SHARIF UNIVERSITY OF TECHNOLOGY
PHYSICS DEPARTMENT

Problem Set 3

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Course: *Network Science - (Fall 2023)*
Due date: *October 22, 2023*

Exercise 3.1

Erdos-Renyi Graphs

Write a program to generate random graphs of mean node degree λ.

(a) How does your algorithm work and how does it save the information about the graph?

(b) Find the distribution of node degrees.

(c) Select a random node. Find the degree distribution of its first neighbors.

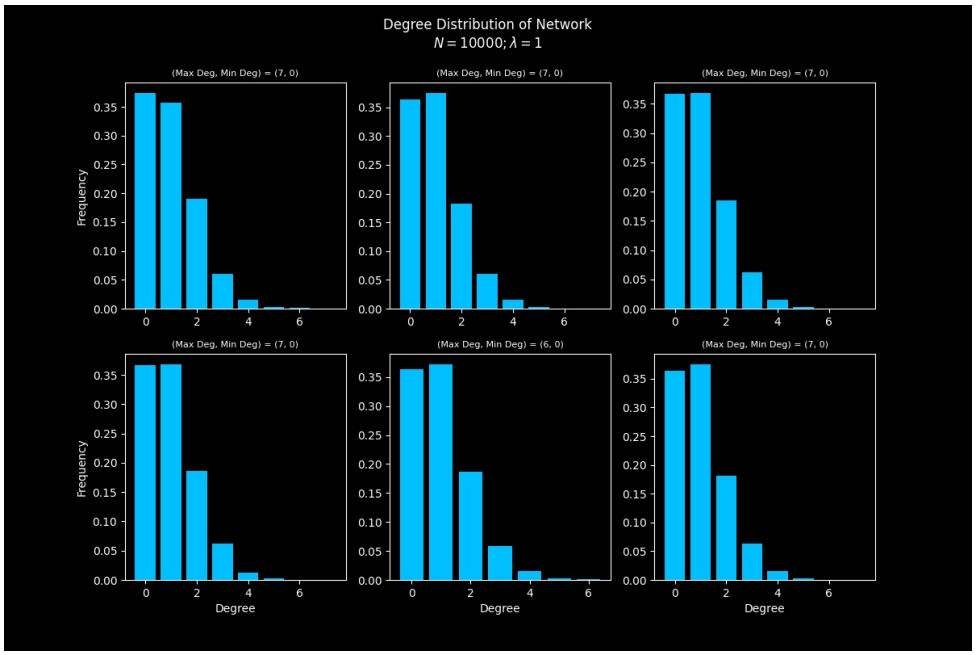
(d) Find the maximum and minimum degree each time you generate a graph.

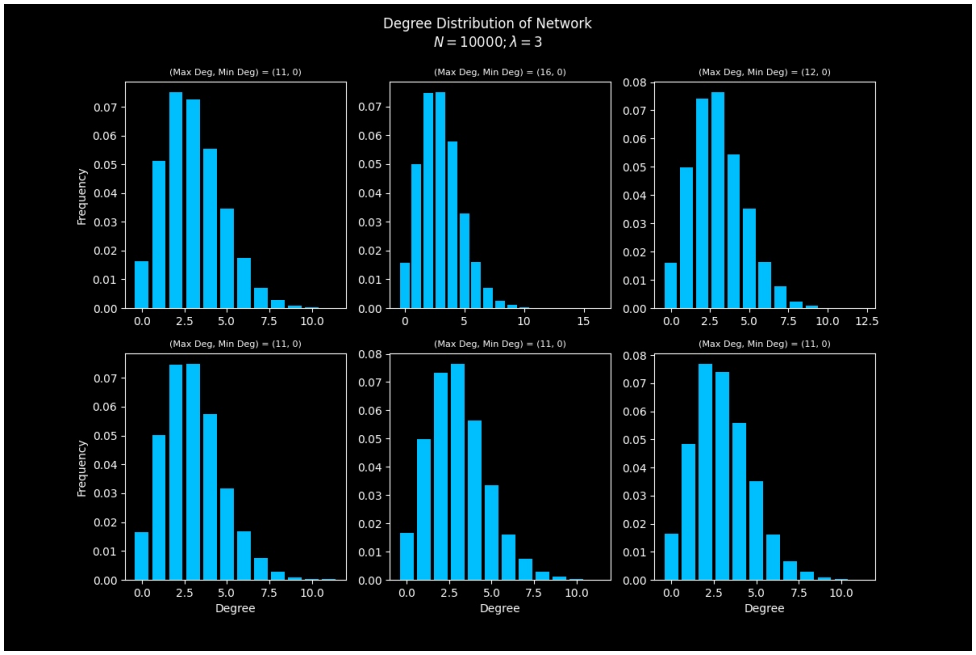
Answer. The algorithm to generate random graphs is implemented in the function **init_rand_graph()**. This function, takes as input the number of nodes, N, and the average node degree λ. It creates a dictionary (hash table) that maps each number in the set $(0, 1, 2, 3, ..., N)$ to an empty list. The number of edges (links) in the network is thus, found to be:

