COSC 302: Analysis of Algorithms — Spring 2018
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Problem Set 2 — Order of Growth and Asymptotic Analysis

HOBBY-MAJORITY(A)
  if $A.length = 1$ **then**
    return $A[1]$
  $x \leftarrow$ Hobby-Majority($[1...n/2]$)
  $y \leftarrow$ Hobby-Majority($A[(n/2)+1...n]$)
  if SameHobby(x, y) **then**
    return x
  else
    if x = NULL **then**
      return y
    if y = NULL **then**
      return x
  return NULL

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1. LINEAR-MEDIAN(A, x)

   $median \leftarrow Median(A)$
   $w_{<M} \leftarrow 0$
   $Array_{<M} \leftarrow \emptyset$
   $Array_{>M} \leftarrow \emptyset$
   if $A.length = 1$ **then**
       return 1
   for $j = 1$ **to** A.length **do**
       if $A[j] \leq median$ **then**
           $Array_{<M} = Array_{<M} + A[j]$
           $w_{<M} = w_{<M} + w(A[i])$
       else
           $Array_{>M} = Array_{>M} + A[j]$
   if $x + w_{<M} > \frac{1}{2}$ **then**
       Linear-Median($Array_{<M}$,x)
   else
       Linear-Median($Array_{>M}$,$w_{<M}$)

2. Invariant: The median is also in the array A and x is the total weight of all elements than the median the previous call's sub-array.

   Initialization: For the first call A is the total array and the weight of elements less than the median is 0.

Maintenance: Suppose that $x + w_{<M} > \frac{1}{2}$ since the median is in A, it must be in either $Array_{<M}$ or $Array_{>M}$ since we iterated through A and placed the median in one of those arrays. Since the total weight of all elements less than the any element is greater than $\frac{1}{2}$ then it must not be in array $Array_{>M}$ and it will be in $Array_{<M}$. Otherwise it will be in $Array_{>M}$ since we do not remove any elements from the array.

Termination: The program terminates when the array is length of 1 since the size of A decreases for every recurse since neither arrays can contain the same elements. The recurrence is $T(n) = T(n/2) + \Theta(n)$ which equals $\Theta(n)$.

3. The trees go left to right starting with the upper right one.

4. Question 3.

   a There is a $\frac{1}{n}$ chance of the number being the maximum.