6.1 Qi's and Vladimirsky's example with two point sources in Fig. 4 in [4]

We consider a 2D example with two point sources presented in [4]. The domain is the unit square $[0,1]^2$. The point sources are located at $(0,0)$ and $(0.8,0)$. The slowness is given by

$$s(x) = \frac{1}{2 + 5x_1 + 20x_2}. \tag{79}$$

The exact solution for this example can be found from the fact that for a positive linear speed function and a point source at a point x^* the ray tracing is analytical and the solution of the eikonal equation is given by [5, 2]

$$s(x) = \frac{1}{\frac{1}{s^*} + v^\top (x - x^*)}, \quad u(x) = \frac{1}{\|v\|}\operatorname{arccosh}\left(1 + \frac{1}{2}s^* s(x)\|v\|^2\|x - x^*\|^2\right). \tag{80}$$

Now, setting up a point source at a different point $x^\star$ can be accounted for by recasting $s(x)$ as

$$s(x) = \frac{1}{\frac{1}{s^*} + v^\top(x - x^\star + x^\star - x^*)} = \frac{1}{\left[\frac{1}{s^*} + v^\top(x^\star - x^*)\right] + v^\top(x - x^\star)}. \tag{81}$$

Therefore, shifting the point source from x^* to $x^\star$ is equivalent to changing the parameter s^* to $s^\star$

$$s^\star = \frac{1}{\frac{1}{s^*} + v^\top(x^\star - x^*)}. \tag{82}$$

Using this recipe, we find two analytical solutions $u^*(x)$ and $u^\star(x)$ using formula (80) for $x^* = (0,0)$ and $x^\star = (0.8,0)$ respectively. The parameters are $s^* = \frac{1}{2}$ and $s^\star = \frac{1}{6}$. Then we set

$$u(x) = \min\{u^*(x), u^\star(x)\}.$$

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