SHARIF UNIVERSITY OF TECHNOLOGY
PHYSICS DEPARTMENT

Problem Set 3: the Sandpile Model

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Course: *Complex Systems - (Spring 2023)*
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Exercise 12

Self-Organized Criticality in the Sandpile Model

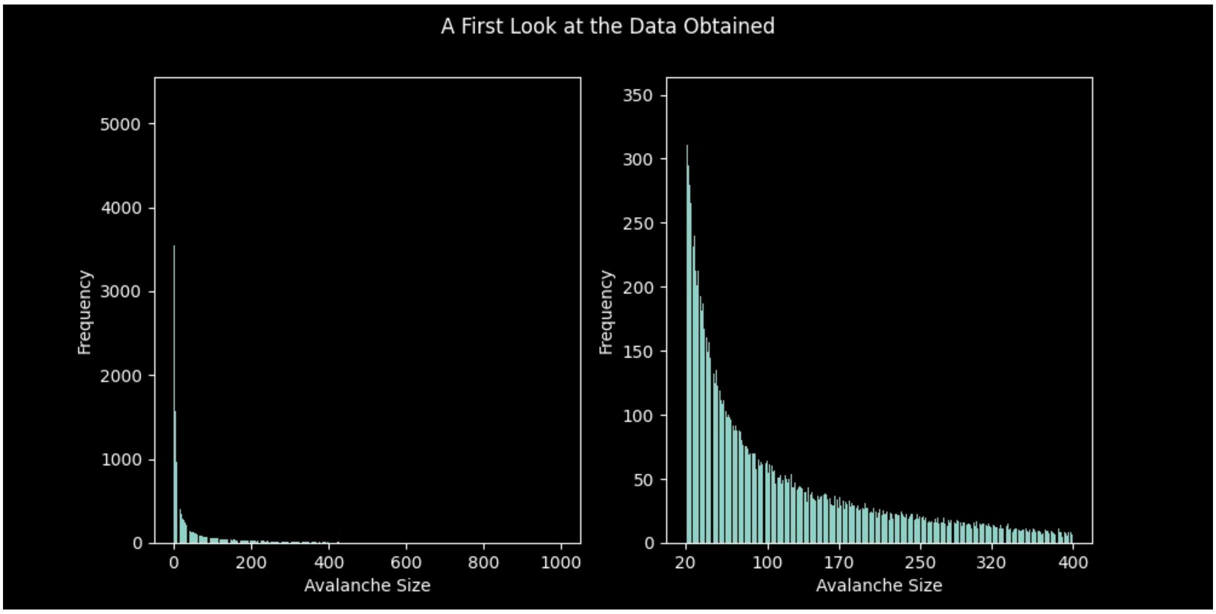
Simulate the sandpile model. Create a lattice of 100 in 100 squares initialized randomly with values from 0 to 3. At each time step, deposit one grain of sand randomly on the lattice. Put a threshold for the height of each cell to be $h_t = 4$. If the height of a cell reaches the threshold, it will deposit one sand from that cell, to each of the cells around it in the 4 primary directions (up, down, left, right). This deposition might lead to the next cell to reach its threshold. This phenomenon forms an avalanche and it keeps on going until all cells are below the threshold. Simulate this model for 10^5 time steps. Draw the log-log plot of the distribution of avalanches with respect to their size. What is the slope of this graph?

