMENT.



# Substitution Property

EXAMPLE 1:

If  $m\angle G = m\angle H$  and we know that

$$m\angle G + m\angle A = m\angle B,$$

We can rewrite the statement as follows:

$$m\angle H + m\angle A = m\angle B.$$

EXAMPLE 2:

$$\text{If } \overline{TV} \cong \overline{WX}$$

and we know that

$$\overline{WX} \cong \overline{YU}$$

We can rewrite the statement as follows:

$$\overline{TV} \cong \overline{YU}$$



THE SUBSTITUTION AND TRANSITIVE PROPERTIES CAN BE USED FOR FULL EXPRESSIONS AS WELL.

Look at the following example.

The following information is GIVEN:

$$d = c,$$

$$a = c$$

$$d + p = n,$$

$$a + c = n$$

**Note:** In order to use the Transitive Property, the equivalent parts must each be an **ENTIRE SIDE** of the equation!



Depending on our goal, we can write quite a few statements from the given equations.

Since two different expressions are equal to  $n$ , they are equal to each other. We can now write a new statement:

$$d + p = a + c.$$

It can help to put boxes around the expressions to clearly see which expressions are equal.

USING THE FOUR GIVEN STATEMENTS ABOVE, WRITE AS MANY NEW STATEMENTS AS YOU CAN. USE ONLY SUBSTITUTION AND THE TRANSITIVE PROPERTY.





# REMINDEERS:

ANGLE MEASURES ARE NUMBERS, AND CAN BE EQUAL.

$$m\angle 1 = m\angle 4$$

ANGLES ARE FIGURES AND CAN BE CONGRUENT.

$$\angle XYZ \cong \angle PQR$$

SEGMENT LENGTHS ARE NUMBERS AND CAN BE EQUAL.

$$MN = ST$$

SEGMENTS ARE FIGURES AND CAN BE CONGRUENT.

$$\overline{GH} \cong \overline{CD}$$



DETERMINE WHETHER EACH  
STATEMENT IS ACCEPTABLE  
AND EXPLAIN.

1.  $\angle ABC = \angle STU$

2.  $VW = VY$

4.  $m\angle 5 = m\angle 8$

3.  $\overline{PR} \cong \overline{GH}$

5.  $\overline{AB} = \overline{CD}$



DETERMINE WHETHER EACH STATEMENT IS ACCEPTABLE AND EXPLAIN.

1.  $\angle ABC = \angle STU$

2.  $VW = VY$

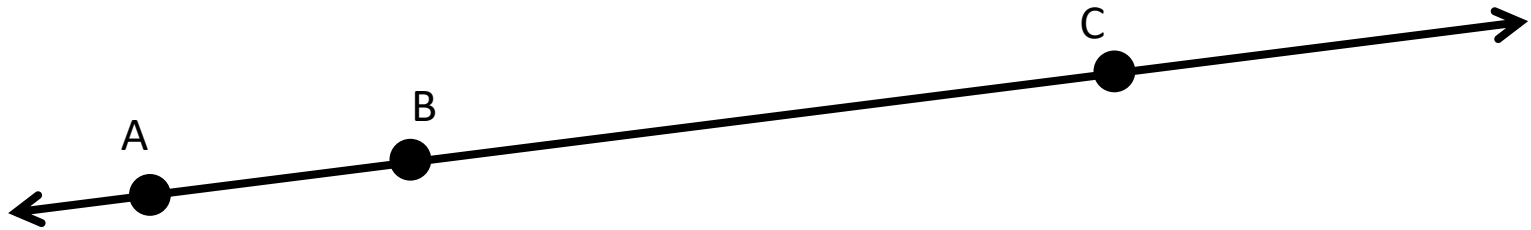
4.  $m\angle 5 = m\angle 8$

3.  $\overline{PR} \cong \overline{GH}$

5.  $\overline{AB} = \overline{CD}$

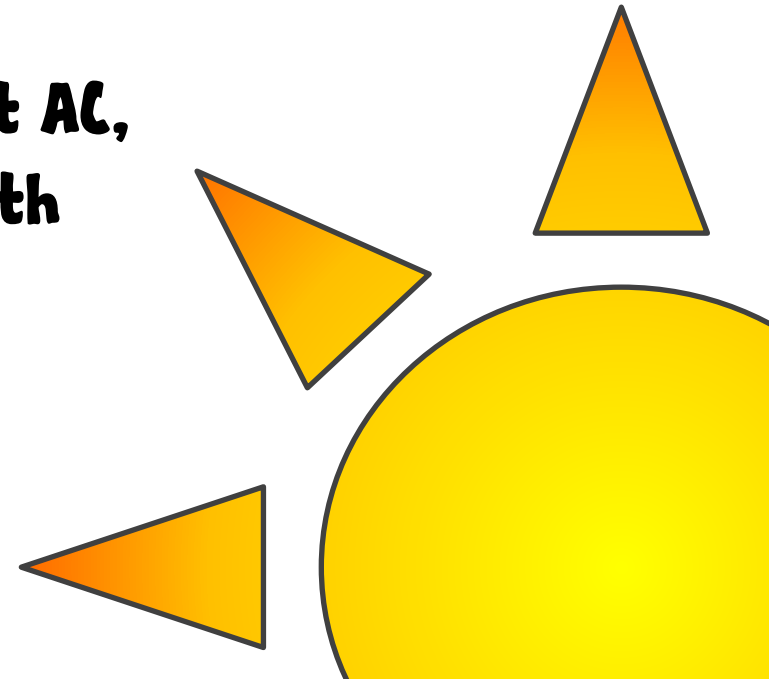


# SEGMENT ADDITION POSTULATE:

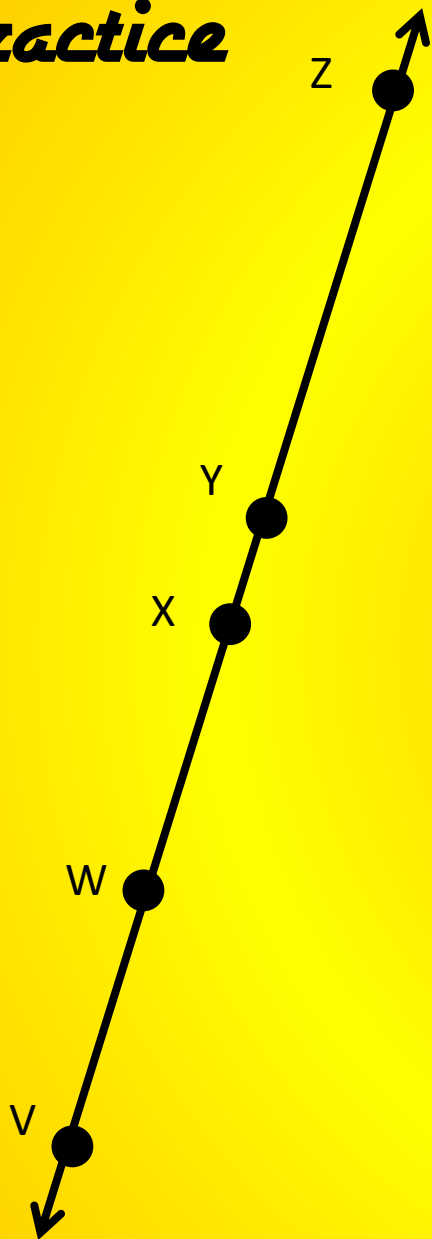


**If B lies between A and C on segment AC,  
then the length of  $\overline{AB}$  plus the length  
of  $\overline{BC}$  is equal to the length of  $\overline{AC}$ .**

$$AB + BC = AC$$



# Practice



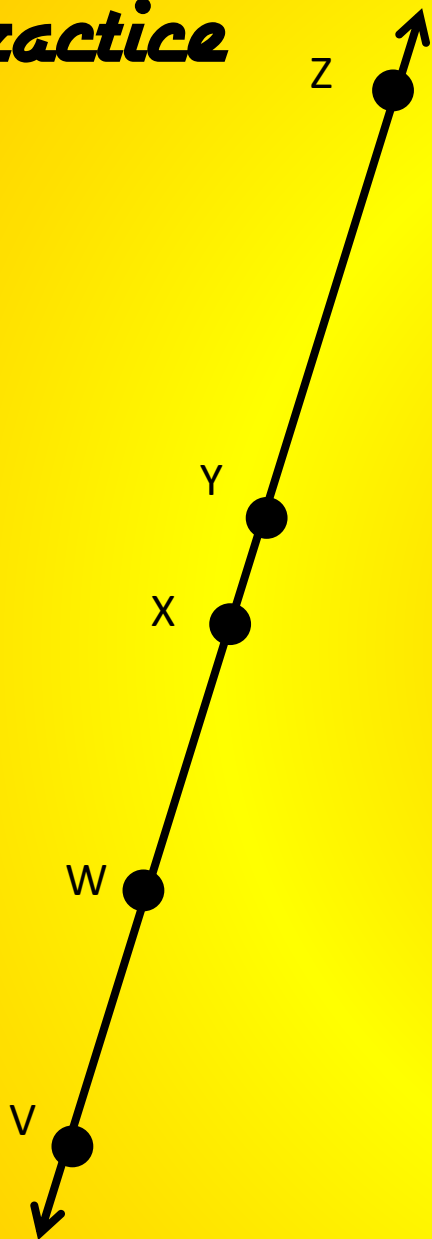
Given:  $X$  is the midpoint of  $\overline{VZ}$ ,  
 $VY = 16$ ,  
 $XY = 2$   
 $WX = 9$

Find each length.

1	$WY =$	
2	$VW =$	
3	$XZ =$	
4	$WZ =$	
5	$YZ =$	
6	$VZ =$	



# Practice



Given:  $X$  is the midpoint of  $\overline{VZ}$ ,  
 $VY = 16$ ,  
 $XY = 2$   
 $WX = 9$

Find each length.

