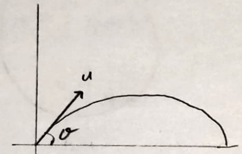
PC1201 Chapter 4
Motion in 2D

Projectile Motion

Derive $y = (\tan\theta)x - \left(\frac{g}{2v_i^2\cos^2\theta_i}\right)x^2$



Along x-direction:

$s = s_0 + ut + \frac{1}{2}at^2$

$x = (u\cos\theta)t$

$t = \frac{x}{u\cos\theta}$ —(1)

Along y-direction:

$y = (u\sin\theta)t + \frac{1}{2}(-g)t^2$ —(2)

Sub (1) → (2):

$y = (u\sin\theta)\cdot\frac{x}{u\cos\theta} - \frac{g}{2}\left(\frac{x}{u\cos\theta}\right)^2$

$= (\tan\theta)x - \left(\frac{g}{2u^2\cos^2\theta}\right)x^2$ //

Max height:

Derive $h_{max} = \frac{v_i^2\sin^2\theta_i}{2g}$

Max height occurs when $v_y = 0$,

$v = u + at$

$0 = u\sin\theta + (-g)t$

$t = \frac{u\sin\theta}{g}$

$s = s_0 + ut + \frac{1}{2}at^2$

$h = u\sin\theta\left(\frac{u\sin\theta}{g}\right) + \frac{1}{2}(-g)\left(\frac{u\sin\theta}{g}\right)^2$

$= \frac{u^2\sin^2\theta}{g} - \frac{u^2\sin^2\theta}{2g}$

$= \frac{u^2\sin^2\theta}{2g}$ //

Derive range (R)

$R = \frac{v_i^2\sin(2\theta_i)}{g}$

Max range when initial & final y same (y=0)

$s = s_0 + ut + \frac{1}{2}at^2$

$0 = 0 + (u\sin\theta)t + \frac{1}{2}(-g)t^2$

$u\sin\theta = +\frac{g}{2}t$

$t = \frac{2u\sin\theta}{g}$

or

$y = (u\sin\theta)t - \frac{1}{2}gt^2$

$0 = (u\sin\theta)t - \frac{1}{2}gt^2$

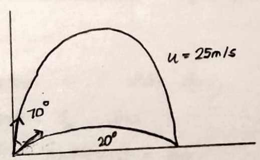
Distance travelled in x-direction in t:

$x = u\cos\theta \cdot \frac{2u\sin\theta}{g}$

$= \frac{2u^2\sin\theta\cos\theta}{g}$ ($2\sin\theta\cos\theta = \sin(2\theta)$)

$= \frac{u^2\sin 2\theta}{g}$

- Maximum range when $\theta_i = 45°$. ($\sin(2\times 45) = 1$)
- Complementary angles produce same range $(90° - \theta_i)$
  - different max height
  - different flight time



$R = \frac{u^2\sin(2\theta)}{g}$

Let 2θ be 2α

$2\alpha \le 90$

$2\theta = 2\alpha$ or $2\theta = 180 - 2\alpha$

$\theta = \alpha$ or $\theta = 90 - \alpha$

For ball thrown at 70°, use $s = s_0 + ut + \frac{1}{2}at^2$

$0 = 0 + (25\sin 70)t_1 + \frac{1}{2}(-9.8)t_1^2$

$t_1 = 4.79s$

For ball at 20°,

$0 = 0 + (25\sin 20)t_2 + \frac{1}{2}(-9.8)t_2^2$

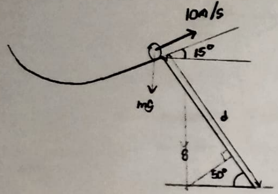
$t_2 = 1.74s$

Time Delay $= t_1 - t_2 = 4.79 - 1.74 = 3.05s$ //

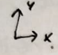
Non Symmetric Trajectory

~~$H_{max} = \ldots$~~
~~$R = \ldots$~~ } Cannot be used

$y = (\tan\theta_i)x - \frac{gx^2}{2v_i^2\cos^2\theta}$ ✓



Method 1



$x = u_x t$

$d\cos50 = (10\cos15)t$

$d = 15.03t \quad -(1)$

$y = u_y t + \frac{1}{2}at^2$

$d\sin50 = (10\sin15)t + \frac{1}{2}(-9.8)t^2$

$-0.766d = 2.588t - 4.9t^2 \quad -(2)$

Sub (1) in (2):

$-0.766(15.03t) = 2.588t - 4.9t^2$

$t = 2.88s \quad -(3)$

$d = 15.03(2.88) = 43.2m$

Method 2

3rd Formula

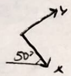
$y = (\tan\theta_i)x - \frac{gx^2}{2v_i^2\cos^2\theta}$

$-d\sin50 = (\tan15)(d\cos50) - \frac{9.8(d\cos50)^2}{2(10)^2\cos^2(15)}$

...