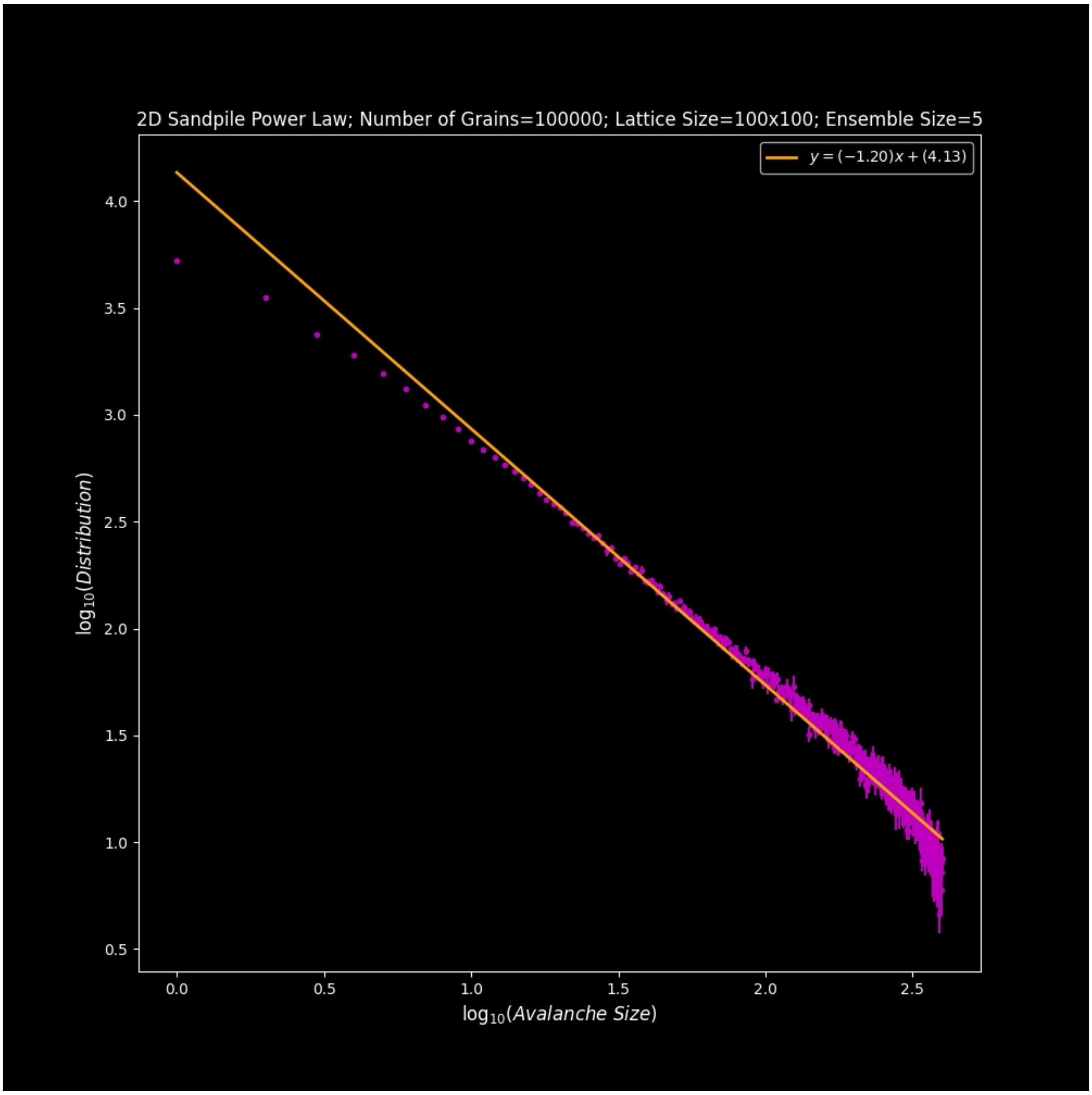
Answer. A first attempt was to define a crude function to implement the sandpile algorithm and check the data generated. Then a better, more efficient function was declared as follows:

- **sanpile():**

  This function takes a 2D matrix, *lattice*, the number of sand grains to be deposited, *num_grains*, and the threshold height of the cells, *h_t*. The algorithm described in the question is then implemented. The size of the avalnches are measured by time steps, such that with every sand grain deposition, if an avalanche occurs, a counter variable is increased by 1. Subsequent avalanches increase the counter variable even further, untill all the cells are below the threshold. Thus the size of the avalnche is calculated and stored in a numpy array. Then another sand grain is deposited and the process repeated.

Using the function, 5 sandpile models were simulated (*ensemble size* $= 5$), and the histogram of the data was plotted. As apparent from the histograms, avalanche sizes above 400 were so rare (for 100,000 sand grains and a 100 by 100 lattice) that were considered outlier data and disregarded. The log-log errorbar-plot was then graphed with a slope of (-1.20 ± 0.01).


A First Look at the Data Obtained
Frequency
Avalanche Size
0
1000
2000
3000
4000
5000
200
400
600
800
50
100
150
250
300
350
20
170
320


2D Sandpile Power Law; Number of Grains=100000; Lattice Size=100x100; Ensemble Size=5
$y = (-1.20)x + (4.13)$
$\log_{10}(Distribution)$
$\log_{10}(Avalanche\ Size)$
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0