experiments

April 12, 2024

1 A first possible experiment

We consider a closed chain of length $2L$, positions are labelled by $x = -L, \dots, L-1$. The QW walk is given by $U = WV$ with

$$W = \begin{pmatrix} \alpha T^2 & i\beta T \\ i\beta T^\dagger & \alpha (T^2)^\dagger \end{pmatrix},$$

and

$$V = \mathbb{I} + \begin{pmatrix} c-1 & ise^{-i\gamma} \\ ise^{i\gamma} & c-1 \end{pmatrix} \otimes |0\rangle\langle 0|\,.$$

Above, $T|x\rangle = |x+1\rangle$ is the translation operator, $\alpha = \cos\theta$, $\beta = \sin\theta$, and $c = \cos\phi$, $s = \sin\phi$

Consider the eigenproblem $U|\psi\rangle = e^{i\omega}|\psi\rangle$. Generically, we have 4 solutions to this equation labelled by momenta $(k, k-\pi, -k, -k+\pi)$.

1.1 Unreasonable resources

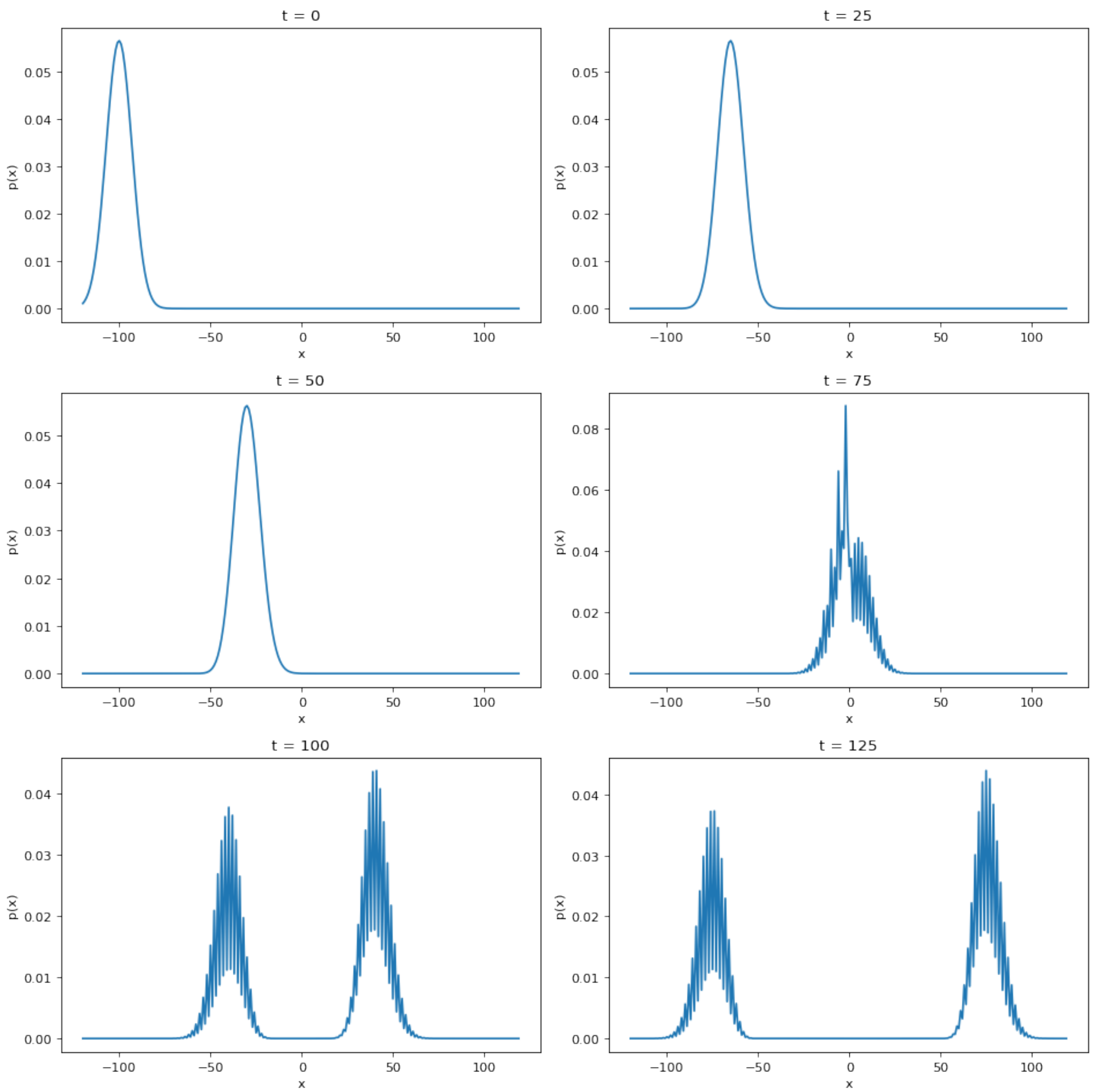
First, we consider the simulation of a scattering through the potential without thinking about experimental constraints. In particular, we consider long evolution times and rather broad wavepackets. The goal is to show the phenomenon we'd like to see in the lab.