$d = 0$ or $d = 43.2m$

Method 3



$S = ut + \frac{1}{2}at^2$

Along y-direction:

$0 = (10\sin65)t + \frac{1}{2}(-9.8\cos50)t^2$

$t = 2.8775s$

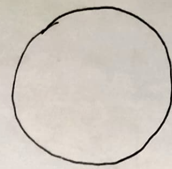
Along x-direction:

$d = (10\cos65)t + \frac{1}{2}(9.8\sin50)t^2$

sub t in d

$d = 43.2m$

Uniform Circular Motion

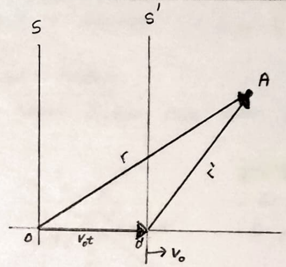


- Occurs when an object moves in a circular path with constant speed
- Acceleration exists as direction of motion is changing

Magnitude: $a_c = \frac{v^2}{r}$

Period (for one cycle): $T = \frac{2\pi r}{v}$

Galilean Transformation Equations



$V_o t + r' = r$

$r' = r - V_o t$

Derivative of position eqn will give velocity eqn

$v' = v - V_o$

Derivative of velocity eqn will give acceleration eqn

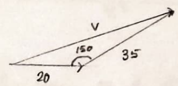
$a' = a$ (V_o is constant)

Examples

Let v be velocity of stone observed by stationary man.

$v' = v - V_o$

$v = v' + V_o$



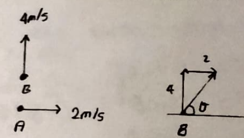
$a^2 = b^2 + c^2 - 2bc(\cos A)$

$v^2 = 20^2 + 35^2 - 2(20)(35)(\cos150)$

$v = 53.27m/s$

$\frac{v}{\sin150} = \frac{35}{\sin A}$

$A = 19.18°$



i) $V_B = \sqrt{4^2 + 2^2} = 4.47m/s$

$\theta = \tan^{-1}\left(\frac{4}{2}\right) = 63.43°$

ii) $\frac{1800}{4} = 450s$

PC1201 Chapter 5

Laws of motion

Zero Net Force

When net force is zero, ($\Sigma F = 0$)

- $a = 0$
- v is constant

Equilibrium occurs when $\Sigma F = 0$

- magnitude of velocity will not change

Inertia Frames

- An inertia frame can be stationary or moving with constant velocity
- Any reference frame that moves with constant velocity relative to an inertia frame is itself also an inertia frame

Newton Laws

First Law (also known as law of inertia)

- In the absence of net force ($\Sigma F = 0$), when viewed from an inertia reference frame, an object at rest remains at rest and an object in motion continues with constant velocity.

$\rightarrow (\Sigma F = 0) \rightarrow a = 0$

Inertia & Mass

- Tendency of an object to resist change in velocity is called Inertia
- Mass is that property of an object that specifies how much resistance an object exhibits to changes in velocity

Second Law

- When viewed from an inertia frame, the acceleration of an object is directly proportional to net force acting on it and inversely proportional to its mass.

$$\Sigma F = ma$$

- If an object experiences acceleration, there must be a nonzero net force acting on it.

Gravitation Force

$|F_g| = mg$ (weight)

Weight is dependent on g

Third Law

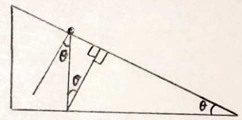
- If two force interact, the force F_{12} exerted by obj 1 on obj 2 is equal in magnitude and opposite in direction, to the force F_{21} exerted by obj 2 on obj 1.

$$F_{12} = -F_{21}$$

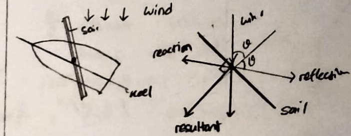
(F_{AB} is the force exerted by A on B)

- Forces always occur in pairs (Action - Reaction)
- Single isolated force cannot exist
- The action and reaction forces must act on different objects and be of the same type

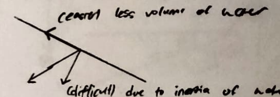
Geometry



Sailing against wind

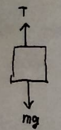


- Net force produced by wind on sail is perpendicular to the sail



Example 1a

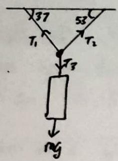
Lamp suspended on chain



$\Sigma F = 0 = T - mg$

$T = mg$

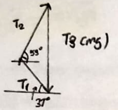
Example 2



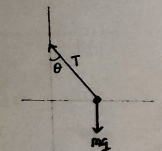
$T_3 = mg$

$T_3 = T_1 \sin 37 + T_2 \sin 53$

$T_1 \cos 37 = T_2 \cos 53$



Ex.



vertical : $T\cos\theta = mg$ — (1)

Horizontal : $T\sin\theta = ma$ — (2)

(2) / (1) : $\tan\theta = \frac{a}{g}$

$a = g\tan\theta$

$= 9.8 \tan 25$

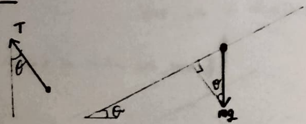
$= 4.6 \ m/s^2$

$v = u + at$

$= 0 + 4.6 \times 18$

$= 82 m/s$

Ex.



a) Along slanted x :

$V = ut + at$

$30 = 0 + a(6)$

$a = 5$

$\Sigma F_x = ma$

$\cancel{m}g\sin\theta = \cancel{m}a$

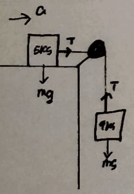
$\theta = \sin^{-1}\left(\frac{a}{g}\right)$

$= 30.68°$

b) $T = mg\cos\theta$

$= (0.1)(9.8)\cos 30.68 = 0.84N$ //

Ex.



on 9kg block :

$m_2 g - T = m_2 a$

$T = m_2(g - a)$

$T = 9(9.8 - a)$

$T = 88.2 - 9a$ —(1)

on 5kg block :

$T = m_1 a$

$T = 5a$ —(2)

(1) in (2)

$5a = 88.2 - 9a$

$a = 6.3 m/s^2$ //

$T = 5(6.3)$

$= 31.5N$ //

Forces of friction

$\mu_k < \mu_s$, kinetic friction is smaller than static friction

- Size of contact area will not change the magnitude of friction

Friction is proportional to the normal force.

