lecture 15 :- Ex 2. 477.

$$
M_R = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \end{array}
\begin{array}{c}
\begin{array}{ccccc} b_1 & b_2 & b_3 & b_4 & b_5 \end{array} \\
\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}
\end{array}
$$

$A = \{a_1, a_2, a_3\}$ $B = \{b_1, b_2, b_3, b_4, b_5\}$.

$R = \{(a_1,b_2), (a_2,b_1), (a_2,b_3), (a_2,b_4), (a_3,b_1), (a_3,b_3), (a_3,b_5)\}$.

PROPERTIES OF RELATIONS. (How to determine the Properties in terms of MATRICES).

1- REFLEXIVE :- $\forall a \in A \quad (a,a) \in R$.

$\forall a_i \in A \quad (a_i, a_i) \in R$. $A = \{a_1, a_2, a_3, \dots a_n\}$.

$\forall i \in \{1, 2, 3 \dots n\} \quad m_{ii} = 1$.

$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $[0]$. $[1]$. $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

2- Symmetric $\forall a,b \in A$ if $(a,b) \in R \rightarrow (b,a) \in R$.

$\forall a_i, b_j \in A, B$ if $(a_i, b_j) \in R \rightarrow (b_j, a_i) \in R$. $A = \{a_1, a_2, \dots a_n\}$ $B = \{b_1, b_2 \dots b_m\}$.

$\forall i,j \in \{1, 2, \dots n\}$. if $m_{ij} = 1 . \rightarrow m_{ji} = 1$.

$m_{12} = 1 \rightarrow m_{21} = 1$.

$[0]$. $[1]$. $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ $m_{12} = 1$. $m_{21} = 0$. $M^T = M$.

$[\ ]$

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

3- Anti Symmetric :- $\forall a,b \in A$ if $(a,b) \in R \land (b,a) \in R \rightarrow a = b$

$A = \{a_1, a_2, \dots a_n\}$.

$\forall a_i, b_j \in A, B$ if $(a_i, b_j) \in R \land (b_j, a_i) \in R \rightarrow a_i = b_j$. $B = \{b_1, b_2, \dots b_n\}$.

$\forall a_i, b_j \in A, B$ if $(a_i, b_j) \in R \wedge (b_j, a_i) \in R \rightarrow a_i = b_j$. $B = \{b_1, b_2, \cdots b_n\}$.

$\forall i,j \in \{1, 2, \cdots n\}$ if $\underbrace{m_{ij} = 1 \wedge m_{ji} = 1}_{\times} \rightarrow i = j$.

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$m_{12} = 1 \wedge m_{21} = 1 \rightarrow 1 \neq 2$.

$[1]$. $[0]$.

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

m_{12}.

m_{21} $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Ex 4.
478.

$$M_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad M_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}.$$

$$M_{R_1 \cap R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad M_{R_1 \cup R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

$M_{R_1 - R_2}$ = HW $\qquad$ $M_{R_2 - R_1}$ = HW.

COMPOSITE OF A RELATION.

R	(a, b)	$A \times B$	$a \in A$	$b \in B$.
S	(b, c)	$B \times C$.	$b \in B$	$c \in C$.

$S \circ R$ (a, c).

$(a, c) \in S \circ R$ if $\exists_b$ $(a, b) \in R \wedge (b, c) \in S$.

$M_R = [r_{ij}]$. $m \times n$.

$M_S = [s_{ij}]$. $n \times p$.

$M_{S \circ R} = [t_{ij}]$. $m \times p$.

R (a_i, b_k).

$S \quad (b_k, c_j)$.

$S \circ R \quad (a_i, c_j)$.

$(a_i, c_j) \in S \circ R \quad \text{iff } \exists b_k \quad (a_i, b_k) \in R \wedge (b_k, c_j) \in S$.

$t_{ij} = 1 \quad \text{iff } \exists k \quad r_{ik} = 1 \wedge s_{kj} = 1$.

$t_{13} = 1 \quad \text{iff } \exists k \quad r_{1k} = 1 \wedge s_{k3} = 1$.

Ex 5
498.

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad M_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$M_{S \circ R} = \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ HW & HW & HW \\ HW & HW & HW \end{bmatrix}$$

Ex 1–20. P 481–482.

Quiz #1:- $A = \{1, 2, \dots, 100\}$.
$R = \{(a,b) \mid a > b\}$.

$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & \cdots & 100 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ \vdots \\ 100 \end{matrix} & \begin{bmatrix} 0 & 0 & \cdots & 0 \\ & \ddots & & \\ & & \ddots & \\ & & & 0 \end{bmatrix} \end{array}$$

How many Non Zero entries in the Matrix M_R.

Total $= 10{,}000 - 100 = \underline{9900}$

$\frac{9900}{2} = 4950$ (ones)

$R = \{(a,b) \mid a \neq b\}$.

$= \frac{0}{10}$

COMPLEMENT - $M_{\bar{R}}$ = By Subtracting Each element from 1.

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$M_{R^{-1}}$ = By taking transpose of M_R.

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

transpose.