SHARIF UNIVERSITY OF TECHNOLOGY
PHYSICS DEPARTMENT

Problem Set 10

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Course: *Computational Physcis - (Spring 2023)*
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Exercise 10.1

Chaos

Draw the bifurcation diagram for the logistic map:

$$x_{n+1} = 4rx_n(1 - x_n)$$

Find r for the beginning of bifurcations and the beginning of the chaotic phase. Find the Feigenbaum constants.

Answer. An imperative approach was taken to solve the problem. Since the effect of the initial value will fade after many iterations, x_0 can be any random number in the interval $(0, 1)$ and so was taken to be 0.5. For each $r \in [0, 1)$, with step size of 0.001, 1000 iterations of the logistic function were made. The result of each iteration was stored in an array, and the last 100 elements of the said array were plotted.

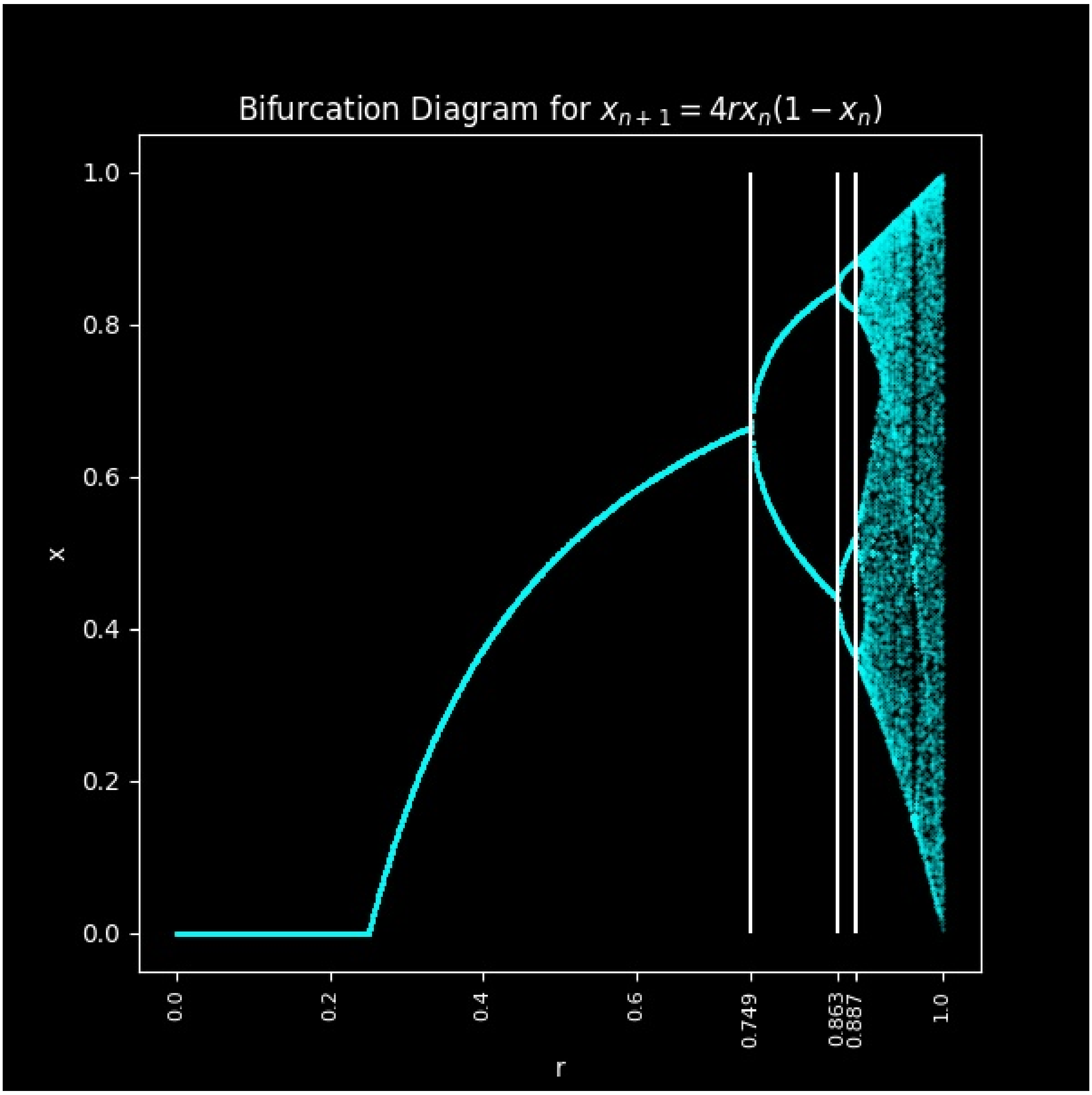
To find the Feigenbaum constants, one must find the values of r where bifurcations happen, and the corresponding values of x, which are indeed the attracting points of the equation. To do so, after the values of x were stored in the aforementioned array for each r, the last 20 entries were examined to find the number of unique values (to the 3rd decimal place, consistent with the step size of r, equal to 0.001) that keep on repeating. If there are 2, 4 or 8 repeating values, it means the system has found the attracting points. Thus, they were recorded.

According to Wikipedia, Feigenbaum constant δ expresses the limit of the ratio of distances between consecutive bifurcations; whereas Feigenbaum's alpha constant is the ratio between the width of a tine and the width of one of its two subtines. In the obtained graph, the upper and lower subtines were not of equal widths; the upper one yielded better results in calculating α, and so was the one used.

The constants are as follows, compared to the values provided on Wikipedia, to the third decimal place:

$$\delta_{calculated} = 4.750 \qquad \delta_{Wikipedia} = 4.670 \qquad RelativeError = 1.735\%$$

$$\alpha_{calculated} = 2.551 \qquad \alpha_{Wikipedia} = 2.503 \qquad RelativeError = 1.934\%$$


Bifurcation Diagram for $x_{n+1} = 4rx_n(1 - x_n)$
1.0
0.8
0.6
0.4
0.2
0.0
x
0.0
0.2
0.4
0.6
0.749
0.863
0.887
1.0
r