- $f_s \leq \mu_s n$ and $f_k = \mu_k n$ (magnitude, not vector)

Static Friction

- Acts to keep object from moving.
- If F increases, so does f_s
- If F decreases, so does f_s

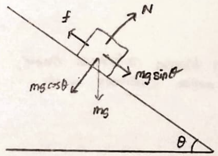
$f_s \leq \mu_s n$ where the equality holds when surfaces on verse of slip

- Called Impending motion

Kinetic Friction

- Kinetic friction acts when object in motion

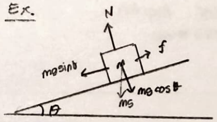
$f_k = \mu_k n$



$\mu = \tan\theta$

- for μ_s, use the angle where the block just slips
- for μ_k, use the angle the block slides down at cont speed

Ex.



i) Angle as the block starts to slide,

$f_s = mg\sin\theta \rightarrow \sin\theta = \frac{f_s}{mg} = \frac{\mu_s N}{mg}$

$\Sigma F_y = 0, \quad N = mg\cos\theta$

$\sin\theta = \frac{\mu_s \cancel{mg}\cos\theta}{\cancel{mg}} \rightarrow \tan\theta = \mu_s \rightarrow \theta = \tan^{-1}(\mu_s)$

ii) Acceleration as it slides down,

$mg\sin\theta - f_k = ma$

$\cancel{m}g\sin\theta - \mu_k \cancel{m}g\cos\theta = \cancel{m}a$

$a = g(\sin\theta - \mu_k\cos\theta)$ //

iii) How to make velocity constant ?

$mg\sin\theta - f_k = 0$

$\cancel{mg}\sin\theta = \mu_k \cancel{mg}\cos\theta$

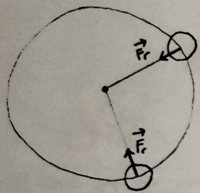
$\mu_k = \tan\theta$

$\theta = \tan^{-1}(\mu_k)$ //

PC1201 Chapter 6 Part 1

Circular motion

Uniform Circular Motion (Horizontal Plane)



- A force F_r is directed towards the center of the circle
- Force associated with acceleration a_c
- Apply Newton's 2^{nd} Laws

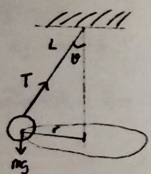
$$\Sigma F = ma_c = m\frac{v^2}{r} \qquad (\Sigma F_r)$$

The force causing centripetal acceleration is called the centripetal force.

- Centripetal force causes circular motion

Conical Pendulum

*(Don't need derive in exam)



- Equilibrium in vertical direction
- U.C.M. in horizontal direction

$v = \sqrt{Lg\sin\theta\tan\theta}$

- v is independent of m

centripetal force $= ma = \frac{mv^2}{r} = T\sin\theta \quad -(1)$

$T\cos\theta = mg$

$\Rightarrow T = \frac{mg}{\cos\theta} \quad -(2)$

$\frac{mv^2}{r} = \frac{mg}{\cos\theta}\sin\theta$

$\frac{v^2}{r} = g\tan\theta \quad -(3)$

$\sin\theta = \frac{r}{L}$

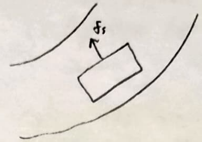
$r = L\sin\theta \quad -(4)$

$\frac{v^2}{L\sin\theta} = g\tan\theta$

$v^2 = Lg\sin\theta\tan\theta$

$v = \sqrt{Lg\sin\theta\tan\theta}$

Horizontal Curve (Flat)



- Static friction supplies centripetal force
- Max speed the car can negotiate the curve is $v = \sqrt{\mu_s g r}$
- Mass does not affect v

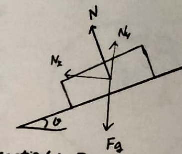
$f_s = \mu_s n = \mu_s mg$

$\mu_s \cancel{m} g = \frac{\cancel{m}v^2}{r}$

$v = \sqrt{\mu_s g r}$

Banked Curve

- Very small or no friction situation
- Normal force supplies the centripetal force



$\tan\theta = \frac{v^2}{rg}$

Vertically: $n\cos\theta = mg$

$n = \frac{mg}{\cos\theta} \quad -(1)$

Horizontally: $n\sin\theta$ is centripetal F

$n\sin\theta = m\frac{v^2}{r} \quad -(2)$

sub (1) → (2):

$\frac{mg}{\cos\theta}\sin\theta = \frac{\cancel{m}v^2}{r}$

$\tan\theta = \frac{v^2}{rg}$

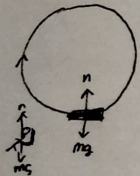
$v = \sqrt{rg\tan\theta}$

Loop - the - Loop in Vertical Plane

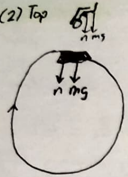
(1) Bottom

$n - mg = \frac{mv^2}{r}$

$n = mg\left(1 + \frac{v^2}{rg}\right)$ //



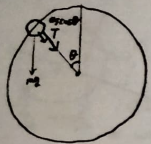
(2) Top



$n + mg = \frac{mv^2}{r}$

$n = mg\left(\frac{v^2}{rg} - 1\right)$ //

• But there is a condition to be fulfilled before reaching top of circle



Net force towards center of circle $= ma$

$T + mg\cos\theta = m\frac{v^2}{r}$

$T = m\left(\frac{v^2}{r}\right) - mg\cos\theta$

In order to complete the upper circle, the string must be tight at all times. So $T > 0$ for all θ.