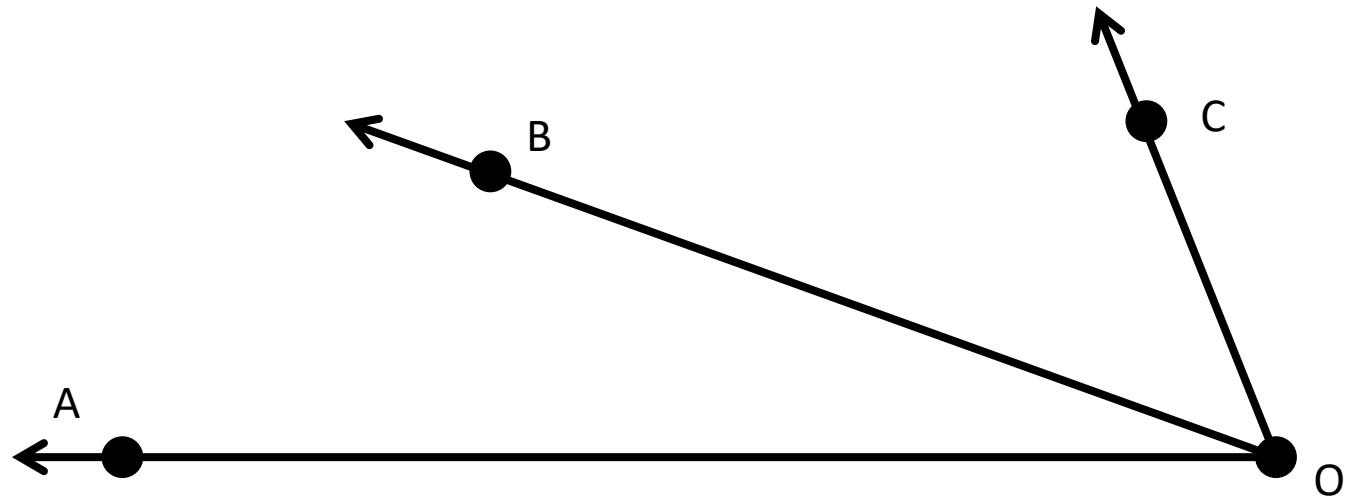
1	$WY =$	11
2	$VW =$	5
3	$XZ =$	14
4	$WZ =$	23
5	$YZ =$	12
6	$VZ =$	28



# ANGLE ADDITION POSTULATE:

If B is in the interior of angle AOC,  
then the measure of angle AOB plus  
the measure of angle BOC is equal  
to the measure of angle AOC.

$$m\angle AOB + m\angle BOC = m\angle AOC$$

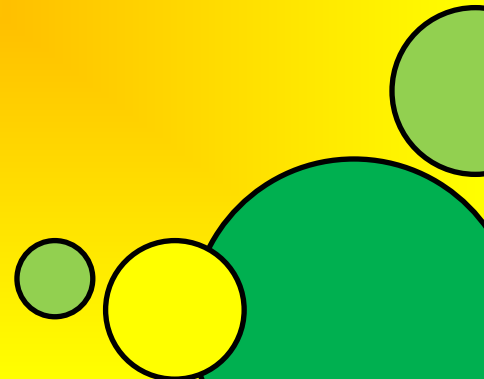
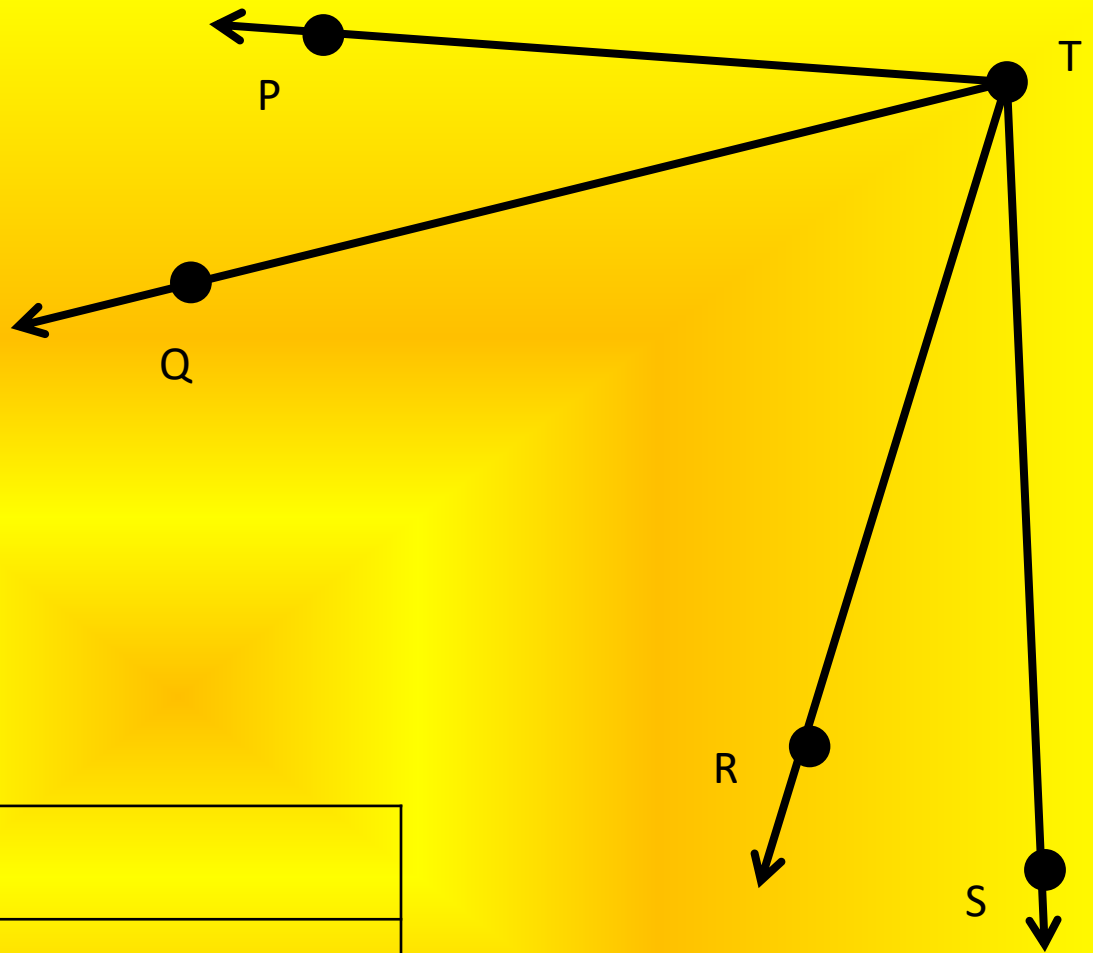


# Practice

Given:  $m\angle STQ = 78^\circ$   
 $m\angle QTP = 31^\circ$   
 $m\angle STR = m\angle QTP$

Find each angle measure.

1	$m\angle STP =$	
2	$m\angle STR =$	
3	$m\angle RTQ =$	
4	$m\angle RTP =$	

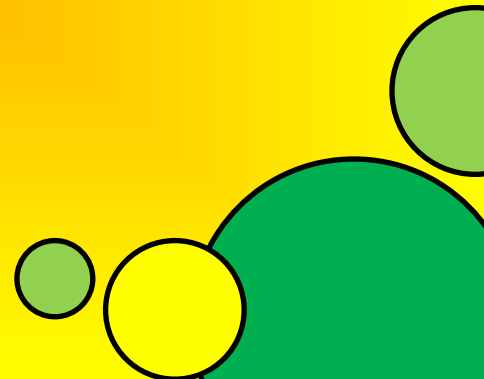
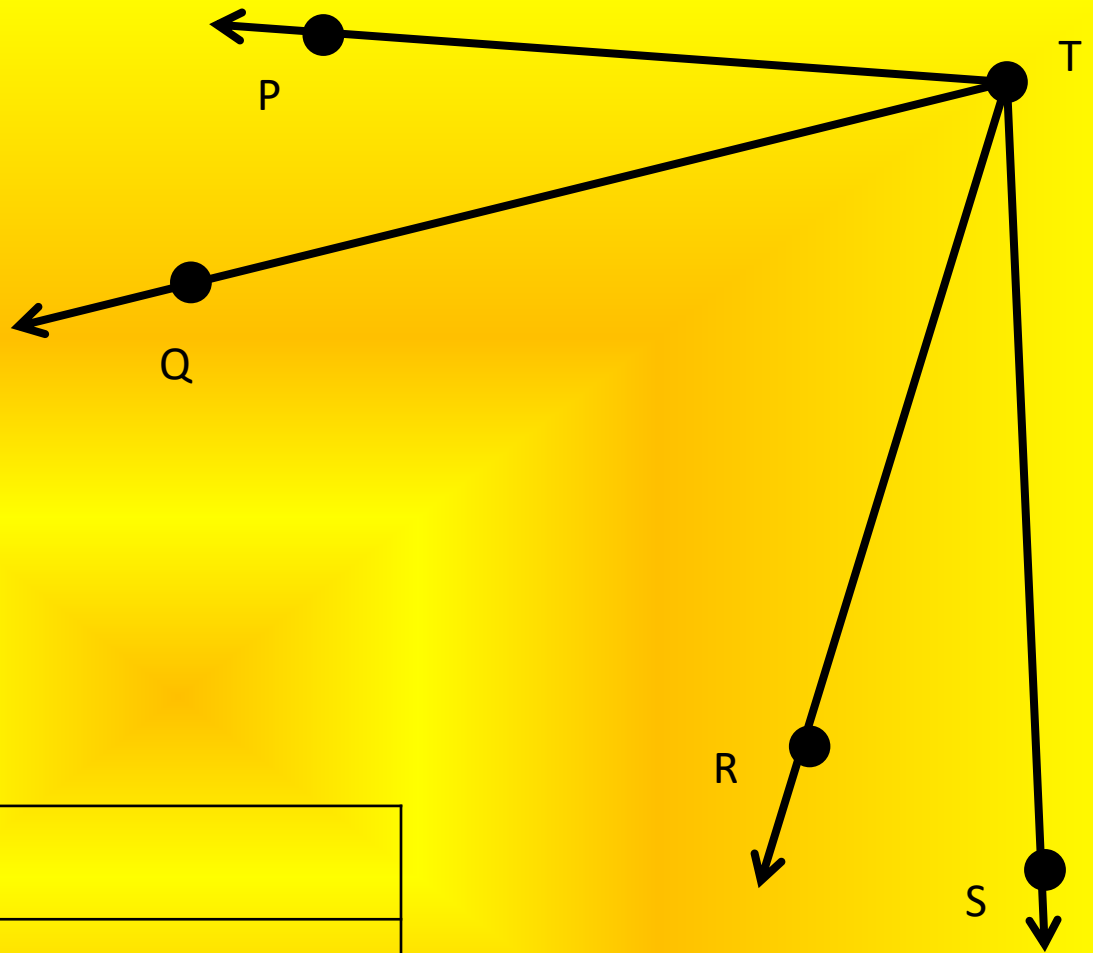


