lecture 6:- Q31- Ex. P44. $\underline{\forall y\ Q(0,y,0)}$

(a) $\forall y Q(0, y, 0) = Q(0,0,0) \wedge Q(0,1,0)$.

x y. $\rightarrow$ T.

$x = \{0,1,2\}$

$\rightarrow y = \{0,1\}$

$z = \{0,1\}$.

$\forall x P(x) = P(0) \wedge P(1) \wedge \cdots \wedge P(N)$. $x = \{0,1,2 \cdots N\}$

(d) $\exists x \neg Q(x, 0, 1)$. (with y over 0 and z over 1)

$= \neg Q(0,0,1) \vee \neg Q(1,0,1) \vee \neg Q(2,0,1)$.

$\exists x P(x) = P(0) \vee P(1) \vee P(2) \vee \cdots \vee P(N)$.

(c) and (b) Do it yourself.

Ex32 P51. All dogs have fleas.

$\forall$ $\underbrace{\text{dogs}}_{\text{Variable.}}$ $\underbrace{\text{have fleas.}}_{\text{predicate.}}$

for all x, x is a dog, x have flees.

$\forall x P(x)$. let $P(x) = \underline{x \text{ have fleas.}}$

$\neg(\forall x P(x)) = \exists x \underline{\neg P(x)}$. $x \in$ Dogs.

There exist x, x is a dog, x does not have fleas.

Nested Quantifiers:- $x, y \in \{1,2,3, \cdots N\}$

$\underline{\forall x}_{x} \forall y\ P(x,y)$.

$\forall x\ [\ P(x,1) \wedge P(x,2) \wedge P(x,3) \wedge \cdots P(x, \cdots)]$

$$= \forall x \left[P(x,1) \wedge P(x,2) \wedge P(x,3) \wedge \cdots \wedge P(x,N) \right]$$

$$= \underline{\forall x P(x,1)} \wedge \forall x P(x,2) \wedge \forall x P(x,3) \wedge \cdots \wedge \forall x P(x,N).$$

$$= (P(1,1) \wedge P(2,1) \wedge (3,1) \wedge \cdots \wedge P(N,1)) \wedge$$
$$(P(1,2) \wedge P(2,2) \wedge P(3,2) \wedge \cdots \wedge P(N,2)) \wedge$$
$$\vdots$$
$$(P(1,N) \wedge P(2,N) \wedge P(3,N) \wedge \cdots \wedge P(N,N)).$$

$\forall x \exists y P(x,y).$

$$= \forall x \left[P(x,1) \vee \underline{P(x,2)} \vee P(x,3) \vee \cdots \vee P(x,N) \right).$$

$$= \quad ?$$

$$= \left[P(1,1) \wedge P(2,1) \wedge P(3,1) \wedge \cdots \wedge P(N,1) \right] \vee$$
$$\left[P(1,2) \wedge P(2,2) \wedge P(3,2) \wedge \cdots \wedge P(N,2) \right] \vee$$
$$\vdots$$
$$[$$

$\exists x \forall y P(x,y) \quad =$

$\exists x \exists y P(x,y) \quad = ?$

$$\neg(\forall x \forall y P(x,y)) = \exists x \exists y \neg P(x,y).$$

$$\neg(\forall x \exists y P(x,y)) = ?$$

$\neg(\forall x \exists y\, P(x,y)) = ?$

$\neg(\exists x \forall y\, P(x,y)) = ?$

$\neg(\exists x \exists y\, P(x,y)) = ?$

Ex1 - P47:- $\forall x \forall y\, (x+y = y+x)$ $x, y \in \mathbb{R}$.

$= T$.

Ex4: P48:- $Q(x,y) = x+y=0$.

$\exists y \forall x\, Q(x,y) = F$.

$\forall x \exists y\, Q(x,y) = T$.

Ex5 · P49:- $Q(x,y,z) = x+y=z$.

$\forall x \forall y \exists z\, Q(x,y,z) = T$.

$\exists z \forall x \forall y\, Q(x,y,z) = F$

Ex6:- the sum of two positive Integers is always positive.

$\underline{\text{for all } x,}$ $\underline{\text{for all } y,}$ $\underline{x \text{ and } y \text{ are positive Integers,}}$

$\underline{x+y>0.}$

let $P(x,y) = x+y>0$.

$\forall x \forall y\, P(x,y)$

$x, y \in \mathbb{Z}^+$.

Ex-9:- $\forall x\,(C(x)) \vee \exists y\,(C(y) \wedge F(x,y))$.

$x,y \in$ persons.

$\rightarrow$ $C(x)$ = x has a Computer.

$F(x,y)$ = x is the friend of y.

for all x, x is a person. x has a Computer, or there exists y, y is a person, y has a Computer and x is a friend of y.

Ex 11:- "If a $\forall$ person is a female, and is a parent then this person is Someones mother".

$\Downarrow$

$\exists$

for all x, x is a person, if x is female and x is a parent then There exist y, such That x is The mother of y. y is a person.

Let $F(x)$ = X is female. $x,y \in$ persons.

$P(x)$ = X is parent.

$M(x,y)$ = x is the mother of y.

$$\forall x \exists y (F(x) \wedge P(x)) \longrightarrow M(x,y).$$

Ex 6. "There is a student in the class who is junior."

There is = Quantifier. $\exists$

student in the class = Domain

who is junior = Predicate.

Quantifier. Domain Predicate.

$\exists$

$\underbrace{\text{There exists } x,}$ x is a $\underbrace{\text{student in this class}}_{x}$,

$\underbrace{x \text{ is junior.}}$

Let $J(x) = x$ is junior.

$\exists x\, J(x)$.

"Every student in this class is either a sophomore or a CS major"

for all x, x is a student in this class.
x is a sophomore or x ~~has~~ a CS major.

Let $S(x) = x$ is a sophomore.
$M(x) = x$ has a CS major.

$\forall x\,(S(x) \vee M(x))$.

Ex28:- $\forall x \exists y\,(x = y^2)$ $x, y \in \mathbb{R}$.

$\exists x \forall y\,(xy = 0)$ $x, y \in \mathbb{R}$.

$\forall x \exists y\,(x + y = 1)$. "

$\exists y \forall x\,(x + y = 1)$.

Ex9 (b) "Every body loves somebody."

for all x, x is a person, there exist y, y is a person. x loves y.

$x, y \in$ persons.

let $L(x,y) =$ x loves y.

$\forall x \exists y \ L(x,y)$.

"Every body loves himself"

for all x, x is a person. x loves x.

$L(x) =$ x loves x.