$T > 0$

$m\frac{v^2}{r} - mg\cos\theta > 0$

$\frac{v^2}{r} - g\cos\theta > 0$

$v^2 > rg\cos\theta$

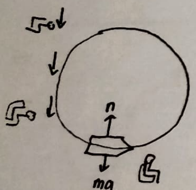
Max height when $\theta = 0$

$v^2 > rg$ (to satisfy all angles)

$v > \sqrt{rg}$

Black Out and Red Out of Pilot

Black Out Case 1: Pilot pulls out from accelerated dive (upright)



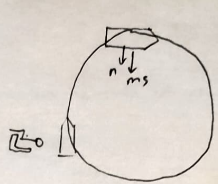
Before the pilot pulls out from an accelerated dive, the blood in his brain has downward velocity. When the pilot pulls out of the dive, the blood still continues to move downwards due to inertia.

As the pilot is upright at the bottom of circle, the blood in his brain has flow downwards to his legs. Thus the pilot can suffer a black-out.

$n_{bottom} = mg\left(1 + \frac{v^2}{rg}\right) > 0$, $> mg$

$G\text{-force} = \frac{n}{mg}$

Black-Out Case 2: Pilot pulls out from an accelerated climb (upside-down)

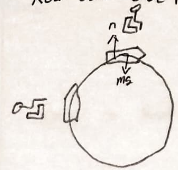


• $v^2 > rg$ must hold for aircraft to reach peak of circle.

$n_{top} = mg\left(\frac{v^2}{rg} - 1\right) > 0$

$G\text{-force} = \frac{n_{top}}{mg} > 0$ (+ve)

Red-Out Case 1: Pilot pulls out of accelerated climb (upright)



• $v^2 > rg$ must hold for aircraft to reach top of circle. $\rightarrow \frac{v^2}{r} > g$

$mg - n = \frac{mv^2}{r}$

$n = mg - \frac{mv^2}{r}$

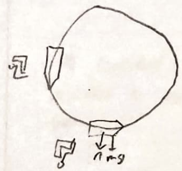
$= mg\left(1 - \frac{v^2}{rg}\right) < 0$

∴ G-force is -ve

Red-Out Case 2: Pilot pulls out of accelerated dive (upside-down)

$-n - mg = \frac{mv^2}{r}$

$n = -\frac{mv^2}{r} - mg < 0$



Rule of thumb about +ve and -ve G-force

+ve G: $n_1 > 0$ and $n_2 = 0$ (possible black-out)

1G: $n_1 = mg$ and $n_2 = 0$ (1G is +ve G)

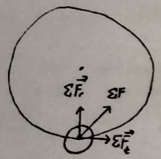
-ve G: $n_1 = 0$ and $n_2 > 0$ (possible red-out)

0G: $n_1 = 0$ and $n_2 = 0$ (free fall) (weightless situation)

PC1201 Chapter 6 Part 2

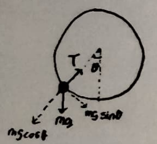
Non-Uniform Circular Motion

- F_r produces the centripetal acceleration
- F_t produces tangential acceleration



$$\Sigma F = \Sigma F_r + \Sigma F_t$$

Vertical Circle with non-uniform speed



$$\text{Centripetal } F = ma = m\frac{v^2}{r}$$

$$T - mg\cos\theta = \frac{mv^2}{r}$$

$$T = \frac{mv^2}{r} + mg\cos\theta$$

$$= mg\left(\frac{v^2}{rg} + \cos\theta\right)$$

$$= m\left(\frac{v^2}{r} + g\cos\theta\right) \quad \leftarrow \text{Tension at any point}$$

when $\theta = 0°$ (bottom):

$$T = mg\left(\frac{v^2}{rg} + 1\right) \quad (T_{max})$$

when $\theta = 180°$ (top),

$$T = mg\left(\frac{v^2}{rg} - 1\right) \quad (T_{min})$$

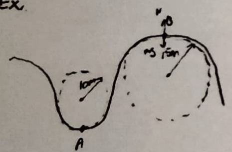
If $T_{top} = 0$, then $T_{top} = m\left(\frac{v^2}{r} + g\cos(180)\right) = 0$

$$\frac{v^2}{r} - g = 0$$

$$V_{top} = \sqrt{gR}$$

- If v less than this value, object will become a projectile.

Ex. $m = 500kg$



a) $N - mg = \frac{mv^2}{r}$

$$N = mg + \frac{mv^2}{r}$$

$$= (500 \times 9.8) + \frac{500 \times 20^2}{10}$$

$$= 2.49 \times 10^4 \ N$$

b) At b: (Normal)

$$mg - N = \frac{mv^2}{r}$$

Max speed corresponds to $n = 0$

$$mg = \frac{mv^2}{r}$$

$$v = \sqrt{gr} = \sqrt{15(9.8)} = 12.1 \ m/s$$

Motion in Accelerated Force

- A fictitious force results from an accelerated frame of reference.
- Observer in accelerated frame (non-inertia)

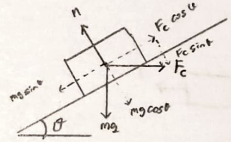
Centrifugal Force

- Fictitious force due to acceleration associated w/ the car's change in direction.

