lecture 1o.   RELATIONS.   العلاقات

   Q:- Why we skip the Chapters.

Ans:- Discrete Mathematics.   Vs Discrete Structures.

       Counting   Discrete Vs Continous.

                                                   أي أي

                                                 Countable.

RECAP OF PREVIOUS CONCEPTS.

SETS:-   { } → Syntax.

Collection of distinct objects ──→ Semantics

{ Tomato, Onion, potato, mango } ✓ .

{ Tomato, Onion, potato, mango, Onion} → Error Semantics

[Tomato, Onion, potato, mango] x──→ Error in Syntax

CARDINALITY OR A SET.

1) A → $|A|$ → Sentax.

2)   The number of elements in a Set ──→ Semantics.

$A = \{1, 2, 3\}$ $B = \{a, b\}$

$|A| = 3$ $|B| = 2$.

POWER SET OF A SET.

    $Pow(A)$ → Syntax,

All possible subsets of a Set. → Symantics.

Ex. $A = \{1, 2, 3\}$. $B = \{a, b\}$.

$pow(A) = \{\Phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, \{1,2,3\}\}$.

$pow(B) = \{\Phi, \{a\}, \{b\}, \{a,b\}\}$.

CARDINALITY OR → $|Pow(A)| = 2^{|A|} = 2^3 = 8$.
A POWER SET

$|pow(B)| = 2^{|B|} = 2^2 = 4$.

MULTIPLICATION OF SETS.

$A = \{1, 2, 3\}$ $B = \{a, b\}$.

MULTIPLICATION OF SETS.

$A = \{\boxed{1}, 2, 3\}$ $B = \{a, b\}$.

$A \times B = \{(1,a), (1,b), (2,a), (2,b), (3,a), (3,b)\}$

Find Subsets of $A \times B$.

$pow(A \times B) = \{\emptyset, \{(1,a)\}, \{(1,b)\}, \{(2,a)\}, \{(2,b)\}, \{(3,a)\}, \{(3,b)\},$
$\{(1,a),(1,b)\}, \{(1,a),(2,a)\}, \{(1,a),(2,b)\} \cdots \cdots$
$\overline{}$
$\overline{}$
$\overline{}$
$\underset{2}{\equiv}$
$\}$.

$|pow(A \times B)| = 2^{|A \times B|} = 2^{|A| \times |B|} = 2^{3 \times 2} = 2^6 = 64.$

RELATION: BINARY RELATIONS.

SET A & B.   A binary relation from A to B
is a subset of $A \times B$.

$|A| = 4$ $|B| = 3$.
How many relations on $A \times B$ is possible.
$|pow(A \times B)| = 2^{|A \times B|} = 2^{|A| \times |B|} = 2^{4 \times 3} = 2^{12}$.

P460 Ex. $A = \{0, 1, 2\}$ $B = \{a, b\}$.

$\{(0,a), (0,b), (1,a), (2,b)\}$.   Is it a Relation from A to B.
is it a Relation from B to A.

Ex4. P461: $A = \{1, 2, 3, 4\}$.

$R = \{(a,b) \mid a \text{ divides } b\}$.

$A \times A = \{(1,1)^{\checkmark}, (1,2)^{\checkmark}, (1,3)^{\checkmark}, (1,4)^{\checkmark}, (2,1)^{x}, (2,2)^{\checkmark}, (2,3)^{x}, (2,4)^{\checkmark}, (3,1)^{x}, (3,2)^{x}, (3,3)^{\checkmark}, (3,4)^{x},$
$(4,1)^{x}, (4,2)^{x}, (4,3)^{x}, (4,4)^{\checkmark}\},$

$a \text{ divides } b = b \div a = \frac{b}{a}$

$R = \{(1,1), (1,2), (1,3), (1,4), (2,2),$

$R = \{(1,1), (1,2), (1,3), (1,4), (2,2),$
$\quad (2,4), (3,3), (4,4)\}.$

a divides $b = b \div a = \dfrac{b}{a}$

a divided by $b = a \div b = \dfrac{a}{b}$

Ex5 462 $R_1 = \{(a,b) \mid a \leq b\}.$ $(1,1), (1,2), (2,1), (1,-1), (2,2)$

$R_2 = \{(a,b) \mid a > b\}.$

$R_3 = \{(a,b) \mid a = b \text{ or } a = -b\}.$

R_4
$R_5 \quad \}$ Do it Yourself.
R_6

PROPERTIES OF RELATIONS.
1- REFLEXIVE   2- SYMMETRIC   3- ANTI SYMMETRIC   4- TRANSITIVE.

REFLEXIVE:-
A relation R on SET A is reflexive.

if $\underline{\forall a \in A \quad (a,a) \in R}.$ $(1,1) \in R \wedge (2,2) \in R \wedge (3,3) \in R \wedge (4,4) \in R$

$A = \{1, 2, 3, 4\}.$

Ex7 P462:- $R_1 = \{(1,1), (1,2), (2,1), (2,2), (3,4), (4,1), (4,4)\}$ X.

$R_2 = \{\Phi\}$ X.

$R_3 = \{(1,1), (1,2), (2,1)\}.$

R_4
$R_5 \quad \}$ Do it yourself at home.
R_6

Ex9.   Is the divides relation on set of integers $\mathbb{Z}$. reflexive.

$R = \{(a,b) \mid a \text{ divides } b\}.$

$\forall a \in A \quad (a,a) \in R.$
$\forall a \in \mathbb{Z} \quad a \text{ divides } a. \quad \} \text{ Reflexive.}$

→ Symmetric, Antisymmetric. Transitive (Next class).