Lecture 17:- Ex - P481-482 Q1-30.

CLOSURE OF RELATIONS.

→ REFLEXIVE CLOSURE.

$R = \{(a,a)\} \cup \boxed{\{(a,a),(b,b)\}}$

$R = \{(a,b)\} \cup \{(a,a),(b,b)\}$

$= \{(a,b),(a,a),(b,b)\}.$

$\Delta = \{(a,a) \mid a \in A\}.$

$R \cup \Delta$ = Reflexive.

Ex1:- $R = \{(a,b) \mid a < b\}.$ $A = \mathbb{Z}.$

$R \cup \Delta = \{(a,b) \mid a < b\} \cup \{(a,a) \mid a \in A\}.$

$= \qquad \qquad \cup \{(a,a) \mid a \in \mathbb{Z}\}.$

$= \{(a,b) \mid a \leq b\}.$

Symmetric CLOSURE.

$R^{-1} = \{(b,a) \mid (a,b) \in R\}.$

$R \cup R^{-1}$ = symmetric.

$A = \{a,b\},$

$R = \{(a,b)\}.$

$R^{-1} = \{(b,a)\}.$

$R \cup R^{-1} = \{(a,b),(b,a)\},$

Ex 2:- $R = \{(a,b) \mid a > b\}$ $A = \mathbb{Z}.$

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$R^{-1} = \{(b,a) \mid (a,b) \in R\}. = \{(b,a) \mid a > b\}.$

a b. b a.

$= \{(a,b) \mid b > a\}.$

$R \cup R^{-1} = \{(a,b) \mid a > b\} \cup \{(a,b) \mid b > a\}.$

$= \{(a,b) \mid a > b \text{ or } b > a\} = \{(a,b) \mid a \neq b\}.$

$K \cup R^{-1} = \{(a,b) \mid a > b\} \cup \{(a,b) \mid b > a\}$
$= \{(a,b) \mid a > b \text{ or } b > a\} = \{(a,b) \mid a \neq b\}$.

TRANSITIVE CLOSURE.

$R = \{(1,3), (1,4), (2,1), (3,2)\}$. $A = \{1,2,3,4\}$.

Missing:
$\cup \{(1,2), (2,3), (2,4), (3,1)\}$.

$R' = \{(1,3), (1,4), (2,1), (3,2), (1,2), (2,3), (2,4), (3,1)\}$.

b c. a b.

$(3,4) \notin R'$.

PATHS IN A GRAPH.

a path exist between two Vertices if $\exists$ a Sequence of edges. Such that $(a, x_1), (x_1, x_2), (x_2, x_3), \ldots (x_{n-1}, x_n), (x_n, b)$.

Ex3 :-
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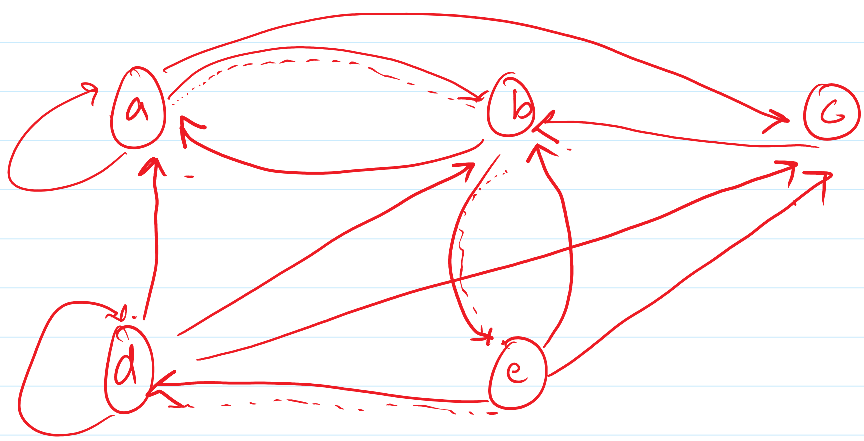


a to d. a b e d by mentioning Vertices

(a,b) (b,e) (e,d). by " Edges.

length = Vertices − 1.
= Number of edges.

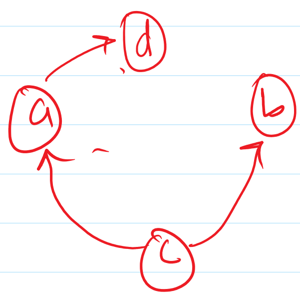
theorem 1:- R be a Relation on A.

485 $\exists$ a path of length n $(n > \mathbb{Z}^+)$. from a to b. when $(a,b) \in R^n$.

Definition:- 485 the Connectivity Relation R^* contain pairs (a,b) such that $\exists$ a path from a to b in R.

$R = \{ \}$. $R^* = \{ \}$.

$R = \{(a,a), (b,b)\}$ $R^* = \{(a,a), (b,b)\}$.