Magnitude of centrifugal force = centripetal force $= m\left(\frac{v^2}{r}\right)$

$$|F_{centrifugal}| = |F_{centripetal}|$$

Revisit to Bank Curved (from non-inertia frame):



∴ Centrifugal force is explained by Newton's first laws

Along inclined plane:

$$mg\sin\theta = \frac{mv^2}{r}\cos\theta$$

$$\tan\theta = \frac{v^2}{rg} \rightarrow \frac{v^2}{r} = g\tan\theta = \frac{g\sin\theta}{\cos\theta}$$

Perpendicular to inclined plane:

$$n = mg\cos\theta + \frac{mv^2}{r}\sin\theta$$

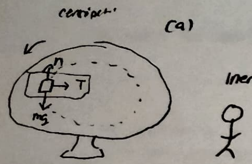
$$= mg\cos\theta + \frac{mg \times \sin\theta}{\cos\theta} \times \sin\theta$$

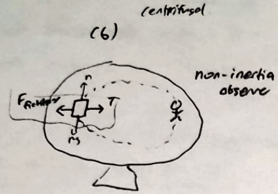
$$= mg\left(\frac{\cos^2\theta + \sin^2\theta}{\cos\theta}\right)$$

$$= \frac{mg}{\cos\theta}$$

$$n \times \cos\theta = mg$$

Fictitious Force in a rotating System

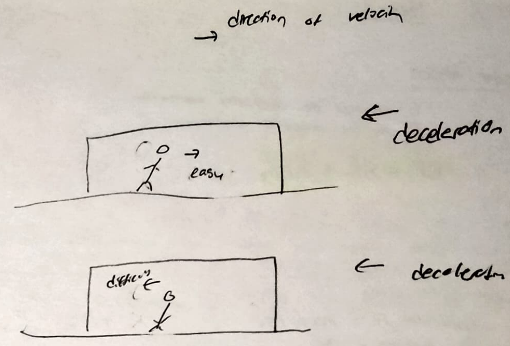




According to observer A, the block is circulating, the tension is the centripetal force. $T = \frac{mv^2}{r}$

According to noninertia observer (b), object is not moving.

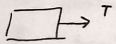
$T - F_{fictitious} = T - \frac{mv^2}{r} = 0$



Ex.

a) $\Sigma F_x = ma$, $T = ma$,

$a = \frac{T}{M} = \frac{18}{5} = 3.6 m/s^2$ to right



b) If $v = const$, $a = 0$, so $T = 0$ (equilibrium)

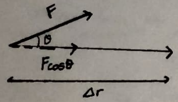
c) Someone in the car (non-inertial observer) claims that the force are T and $F_{fictitious}$.

Someone at rest outside the car (inertia observer) claims that T is the only force on M in the x-direction.

PC1201 Chapter 7
Energy

Work

The work, W, done on a system by an agent exerting a constant force on the system is..



$$W = F \Delta r \cos\theta$$

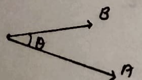
- Displacement refers to distance where force is applied
- Work is a scalar quantity, and is an accumulated sum.
- If displacement is 0, $\Sigma W = 0$.

Unit : J = nm

Work is an energy transfer

- If work done on a system is +ve, energy is transferred to the system
- If work done on a system is -ve, energy is transferred out from the system.

Scalar Product of Two Vectors



$$A \cdot B = AB\cos\theta$$

$A \cdot B = |A|\,|B| \cos\theta$

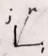
$A \cdot B = |A| \cos\theta \, |B|$

- Commutative
  - $A \cdot B = B \cdot A$
- Distributive law of multiplication
  - $A \cdot (B + C) = A \cdot B + A \cdot C$

Dot Product of Unit Vectors

$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1 \times 1 \times \cos 0 = 1 \times 1 \times 1 = 1$

$\hat{i} \cdot \hat{j} = \hat{i} \cdot \hat{k} = \hat{j} \cdot \hat{k} = 1 \times 1 \times \cos 90 = 1 \times 1 \times 0 = 0$

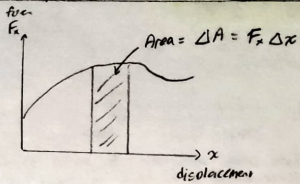


$A = 2\hat{i} + 3\hat{j}$

$B = 4\hat{i} + 5\hat{j}$

$A \cdot B = (2,3) \cdot (4,5) = (2 \times 4) + (3 \times 5) = 23$

Work produced by a varying force



$W \sim F_x \Delta x$

$W \approx \sum_{x_i}^{x_f} F_x \Delta x$

$W = \int_{x_i}^{x_f} F_x \, dx$

- The work is equal to the area under the curve of force vs displacement.
- If more than one force acts on a system and the system can be modelled as a particle.

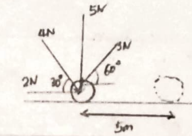
$$\Sigma W = W_{net} = \int_{x_i}^{x_f} \Sigma F_x \, dx$$

or

- If system cannot be modeled as a particle, then total W done is equal to algebraic sum of work produced by individual forces

$$W_{net} = \Sigma W_{\text{by individual forces}}$$

Ex.