# Practice

Given:  $m\angle STQ = 78^\circ$   
 $m\angle QTP = 31^\circ$   
 $m\angle STR = m\angle QTP$

Find each angle measure.

1	$m\angle STP =$	$109^\circ$
2	$m\angle STR =$	$31^\circ$
3	$m\angle RTQ =$	$47^\circ$
4	$m\angle RTP =$	$78^\circ$





WE ALREADY KNOW HOW TO SOLVE THE FOLLOWING PROBLEM. SOLVE FOR  $x$ , AND SHOW EVERY STEP OF YOUR WORK.

$$x + 3 = 5x - 8$$

Show all  
work!

Note that you  
use the  
Properties of  
Equality to solve  
for  $x$ .

NOW, WE WILL ARRANGE THESE STEPS INTO A TWO-COLUMN PROOF TO PROVE THAT THE SOLUTION IS  $x = \frac{11}{4}$  AND JUSTIFY WHAT WE DID IN EACH STEP OF THE WORK.

# The Two-Column Structure for Proofs

LET'S REVIEW THIS SAMPLE PROOF TOGETHER. IT IS BASED ON THE SAME EQUATION WE JUST SOLVED.



The problem will contain at least one "Given" statement. Your end result must be the statement you are asked to "Prove."

## Sample Proof #1

**Given:**  $x + 3 = 5x - 8$

**Prove:**  $x = \frac{11}{4}$

The first column contains a series of statements that leads us logically from the given statement(s) to the fact that we are proving.

Line one always contains the first given statement.

	STATEMENT	JUSTIFICATION
1	$x + 3 = 5x - 8$	Given
2	$3 = 4x - 8$	Subtraction Prop. of Eq.
3	$11 = 4x$	Addition Prop. of Eq.
4	$\frac{11}{4} = x$	Division Prop. Of Eq.
5	$x = \frac{11}{4}$	Symmetric Prop. Of Eq.
6		

The second column contains the justifications for each statement.

Justifications can include definitions, properties, postulates, and theorems that have already been accepted as true.



# Sample Proof #2

THE FIRST GIVEN STATEMENT WILL ALWAYS BE COPIED EXACTLY ONTO LINE ONE. IF THERE ARE MULTIPLE GIVEN STATEMENTS, THE ADDITIONAL STATEMENTS EACH WILL HAVE THEIR OWN LINE.

**Given:**  $\angle ABC \cong \angle DEF$   
 $\angle DEF \cong \angle JKL$

**Prove:**  $\angle ABC \cong \angle JKL$

THE JUSTIFICATION FOR BOTH GIVEN STATEMENTS IS SIMPLY THAT WE WERE "GIVEN" THE STATEMENTS AND THUS MAY ASSUME THEY ARE TRUE.

	STATEMENT	JUSTIFICATION
1	$\angle ABC \cong \angle DEF$	Given
2	$\angle DEF \cong \angle JKL$	Given
3	$\angle ABC \cong \angle JKL$	Transitive Prop. Of Congruence (1, 2)
4		
5		
6		

WE DO NOT ALWAYS NEED ALL OF THE PROVIDED SPACE. THIS PROOF WAS SHORT.

SINCE SUBSTITUTION AND THE TRANSITIVE PROPERTY ARE ALWAYS USED TO COMBINE TWO PREVIOUS STATEMENTS, WE MUST SPECIFY WHICH PREVIOUS LINES WE ARE USING.



# Let's complete the proof below together.

**Given:**  $\frac{x}{10} = 3$ ,  
 $y - 5 = 25$

**Prove:**  $x = y$

Hint: You will need to use the properties of equality to solve the equations, then use the transitive property.

	STATEMENT	JUSTIFICATION
1		
2		
3		
4		
5		

Start by copying  
**BOTH** given  
statements.



(continued on next page)

# Let's complete the proof below together.

**Given:**  $\frac{x}{10} = 3$ ,  
 $y - 5 = 25$

**Prove:**  $x = y$

Next, solve the first equation!



	STATEMENT	JUSTIFICATION
1	$\frac{x}{10} = 3$	Given
2	$y - 5 = 25$	Given
3		
4		
5		

(continued on next page)

Hint: You will need to use the properties of equality to solve the equations, then use the transitive property.



# Let's complete the proof below together.

**Given:**  $\frac{x}{10} = 3$ ,  
 $y - 5 = 25$

**Prove:**  $x = y$

Now, solve the  
second equation.



	STATEMENT	JUSTIFICATION
1	$\frac{x}{10} = 3$	Given
2	$y - 5 = 25$	Given
3	$x = 30$	Multiplication Prop. of Equality
4		
5		

(continued on next page)

Hint: You will need to use the properties of equality to solve the equations, then use the transitive property.

# Let's complete the proof below together.

**Given:**  $\frac{x}{10} = 3,$   
 $y - 5 = 25$

**Prove:**  $x = y$

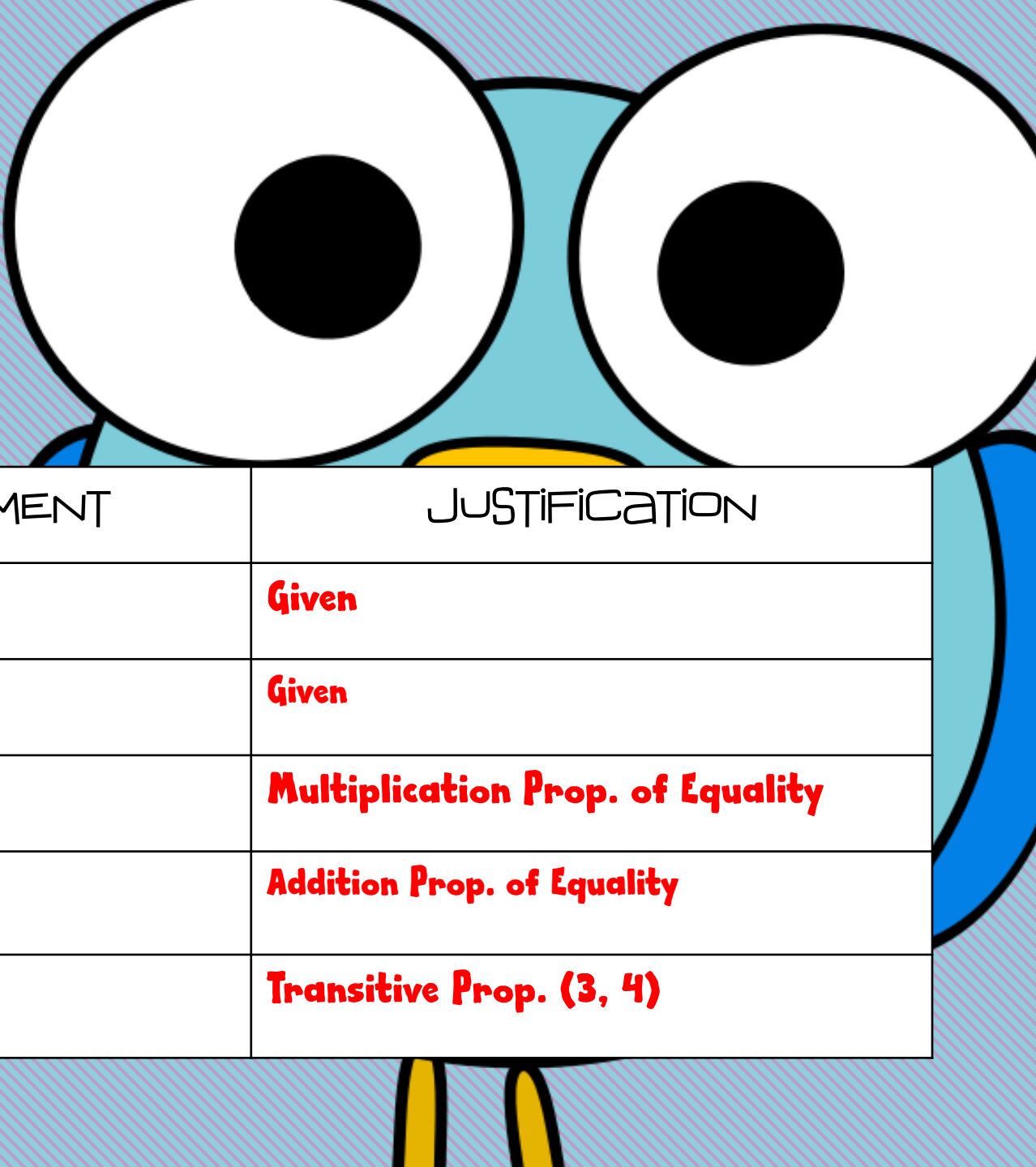
**Almost done!  
Finish up with the  
Transitive  
Property.**



	STATEMENT	JUSTIFICATION
1	$\frac{x}{10} = 3$	<b>Given</b>
2	$y - 5 = 25$	<b>Given</b>
3	$x = 30$	<b>Multiplication Prop. of Equality</b>
4	$y = 30$	<b>Addition Prop. of Equality</b>
5		

(continued on next page)

Hint: You will need to use the properties of equality to solve the equations, then use the transitive property.



