Ayoola Etiko, STATS 3093, 3598133

Question 2

↳ page 391 excersise 8

a

$$P\left(-Z_{a_1} < \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} < Z_{a_2}\right) = 1-\alpha$$

$$P\left(-Z_{a_1}\frac{\sigma}{\sqrt{n}} < \bar{X}-\mu < Z_{a_2}\frac{\sigma}{\sqrt{n}}\right) = 1-\alpha$$

multiplying both $-Z_{a_1}$ & Z_{a_1} by $\frac{\sigma}{\sqrt{n}}$

$$P\left(-Z_{\alpha_1}\frac{\sigma}{\sqrt{n}} - \bar{X} < -\mu < Z_{\alpha_2}\frac{\sigma}{\sqrt{n}} - \bar{X}\right)$$

$$\cancel{P\left(-Z_{\alpha_2}\frac{\sigma}{\sqrt{n}} + \bar{X} > \mu >\right.}$$

$$P\left(Z_{\alpha_1}\frac{\sigma}{\sqrt{n}} + \bar{X} > \mu > -Z_{\alpha_2}\frac{\sigma}{\sqrt{n}} + \bar{X}\right) = 100(1-\alpha)$$

our general formula for $100(1-\alpha)$

is $P\left(Z_{\alpha_1}\frac{\sigma}{\sqrt{n}} + \bar{X},\ -Z_{\alpha_2}\frac{\sigma}{\sqrt{n}} + \bar{X}\right)$

$$= P\left(\bar{X} - Z_{\alpha_2}\frac{\sigma}{\sqrt{n}},\ \bar{x} + Z_{\alpha_1}\frac{\sigma}{\sqrt{n}}\right)$$

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(b) $\alpha = .05$ $\quad \alpha_1 = \alpha/4 = 0.0125 \quad \alpha_2 = 3\alpha/4 = 0.0375$

$\alpha = 0.05 \quad Z_\alpha = 1.96$
$\alpha_1 = 0.0125 \quad Z_{\alpha_1} = 2.24$
$\alpha_2 = 0.0375 \quad Z_{\alpha_2} = 1.78$

using the general formula:

$$\text{width} = \frac{2 z_\alpha \sigma}{\sqrt{n}} = 2 \cdot 1.96 \times \frac{\sigma}{\sqrt{n}}$$

$$= 3.92 \frac{\sigma}{\sqrt{n}}$$

Width with $100(1-\alpha)$

$$\left(-z_{\alpha_2}\frac{\sigma}{\sqrt{n}},\ z_{\alpha_1}\frac{\sigma}{\sqrt{n}}\right)$$

$\cancel{-1.78\frac{\sigma}{\sqrt{n}}}$

$$2.24\frac{\sigma}{\sqrt{n}} - \left(-1.78\frac{\sigma}{\sqrt{n}}\right) = 4.02\frac{\sigma}{\sqrt{n}}$$

hence with the general formula of $100(1-\alpha)$ gives a wider intervals in the specific formula.