lecture 18:- Equivalence Classes.

$[a]_R = \{ \underline{s} \mid (a,s) \in R \}$.

<u>Ex8</u> :- 496.

$R = \{(a,b) \mid a = b \text{ or } a = -b\}$.

$[1]_R = \{1, -1\}$ $= (1, 1), (1,-1), (-1,1)$

$[-1]_R = \{1, -1\}$. $(-1,-1)$.

<u>Ex9</u>:- 496 Equivalence classes of $\underline{0}, 1$ Congruent module 4?

$R = \{(a,b) \mid a \equiv b \mod 4\}$.

$[0]_4 = \{0, \pm 4, \pm 8, \pm 12, \pm 16 \text{ ------ } \infty\}$.

$[1]_4 = \{1, 5, 9, 13, 17, 21 \text{ ------ } \infty, -3, -7, -11, -15, -19 \text{ ---- } -\infty\}$.

$4\overline{)-1}$ quotient -1, -4, $+$, remainder 3.

$4\overline{)-3}$ quotient -1, ∓ 4, remainder 1.

$4\overline{)-5}$ quotient -2, ∓ 8, remainder 3.

$0 \equiv b \mod 4$.

0

$[2]_4$ = HW.

$[3]_4$ = HW.

PARTITION.

Let S be a set. $P = \{P_1, P_2, \dots P_n\}$ be the partition of S when.

i). $\forall i \; P_i \neq \Phi$.

(ii) $\forall i,j \; P_i \cap P_j = \Phi$.

(iii) $\bigcup_{i=1}^{n} P_i = S$.

<u>Ex12</u> :- 498 $S = \{1, 2, 3, 4, 5, 6\}$.

P.

$A_1 = \{1, 2, 3\}$ P_1

$A_2 = \{4, 5\}$. P_2

$A_3 = \{6\}$. P_3.

i) $A_1 \neq \Phi$ (i) Holds.

$A_2 \neq \Phi$

$A_3 \neq \Phi$.

$A_2 \neq \phi$
$A_3 \neq \phi$.

(ii) $A_1 \cap A_2 = \phi$ $A_2 \cap A_3 = \phi$ $A_1 \cap A_3 = \phi$.

(ii) Holds.

(iii) $A_1 \cup A_2 \cup A_3 = \{1,2,3\} \cup \{4,5\} \cup \{6\} = S$.

★ the equivelence classes creates a partition.

$ER \rightarrow EC \rightarrow P$

$ER \leftarrow EC \leftarrow P$

Ex 13 / 499 :- Find out the tuples in Equivalence Relation produced by the partition $A_1 = \{1,2,3\}$ $A_2 = \{4,5\}$ $A_3 = \{6\}$.

$TA_1 = \{ (1,1), (1,2), (1,3), (2,1), (2,2), (2,3),$
$(3,1), (3,2), (3,3), \}$.

$TA_2 =$? HW
$TA_3 =$? HW.

$R = TA_1 \cup TA_2 \cup TA_3$.

Ex 14 / 499. : What are the sets in the partition of Integers arising from congruent modulo 4?

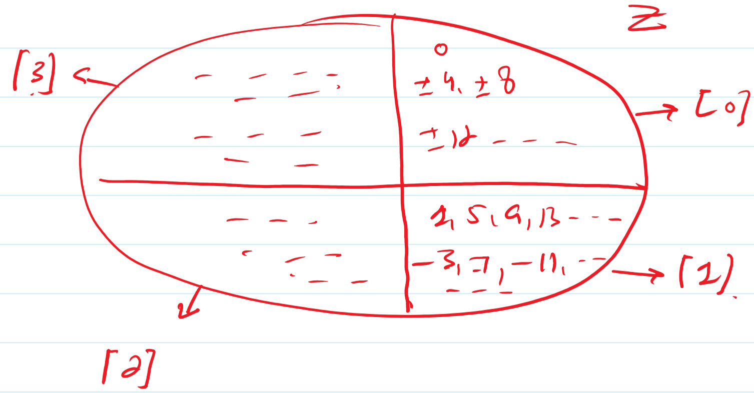
$[0] = \{ 0, \pm 4, \pm 8, \pm 12, \pm 16, \ldots \}$.