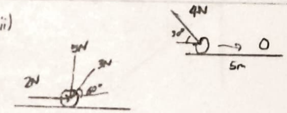
Net horizontal force is :

$2 + 4\cos 30 - 3\cos 60 = 3.96N$

Net work done :

$3.96N \times 5m = 19.82J$

ii)

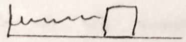


Net work is sum of work by individual forces.

$(2 \times 5 - 3\cos 60 \times 5) + 4\cos 30 \times 5$

$= 19.82J$

Hooke's Law



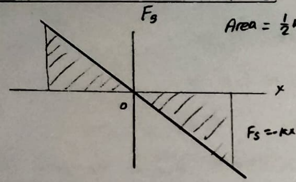
- Force exerted by spring is, $F_s = -kx$
- x is the position of the block w.r.t. equilibrium ($x = 0$)
- k is the spring constant (measures stiffness of spring)

- When x is +ve (spring is stretched), F is -ve.
  x is -ve (spring is compressed), F is +ve.

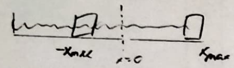
- The force exerted by the spring is always directed opposite to the displacement from equilibrium.
- F is called the restoring force.
- If the block is released, it will oscillate between x and $-x$.

Work produced by a spring

Area = $\frac{1}{2}kx^2$



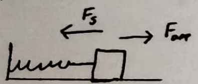
$W_s = \frac{1}{2}kx^2$



- Total work produced by spring as the block moves from $-x_{max}$ to x_{max} is:

$$\left[\frac{-kx^2}{2}\right]_{-x_{max}}^{x_{max}} = \frac{-kx_{max}^2}{2} - \left(\frac{-k(-x_{max})^2}{2}\right) = 0$$

Spring w/ an Applied Force



$\boxed{F_{app} = -F_s} = -(-kx) = kx$

- The applied force is equal and opposite to the spring force
- Work produced by F_{app} is equal to $\frac{1}{2}kx_{max}^2$

Ex. $W_s = \frac{1}{2}kx^2$

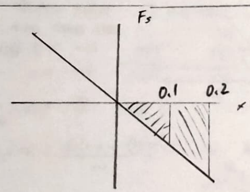
$4 = \frac{1}{2}k(0.1)^2$

$k = 800$ N/m

work to stretch from 10 cm to 20 cm:

$\Delta W = \frac{1}{2}(800)(0.2)^2 - 4$

$= 12J$



Kinetic Energy

- Kinetic Energy is the energy of a particle due to its motion.

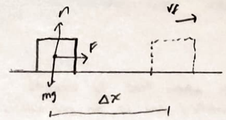
$K = \frac{1}{2}mv^2$

$\Sigma W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

if $v_i = 0$, $\Sigma W = \frac{1}{2}mv_f^2$

Work - Kinetic Energy Theorem

- $\Sigma W = K_f - K_i = \Delta K$
- If work is done on a system and the only change in the system is in its speed, the W done by the net force equals change in kinetic energy.



- normal & gravitational forces do not work

$W = F\Delta x = \Delta K$

$= \frac{1}{2}mv^2 - 0$

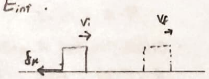
Nonisolated System

- System interacts with or is influenced by its environment.

(If you add the loss of energy to find sum, you will get back the initial sum.

Internal Energy

- The energy associated w/ an object's temperature is called internal energy E_{int}.



- The friction does work and increases the internal energy of the surface.

Potential Energy

- Potential energy is energy related to the configuration of a system

Examples

- Elastic Potential Energy
- Gravitational Potential Energy
- Electrical Potential Energy

Work : Force × Displacement along same direction

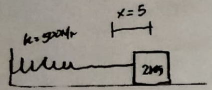
KE : $\frac{1}{2}mv^2$

GPE : mgh

EPE : $\frac{1}{2}kx^2$

Internal Energy : Associated w/ temperature

Ex.



a) when block is pulled back to equilibrium, $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

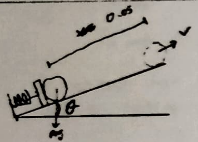
$$\frac{1}{2}(500)(0.05)^2 = \frac{1}{2}(2)v^2$$

$$v = 0.79 m/s$$

b) $\frac{1}{2}kx^2 - (f_s \times d) = \frac{1}{2}mv^2$

$$\frac{1}{2}kx^2 - (\mu_s mg \times 0.05) = \frac{1}{2}mv^2$$

$$v = 0.53 m/s$$



$K = 1.2 N/cm$

Initial Energy in spring $= \frac{1}{2}\left(\frac{1.2}{10^{-2}}\right)(5 \times 10^{-2})^2$

$$= 0.15 J$$

Energy when spring extends to equilibrium:

$$0.15 - (mg\sin\theta \times 0.05) = \frac{1}{2}mv^2$$

$$0.15 - (0.1 \times 9.8 \sin 10 \times 0.05) = \frac{1}{2}(0.1)v^2$$

$$v = 1.68 m/s$$

Conservation of Energy

- Energy cannot be created or destroyed
- If total energy in a system changes, it can only be due to the fact that energy has crossed the boundary of the system by some method of energy transfer

Power

- The time rate of energy transfer is called power
- Average power is given by: $P = \frac{W}{\Delta t}$ when the method of energy transfer is work

Instantaneous Power

$$P = F \times v$$

Power = Force × velocity

Ex.

$$KE = \frac{1}{2}mv^2$$

To double the speed: $\frac{1}{2}m(2v)^2$

$$= \cancel{\tfrac{1}{2}}\, 2mv^2$$

$$P = \frac{W}{\Delta t} = \frac{dW}{dt} \quad \begin{matrix} \leftarrow \times 4 \\ \leftarrow \times 0.5 \end{matrix}$$

Power required is eight times.