We consider a QW with parameters $L = 300$, $\theta = \pi/4$, $\phi = \pi/3$, and $\gamma = 3\pi/4$.

We prepare an initial gaussian packet localized at $x_0 = -100$ travelling to the right with momentum close to $k_0 = \pi/4$ ($\sigma_k = 0.1$). We will track the evolution of the wavepacket as it scatters on the potential.

We will see that the scattering excites modes with momenta $k_0 - \pi$ and $\pi - k_0$, not only $-k_0$. This is the same physics found for the Thirring QCA [Bisio et al. 2018], and it's what we'd like to observe.

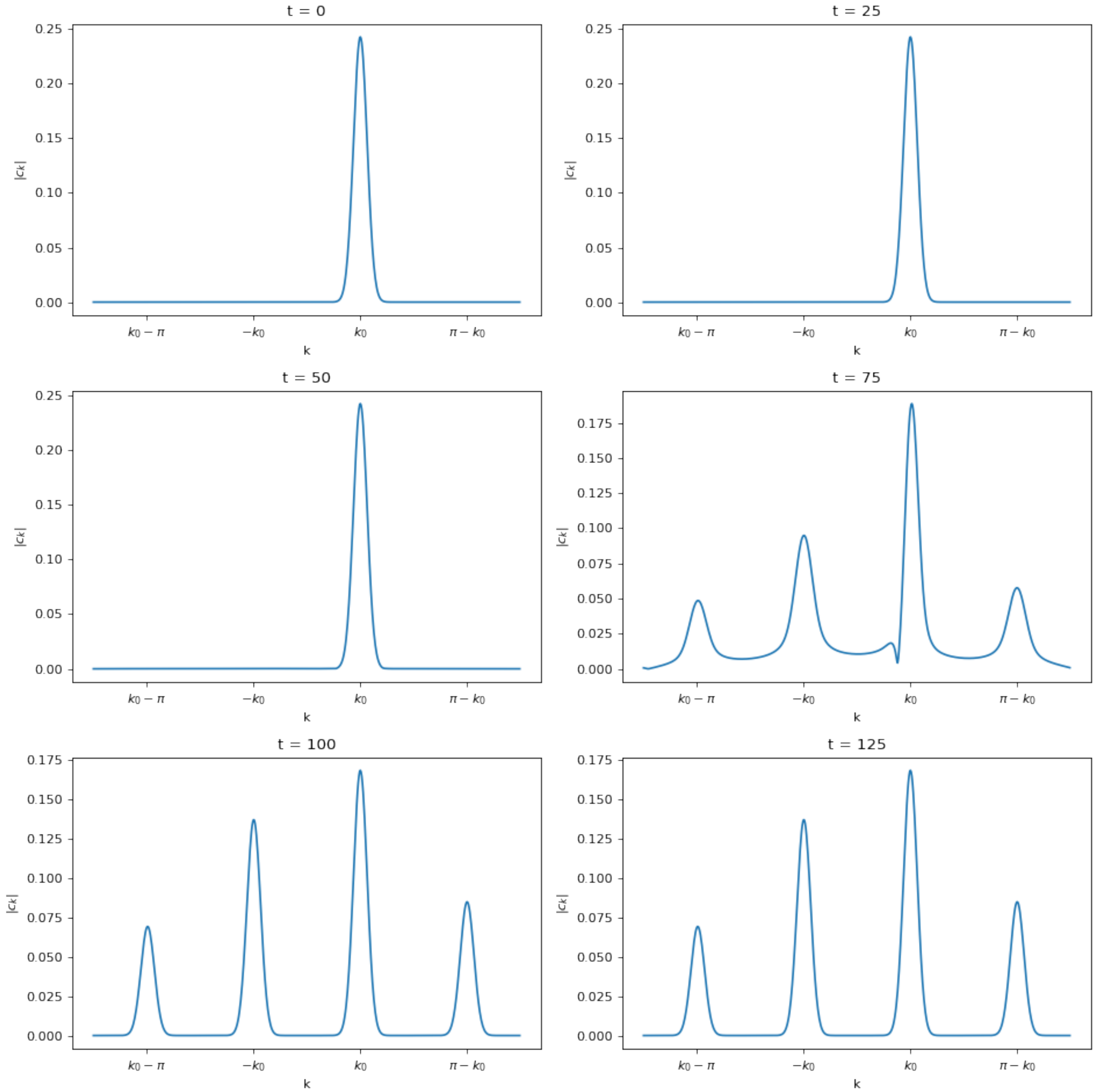
The evolution looks like this.



