SHARIF UNIVERSITY OF TECHNOLOGY
PHYSICS DEPARTMENT

Problem Set 8 (the Ising Model)

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Course: *Computational Physcis - (Spring 2023)*
Due date: *June 23, 2023*

Exercise 9.1

Simulate the 2D Ising model with the Metropolis Algorithm. Examine the behavior of C_v, χ, $|\langle M \rangle|$ and ξ, especially around the critical temperature.

Answer. The following functions were defined:

- **initilize_system():**

  A basic function to construct a random matrix of 1's and -1's of side length L.

- **Metropolis():**

  Takes a lattice, a value of β and the number of Metropolis steps to be taken. To make the function faster, a look-up table of transition probabilities was implemented. In each iteration, a spin is chosen at random and flipped according to the Metropolis algorithm.

- **energy():**

  This computes the energy of the system based on the following relation, as described in the textbook:

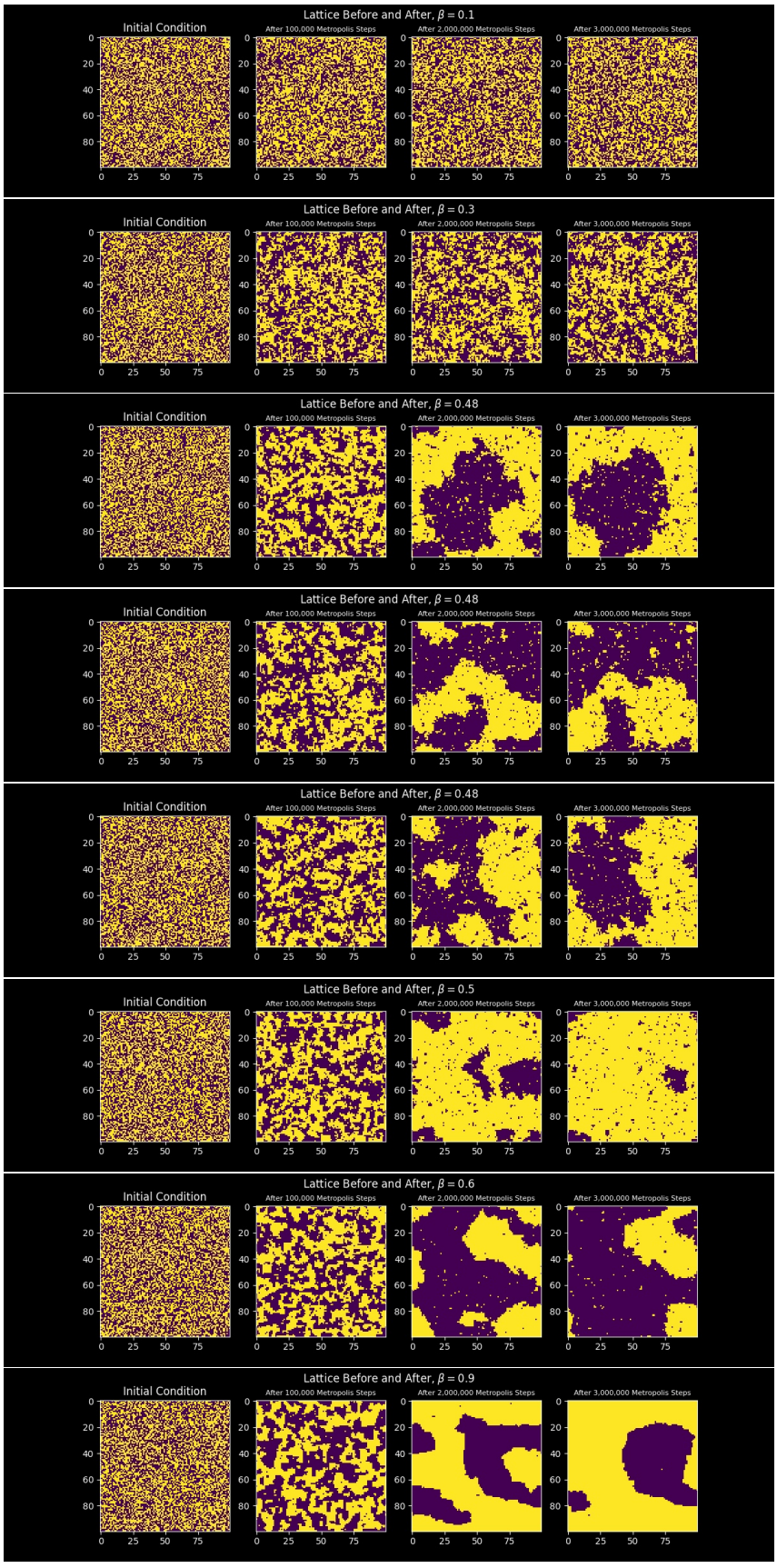
  $$E(\overrightarrow{s}) = -J \sum_{\langle i,j \rangle} s_i s_j - h \sum_i s_i$$