Units of Power

S.I. unit of power is called Watt.

$$1 \text{ watt} = 1 J/s$$

$$= ((kg \times m/s^2) \times m)/s = 1 kg\, m^2/s^3$$

- Units of power can be used to express units of work or energy

$$1 kWh = (1000 W)(3600 s) = 3.6 \times 10^6 J$$

Energy transferred in 1hr at a constant rate.

Ex

Total energy in kWh for one month

$(1.5 \times 8 + 4 \times 1 + 0.5 \times 24) \times 30 = 840\,kWh$

In term of Joules,

$840 \times 10^3 \times 3600 = 3.024 \times 10^9\,J$

Monthly Cost $= 840\,kWh \times \$0.25/kWh = \underline{\$210}$

Chapter 8 - PC1201

Isolated System is one for which there are no energy transfers across the boundary. The energy is conserved.

→ If there is friction on table, the block is not sliding in an isolated system anymore.

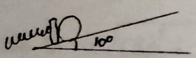
Conservative Forces in Isolated System

- The work done by a conservative force on a particle moving between any two points is independent of the path taken by the particle.
- The work done by a conservative force on a particle moving through any closed path is zero.

Ex: Gravitational Force, Spring Force

Conservation of Mechanical Energy

$E_{mech} = K + U_g$ $\qquad K_f + U_f = K_i + U_i$



Initial energy stored in spring

$= \frac{1}{2}kx^2 = \frac{1}{2}(1.2/10^{-2}) \times (5\times10^{-2})^2 = 0.15J$

0.15J used to move ball up slope of 10°

for vertical distance of 5cm sin10 = 0.87cm

$mgh + \frac{1}{2}mv^2 = 0.15J$

$(0.1)(9.8)(0.87\times10^{-2}) + \frac{1}{2}(0.1)v^2 = 0.15$

$v = 1.68m/s$

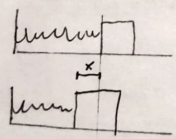
Elastic Potential Energy

The force the spring exerts is: $F_s = -kx$

Work done by external applied force on spring block system:

$W = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$ (x measured from equilibrium)

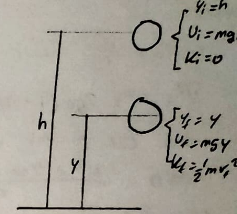
$U_s = \frac{1}{2}kx^2$ (when x is distance from natural position)



Potential Energy
↓ convert to
Kinetic Energy

- EPE is max when spring reach max extension or compression
- U_s always +ve because x^2 is +ve

Conservation of energy (Drop ball)

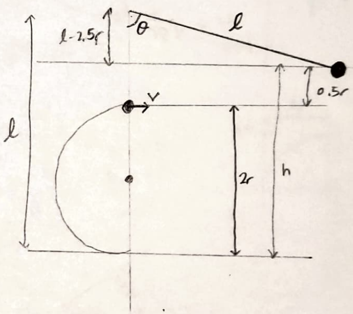


Initial Condition: $E_i = K_i + U_i = mgh$

- Ball dropped from rest, $K_i = 0$

$\frac{1}{2}mv_f^2 + mgy = mgh$

$\Delta KE + \Delta PE = 0$



Conservation of mech energy: $\cancel{m}gh = \frac{1}{2}\cancel{m}v^2 + \cancel{m}g(2r)$

$h = \frac{v^2}{2g} + 2r$ — (1)

Centripetal Acceleration:

$mg + \cancel{T}^{\to 0} = \frac{mv^2}{r}$ (just clear half circle) (T very small → 0)

$v^2 = gr$ — (2)

Sub (2) to (1):

$h = \frac{gr}{2g} + 2r$

$= \frac{r}{2} + 2r$

$= \frac{5r}{2} = 2.5r$

$\cos\theta = \frac{l - 2.5r}{l}$

Centripetal acceleration:

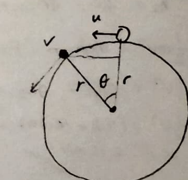
$mg\cos\theta - n = \frac{mv^2}{r}$

When the ball leaves the sphere, (n=0)

$mg\cos\theta - 0 = \frac{mv^2}{r}$

$g\cos\theta = \frac{v^2}{r}$

$\cos\theta = \frac{v^2}{rg}$ — (1)



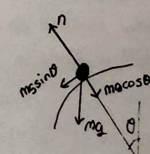
By conservation of energy:

$(KE + PE)_i = (KE + PE)_f$

$\frac{1}{2}mu^2 + mgr = \frac{1}{2}mv^2 + mg(r\cos\theta)$

$\frac{1}{2}u^2 + gr = \frac{1}{2}v^2 + gr\cos\theta$

$v^2 = u^2 + 2gr(1 - \cos\theta)$ — (2)



Cont.