**Given:**  $\frac{x}{10} = 3$ ,  
 $y - 5 = 25$

**Prove:**  $x = y$

	STATEMENT	JUSTIFICATION
1	$\frac{x}{10} = 3$	<b>Given</b>
2	$y - 5 = 25$	<b>Given</b>
3	$x = 30$	<b>Multiplication Prop. of Equality</b>
4	$y = 30$	<b>Addition Prop. of Equality</b>
5	$x = y$	<b>Transitive Prop. (3, 4)</b>



**Given:**     $m = n$ ,  
               $n + p = 2t$ ,  
               $t = m$

**Prove:**     $p = t$

*Complete the  
proof.*



	STATEMENT	JUSTIFICATION
1		
2		
3		
4		
5		
6		
7		



**Given:**     $m = n$ ,  
                $n + p = 2t$ ,  
                $t = m$

**Prove:**     $p = t$

*Complete the*



*proof.*

	STATEMENT	JUSTIFICATION
1	$m = n$	Given
2	$n + p = 2t$	Given
3	$t = m$	Given
4	$n = t$	Transitive Prop. (1, 3)
5	$n + p = 2n$	Subst. (2, 4)
6	$p = n$	Subtraction Prop. of Equal. (Subtract $n$ from both sides.)
7	$p = t$	Transitive Prop. (4, 6)



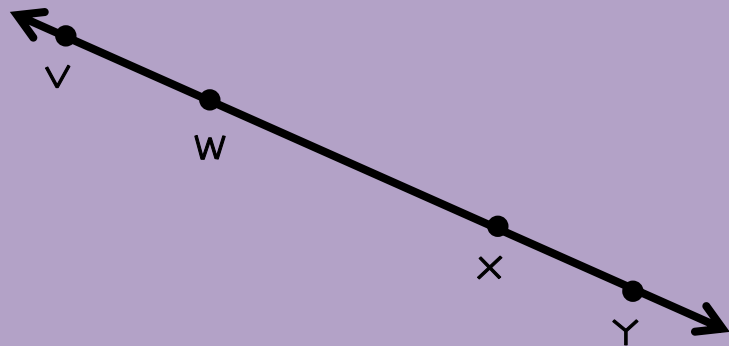


**Some proofs require additional justifications. We will now focus on proofs that use *definitions* and *postulates*.**

Here are a few justifications that will be used frequently in proofs.

ANGLE ADDITION POSTULATE  
CONGRUENT  
MIDPOINT  
SEGMENT ADDITION POSTULATE  
BISECTOR

# EXAMPLE 1



Before beginning a geometry proof, take a minute to look and think. Figure out what you know and what you are trying to prove! Use the diagram.

**Given:**  $W$  is the midpoint of  $\overline{VX}$ .  
 $XY = VW$

**Prove:**  $\overline{WX} \cong \overline{XY}$

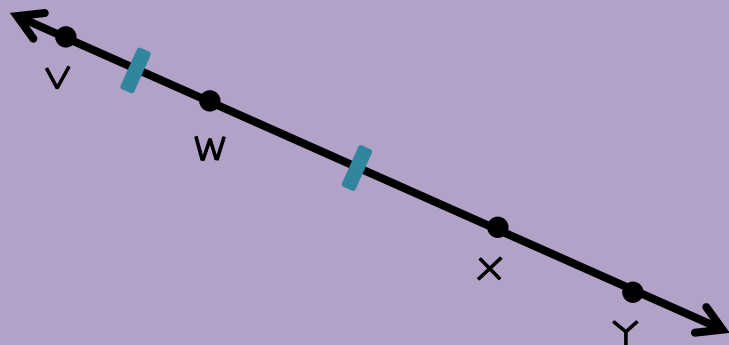
	STATEMENT	JUSTIFICATION
1		
2		
3		
4		
5		
6		
7		
8		



Start by copying the given information and marking it on the diagram.

(continued on next page)

# EXAMPLE 1



**Given:**  $W$  is the midpoint of  $\overline{VX}$ .  
 $XY = VW$

**Prove:**  $\overline{WX} \cong \overline{XY}$

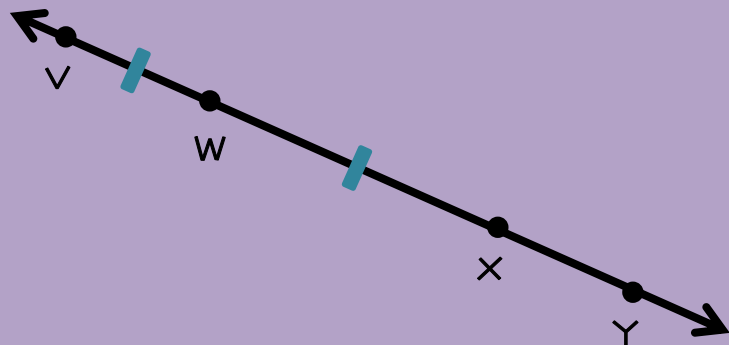
	STATEMENT	JUSTIFICATION
1	$W$ is the midpoint of $\overline{VX}$ .	Given
2	$XY = VW$	Given
3		
4		
5		
6		
7		
8		

Next, it helps to write any equations that we can (use definitions).



(continued on next page)

# EXAMPLE 1



**Given:**  $W$  is the midpoint of  $\overline{VY}$ .  
 $XY = VW$

**Prove:**  $\overline{WX} \cong \overline{XY}$

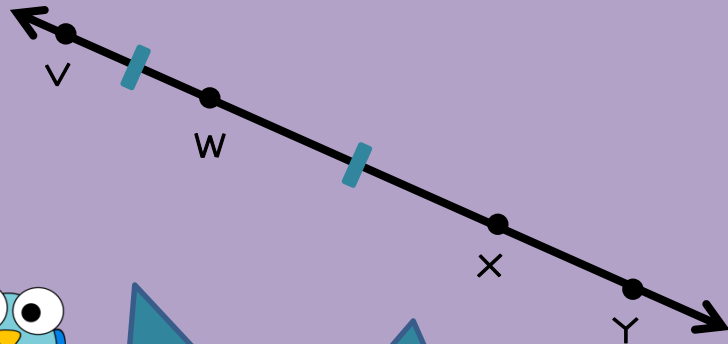
	STATEMENT	JUSTIFICATION
1	$W$ is the midpoint of $\overline{VY}$ .	Given
2	$XY = VW$	Given
3	$VW = WX$	Definition of midpoint
4		
5		
6		
7		
8		

Now, we can use the transitive property.



(continued on next page)

# EXAMPLE 1



**Given:**  $W$  is the midpoint of  $\overline{VY}$ .  
 $XY = VW$

**Prove:**  $\overline{WX} \cong \overline{XY}$

	STATEMENT	JUSTIFICATION
1	$W$ is the midpoint of $\overline{VY}$ .	Given
2	$XY = VW$	Given
3	$VW = WX$	Definition of midpoint
4	$WX = XY$	Transitive Prop. (2, 3)
5		
6		
7		
8		



We are almost there! Our final line does not look QUITE like the goal, so we use one more definition.

(continued on next page)



# EXAMPLE 1

The definition of congruent helps us convert between EQUALITY statements (equations) and CONGRUENCE statements.

When segments have equal lengths, they are congruent.



**Given:**  $W$  is the midpoint of  $\overline{VX}$ .  
 $XY = VW$

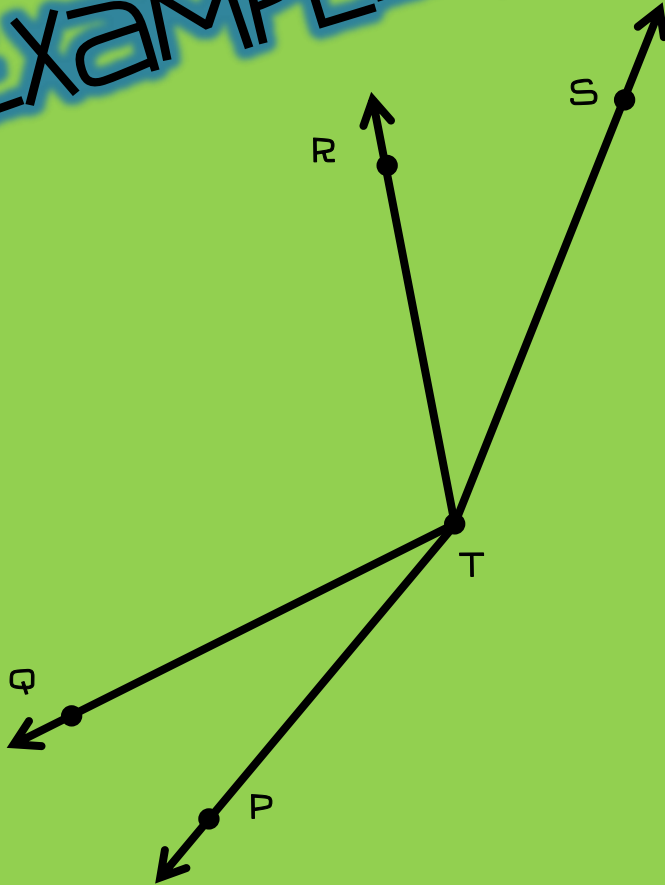
**Prove:**  $\overline{WX} \cong \overline{XY}$