  To do so, periodic boundary conditions are applied and a vectorized energy calculation is implemented.

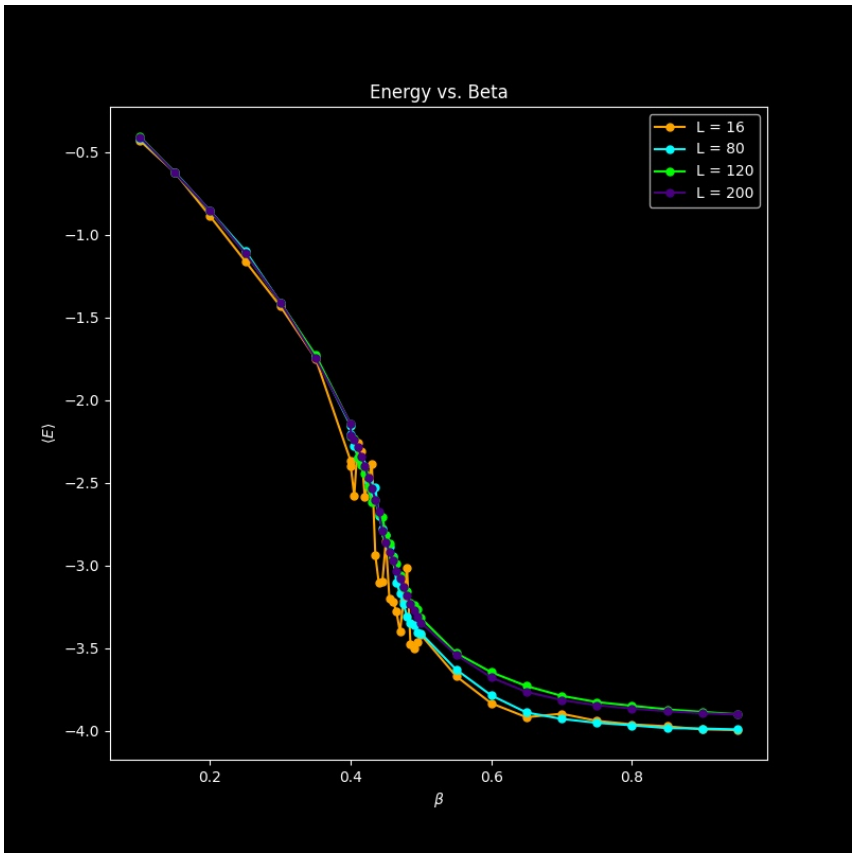
- **magnetization():**

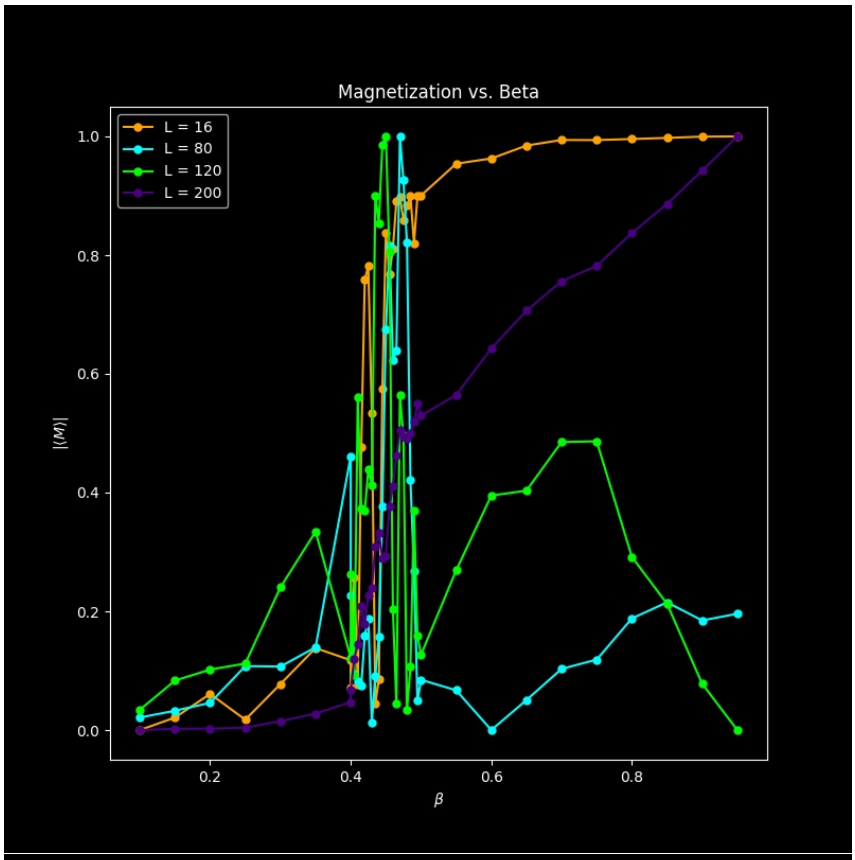
  Returns the total magnetization (basically the absolute sum) of a lattice.

