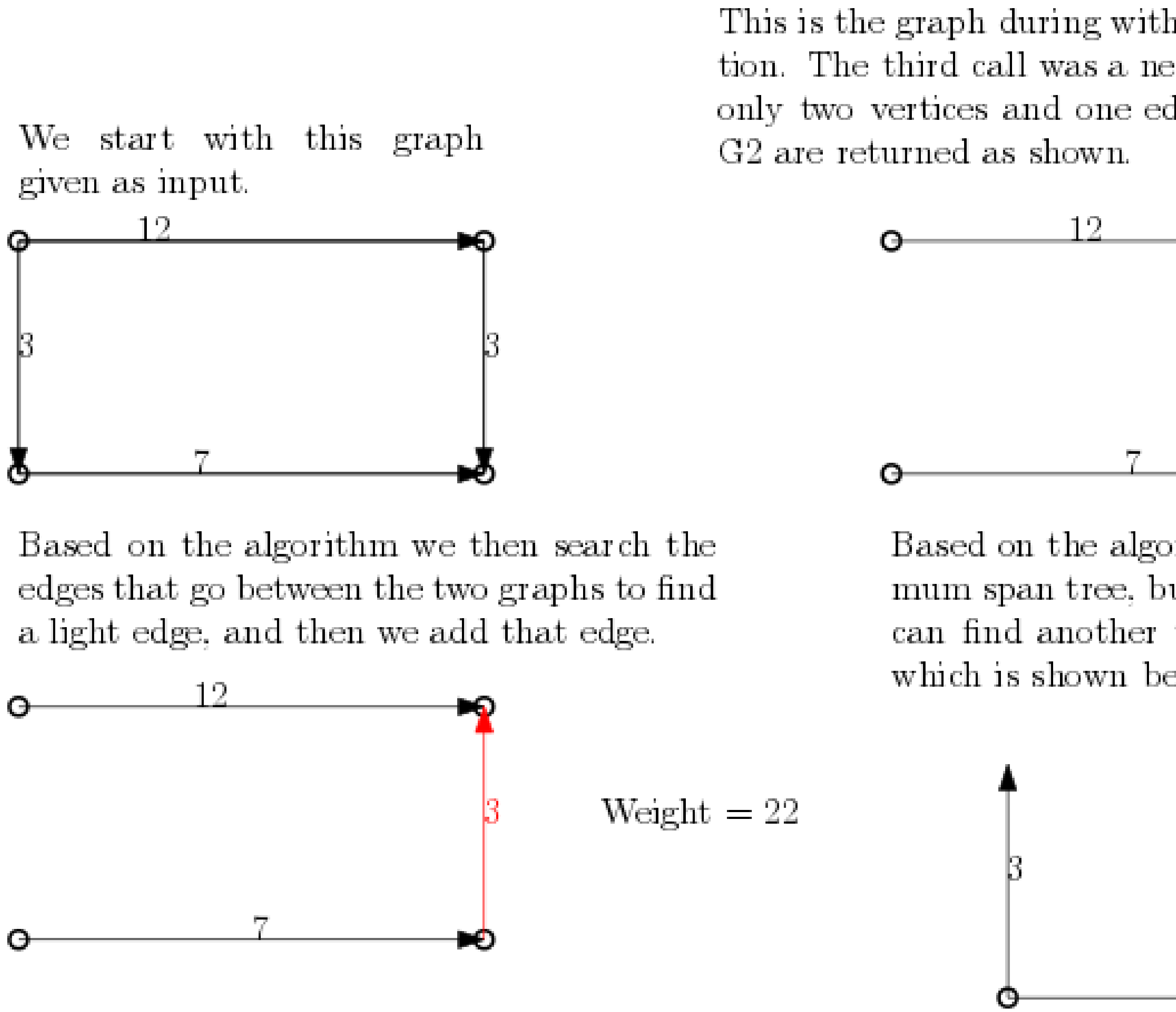
COSC 302: Analysis of Algorithms — Spring 2018

Problem Set 7 — Minimum Spanning Trees and Single Source Shortest Paths Due by 4:30pm Friday, March 30, 2018 as a single pdf via Moodle (either generated via LaTeX, or concatenated photos of your work). Late assignments are not accepted.

This is an *individual* assignment: collaboration (such as discussing problems and brainstorming ideas for solving them) on this assignment is highly encouraged, but the work you submit must be your own. Give information only as a tutor would: ask questions so that your classmate is able to figure out the answer for themselves. It is unacceptable to share any artifacts, such as code and/or write-ups for this assignment. If you work with someone in close collaboration, you must mention your collaborator on your assignment.

1. Problem 23.1-1 from CLRS. Let $G = (V, E)$ be a connected undirected graph. Let (u, v) be a minimum weight edge. Let there be T which is some subset of E that does not contain a vertex u. Since (u, v) is a minimum weight edge, it is a safe edge. And a minimum spanning tree must contain all vertices including v the edge (u, v) will be added.

2. Problem 23.2-8 from CLRS. With the valid input where a tree that has lower weight exists.



3. Problem 24.1-6 from CLRS.
   Invariant: The negative weight cycle is intact right before the start of every loop.
   Initialization: At the start of the loop, no edges have been removed nor recorded.
   Maintanence: At the start of the loop we first check if Bellman-Ford function returns true. If it returns false we can remove an edge from the graph without changing the cycle and thus passes the invariant. Then when Bellman-Ford returns true, it means that the edge u, v was a part of the cycle, thus we add back the edge and using the adjacency list do a breadth first search to find the remaining parts of the cycle. If we find another edge (v, x) when removed causes the BF return true then we change u to equal v and v to equal the current x so that we search it's neighbors for another edge in the cycle.
   Termination: The function terminates if there was no cycle found. It also terminates when $x = u_1$ with u_1 being the first vertex found to the part of the cycle.

```
function NEGATIVE-CYCLE(G)
    Q = ∅
    for all (u, v) ∈ E do
        E = E − (u, v)
        if BELLMAN-FORD(G) == true then
            Q = Q + u_1
            u = u_1
            E = E + (u, v)
            for all x ∈ adj[v] do
                E = E − (v, x)
                if BELLMAN-FORD(G) == true then
                    Q = Q + u
                    E = E + (v, x)
                    u = v
                    v = x
                    if x = u_1 then
                        return Q
    return Q
```

4. Problem 24.3-4 from CLRS.

   (a) We first check if there is one vertex s which $\pi[s] = NIL$ and $dis[s] = 0$.

   (b) We then check if the information given is a tree on the graph given. We first check if $dis.length = \pi.length - 1$. We then do a BFS and check if it visits all = and $\pi[v] \neq \infty$.Then, we check if for each pair (u, v), $dis[u] + w(u, v) \geq dis[v]$

   (c) We check for each $v \in \pi - s$, if $dis[\pi[v]] + w(\pi[v], v) = dis[v]$