	STATEMENT	JUSTIFICATION
1	<b><math>W</math> is the midpoint of <math>\overline{VX}</math>.</b>	<b>Given</b>
2	<b><math>XY = VW</math></b>	<b>Given</b>
3	<b><math>VW = WX</math></b>	<b>Definition of midpoint</b>
4	<b><math>WX = XY</math></b>	<b>Transitive Prop. (2, 3)</b>
5	<b><math>\overline{WX} \cong \overline{XY}</math></b>	<b>Definition of Congruent</b>
6		
7		
8		





# EXAMPLE 2



**Given:**  $\angle PTR \cong \angle QTS$

**Prove:**  $\angle PTQ \cong \angle RTS$

	STATEMENT	JUSTIFICATION
1		
2		
3		
4		
5		
6		
7		
8		

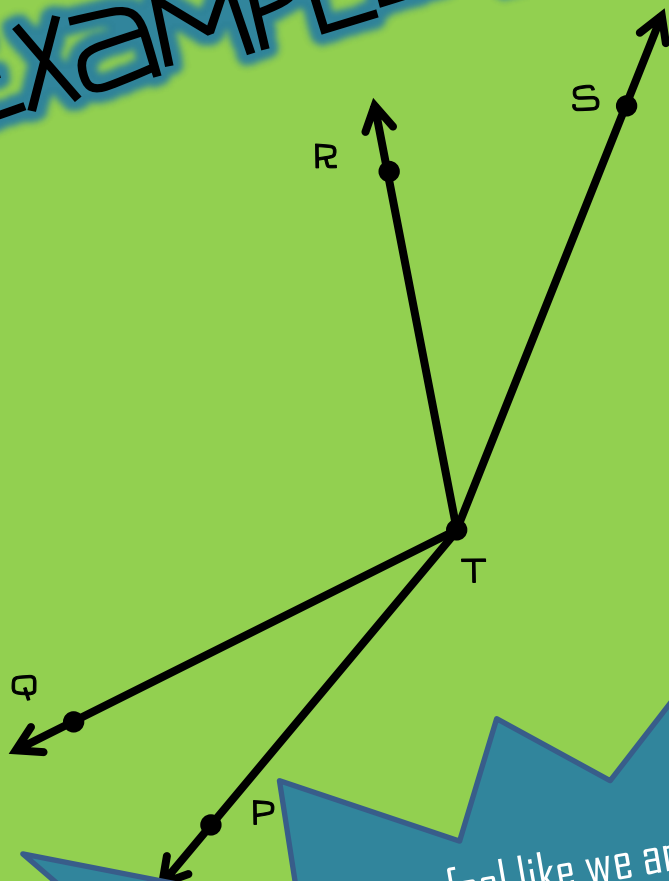
As always, start by filling in any given information. Then, let's see if we can convert it to an equation that we can work with a little more easily.



Before beginning a geometry proof, take a minute to look and think. Figure out what you know and what you are trying to prove! Use the diagram.

(continued on next page)

# EXAMPLE 2



**Given:**  $\angle PTR \cong \angle QTS$

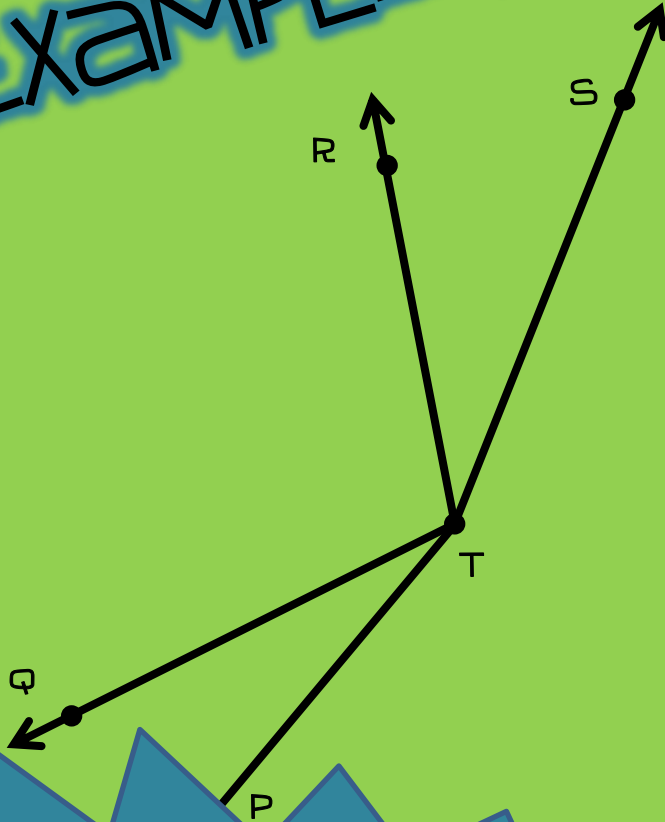
**Prove:**  $\angle PTQ \cong \angle RTS$

	STATEMENT	JUSTIFICATION
1	$\angle PTR \cong \angle QTS$	Given
2	$m\angle PTR = m\angle QTS$	Defn. congruent
3		
4		

Now, we may feel like we are stuck. This is when we use the diagram and think "Do I know anything else?" Sometimes, we can get additional information from the diagram. In this case, we can write some equations of our own using Angle Addition Postulate.

(continued on next page)

# EXAMPLE 2



**Given:**  $\angle PTR \cong \angle QTS$

**Prove:**  $\angle PTQ \cong \angle RTS$

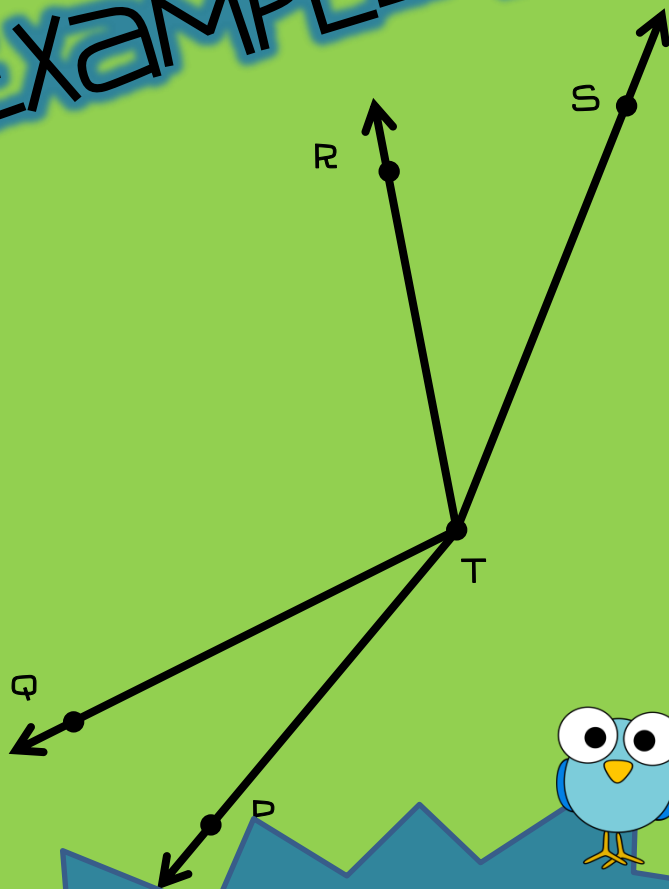
	STATEMENT	JUSTIFICATION
1	$\angle PTR \cong \angle QTS$	Given
2	$m\angle PTR = m\angle QTS$	Defn. congruent
3	$m\angle PTQ + m\angle QTR = m\angle PTR$	Angle Addition Post.
4	$m\angle RTS + m\angle QTR = m\angle QTS$	Angle Addition Post.
5		
6		
7		
8		

The Angle Addition Postulate gives us two new equations to work with. Now that we have a few lines of equations, let's try to combine some to get new statements that will lead us to our goal. Copy line 3, but replace  $m\angle PTR$  with  $m\angle QTS$ .