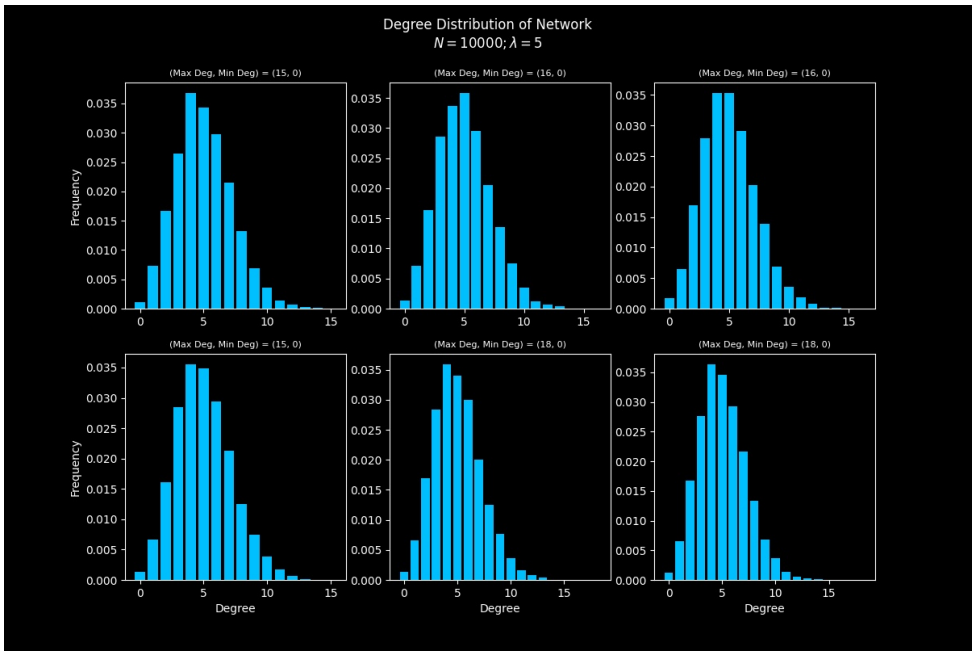
$$\text{number of edges} = \binom{N}{2} \times \frac{\lambda}{N} = \frac{N(N-1)}{2} p$$

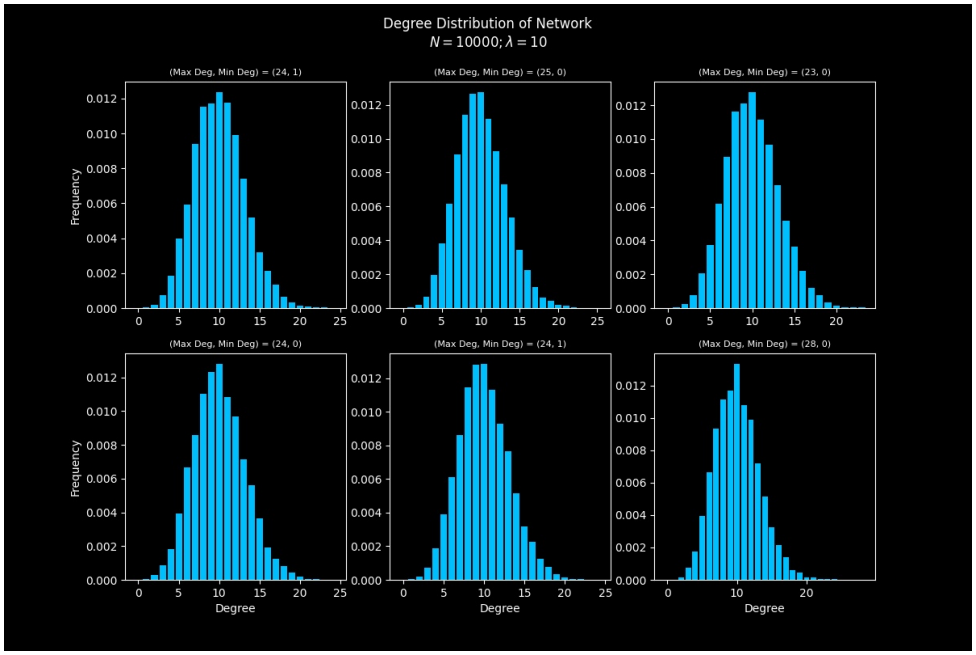
Then, two nodes are randomly chosen, if there is an edge connecting them, they are ignored and two new random nodes are selected. If there is no edge between them, each one is appended to the other's list of neighbors, and a counter (for counting the number of edges) is increased to indicate the addition of a new edge. This process goes on until the counter is equal to the desired number of edges as in the above-mentioned relation.

