lecture 12:- properties of Relations:-

1- Reflexive ✓ (lecture 10)

$A, B$.
$R \subseteq A \times B$.

2- Symmetric.

$|A \times B| = |A| \times |B|$.

$\forall a,b \in A$ if $(a,b) \in R \rightarrow (b,a) \in R$.

Total Subsets of $A \times B$.

$2^{|A \times B|} = 2^{|A| \times |B|}$.

Ex7 P462. $A = \{1,2,3,4\}$. ✗

$R_1 = \{(1,1), (1,2), (2,1), (2,2), (3,4), (4,1), (4,4)\}$.

a b . a b . a b . a b a b . a b .

if $(1,1) \in R \rightarrow (1,1) \in R$. ✓

$(1,2) \in R \rightarrow (2,1) \in R$. ✓

$(2,1) \in R \rightarrow (1,2) \in R$ ✓

$(2,2) \in R \rightarrow (2,2) \in R$,

$(3,4) \in R \rightarrow (4,3) \notin R$ ✗,

$(4,1) \in R \rightarrow (1,4) \notin R$ ✗,

$R_4 = \{(1,1), (3,1), (1,3), (2,3)\}$.

$R_2 = \{ \}$.

$R_3 = \{(1,1)\}$.

3- Anti Symmetric:- $\forall a,b \in A$ if $\underbrace{(a,b) \in R}_{(1,1)} \wedge \underbrace{(b,a) \in R}_{(1,1)} \rightarrow a = b$.

$\rightarrow 1 = 1$.

Ex7 P462. $A = \{1,2,3,4\}$.

$R_1 = \{(1,1), (2,1), (1,2), (3,4), (4,4)\}$.

$\boxed{(2,1) \in R \wedge (1,2) \in R} \rightarrow 2 \neq 1$

$R_2 = \{ \}$.

$R_3 = \{(1,1)\}$.

Ex15 P.464:- Is the divides Relation on $Z^+$

1) Symmetric.
2) Anti Symmetric.

$R = \{(a,b) |$ a divides b$\}$. $Z^+$

a divided by b $= a \div b = \frac{a}{b}$

symmetric:-

$\forall a \in A$ if $(a,b) \in R \rightarrow (b,a) \in R$.

a divides b $= b \div a = \frac{b}{a}$.

$\forall a \in Z^+$ if a divides b $\rightarrow$ b divides a.

$(1,2)$

1 divides 2 $= \frac{2}{1} = 2$.

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$(1,2) \in R \rightarrow (2,1) \notin R$

$(2,4) \in R \rightarrow (4,2) \notin R$.

Symmetric does not hold.

Anti Symmetric:- $\forall a,b \in A$ if $(a,b) \in R \wedge (b,a) \in R \rightarrow a = b$.

$\forall a,b \in Z^+$ if $a$ divides $b$ $\wedge$ $b$ divides $a$ $\rightarrow$ $a = b$.

It is Anti Symmetric.

3- Transitive:- $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.

$A = \{1,2,3,4\}$ Ex 7 p462.

$R_1 = \{(1,1),(1,2),(2,1),(2,2),(3,4),(4,1),(4,4)\}$. X.

$R_2 = \{\ \}$. ✓

$R_3 = \{(1,1),(2,2),(3,3),(1,2)\}$ ✓

Ex. $R = \{(a,b) \mid a \geq b\}$. $A = Z$.

Symmetric:- $\forall a,b \in A$ if $(a,b) \in R \rightarrow (b,a) \in R$.

$\forall a,b \in Z$ if $a \geq b \rightarrow b \geq a$.

$3 \geq 1 \rightarrow 1 \ngeq 3$.

$(3,1) \in R \rightarrow (1,3) \notin R$.

Anti Symmetric:- $\forall a,b \in A$ if $(a,b) \in R \wedge (b,a) \in R \rightarrow a = b$.

$\forall a,b \in Z$ if $a \geq b \wedge b \geq a \rightarrow a = b$

This is Anti Symmetric.

Transitive $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.

$\forall a,b,c \in Z$ if $a \geq b \wedge b \geq c \rightarrow a \geq c$.

$3 \geq 1 \wedge 1 \geq 0 \rightarrow 3 \geq 0$.

Reflexive. $\forall a \in A$ $(a,a) \in R$.

$\forall a \in Z$ $a \geq a$ ✓.

$\forall a,b \in A$ if $(a,b) \in R \wedge (b,a) \in R \rightarrow a = b$.

$a \geq b \wedge b \geq a \longrightarrow a = b.$

Homework $\boxed{\text{To} \leq, <,}$ Ex Question. P 466, 467, 468.

Tommorrow

Morning.