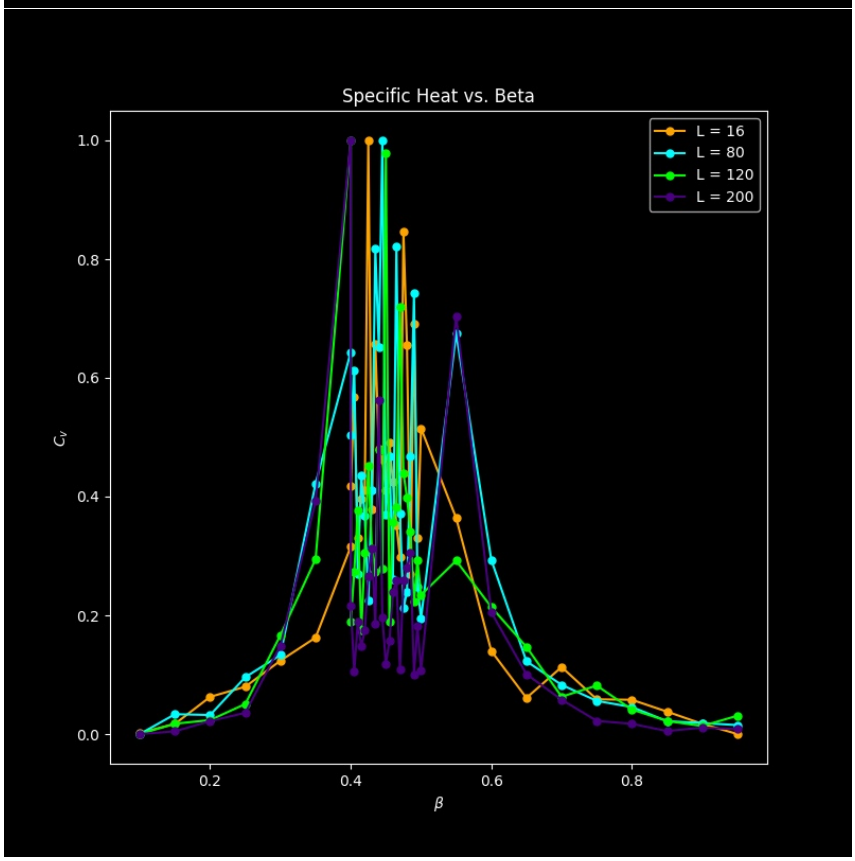
For lattice size of 100, the visualization is shown for different values of β and different number of steps. As expected, for smaller values of β, randomness is evident even after $3,000,000$ Metropolis steps, indicating the high temperature of the system.