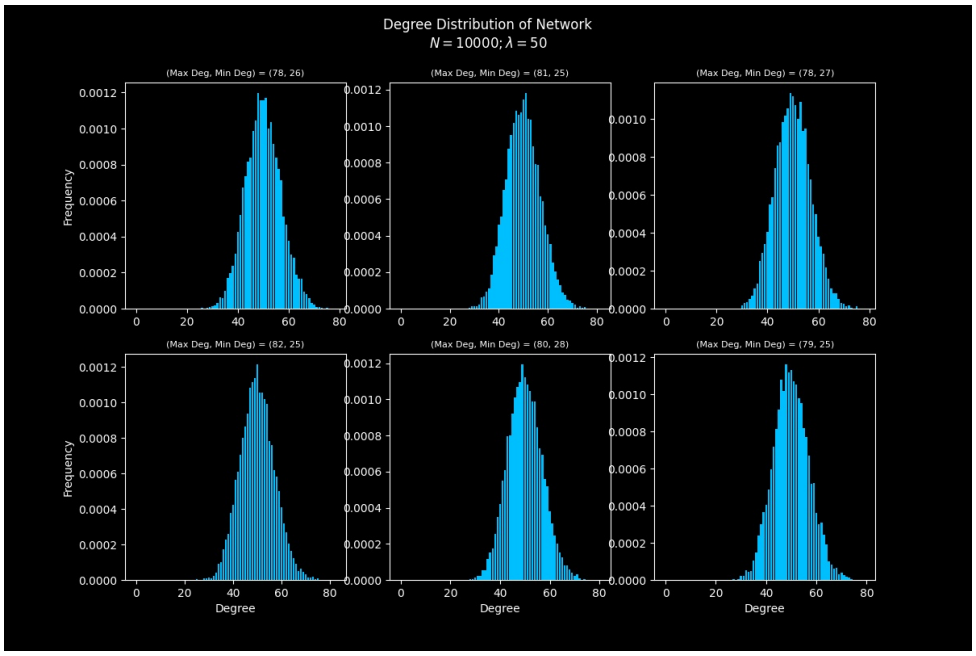
Graphs of $N = 10,000$ are generated 6 times for each $\lambda \in \{1, 3, 5, 10, 50\}$ and the distribution of the degrees is plotted. The maximum and minimum degrees are mentioned on the plots. Then, 1000 random nodes are selected from a graph with a given λ, the distribution of the degree of their first neighbors is then normalized and plotted.