After the scattering with the potential, we have two wavepackets, corresponding to transmitted and reflected waves.

The oscillating behaviour on small scales is due to the interference between modes with momenta k_0 and $k_0 - \pi$, and $-k_0$ and $-k_0 + \pi$. Observing this interference pattern is one possible way to show that modes with momenta $k_0 - \pi$ and $-k_0 + \pi$ are excited.

Another way it to decompose the outcoming signal in the momentum eigenbasis.



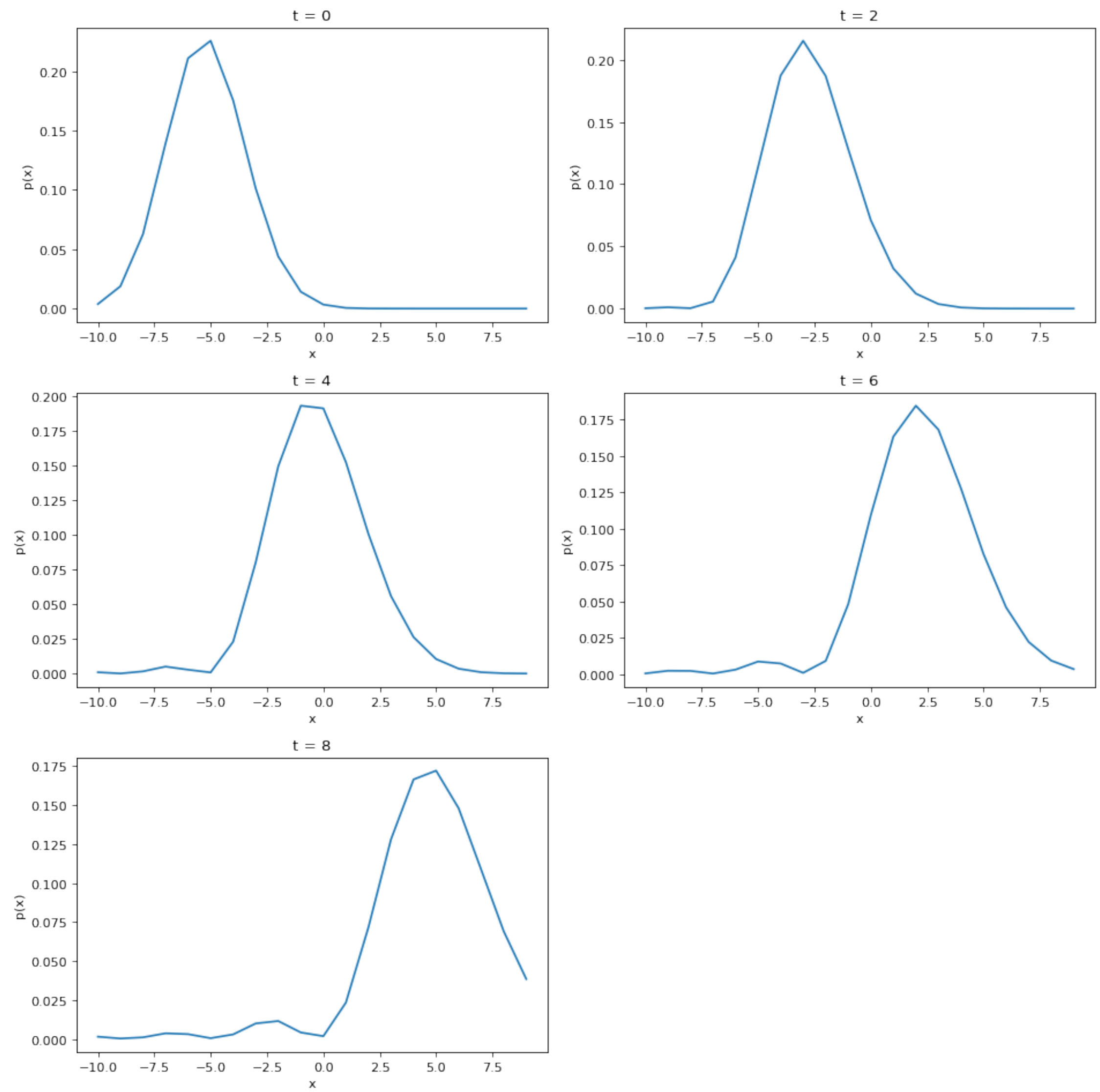
We see that the signal, initially prepared with a momentum picked at k_0, develops 3 other picks after scattering with the potential.

