CPT411 Automata Theory & Formal Languages

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Tutorial II

1. Consider the language $L = \{1^n 2^n : n > 0\}$. Is the string 122 in L?

   L consists of strings where the number of 1s = the number of 2s, in that order. String 122 has one 1 and two 2s.
   $\therefore$ Since $1 \neq 2, 122$ is not in L.

2. Let $L_1 = \{a^n b^n : n > 0\}$ and $L_2 = \{c^n : n > 0\}$. For each of the following strings, state whether or not it is an element of $L_1 L_2$:

   (a) No. Concatenation of non-empty strings from L_1 and L_2 cannot yield ε.

   (b) Yes. $aabb \in L_1$ for $n = 2$, $cc \in L_2$ for $n = 2$.

   (c) No. abb is not in L_1 as $n_1 \neq n_2$.

   (d) No. cce is not in L_2 as it ends with e.

3. Let $L_1 = \{\text{peach, apple, cherry}\}$ and $L_2 = \{pie, cobbler, \varepsilon\}$. List the elements of $L_1 L_2$ in lexicographic order.

   apple, applecobbler, applepie, cherry, cherrycobbler, cherrypie, peach, peachcobbler, peachpie

4. Let $L = \{w \in \{a, b\}^* : |w| \equiv_3 0\}$. List the first six elements in a lexicographic enumeration of L.

   $\varepsilon, aaa, aab, aba, abb, baa$

5. For each of the following languages L, give a simple English description. Show two strings that are in L and two that are not (unless there are fewer than two strings in L or two not in L, in which case show as many as possible).

   (a) $L = \{w \in \{a, b\}^* \mid \text{exactly one prefix of } w \text{ ends in } a\}$: **Strings where only one prefix ends with a.**

      i. $\{a, ba\} \in w$

      ii. $\{aa, ab\} \notin w$

   (b) $L = \{w \in \{a, b\}^* \mid \text{all prefixes of } w \text{ end in } a\}$: **Strings where every prefix ends with a.**

      i. $\{a, aa\} \in w$

      ii. $\{b, ab\} \notin w$

   (c) $L = \{w \in \{a, b\}^* \mid \exists x \in \{a, b\}^+ (w = axa)\}$: **Strings that start and end with a, with at least one character in between.**

      i. $\{aaa, aba\} \in w$

      ii. $\{a, bb\} \notin w$

6. Let $L = \{w \in \{a, b\}^* : w = w^R\}$. What is chop($L$)?
   (chop(L) is all the odd length strings in L with their middle character chopped out.)

   chop(L) = $\{w \in \{a, b\}^* : w \text{ is odd-length, } w = w^R \text{ without middle char}\}$

   $\therefore$ L consists of palindromes. chop(L) removes the middle character from odd-length palindromes.

7. What is the concatenation of 011 and 1010?

   0111010

8. The length of the string cbccaba is

   7

9. The binary string 1000000 is a member of which of the following problems? State true or false. Remember, a "problem" is a language whose strings represent the cases of a problem that have the answer "yes." In this question, you should assume that all languages are sets of binary strings interpreted as base-2 integers. The exception is the problem of finding palindromes, which are strings that are identical when reversed, like 0110110, regardless of their numerical value.

   $1000000_2 = 64_{10}$

   (a) False. Not a prime. 64 is composite.

   (b) False. Not a palindrome. $1000000 \neq 0000001$

   (c) False. $64 = 8^2$ is a perfect square.

   (d) True. 64 is not a multiple of 3.