(continued on next page)

# EXAMPLE 2



**Given:**  $\angle PTR \cong \angle QTS$

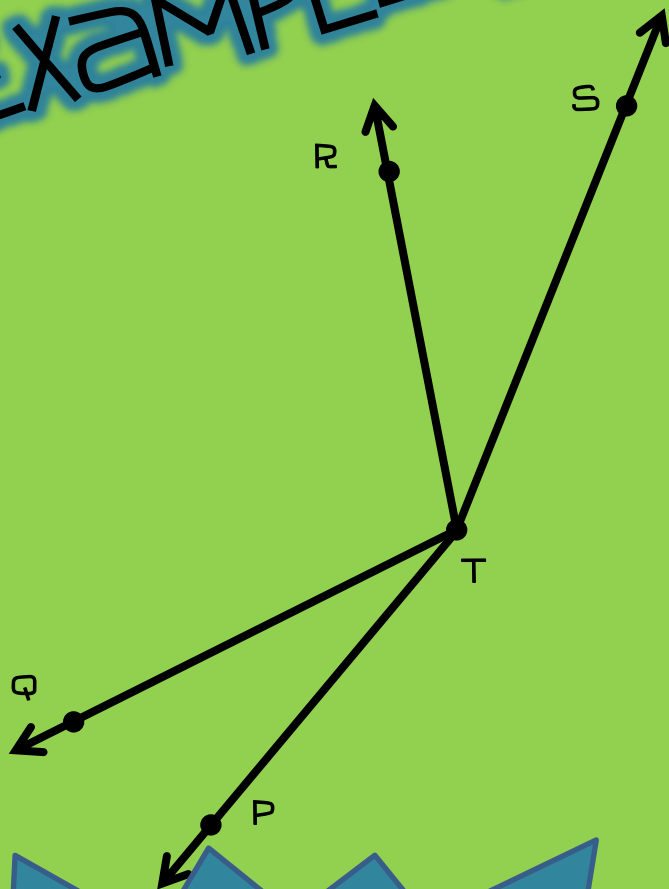
**Prove:**  $\angle PTQ \cong \angle RTS$

	STATEMENT	JUSTIFICATION
1	$\angle PTR \cong \angle QTS$	Given
2	$m\angle PTR = m\angle QTS$	Defn. congruent
3	$m\angle PTQ + m\angle QTR = m\angle PTR$	Angle Addition Post.
4	$m\angle RTS + m\angle QTR = m\angle QTS$	Angle Addition Post.
5	$m\angle PTQ + m\angle QTR = m\angle QTS$	Substitution (2, 3)
6		
7		
8		

Now, look at lines 4 and 5. When you notice that one entire side of the equation matches, that can be a signal to use the Transitive Property.

(continued on next page)

# EXAMPLE 2



**Given:**  $\angle PTR \cong \angle QTS$

**Prove:**  $\angle PTQ \cong \angle RTS$

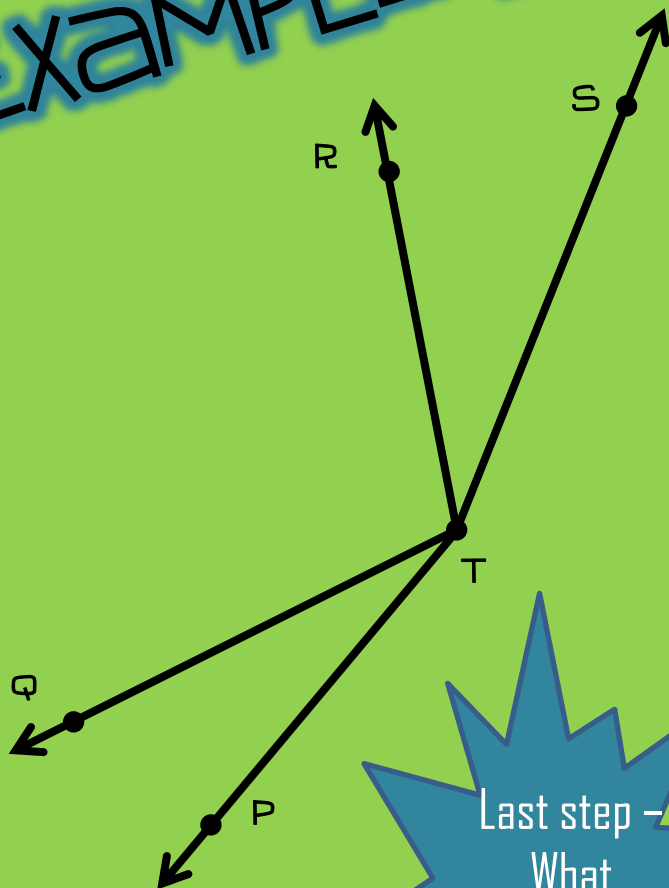
	STATEMENT	JUSTIFICATION
1	$\angle PTR \cong \angle QTS$	Given
2	$m\angle PTR = m\angle QTS$	Defn. congruent
3	$m\angle PTQ + m\angle QTR = m\angle PTR$	Angle Addition Post.
4	$m\angle RTS + m\angle QTR = m\angle QTS$	Angle Addition Post.
5	$m\angle PTQ + m\angle QTR = m\angle QTS$	Substitution (2, 3)
6	$m\angle RTS + m\angle QTR = m\angle PTQ + m\angle QTR$	Transitive Property (4, 5)
7		

We are almost there! Note that if we can get the  $m\angle QTR$  to drop away from both sides, we pretty much have the result we want. What property can make  $m\angle QTR$  "disappear?"



(continued on next page)

# EXAMPLE 2



Last step -  
What  
definition  
will finish up  
this proof?

**Given:**  $\angle PTR \cong \angle QTS$

**Prove:**  $\angle PTQ \cong \angle RTS$

	STATEMENT	JUSTIFICATION
1	$\angle PTR \cong \angle QTS$	Given
2	$m\angle PTR = m\angle QTS$	Defn. congruent
3	$m\angle PTQ + m\angle QTR = m\angle PTR$	Angle Addition Post.
4	$m\angle RTS + m\angle QTR = m\angle QTS$	Angle Addition Post.
5	$m\angle PTQ + m\angle QTR = m\angle QTS$	Substitution (2, 3)
6	$m\angle PTQ + m\angle QTR = m\angle RTS + m\angle QTR$	Transitive Property (4, 5)
7	$m\angle PTQ = m\angle RTS$	Subtraction Prop. of Equal.
8		

(continued on next page)

# EXAMPLE 2

Don't worry – It is normal if you are thinking “I would have no idea how to come up with the sequence of those steps myself!” – It takes practice. Take time to look it over and see how each step led to the next.



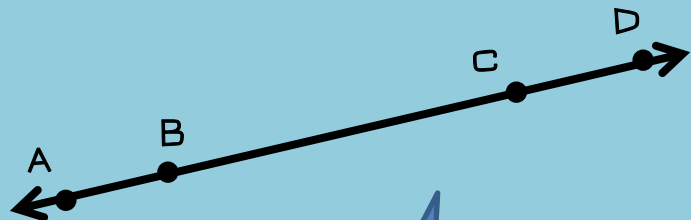
**Given:**  $\angle PTR \cong \angle QTS$

**Prove:**  $\angle PTQ \cong \angle RTS$

	STATEMENT	JUSTIFICATION
1	$\angle PTR \cong \angle QTS$	Given
2	$m\angle PTR = m\angle QTS$	Defn. congruent
3	$m\angle PTQ + m\angle QTR = m\angle PTR$	Angle Addition Post.
4	$m\angle RTS + m\angle QTR = m\angle QTS$	Angle Addition Post.
5	$m\angle PTQ + m\angle QTR = m\angle QTS$	Substitution (2, 3)
6	$m\angle PTQ + m\angle QTR = m\angle RTS + m\angle QTR$	Transitive Property (4, 5)
7	$m\angle PTQ = m\angle RTS$	Subtraction Prop. of Equal.
8	$\angle PTQ \cong \angle RTS$	Defn. congruent



# EXAMPLE 3



**Given:**  $AB = CD$

**Prove:**  $AC = BD$

Start this one off. You should have a good idea how to begin.

Once you've got the given statement, then start writing some equations using Segment Addition Postulate. See if you can then combine your equations to write new ones.



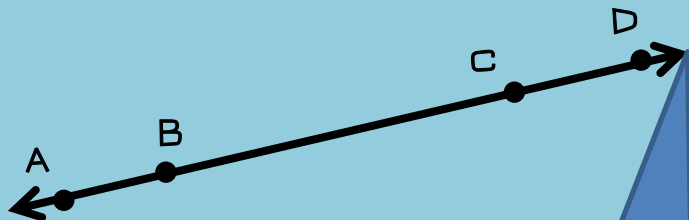
	STATEMENT	JUSTIFICATION
1		
2		
3		
4		
5		
6		
7		
8		



# EXAMPLE 3

**Given:**  $AB = CD$

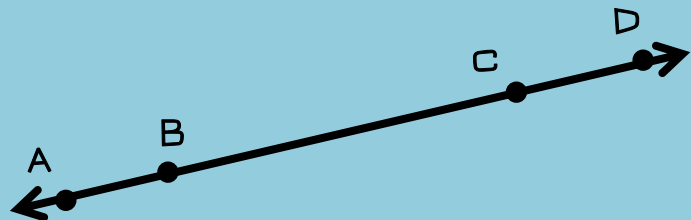
**Prove:**  $AC = BD$



How far were you able to get on your own? We have to use substitution to get one side of the equation to match. We have a few options – there can be more than one correct proof. Here is one substitution you can do.

	STATEMENT	JUSTIFICATION
1	$AB = CD$	Given
2	$AB + BC = AC$	Segment Add. Post.
3	$CD + BC = BD$	Segment Add. Post.
4	$AB + BC = BD$	Substitution (1, 3)
5		
6		
7		
8		

# EXAMPLE 3



The transitive property is all we need to finish it up!