1.2 Reasonable resources (?)

We try to find parameters for which the interference pattern is still visible, while satisfying the experimental constraints. In particular, we try to minimize the number of time steps necessary to see the interference pattern.

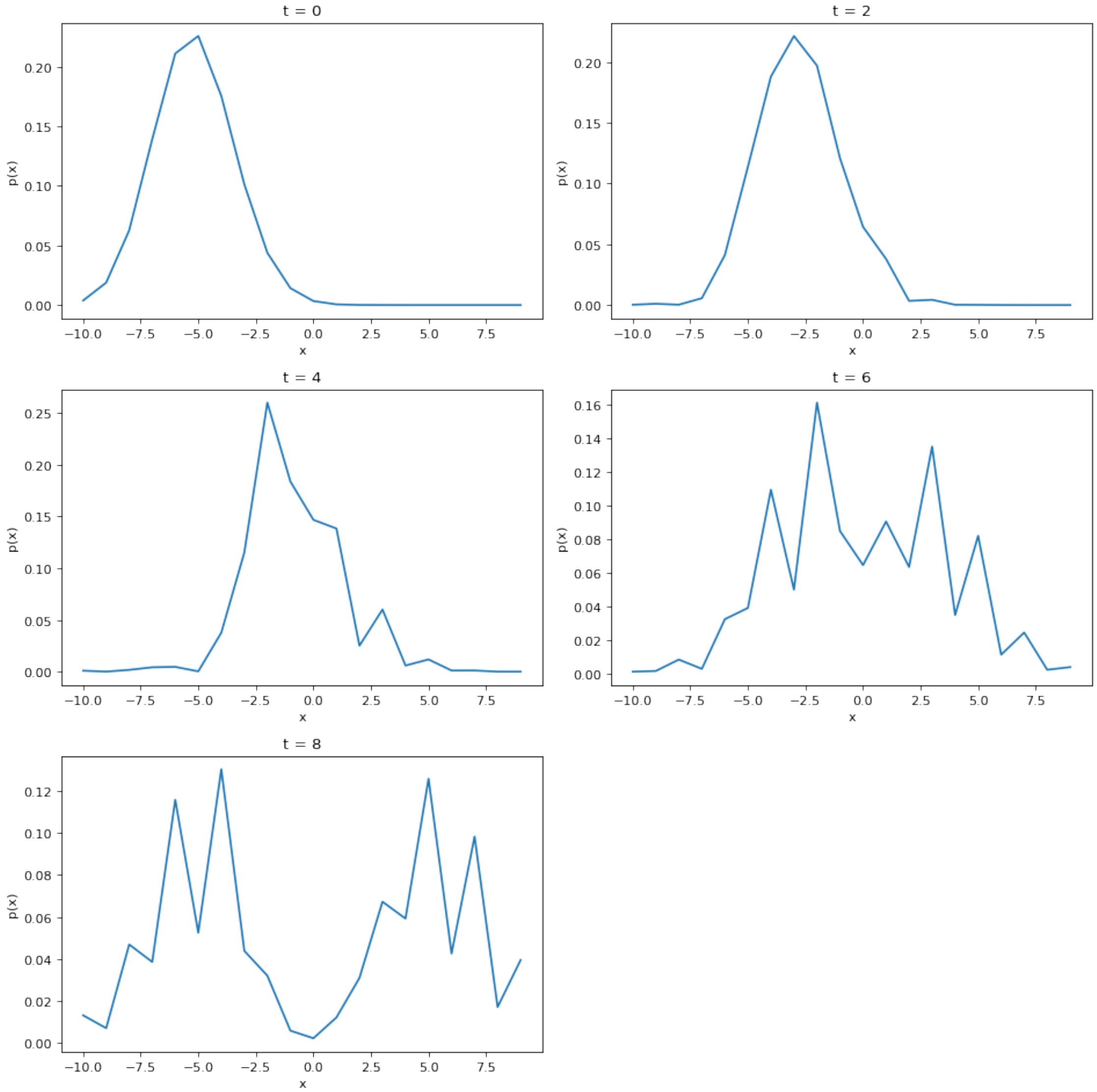
We choose the following parameters: $L = 100$, $\theta = \pi/4$, $\gamma = 3\pi/4$, $k_0 = \pi/4$, $\sigma_k = 0.4$, and $x_0 = -5$.