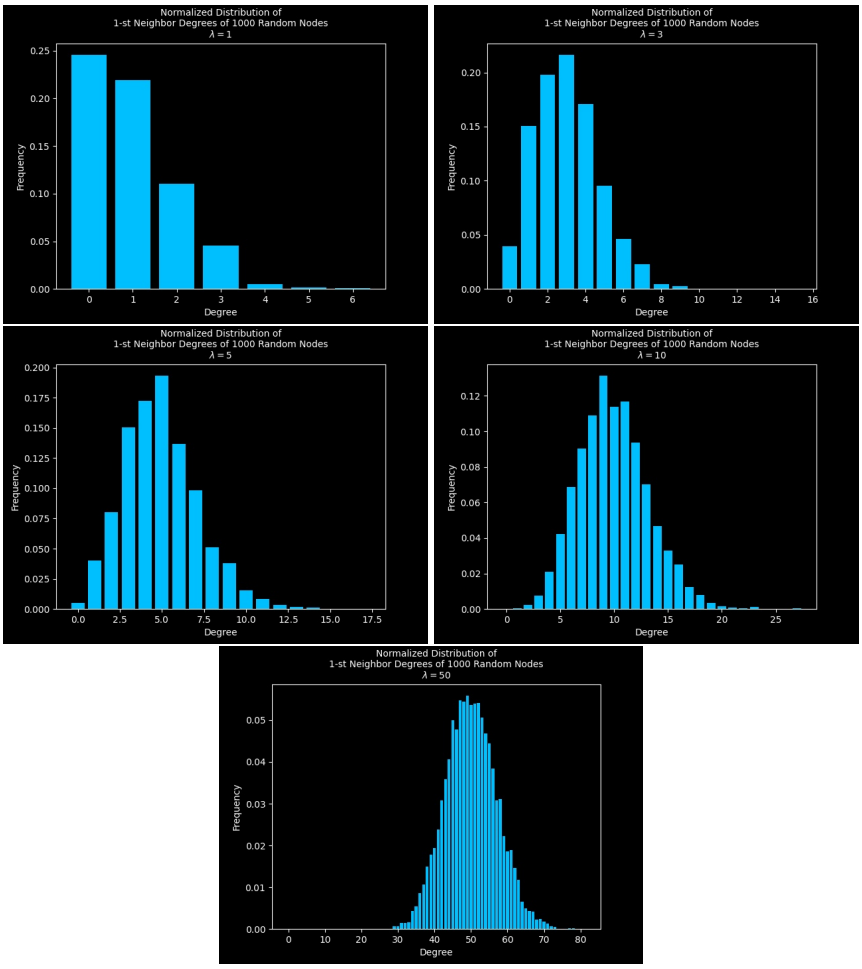

Degree Distribution of Network
$N = 10000; \lambda = 1$
(Max Deg, Min Deg) = (7, 0)
(Max Deg, Min Deg) = (7, 0)
(Max Deg, Min Deg) = (7, 0)
(Max Deg, Min Deg) = (7, 0)
(Max Deg, Min Deg) = (6, 0)
(Max Deg, Min Deg) = (7, 0)
Frequency
Degree


Degree Distribution of Network
$N = 10000; \lambda = 3$
(Max Deg, Min Deg) = (11, 0)
(Max Deg, Min Deg) = (16, 0)
(Max Deg, Min Deg) = (12, 0)
(Max Deg, Min Deg) = (11, 0)
(Max Deg, Min Deg) = (11, 0)
(Max Deg, Min Deg) = (11, 0)
Frequency
Degree


Degree Distribution of Network
$N = 10000; \lambda = 5$
(Max Deg, Min Deg) = (15, 0)
(Max Deg, Min Deg) = (16, 0)
(Max Deg, Min Deg) = (16, 0)
(Max Deg, Min Deg) = (15, 0)
(Max Deg, Min Deg) = (18, 0)
(Max Deg, Min Deg) = (18, 0)
Frequency
Degree


Degree Distribution of Network
$N = 10000; \lambda = 10$
(Max Deg, Min Deg) = (24, 1)
(Max Deg, Min Deg) = (25, 0)
(Max Deg, Min Deg) = (23, 0)
(Max Deg, Min Deg) = (24, 0)
(Max Deg, Min Deg) = (24, 1)
(Max Deg, Min Deg) = (28, 0)
Frequency
Degree


Degree Distribution of Network
$N = 10000; \lambda = 50$
(Max Deg, Min Deg) = (78, 26)
(Max Deg, Min Deg) = (81, 25)
(Max Deg, Min Deg) = (78, 27)
(Max Deg, Min Deg) = (82, 25)
(Max Deg, Min Deg) = (80, 28)
(Max Deg, Min Deg) = (79, 25)
Frequency
Degree


Normalized Distribution of
1-st Neighbor Degrees of 1000 Random Nodes
$\lambda = 1$
$\lambda = 3$
$\lambda = 5$
$\lambda = 10$
$\lambda = 50$
Frequency
Degree