$\cos\theta = \frac{v^2}{rg}$ — (1)

$v^2 = u^2 + 2gr(1-\cos\theta)$ — (2)

sub (2) to (1):

$\cos\theta = \frac{u^2 + 2gr(1-\cos\theta)}{rg}$

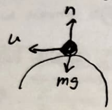
$rg\cos\theta = u^2 + 2gr(1-\cos\theta)$

$rg\cos\theta = u^2 + 2gr - 2gr\cos\theta$

$3rg\cos\theta = u^2 + 2gr$

$\cos\theta = \frac{u^2 + 2gr}{3gr}$

$\theta = \cos^{-1}\left(\frac{u^2 + 2gr}{3gr}\right)$



- Impossible to slide object down hemisphere

$\cos\theta = \frac{u^2 + 2gr}{3gr}$

$\cos(90) = \frac{u^2 + 2gr}{3gr}$

$u^2 = -2gr$

Let $\theta = 0°$ (ball leave sphere straightaway at highest pt)

$\cos(0) = \frac{u^2 + 2gr}{3gr}$

$1 = \frac{u^2 + 2gr}{3gr}$

$3gr = u^2 + 2gr$

$u = \sqrt{gr}$ //

Another approach by newton's second law:

$mg - \cancel{n}^{0} = \frac{mu^2}{r}$

$g = \frac{u^2}{r}$

$u = \sqrt{gr}$

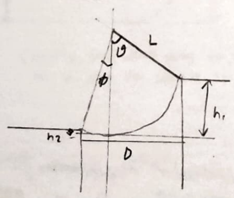
a) $D = L\sin\theta + L\sin\phi$

$50 = 40(\sin 50 + \sin\phi)$

$\phi = 28.9°$

$h_1 = 40 - 40\cos 50 = 14.29m$

$h_2 = 40 - 40\cos 28.9 = 4.98m$



$\frac{1}{2}mv^2 + mgh_1 - (F\times D) = mgh_2 + 0$

$\frac{1}{2}v^2 + gh_1 = gh_2 + \frac{FD}{m}$

$\frac{1}{2}v^2 + (9.8\times 14.29) = (9.8\times 4.98) + \frac{110\times 50}{50}$

$v = 6.13\ m/s$ //

b) Total mass = 80 + 50 = 130kg

$\frac{1}{2}Mv^2 + Mgh_2 + (F\times D) = mgh_1 + 0$

...

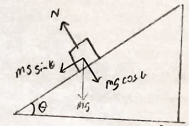
$v = 9.89\ m/s$

Nonconservative Force

In general, if friction is acting in a system: $\Delta E_{mech} = \Delta K + \Delta U = -$

- Nonconservative forces acting in a system cause a change in the mechanical energy of the system.

Ex.



a) $\Delta K = \frac{1}{2}m(\cancel{v_f^2}^{0} - v_i^2)$

$= -\frac{1}{2}mv_i^2$

$= -160J$

b) $\Delta U = mg\, 3\sin 30$

$= 73.5J$

c) Mechanical energy converted to heat due to friction is:

$160 - 73.5 = 86.5J$ //

$W = f\times d$

$86.5 = f\times 3$

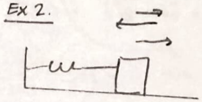
$f = 28.8N$

d) $f_k = \mu_k n$

$28.8 = \mu_k\, mg\cos 30$

$\mu_k = 0.679$ //

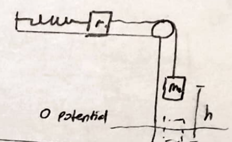
Ex 2.



- Without friction, the energy continues to be transferred between kinetic and elastic potential energy. The total energy remain the same
- If friction is present, the energy decreases

$\Delta E_{mech} = -f_k d$

Ex 3 (connected blocks)



- The system consists of two blocks, the spring, and the earth.
- Gravitational and potential energy are not involved. friction $\neq 0$.
- Kinetic energy is 0 if initial and final configurations are at rest.

Cont.

- Block 2 undergoes change in G.P.E
- Spring undergoes a change in E.P.E

(Loss in GPE of m_2) − (Heat produced) = Elastic Potential Energy given to Spring

$m_2gh - (\mu_k m_1 g)\times h$

$= \frac{1}{2}kh^2$

(Chapter 8)

a) Initial amount of energy in the system:

$$\frac{1}{2}mv_i^2$$

This energy is given to EPE in the spring when compressed and work done by friction

$$\frac{1}{2}mv_i^2 - (\mu_k mg \times d) = \frac{1}{2}kd^2$$

$$\frac{1}{2}(1)(3)^2 - (0.25(1)(9.8) \times d) = \frac{1}{2}(50)d^2$$

$$25d^2 + 2.45d - 4.5 = 0$$

$$d = 0.378m \quad \text{or} \quad d = -0.476 \text{ (rej)}$$

b)

$$\frac{1}{2}mv_i^2 - f(2d) = \frac{1}{2}mv_f^2$$

$\times 2/m$

$$v_i^2 - \frac{2}{m}(f)(2d) = v_f^2$$

$$v_f = \sqrt{(3)^2 - \frac{2}{1}(0.25 \times 1 \times 9.8)(2 \times 0.378)}$$

$$= 2.30 \text{ m/s}$$

c) 1st Method:

$$\vec{d} + \overleftarrow{d} + \overleftarrow{D}$$

$$\frac{1}{2}(1)(3)^2 = f \times (d + d + D)$$

$$D = 1.08m$$

2nd Method:

friction

$$f = \mu_k mg$$

$$= 0.25 \times 1 \times 9.8$$

$$= 2.45N$$

$$f \times D = \frac{1}{2}(1)(2.3)^2$$

$$D = 1.08m$$