First consider the model without interaction.



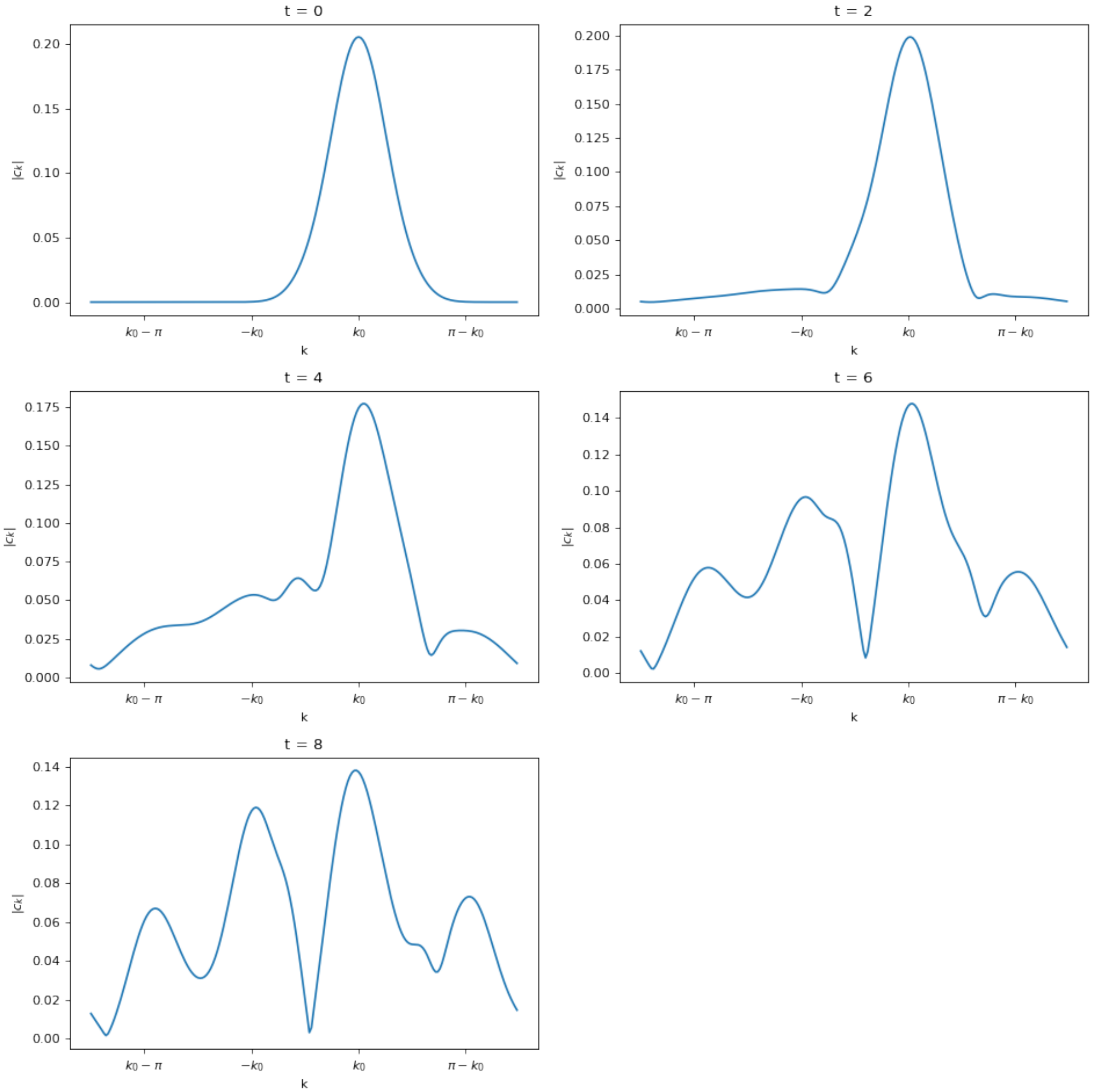
The point here is that the wavepacket is small enough (and fast enough), that it has the time to go completely from one side of $x = 0$ to the other.

