

lecture 14:- ADDITIONAL OPERATIONS ON RELATIONS.

1. COMPLEMENT OF RELATION.

$$\bar{R} = \{ (a,b) \mid (a,b) \notin R \}$$

$$\bar{R} = A \times A - R$$

$$|A \times A| - |R|$$

$$R' = \{ (b,a) \mid (a,b) \in R \}$$

Ex:- $A = \{1, 2, 3, 4\}$

$$A \times A = \{ (1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4) \}$$

$$R = \{ (a,b) \mid a < b \}$$

$$R = \{ (1,2), (1,3), (1,4), (2,3), (2,4), (3,4) \}$$

$$\bar{R} = \{ (1,1), (2,1), (2,2), (3,1), (3,2), (3,3), (4,1), (4,2), (4,3), (4,4) \}$$

$$R' = \{ (2,1), (3,1), (4,1), (3,2), (4,2), (4,3) \}$$

$$A = \{1, 2\}$$

$$A \times A \times A = \{ (1,1,1), (1,1,2), (1,2,1), (1,2,2), (2,1,1), (2,1,2), (2,2,1), (2,2,2) \}$$

Q:- How many binary relations on A.

$$A:- 2^{|A| \times |A|}$$

Q:- How many ternary relations on A.

$$A:- 2^{|A| \times |A| \times |A|}$$

$$R \subseteq A \times A \times A$$

$$A = n$$

$$2^{|A| \times |A| \times |A|}$$

$$= 2^{n^3}$$

$$2^{2 \times 2 \times 2}$$

$$= 2^8 = 256$$

Q:- How many 4ary relations on A.

$$A:- 2^{|A| \times |A| \times |A| \times |A|}$$

Ex1:-
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$$R \quad N \times N \times N$$

$$R = \{ (a,b,c) \mid a < b < c \}$$

$$(1,2,3) \in R$$

$$\downarrow \downarrow \downarrow$$

$$a \quad b \quad c$$

$$(2,4,3) \notin R$$

Ex2:-
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$$R \quad N \times N \times N$$

$$R = \{ (a,b,c) \mid b = a+k, c = a+2k \}$$

$$k \in \mathbb{Z}$$

$$(1,3,5) \in R$$

$$3 = 1+k \Rightarrow k=2$$

$$k=2 \in \mathbb{Z}$$

$$\downarrow \downarrow \downarrow$$

$$a \quad b \quad c$$

$$5 = 1+2 \cdot 2 \Rightarrow 5 = 1+4 \Rightarrow 5=5$$

$$(2,5,9) \in R ?$$

Ex3:-
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$$R \quad \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}^+$$

$$R = \{ (a,b,m) \mid a \equiv b \pmod{m} \}$$

$$-1 \equiv -9 \pmod{4}$$

$$(8,2,3) \in R$$

$$\downarrow \downarrow \downarrow$$

$$a \quad b \quad m$$

$$(-1,9,5) \in R \quad ? \quad \text{HW.}$$

$$(4,0,7) \in R \quad \checkmark$$

$(8, 2, 3) \in R$

$\downarrow \downarrow \downarrow$
 $a \ b \ m$

$(-1, 9, 5) \in R \ ? \quad \text{HW.}$

$(14, 0, 7) \in R \quad \checkmark$

$(11, 0, 6) \in R \ ? \quad \text{HW.}$

$(-2, -8, 5) \in R \ ? \quad \text{HW.}$

Ex 4:- R

A, N, S, D, T

A = Set of Airlines.

N = Set of flight Numbers.

S = Set of starting points.

D = Set of Destinations.

T = Set of Departure times.

$= (PIA, PK204, PW, SKI, 12:00am)$

REPRESENTING RELATIONS Using Matrices.

M

m_{ij}

$R \quad A \times B$

$A = \{a_1, a_2, a_3, \dots, a_m\}$

$B = \{b_1, b_2, b_3, \dots, b_n\}$

$M_R = [m_{ij}]$

$m_{ij} = \begin{cases} 1 & \text{if } (a_i, b_j) \in R \\ 0 & \text{if } (a_i, b_j) \notin R \end{cases}$

Row = $|A| = m$

Col = $|B| = n$

Ex 1:-
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$A = \{1, 2, 3\}$
 $\uparrow \quad \uparrow \quad \uparrow$
 $a_1 \ a_2 \ a_3$

$B = \{1, 2\}$
 $\uparrow \quad \uparrow$
 $b_1 \ b_2$

$R = \{(2, 1), (3, 1), (3, 2)\}$

$M_R = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \\ m_{31} & m_{32} \end{bmatrix} = \begin{matrix} & \begin{matrix} b_1 & b_2 \end{matrix} \\ \begin{matrix} a_1 \\ a_2 \\ a_3 \end{matrix} & \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \end{matrix}$

Ex 2:- HW.

