## CPT411 Automata Theory & Formal Languages

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## Tutorial I

- 1. Examine the following formal description of sets so that you understand which member they contain. Write a short informal English description of each set.
  - (a) Set containing all positive odd numbers.
  - (b) Set containing all even integers.
  - (c) Set containing all positive even numbers/multiples of 2.
  - (d) Set containing numbers of multiples of 2 and 3/all multiples of 6.
  - (e) Set containing all binary palindromes.
  - (f) Set is empty. No number can be equal to itself plus one.
- 2. Write formal descriptions of the following sets:
  - (a) {1, 10, 100}
  - (b)  $\{n \mid n \in \mathbb{Z}, n > 5\}$
  - (c)  $\{n \mid n \in \mathbb{N}, n < 5\}$
  - (d) {"aba"}
  - (e)  $\{\varepsilon\}$
  - $(f) \{\emptyset\}$
- 3. Let A be the set  $\{x, y, z\}$  and B be the set  $\{x, y\}$ .
  - (a) No. A contains z, which is not in B.
  - (b) Yes. Every element in B is also in A.
  - (c)  $\{x, y, z\}$
  - $(d) \{x, y\}$
  - (e)  $\{(x,x),(x,y),(y,x),(y,y),(z,x),(z,y)\}$
  - (f)  $P(B) = \{\emptyset, \{x\}, \{y\}, \{x, y\}\}\$
- 4. If A has a elements and B has b elements, how many elements are there in  $A \times B$ ?

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If |A| = a elements and |B| = b elements,
|A \times B| = a \times b
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5. If C is a set with c elements, how many elements are there in the power set of C?

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|P(C)| = 2^c
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- 6. Let X be the set  $\{1, 2, 3, 4, 5\}$  and Y be the set  $\{6, 7, 8, 9, 10\}$ . The unary function  $f: X \to Y$  and the binary function  $g: X \times Y \to Y$  are described in the following tables.
  - (a) f(2) = 7
  - (b)  $\forall x \in X : x \in D_f, R_f[6, 7]$
  - (c) g(2, 10) = 6
  - (d)  $\forall (x,y) \in X \times Y : (x,y) \in D_q, R_q[6,10]$
  - (e) g(4,7) = 8
- 7. Consider the domain and range values in following table. State whether it is a function or a relation.

R at 0 has multiple possible outputs.

- $\therefore$  It is a relation.
- 8. Give an example of a relation that you have encountered in:
  - (a) Daily life: "is a fan of football club of"
  - (b) School of Computer Sciences: "instructs modules of"
- 9. Which method is better to describe a person and his/her age? Function or relation?

Each person has exactly one age at a given time.

- : Function is better for the description.
- 10. Give examples to show that:
  - (a) Let  $A = \{2n \mid n \in \mathbb{N}\}$  and  $B = \{3n \mid n \in \mathbb{N}\}$ ,  $A \cap B = \{6k \mid k \in \mathbb{N}\}$  and only 0 is divisible by 2 and 3.: finite.
  - (b) Let  $A = \mathbb{N}$  and  $B = \{n \mid n \geq 5\}, A \cap B = \{5, 6, 7, 8, \ldots\}$ : countably infinite.
  - (c) Let  $A = \mathbb{R}$  and  $B = \mathbb{Q}$ ,  $A \cap B =$  Some isolated points : finite.