We now introduce the interaction, with $\phi = \pi/3$ and $\gamma = 3\pi/4$.



We see that the packet has enough time to split in two separate wavepackets (reflected and transmitted), and the interference patterns are clearly visible.

We also consider the same scattering process in momentum space.

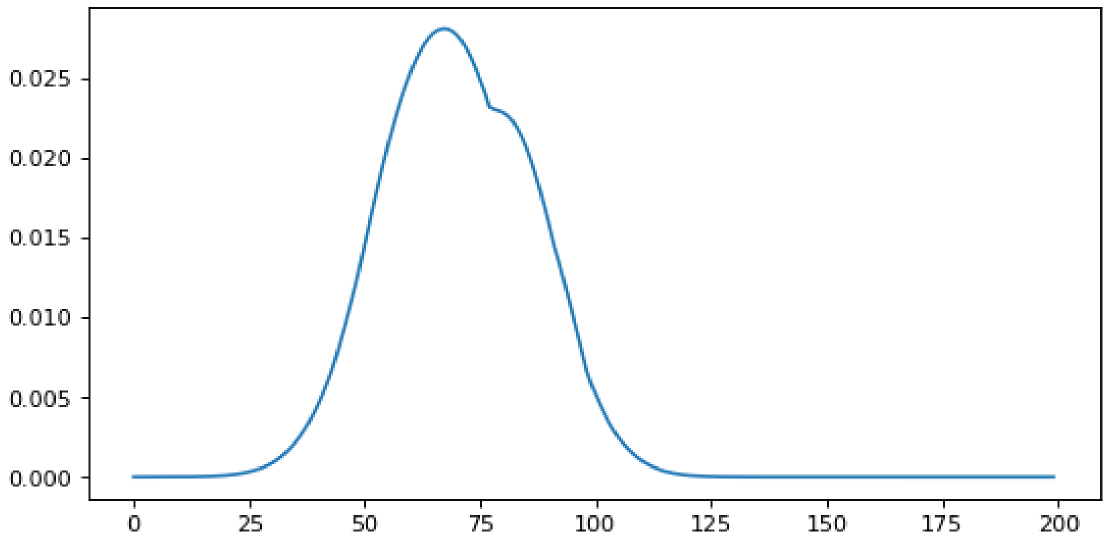


1.3 Other things

1.3.1 Convergence rate

We check at what times the momentum distribution changes appreciably.

[9]: [<matplotlib.lines.Line2D at 0x11d648250>]



1.3.2 Comparison with analytical result

We check that the numerics agree with the analytical solution.

[14]:
```
array([0.48      , 0.08      , 0.31797959, 0.12202041])
```

Find the relative size of the 4 picks in momentum distribution found with the numerics and check that they agree with the analytical result.

[17]:
```
0.47948538166634874
```

[18]:
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0.08019128144422993
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[19]:
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0.3182514494529883
```

[20]:
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0.12207188743631182
```

We find agreement between the simulation and the analytical result.