**Given:**  $AB = CD$

**Prove:**  $AC = BD$

	STATEMENT	JUSTIFICATION
1	$AB = CD$	Given
2	$AB + BC = AC$	Segment Add. Post.
3	$CD + BC = BD$	Segment Add. Post.
4	$AB + BC = BD$	Substitution (1, 3)
5	$AC = BD$	Transitive Prop. (2, 4)
6		
7		
8		

# Review of steps

IN GENERAL, WHEN WRITING A TWO- COLUMN PROOF,  
WE MUST:

1. COPY ALL GIVEN INFORMATION
2. PAUSE TO GET AN UNDERSTANDING OF WHAT YOU KNOW AND WHAT YOU ARE TRYING TO PROVE. MAKE MARKS ON THE DIAGRAM.
3. WRITE AND WORK WITH EQUATIONS BASED ON THE GIVEN STATEMENTS IF POSSIBLE. (SOMETIMES YOU CAN CONVERT STATEMENTS INTO EQUATIONS USING DEFINITIONS)
4. DEVELOP NEW EQUATIONS FROM THE DIAGRAM IF POSSIBLE.
5. MANIPULATE AND COMBINE YOUR EQUATIONS, ALWAYS KEEPING YOUR GOAL IN MIND. (YOU CAN SOMETIMES USE THE TRANSITIVE PROPERTY OR SUBSTITUTION TO COMBINE TWO LINES)



# Special Angle Pairs

## REVIEW THESE ANGLE PAIRS:

COMPLEMENTARY ANGLES (2 & 3)

SUPPLEMENTARY ANGLES (4 & 5)

ADJACENT ANGLES (1 & 2)

VERTICAL ANGLES (6 & 8)

LINEAR PAIR (6 & 7)

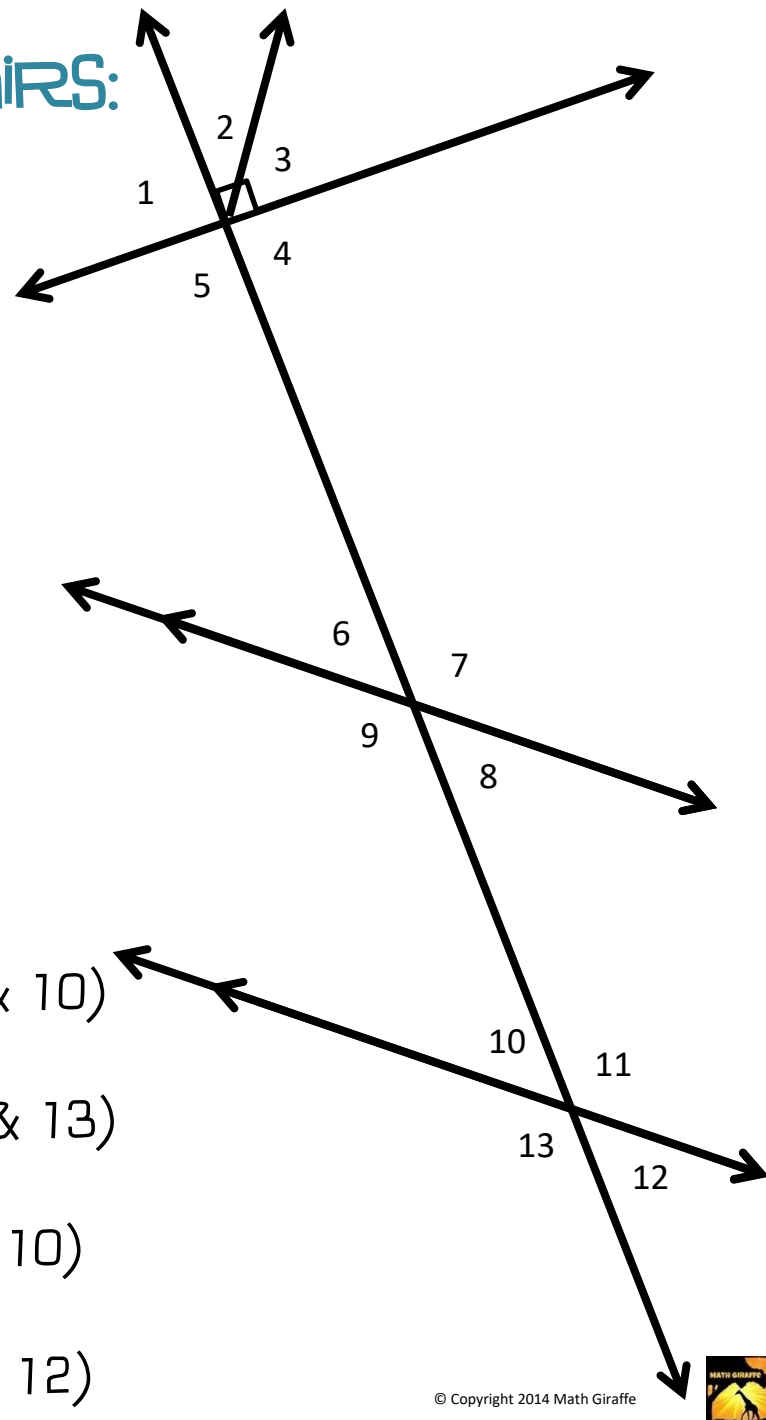
CORRESPONDING ANGLES (6 & 10)

ALTERNATE INTERIOR ANGLES (8 & 10)

ALTERNATE EXTERIOR ANGLES (7 & 13)

SAME-SIDE INTERIOR ANGLES (9 & 10)

SAME-SIDE EXTERIOR ANGLES (7 & 12)



# THEOREMS THAT YOU CAN USE AS JUSTIFICATIONS:

Vertical Angles Theorem: Vertical angles are congruent.

Right angles Theorem: All right angles are congruent.

Linear Pair Theorem: Angles in a linear pair are supplementary.

YOU MIGHT ALSO USE SOME NEW  
DEFINITIONS.  
(SUCH AS THE FOLLOWING SAMPLES):

Definition of perpendicular

Definition of right angle

## For Angles Along a Transversal:

Corresponding Angles Postulate: If lines are parallel, then corresponding angles are congruent.

Alternate Interior / Alternate Exterior Angles Theorems: If lines are parallel, then alternate interior / alternate exterior angles are congruent.

Same- Side Interior / Same- Side Exterior Angles Theorems: If lines are parallel, then same-side interior / same-side exterior angles are supplementary.

(We also often use the CONVERSES of the theorems regarding angles along a transversal.)

**Given:** Angle 2 is a right angle.  
Lines  $m$  and  $n$  are parallel.

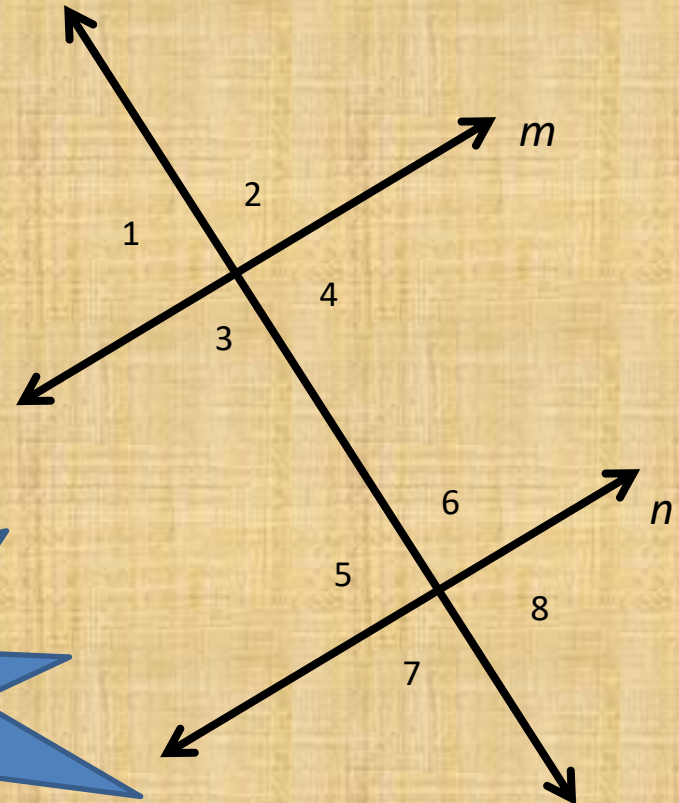
**Prove:**  $m\angle 7 = 90^\circ$

LET'S TRY  
THIS PROOF

	STATEMENT	JUSTIFICATION
1		
2		
3		
4		
5		
6		
7		
8		



After copying the given statements, write an equation using the definition of right angle.

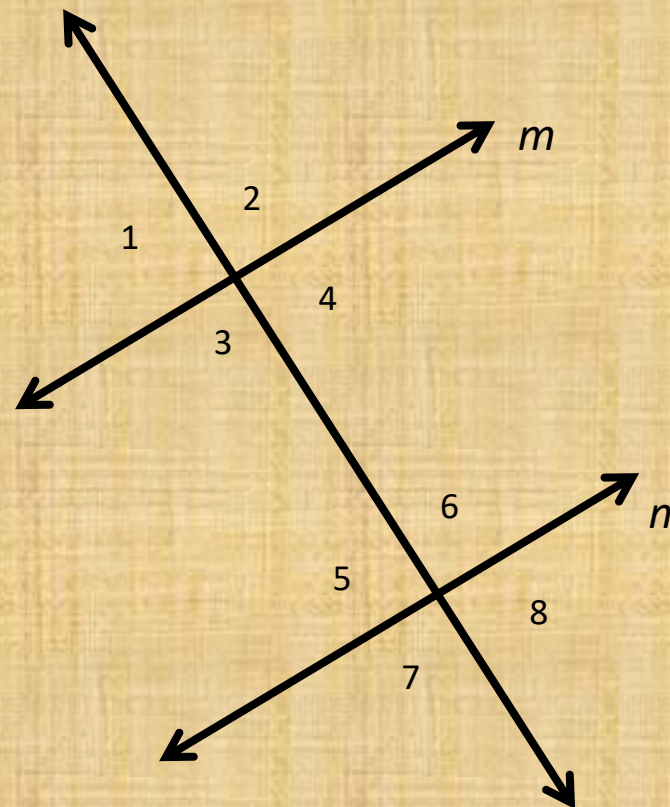


**Given:** Angle 2 is a right angle.  
Lines m and n are parallel.

**Prove:**  $m\angle 7 = 90^\circ$

	STATEMENT	JUSTIFICATION
1	Angle 2 is a right angle.	Given
2	Lines m and n are parallel.	Given
3	$m\angle 2 = 90$	Defn. right angle
4		
5		
6		
7		
8		

Choose the correct theorem for angles along a transversal. Remember our goal is to incorporate angle 7.