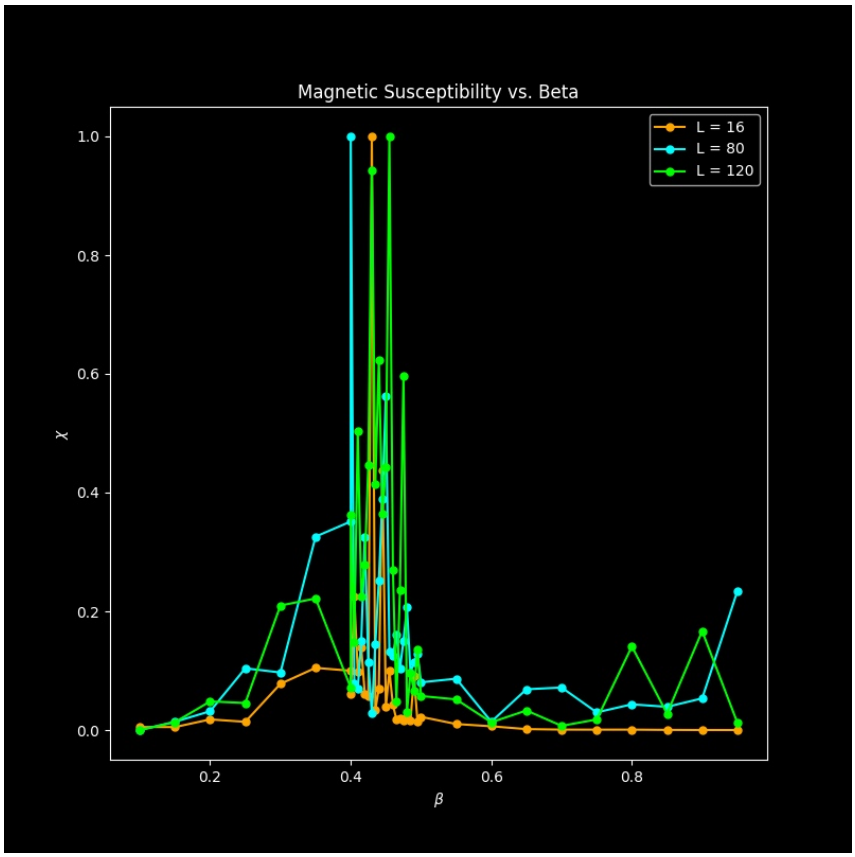

Lattice Before and After, $\beta = 0.1$
Lattice Before and After, $\beta = 0.3$
Lattice Before and After, $\beta = 0.48$
Lattice Before and After, $\beta = 0.48$
Lattice Before and After, $\beta = 0.48$
Lattice Before and After, $\beta = 0.5$
Lattice Before and After, $\beta = 0.6$
Lattice Before and After, $\beta = 0.9$
Initial Condition
After 100,000 Metropolis Steps
After 2,000,000 Metropolis Steps
After 3,000,000 Metropolis Steps

For values of $L \in \{16, 80, 120, 200\}$, C_v, χ, $|\langle M \rangle|$, and ξ were calculated for ensemble sizes of 50, and noramalized and plotted as follows (More Metropolis steps were needed for the second and the fourth plots to obtain cleaner data points; due to the tight deadline and the time-consuming process of the program, this was not feasible):




Magnetization vs. Beta
L = 16
L = 80
L = 120
L = 200
1.0
0.8
0.6
0.4
0.2
0.0
|⟨M⟩|
0.2
0.4
0.6
0.8
β


Specific Heat vs. Beta
L = 16
L = 80
L = 120
L = 200
1.0
0.8
0.6
0.4
0.2
0.0
C_v
0.2
0.4
0.6
0.8
β



Sources:
1. The Metropolis Algorithm
2. Ising model