$[1] = \{ 1, 5, 9, 13, 17, \ldots, -3, -7, -11, -15, \ldots \}$.

$[2] =$ HW

[2] = HW

[3]. = HW.



Ex 500-503. Q 1-30.

PARTIAL ORDERINGS.

Partial Order.
1 - Reflexive.
2 - Antisymmetric.
3 - Transitive.

$a \sim_R b \iff (a,b) \in R$.
$a \not\sim_R b \iff (a,b) \notin R$.

Ex 1: 504

$R = \{(a,b) \mid a \geq b\}$. $A = \mathbb{Z}$.

1 - Reflexive $\forall a \in A \quad (a,a) \in R$.

$\forall a \in \mathbb{Z} \quad a \geq a. \quad \checkmark$.

2 - Anti Symmetric $\forall a,b \in A$ if $(a,b) \in R \wedge (b,a) \in R \rightarrow a=b$.

$\forall a,b \in \mathbb{Z}$ if $a \geq b \wedge b \geq a \rightarrow a=b \quad \checkmark$.

3 - Transitive $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.

$\forall a,b,c \in \mathbb{Z}$ if $a \geq b \wedge b \geq c \rightarrow a \geq c$.

Ex2 : $R = \{(a,b) \mid a \mid b\}$ $A = Z^+$.
504

1- Reflexive $\forall a \in A$ $(a,a) \in R$.

$\forall a \in Z^+$ a/a ✓

2- Anti Symmetric $\forall a,b \in A$ if $(a,b) \in R \wedge (b,a) \in R \to a = b$.

$\forall a,b \in Z^+$ if a/b and $b/a \to a = b$. ✓

3- Transitive $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \to (a,c) \in R$. ✓

$\forall a,b,c \in Z^+$ if $a/b \wedge b/c \to a/c$ ✓.

it is PO.

POSET. $(S, \preceq)$. $\preceq = \leq$

$(Z, \leq)$. $= \geq$

$= \;=$

$(Z^+, /)$. same. $= /$

$R = \{(a,b) \mid a \mid b\}$ $A = Z^+$ $= \subseteq$

Definitions. Comparable:- Two elements a and b in the Poset. $(S, \preceq)$ are Comparable if $a \preceq_R b$ or $b \preceq_R a$.

$\Updownarrow$ $\Updownarrow$

$(a,b) \in R$ or $(b,a) \in R$.

Ex5 : 5 and 3 7 and 9 3 and 9.
504

are these Comparable in $(Z, /)$

504 are these Comparable in $(\mathbb{Z}, |)$.

$(5,3) \in R$ or $(3,5) \in R$.

$5|3$ or $3|5$

Ex 504:- Show that Inclusion or $\subseteq$ is a partial order on the powerset of any set.

$(P(S), \subseteq)$.

$R = \{(a,b) \mid a \subseteq b\}$.

$S = \{a,b,c\}$.

$P(S) = \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}, \{a,b,c\}\}$.

$R = \{(\phi,\phi), (\phi,\{a\}), (\phi,\{b\}), (\phi,\{c\}), (\phi,\{a,b\}), (\phi,\{b,c\}), (\phi,\{a,c\}), (\phi,\{a,b,c\}),$

$(\{a\},\{a\}), (\{a\},\{a,b\}), (\{a\},\{a,c\}), (\{a\},\{a,b,c\}),$

$(\{b\},\{b\}),$ - - - - - - -

$R = \{(a,b) \mid a \cap b = \phi\}$. $R = \{a,b,c\}$.

$(P(S), a \cap b = \phi)$.

$\left(P(S), \frac{|a \cap b|}{|a \cup b|} \geq 1\right)$.