$R = \{(c,a), (c,b), (a,d)\}$. $R^* = \{(c,a), (a,d), (c,b), (c,d)\}$.

$$R^* = \bigcup_{n=1}^{\infty} R^n .$$

Ex4 :- 485 $R = \{(a,b) \mid a \text{ has met } b\}$. A = Set of people.

What is $R^n = ?$ what is R^*.

$R^2 = R \circ R$.

$R^* =$ $(a,b) \in R^*$.
iff $\exists$ a sequence.
starting with a
and ending with b.

Revision.
R $(a,b) \in R$. $a \in A$ $b \in B$.
S $(b,c) \in S$ $b \in B$ $c \in C$.
$(a,c) \in S \circ R$ iff $\exists b$ $(a,b) \in R \wedge (b,c) \in S$.

$(a,c) \in R \circ R$ iff $\exists b$ $(a,b) \in R \wedge (b,c) \in R$.

$(a, b) \in R^2$ if $\exists x_1$ $(a, x_1) \in R \wedge (x_1, b) \in R$.

a has met b if a has met x_1 $\wedge$ x_1 has met b.

Ex6 :- $R = \{(a,b) \mid$ a and b has a Common boundary$\}$.
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What is R^n = ?
" " R^* = ?

A = Set of States in US.

$R^2 = R \circ R$.

$(a, c) \in R \circ R$ if $\exists_b$ $(a,b) \in R \wedge (b, c) \in R$.

$(a, b) \in R^2$ if $\exists x_1$ $(a, x_1) \in R \wedge (x_1, b) \in R$.

theorem 2:- the transitive closure of R equals the R^*.

WARSHAL ALGO:-

EQUIVELENCE RELATION.
1- Reflexive.
2- Symmetric.
3- Transitive.

$\boxed{a \sim_R b} \longleftrightarrow \boxed{(a,b) \in R}$

Ex2:- $R = \{(a,b) \mid a - b \in \mathbb{Z}\}$ $A = \mathbb{R}$.

Reflexive:- $\forall a \in A$ $(a, a) \in R$. ✓
$\forall a \in \mathbb{R}$ $a - a \in \mathbb{Z}$.

Symmetric $\forall a, b \in A$ if $(a,b) \in R \rightarrow (b,a) \in R$.
$\forall a, b \in \mathbb{R}$ if $a - b \in \mathbb{Z} \rightarrow b - a \in \mathbb{Z}$. ✓

Transitive $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.
$\forall a,b,c \in R$ if $a-b \in Z \wedge b-c \in Z \rightarrow a-c \in Z$ ✓.

Hence Equivalence Relation.

Ex3:- $R = \{(a,b) \mid a \equiv b \bmod m\}$ $m > 1$ $(m \in Z^+)$.
$A = Z$.

Reflexive:- $\forall a \in A$ $(a,a) \in R$.
$\forall a \in Z$ $a \equiv a \bmod m$. ✓

Symmetric $\forall a,b \in A$; if $(a,b) \in R \rightarrow (b,a) \in R$.
$\forall a,b \in$ if $a \equiv b \bmod m \rightarrow b \equiv a \bmod m$ ✓

Transitive $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.
$\forall a,b,c \in$ if $a \equiv b \bmod m \wedge b \equiv c \bmod m \rightarrow a \equiv c \bmod m$ ✓.

Hence Equivalence

Ex:-6. $R = \{(a,b) \mid a$ divides $b\}$. $A = Z$.

Reflexive:- $\forall a \in A$ $(a,a) \in R$.
$\forall a \in Z$ a divides a ✓

Symmetric $\forall a,b \in A$; if $(a,b) \in R \rightarrow (b,a) \in R$.
$\forall a,b \in Z$ if a divides b $\rightarrow$ b divides a ✗.

Transitive $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.
$\forall a,b,c \in Z$ if

Not Equivalence.

Ex7:- $R = \{(x,y) \mid |x-y| < 1\}$. $A = R$.

Reflexive:- $\forall a \in A$ $(a,a) \in R$.

Reflexive :- $\forall a \in A \quad (a,a) \in R$.

$\forall a \in \mathbb{R} \quad |a-a| < 1$ ✓.

Symmetric $\forall a,b \in A$ if $(a,b) \in R \to (b,a) \in R$.

$\forall a,b \in \mathbb{R}$ if $|a-b| < 1 \to |b-a| < 1$ ✓.

Transitive $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \to (a,c) \in R$.

$\forall a,b,c \in \mathbb{R}$ if $|a-b| < 1 \wedge |b-c| < 1 \to |a-c| < 1$.

$a = 0.4$ $\quad |0.4-1.3| < 1 \wedge |1.3-1.8| < 1 \to |0.4-1.8| < 1$.

$b = 1.3$ $\quad \times$.

$c = 1.8$. $\quad$ Not Equivalence.

Ex Q 1-30. 500 - 502.