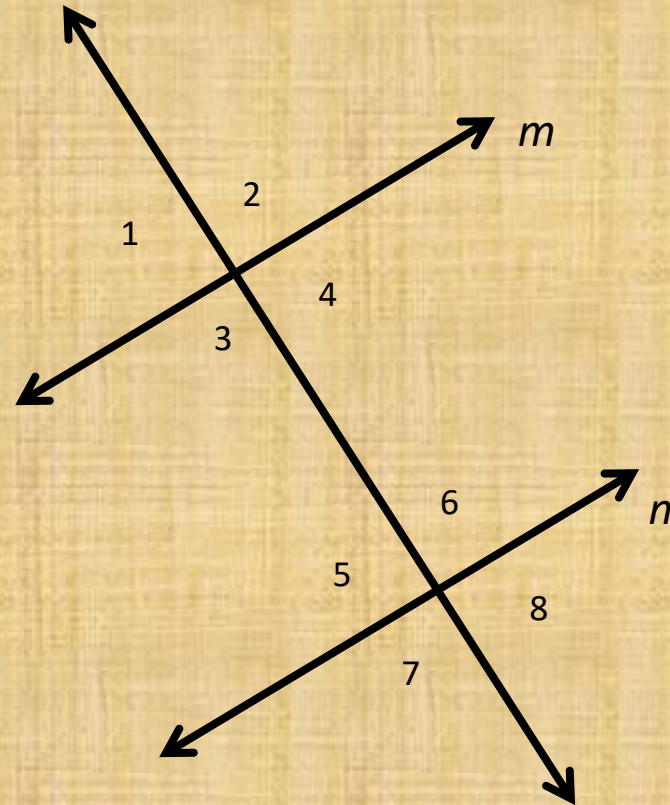
**Given:** Angle 2 is a right angle.  
Lines  $m$  and  $n$  are parallel.

**Prove:**  $m\angle 7 = 90^\circ$



Almost there!  
Finish it up.

	STATEMENT	JUSTIFICATION
1	Angle 2 is a right angle.	Given
2	Lines $m$ and $n$ are parallel.	Given
3	$m\angle 2 = 90$	Defn. right angle
4	$m\angle 2 = m\angle 7$	Alternate Exterior Angles Theorem
5		
6		
7		
8		

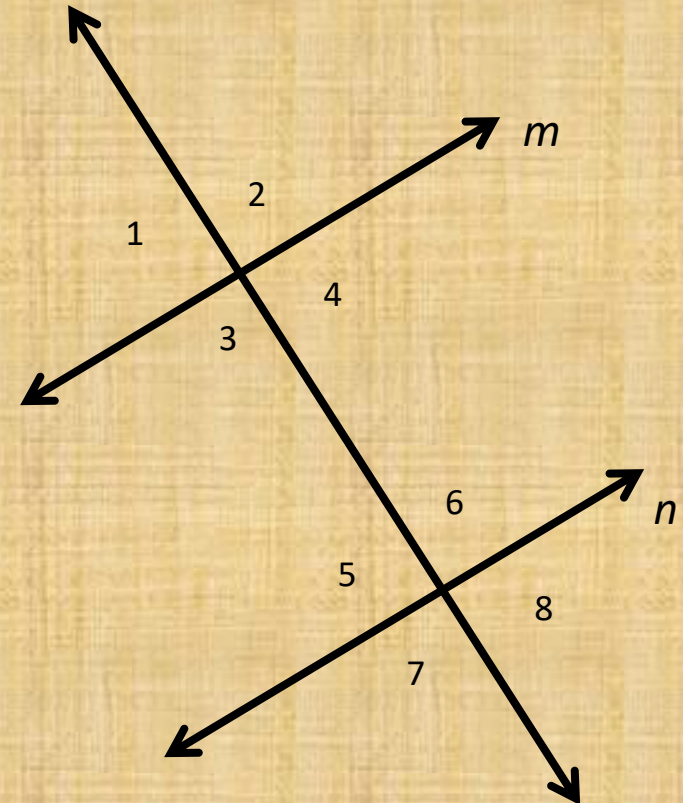




**Given:** Angle 2 is a right angle.  
Lines  $m$  and  $n$  are parallel.

**Prove:**  $m\angle 7 = 90^\circ$

	STATEMENT	JUSTIFICATION
1	Angle 2 is a right angle.	Given
2	Lines $m$ and $n$ are parallel.	Given
3	$m\angle 2 = 90$	Defn. right angle
4	$m\angle 2 = m\angle 7$	Alternate Exterior Angles Theorem
5	$m\angle 7 = 90$	Substitution (3, 4)
6		
7		
8		

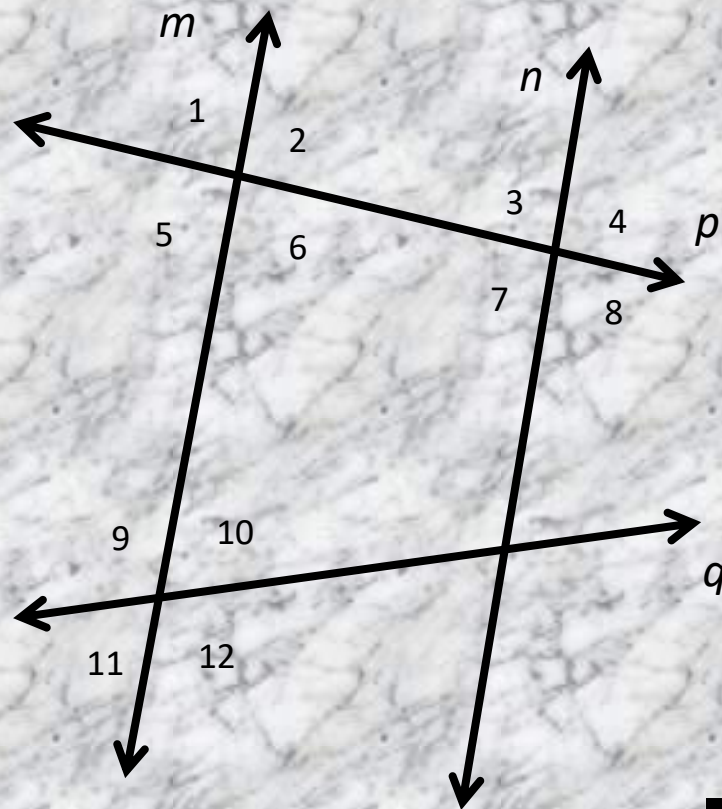


**Given:**  $\angle 9 \cong \angle 3$   
 $m \parallel n$

**Prove:**  $p \parallel q$

	STATEMENT	JUSTIFICATION
1		
2		
3		
4		
5		
6		
7		
8		

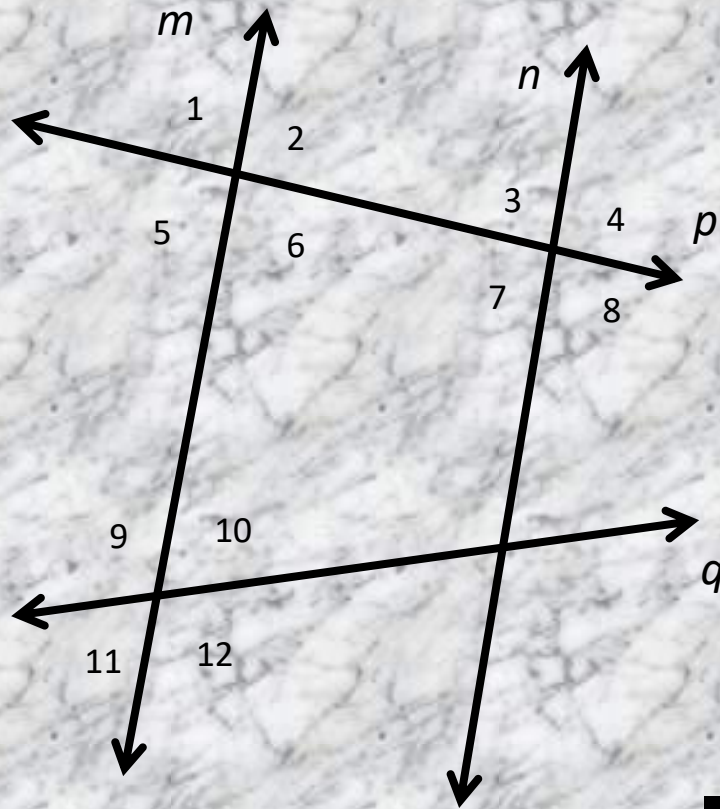
ONE MORE  
 PRACTICE PROOF -  
 TRY THE WHOLE  
 THING!



**Given:**  $\angle 9 \cong \angle 3$   
 $m \parallel n$

**Prove:**  $p \parallel q$

	STATEMENT	JUSTIFICATION
1	$\angle 9 \cong \angle 3$	<b>Given</b>
2	$m \parallel n$	<b>Given</b>
3	$\angle 1 \cong \angle 3$	<b>Corresp. Angles Post.</b>
4	$\angle 1 \cong \angle 9$	<b>Transitive Prop. (1, 3)</b>
5	$p \parallel q$	<b>Converse of Corresp. Angles Post.</b>
6		
7		
8		



# PROOFS WITH TRIANGLES

*A few more theorems to use as justifications*

**TRIANGLE SUM THEOREM:** The sum of the interior angles of a triangle is 180 degrees.

**BASE ANGLE THEOREM FOR ISOSCELES**

**TRIANGLES:** If two sides of a triangle are congruent, then the two angles opposite them are congruent.

**CONVERSE OF BASE ANGLE THEOREM:** If two angles of a triangle are congruent, then the two sides opposite them are congruent.

