Sunday, September 26, 2021 9:41 PM

Lecture 5 :-

A says "
B says "
C says "

" knave: knights.
" Spies.
".

p_1 : A is a knight — $\neg p_1$: A is not a knight
p_2 : A is a knave — $\neg p_2$: A is not a knave
p_3 : A is a Spy. — $\neg p_3$: A is not a Spy.

q_1 :
q_2 :
q_3 :

$\neg q_1$:
$\neg q_2$:
$\neg q_3$:

γ_1 :
γ_2 :
γ_3 :

$\neg\gamma_1$:
$\neg\gamma_2$:
$\neg\gamma_3$:

Case 1 :- A: knight, B: knight, C: knight

○ = T
○ = T ✗
○ = T

$p_1 = T$ — $\neg p_1 = F$
$p_2 = F$ — $\neg p_2 = T$
$p_3 = F$ — $\neg p_3 = T$
$q_1 = T$ — $\neg q_1 =$
$q_2 = F$ — $\neg q_2 =$
$q_3 = F$ — $\neg q_3 =$
$\gamma_1 = T$ — $\neg\gamma_1 =$
$\gamma_2 = F$ — $\neg\gamma_2 =$
$\gamma_3 = F$ — $\neg\gamma_3 =$

Predicates:- $P(x)$: $x > 3$. $x \in \{0, 1, 2, 3, 4\}$.

x: Variable. > 3: Condition.

$P(0)$: $0 > 3$. $= F$ ✓

$P(1)$: $1 > 3$ $= F$ ✓

$P(2)$: $2 > 3$ $= F$ ✓

$P(3)$: $3 > 3$. $= F$ ✓

$P(4)$: $4 > 3$. $= T$. ✓

Quantifiers. Universal :- $\forall$ → Syntax. → Symantics.

$\forall x P(x)$.

↳ : $= P(1) \wedge P(2) \wedge P(3) \ldots\ldots \wedge P(N)$.

$x \in \{1, 2, 3, \ldots N\}$.

all of, for every, for all, given any, for each. ...

Ex 8:- P33:- $P(x)$: $x + 1 > x$ $x \in \mathbb{R}$.

$\forall x P(x)$: T.

Counter Example

Ex 9:- P33. $P(x)$: $x < 2$. $x \in \mathbb{R}$.

$\forall x P(x)$: ?

$P(3)$: $3 < 2$. : F.

↳ F.

Ex10 :- P34 :- $p(x) = x^2 \geq 0$. $x \in \mathbb{Z}$.

Ex11 :- P34 $p(x) = x^2 < 10$ $x \in \{1,2,3,4\}$.

$\forall x\, p(x) = p(1) \wedge p(2) \wedge p(3) \wedge p(4)$.

$= T \wedge T \wedge T \wedge F = F$.

Ex12 :- P34 Do it yourself.

Ex13 :- p34. $\forall x\, (x^2 \geq x)$ $x \in \mathbb{R}$.

let $p(x) = x^2 \geq x$.

$\forall x\, p(x) = \forall x\, (x^2 \geq x)$

$p(0.5) = (0.5)^2 \geq 0.5 = F$.

$0.25 \not\geq 0.5$

Ex 5 P31 :- $R(x,y,z) = x+y=z$.

$R(1,2,3) = 1+2=3 = T$.

Ex 1-4 Do it P 31.

Existential Quantifier $\exists$

$\exists x\, p(x) = p(1) \vee p(2) \vee \cdots \vee p(N)$ $\boxed{x \in \{1,2,\ldots N\}}$.

for atleast one, for some, there exist,

Ex 14. P35 :- $p(x) = x \geq 3$. $x \in \mathbb{R}$.

$\exists x P(x) = ?$ $= T$

EXIS p35 $P(x) = x = x+1$ $x \in \mathbb{R}$.

$\exists x P(x) = ? = F$.

Bound and free Variable.

$\forall x P(x, y, z)$.

Bounded. Free Variables.

$\forall x \forall y P(x, y, z)$. Free.

Bounded

P 37:- $\forall x (Q(x) \wedge R(x))$.

$= \forall x Q(x) \wedge \forall x R(x)$. $x \in \{1, 2, 3, \dots N\}$

$\neg \forall x P(x) = \neg (P(1) \wedge P(2) \wedge P(3) \wedge \dots \wedge P(N))$.

$= \neg P(1) \vee \neg P(2) \vee \neg P(3) \vee \dots \vee \neg P(N)$.

$= \exists x \neg P(x)$. ★ $\neg \forall x P(x) = \exists x \neg P(x)$

$\neg \exists x P(x) = \neg (P(1) \vee P(2) \vee P(3) \vee \dots \vee P(N))$

$= \neg P(1) \wedge \neg P(2) \wedge \neg P(3) \wedge \dots \wedge \neg P(N)$.

$= \forall x \neg P(x)$. ★ $\neg \exists x P(x) = \forall x \neg P(x)$

$\neg\exists x P(x) \equiv \forall x \neg P(x)$

$\neg(\forall x \exists y \forall z\ P(x,y,z))$ find Negation.

$\neg \forall x (\exists y \forall z P(x,y,z))$ $\neg \forall x P(x)$.

$P(x)$. $= \exists x \neg P(x)$.

$\exists x \neg \exists y (\forall z P(x,y,z))\ P(x)$.

$\neg \exists x P(x) \equiv \forall x \neg P(x)$.

$\exists x \forall y \neg \forall z P(x,y,z)$

$\exists x \forall y \exists z \neg P(x,y,z)$. $x,y,z = \{0,1\}$.

Expression?

How to Solve English Statements involving Quantifiers.